Open vs Periodic Boundary Conditions in the Deconfined Phase

A Dream of Spring

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in collaboration with

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Universität Bielefeld

CERN, 14th of June 2018

Topology in a Nutshell

Open-Boundary Conditions

Boundary Region

Topological Susceptibility

Conclusions and Outlooks

Classical gauge theories

↕

Fibre bundles

Classical gauge theories

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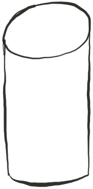
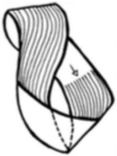
Fibre bundles

→ **Can be 'twisted'!**





Classification

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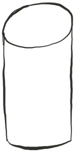

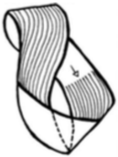

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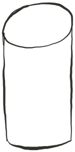

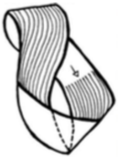

↓ de Rham's Theorem

$$\frac{\partial}{\partial x}$$

↓ Top. invariants

$$\mathbb{P}[\mathcal{F}]$$

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$SU(N)$ bundles in $4D$

2^{nd} Chern class: $q = -\frac{1}{8\pi^2}\mathbf{Tr}\,(F \wedge F)$

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Some Solutions on $\mathbb{T}^4$

$$A_\mu = \frac{2\pi}{a}\sum_\nu x^\nu b_{\mu\nu}\mathbf{N} \qquad b_{\mu\nu} \in \mathbb{N}$$

$$F_{\mu\nu} = \frac{2\pi}{a^2}(b_{\mu\nu} - b_{\nu\mu})\mathbf{N}$$

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Remarks

- For compact spaces $Q \in \mathbb{Z}$
- For non-compact spaces $Q \in \mathbb{R}$
- **Continuum** story

Refs: [Avis,Isham 1978]
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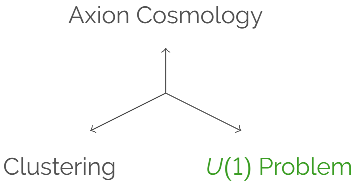

Motivations

Axion Cosmology

Clustering

$U(1)$ Problem

Issues

Physical: High-T Suppression

Algorithmical: Topological Freezing

No more top. transitions!

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Lat. accomodates $<$ and $<$ inst.

↕

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Continuum limit

$\downarrow$

Distinct top. sectors emerge

$\downarrow$

Access. by small disp. in conf. space

$\downarrow$

$t_{autocorrelation}(Q) >> 0$

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Solutions

- Meta-Dynamics
- Reweighting
- Multiscale Equilibration
- Master-Field
- Open-Boundary Conditions
- . . .

Refs: [Laio et al., 2015; Moore et al.,2018]
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$\downarrow$

Open-up the torus!

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OBC

$$F_{0\mu}(x)|_{x_0=0} = F_{0\mu}(x)|_{x_0=l_0} = 0$$

$$\Big\downarrow \; T \neq 0$$

$$F_{i\mu}(x)|_{x_i=0} = F_{i\mu}(x)|_{x_i=l_i} = 0$$

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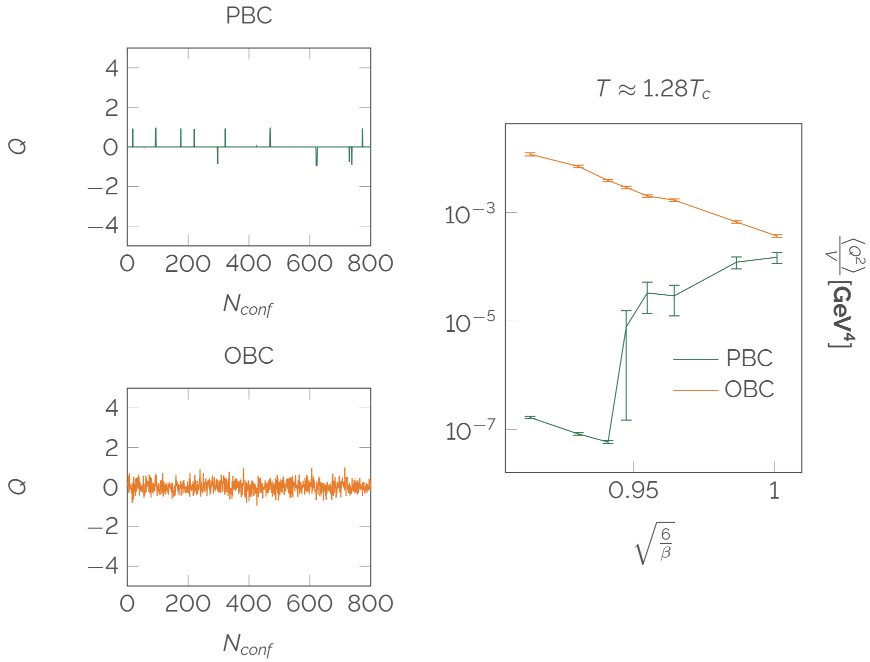
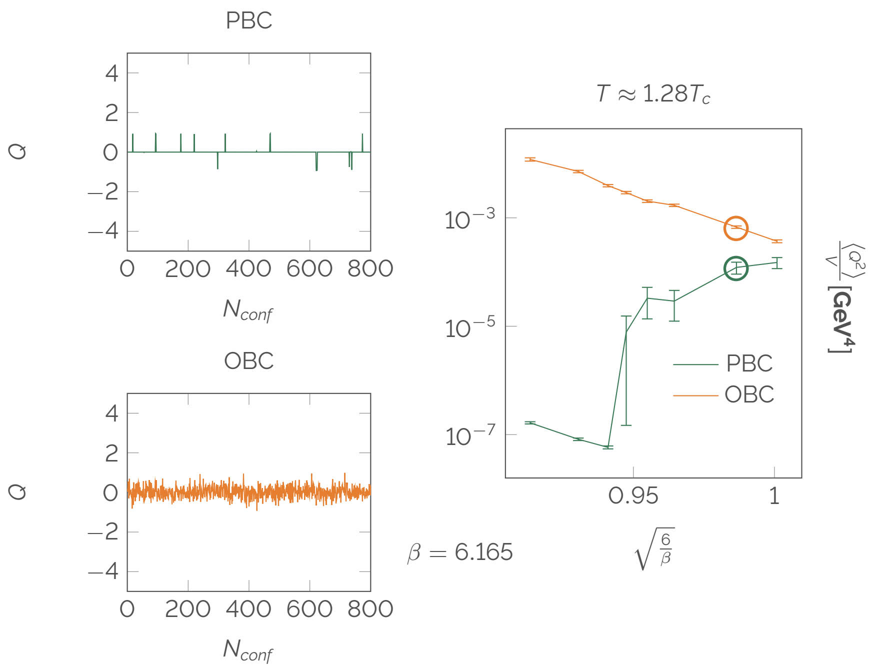
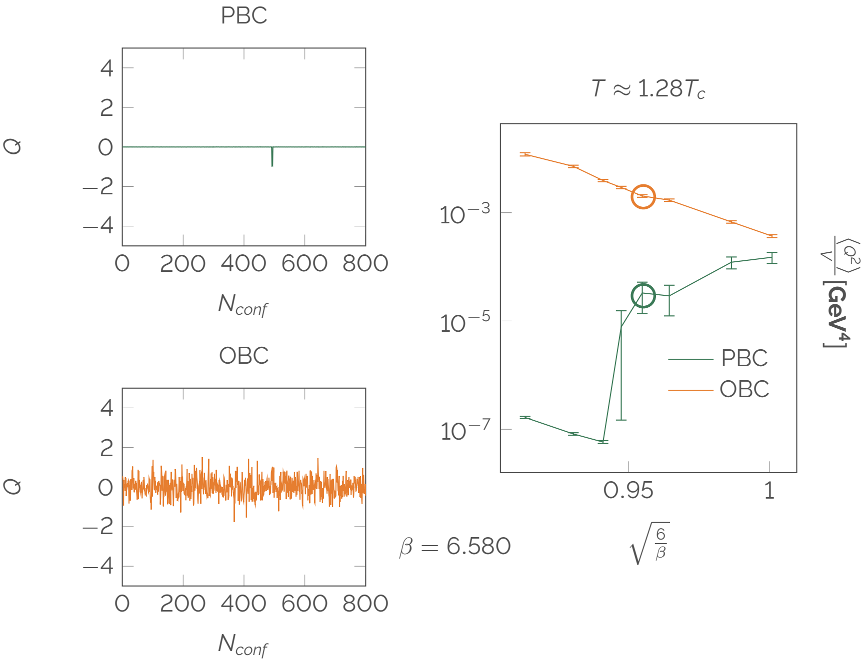


PBC
Q
4
2
0
−2
−4
0 200 400 600 800
N_{conf}
OBC
Q
4
2
0
−2
−4
0 200 400 600 800
N_{conf}
$T \approx 1.28 T_c$
10^{-3}
10^{-5}
10^{-7}
0.95
1
$\sqrt{\frac{6}{\beta}}$
$\frac{\langle Q^2 \rangle}{V}$ [GeV^4]
PBC
OBC


PBC
Q
N_{conf}
OBC
Q
N_{conf}
$T \approx 1.28T_c$
$\frac{\langle Q^2 \rangle}{V}$ [GeV^4]
$\sqrt{\frac{6}{\beta}}$
PBC
OBC
$\beta = 6.165$


PBC
4
2
0
−2
−4
Q
0 200 400 600 800
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PBC
OBC
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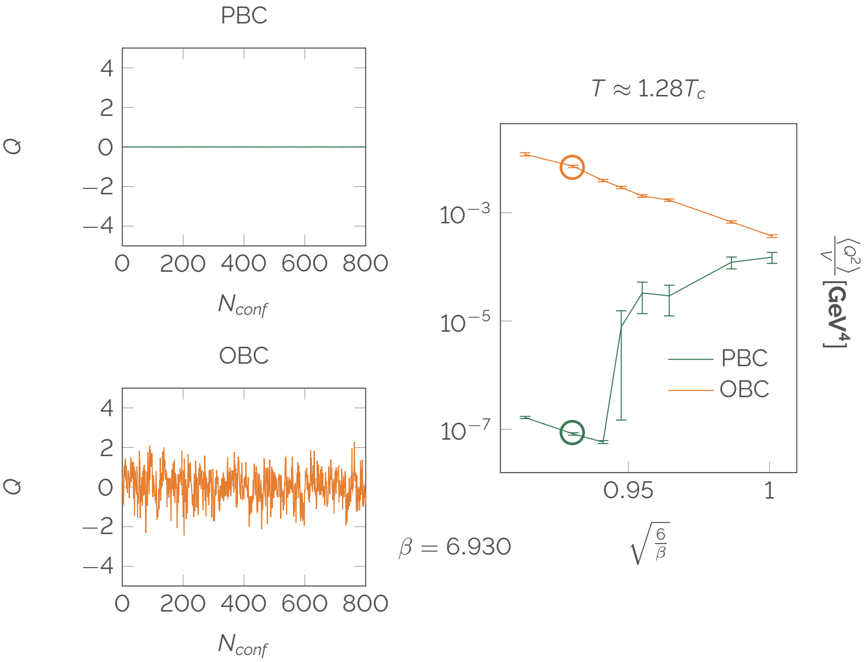
PBC
OBC
Q
N_{conf}
$T \approx 1.28T_c$
PBC
OBC
$\frac{\langle Q^2 \rangle}{V}$ [GeV^4]
$\sqrt{\frac{6}{\beta}}$
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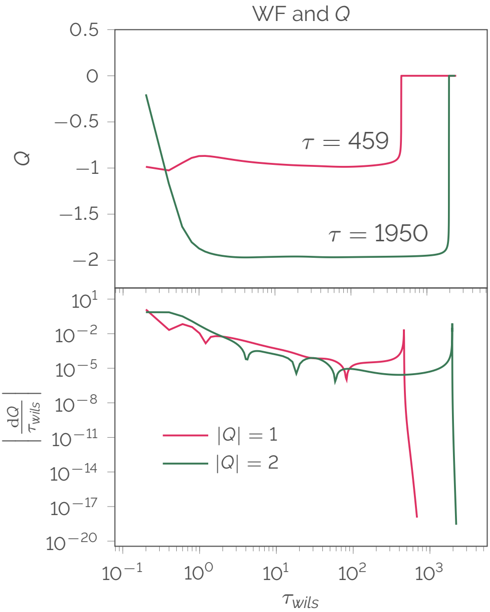
Wilson Flow



WF and Q
$\tau = 459$
$\tau = 1950$
Q
$\left|\frac{dQ}{\tau_{wils}}\right|$
$|Q| = 1$
$|Q| = 2$
τ_{wils}
0.5
0
−0.5
−1
−1.5
−2
10^{1}
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
10^{-20}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

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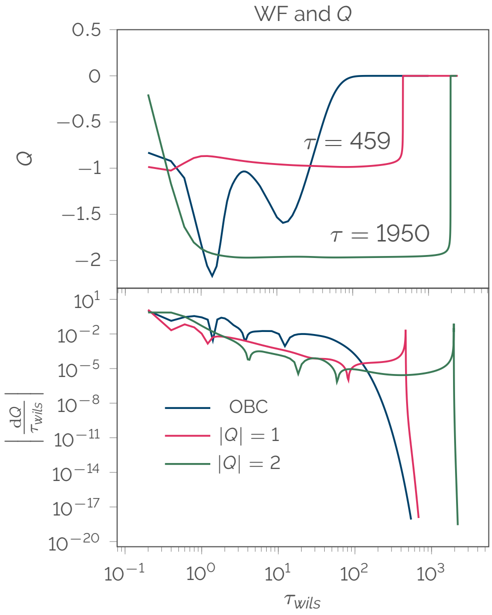

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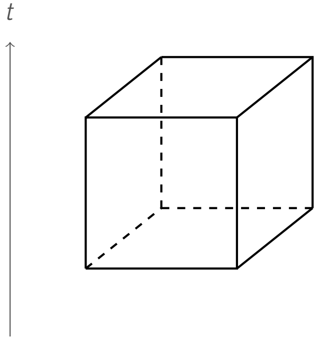
Finite Size Effects



$V_s = L_s^3$

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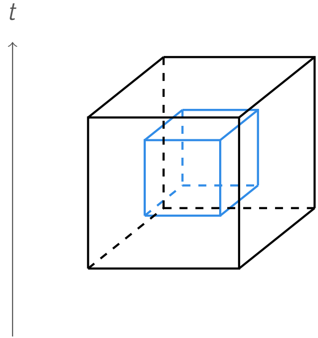

$V_s = L_s^3$ $\qquad$ $V_s' = L_s'^3$

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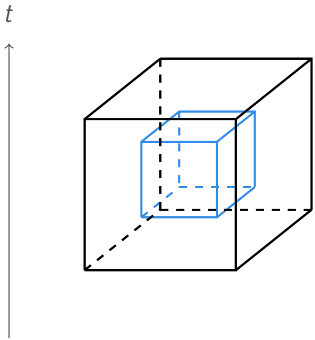

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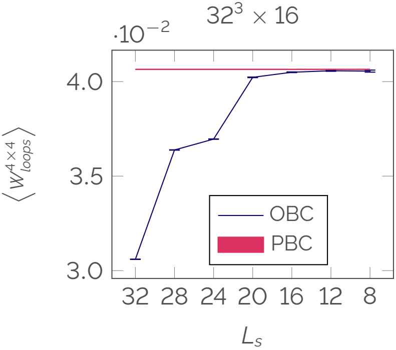
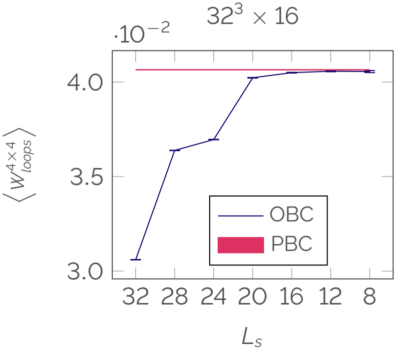


Boundary Region



Discard "boundary region"!

[Husung et al.,2017] at $T = 0$:

$t_w^2 \langle E \rangle$ as a probe

$\rightarrow L_{B0} \approx 0.75[\mathrm{fm}]$

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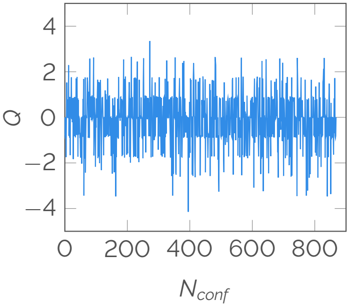
Coarse Lattices

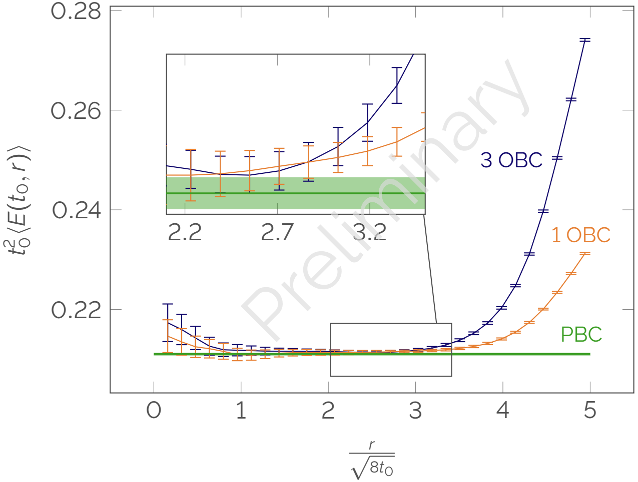
$64^3 \times 6, \beta = 6.139$

$a = 0.074[\mathrm{fm}], T \approx 1.5 T_c$

[Berkowitz et al., 2015]



Boundary Region, $T = 1.5T_c$



$L_{BT} \approx 0.93[\mathrm{fm}]$

χ_t

$$\chi_t = \frac{1}{V}\,\frac{\mathrm{d}^2}{\mathrm{d}\theta^2}\ln \mathcal{Z}(\theta)\Big|_{\theta=0}$$

$$= \frac{1}{V}\left\langle \int \mathrm{d}x q(x) \int \mathrm{d}y q(y)\right\rangle$$

$$= \frac{1}{V}\left\langle \int \mathrm{d}x_0 \int \mathrm{d}y q(x_0) q(x_0+y)\right\rangle$$

$$= \frac{1}{V}\left\langle \int \mathrm{d}x_0 \int \mathrm{d}y G_{qq}(x_0, x_0+y)\right\rangle$$

[Bruno et al, 2014]

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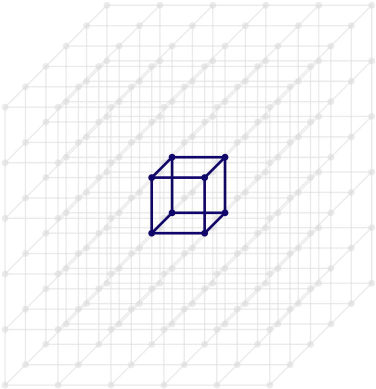

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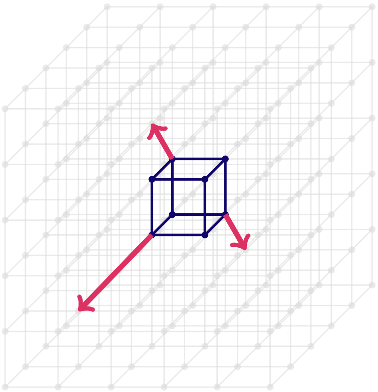

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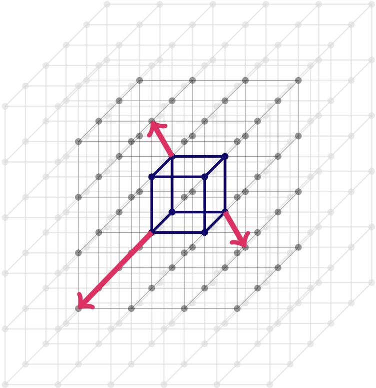

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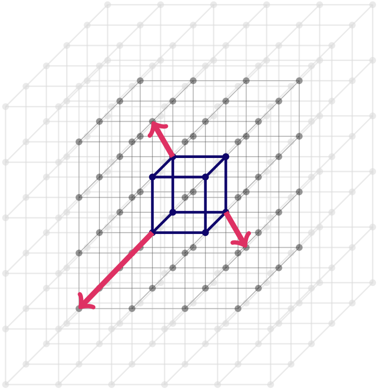


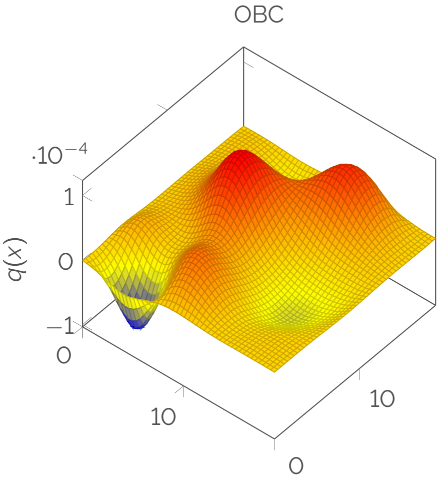
$\mathbb{S}$ to $\mathbb{S}$:

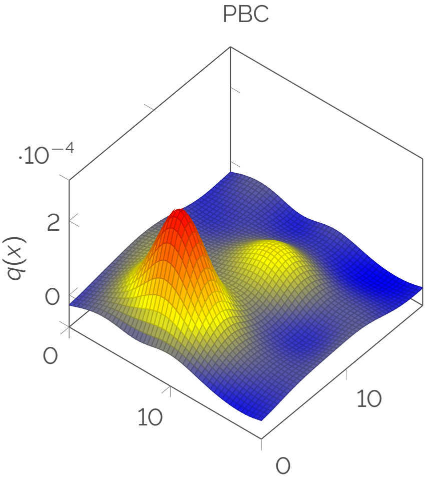
$$\chi_t =$$

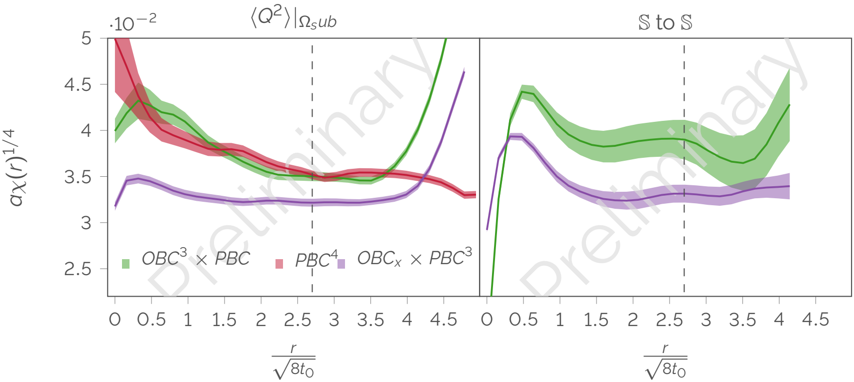
$$\left\langle \frac{1}{V_{mid}} \int \mathrm{d}x_{mid} \int_{\Omega_{cut}} \mathrm{d}y G_{qq}(x_{mid}, x_{mid} + y) \right\rangle$$

Subvolume av.:

$$\chi_t = < Q^2 >\big|_{\Omega_{sub}}$$

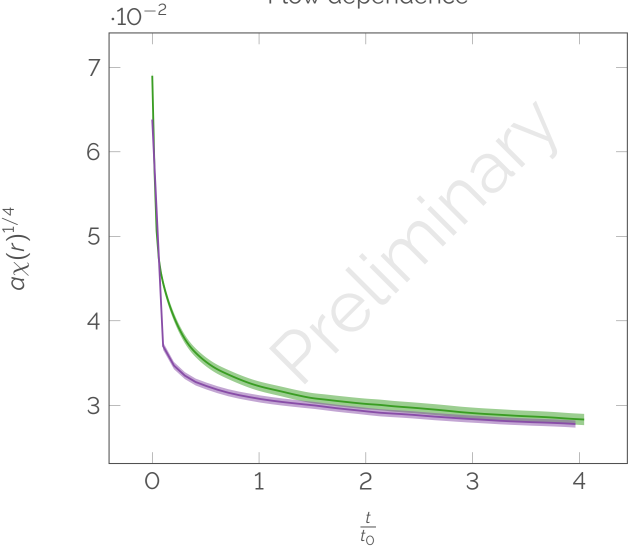

OBC
·10−4
1
0
−1
q(x)
0
10
0
10


PBC
·10−4
2
0
q(x)
0
10
0
10



$\cdot 10^{-2}$
$\langle Q^2 \rangle|_{\Omega_s ub}$
$\mathbb{S}$ to $\mathbb{S}$
$a\chi(r)^{1/4}$
5
4.5
4
3.5
3
2.5
$OBC^3 \times PBC$
PBC^4
$OBC_x \times PBC^3$
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5
$\frac{r}{\sqrt{8t_0}}$
Preliminary

Flow dependence



Todo's

What's next?

- Assess systematics

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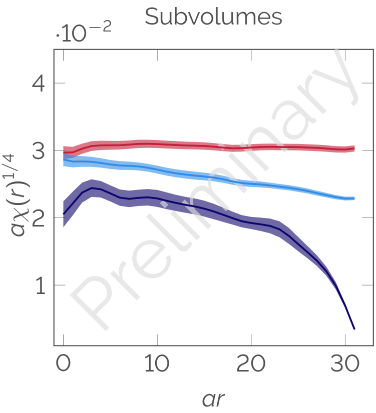

Take Away

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- Similar boundary region for one and three open directions

- χ_t may also be accessible by local measurements

- χ_t at high-T?

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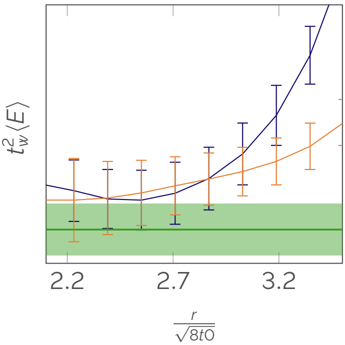

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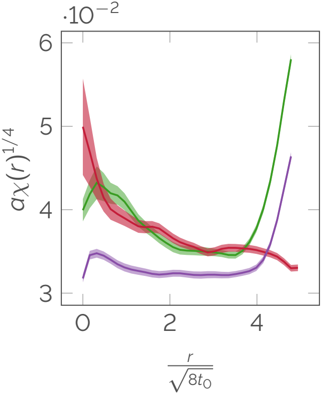

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- χ_t may also be accessible by local measurements
- **χ_t at high-T?**

Take Away

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Thank you!

(... and https://www.pinterest.com/pin/571746115169525297/ for the Moebius strip)

(but more **Yannis Burnier** for his early-on collaboration)