Properties of hadrons in external magnetic fields and polarisabilities from lattice QCD

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In collaboration with Gunnar Bali, Gergely Endrődi and Benjamin Gläßle

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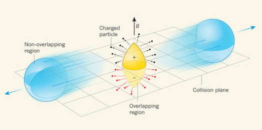
Contents

1. Motivation

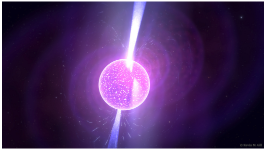
Strong (external) magnetic fields

- non-central heavy Ion collisions
  ($\sim 10^{18}$ G $\quad \rightarrow \quad$ 0.02 GeV^2)



- surface and interior of magnetars
  ($\sim 10^{15} - 10^{20}$ G $\quad \rightarrow \quad \lesssim 2$ GeV^2)

  (also for results of neutron star mergers)



- the early universe
  ($\sim 10^9$ G – at T_C^{QCD})

$\Rightarrow$ Properties of QCD in external magnetic fields are important!

Effects of external magnetic fields

- affect the spectrum of bound states
  - charged particles: direct influence on masses
  - neutral particles: influenced indirectly (subleading effect?)

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[Chernodub, PRL 106 (2011)]

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- influence the thermodynamic properties of QCD!

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Cases where spectrum and decay rates are important

- compact stellar objects:
  - masses of lightest hadrons appear in EOS
  - decay rates important for stability and equilibrium analyses
- cooling mechanisms: (e.g. for magnetars)

  [Duncan, Thompson, Astroph. J 392 (1992)]
  - mainly through weak decays and (inverse) β-decay (Urca processes)
  - of particular relevance: pion decay!
- model building for the phase diagram and EOS:

  need spectrum as input and/or for comparison!

  for instance: Hadron resonance gas at finite magnetic fields

  [Endrődi, JHEP 1304 (2013)]

Current status of lattice studies

- phase diagram:
  rather well understood from LQCD
  - phase diagram
    [Bali, *et al*, JHEP 1202 (2012);
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  - equation of state
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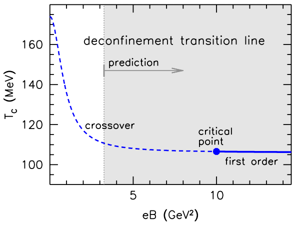

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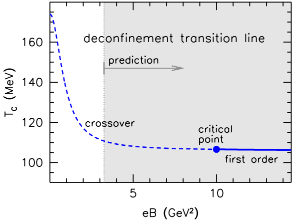

- spectrum: only some first quenched studies (no continuum limit)

Wilson fermions: [Wilson: Hidaka, Yamamoto, PRD 87 (2013)]
Overlap fermions: [Luschevskaya *et al*, JHEP 1709 (2017)]

more studies concerning magnetic moments and polarisabilities

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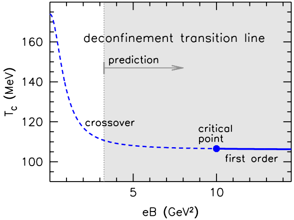

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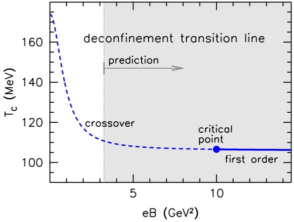

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  actually: no full calculation for decay rates!

  (only investigated changes of decay constants, e.g., in χPT)

Magnetic moments and polarisabilities of hadrons

external B-fields: can be used to probe structure of hadrons

energy (mass) of hadron H with $B = 0$ mass m and charge q:

$$E_{H;n}^2 = m^2 + (1+2n)|qB| - g_H s_z qB - 4\pi m \beta_H |eB|^2 + \ldots \qquad n \in \mathbb{Z}_0^+$$

g_H: magnetic moment $\qquad$ β_H: polarisability

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polarisabilities: important to probe electromagnetic structure of hadrons

- can be compared to experimental data (mostly proton and pion)
  (recent and new experiments for measurements of pion polarisabilities)
- for neutrons and other mesons: results less precise/not available
  $\Rightarrow$ lattice can potentially decrease uncertainties and make predictions
- theoretically not well understood beyond χPT

Lattice setup with quenched Wilson fermions

- use the (unimproved) Wilson discretisation for fermions

  (advantageous for spectroscopy)
- first: work in the quenched approximation

  ($N_f = 1 + 1$ simulations extremely difficult for Wilson fermions)

  quenched spectrum reproduces QCD spectrum typically up to 10%
- 3 different lattice spacings:

a [fm]	0.125	0.093	0.062
lattice	36×12^3	48×16^3	72×24^3

- different pion masses between 417 and 770 MeV

  focus on $m_\pi = 417$ MeV (physical size $m_\pi L \sim 3$)
- different external fields up to 4 GeV^2 (valence)
- will show first results with $N_f = 2 + 1$ in the electro-quenched setup

  (configurations generated with $N_f = 2 + 1$ but $B = 0$ – RQCD)
  $O(a)$-improved; $a = 0.064$ fm; $m_\pi = 418$ MeV; 64×32^3

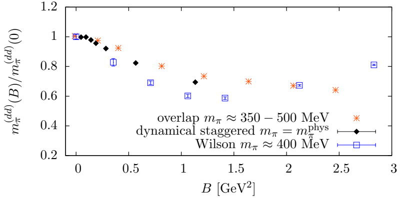
2. Additive quark mass renormalisation at finite B

[PoS LAT2015 (2016), arXiv:1510.03899;
PRD 97 (2018), arXiv:1707.05600]

Importance of additive quark mass renormalisation

Discrepancy for π^0 results in the literature:

- Wilson fermions:
  non-monotonous
  [Hidaka, Yamamoto, PRD 87 (2013)]
  (our results also show this)



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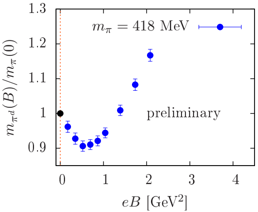

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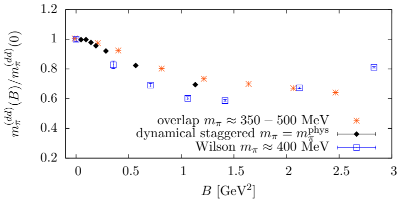
  [Luschevskaya *et al*, NPB 898 (2015)]
  [Bali, Endrődi *et al*, JHEP02 (2012)]



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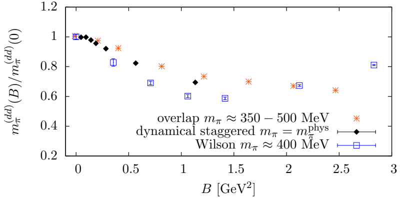

Wilson fermions: explicitly break full chiral symmetry

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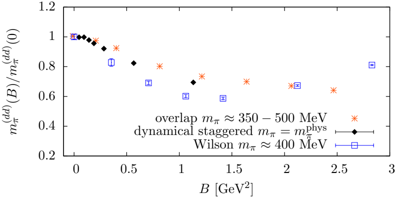

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known from QCD+QED:
additive renormalisation changes with coupling (presence of field)

[e.g. Borsanyi, *et al* Science 347 (2015)]

$\Rightarrow$ something similar happens for external B-fields

[Bali, BB, Endrődi, Gläßle, PoS LAT2015 (2016)]

Tuning of quark masses in the interacting theory

- standard method for QCD+QED:
  adjust $m_{u/d}$ so that pseudo-pions ($\pi^{u/d}$) masses remain constant.
  [BMW, PRL 111 (2013); Science 347 (2015)]
- advantage: no renormalisation needed.
- problem: disconnected diagrams present. $\Rightarrow$ typically neglected

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- use: $m_{\pi^{u/d}}(\mathbf{B}) \to 0$ for $m_{u/d} \to 0$.
- use current quark masses $\widetilde{m}_f$ from Ward identities (WIs)
  determine $m_{c,f}$ via $\widetilde{m}_f \to 0$

for this: need to compute WIs for QCD+QED (not available in literature)

$\Rightarrow$ get new terms (QED: covariant derivative does not commute with τ^i)

Ward identities for QCD+QED [Bali, BB, Endrődi, Gläßle, PRD 97 (2018)]

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- "neutral" WIs ($\bar{u}u$ and $\bar{d}d$): left unchanged

$\Rightarrow$ define "neutral" current quark masses via:

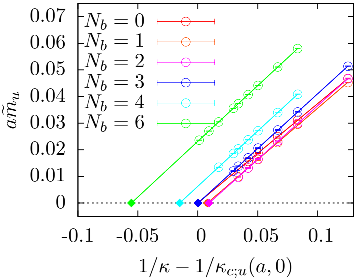
$$a\widetilde{m}_{u/d} = \frac{\partial_0 \left\langle (J_A)_0^{u/d}(x_0) P^{u/d}(0) \right\rangle}{2 \left\langle P^{u/d}(x_0) P^{u/d}(0) \right\rangle}$$

- advantage: easy to compute (standard PCAC masses)
- disadvantage:
  disconnected diagrams are ignored (and anomaly terms)

  $\Rightarrow$ unknown systematic effect in tuning

Determination of $m_{c,u/d}$

perform a (linear) chiral extrapolation of $\widetilde{m}_{u/d}$ to determine $m_{c,u/d}$



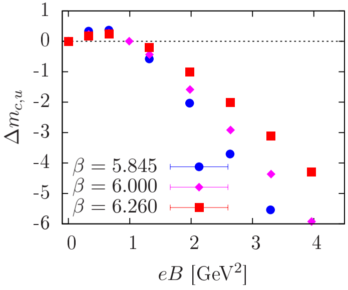
(results have been checked with higher orders in $(\bar{m} - \bar{m}_c)$.)

Determination of $m_{c,u/d}$

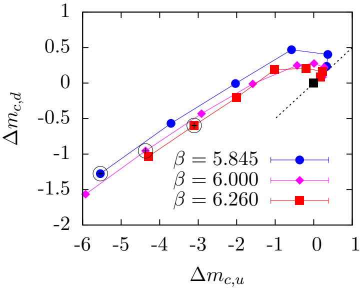
for visualisation use:

$$\Delta m_{c;f}(a,B) = \frac{m_{c;f}(a,B) - m_c(a,0)}{m_f(a,0;m_\pi = 415\,\mathrm{MeV})}$$

results for $\Delta m_{c;u}$:

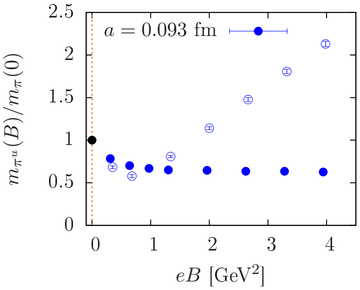


results in the $(\bar{m}_u, \bar{m}_d)$-plane:

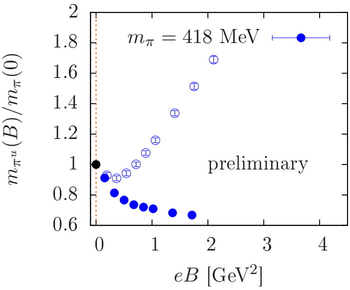


Neutral pions on LCP(B)s

quenched setup:



$N_f = 2 + 1$ setup:



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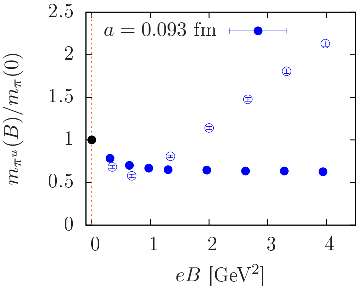

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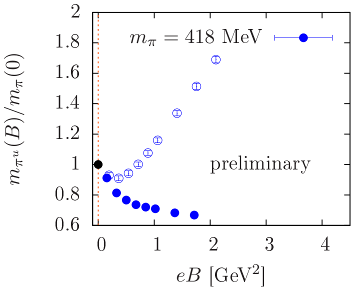

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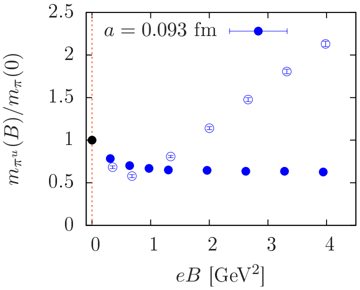

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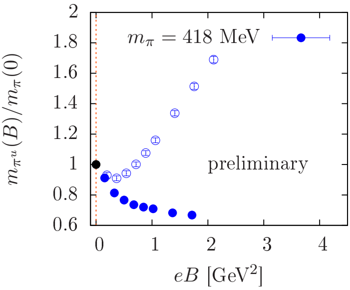

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result of tuning: removal of a particular type of B-dependent lattice artefacts
(improvement scheme – but no Symanzik improvement!)

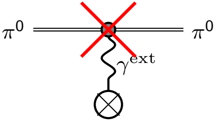
tuning of the quark mass with B:
lines of constants physics become B-dependent (LCP(B)s)

3. Meson spectrum and polarisabilities

[PRD 97 (2018), arXiv:1707.05600]

Results for the spectrum – Pions

π^0: (spin 0, $q = 0$)



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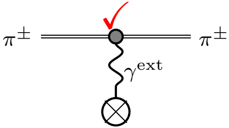

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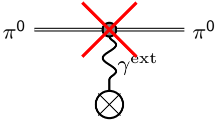

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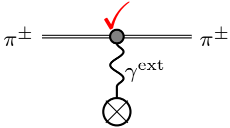

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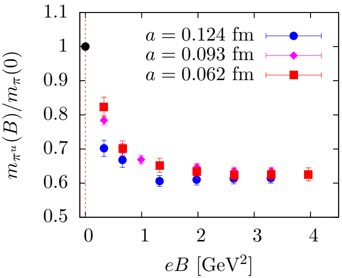

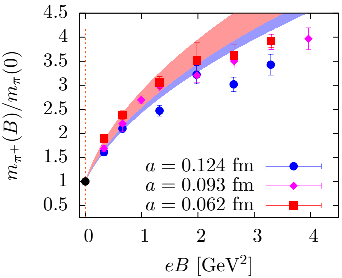

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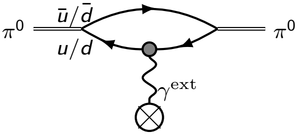

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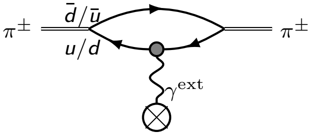
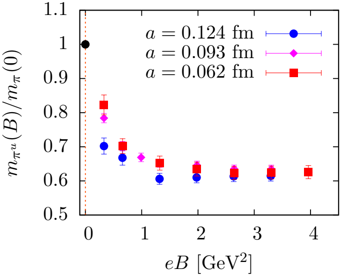
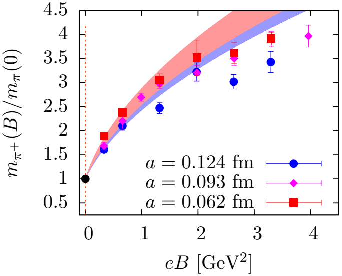


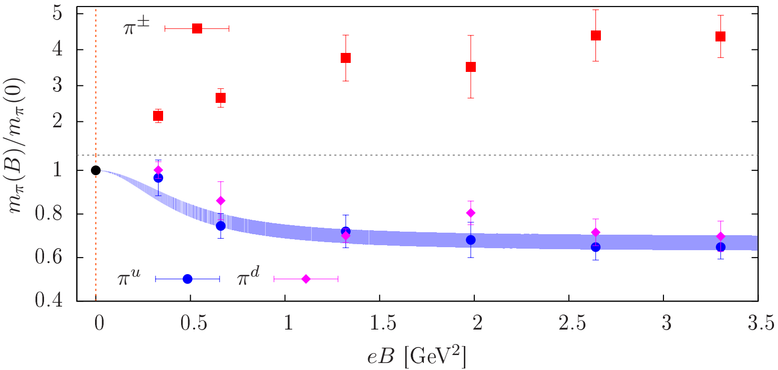
⇒ affected by subleading effects

⇒ additional subleading effects



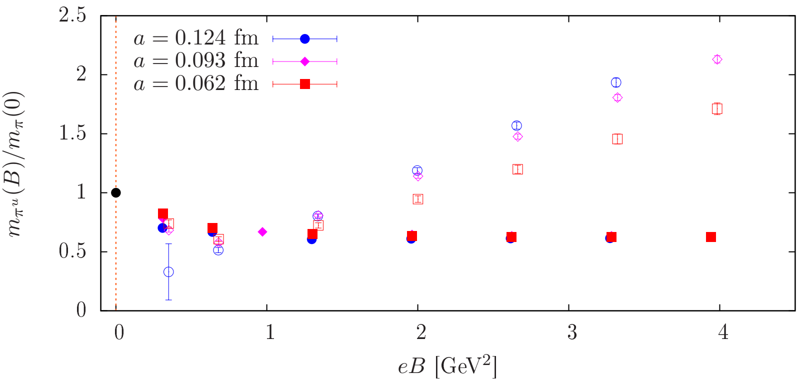


Results for the spectrum – pions continuum



Interpolating curve: $$\frac{m_{\pi^u}(B)}{m_\pi(0)} = \frac{1 + 3.2(8)\ \text{GeV}^{-2}\ (eB)^2}{1 + 4.8(1.2)\ \text{GeV}^{-2}\ (eB)^2}$$

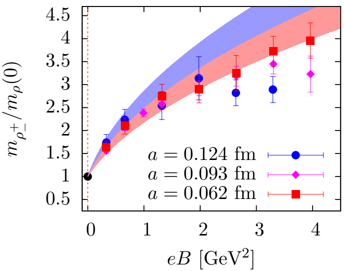
LCP(B) – only a removal of lattice artefacts?

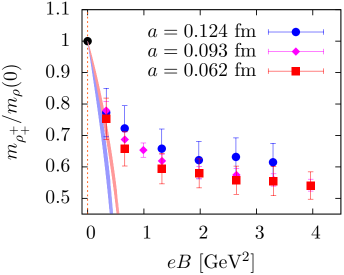


Results for the spectrum – charged ρ-mesons $s_z = \pm 1$

$\rho^{\pm}_{s_z=\pm1}$: (spin 1, $q = \pm 1$)

$E^2 = m_\rho^2 + (2n+1)|eB| - 2 s_z B$





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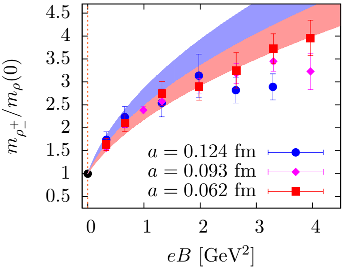

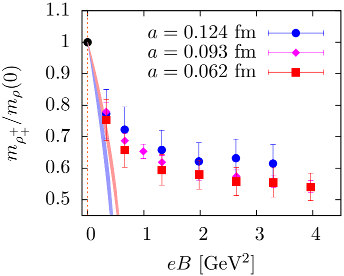

ρ-meson condensation: naively $E = 0$ when $q = s_z = 1$ and $eB = m_\rho^2$.

$\Rightarrow$ system could become superconducting [Chernodub, PRL 106 (2011)]

QCD inequalities: condensation cannot occur [Hidaka, Yamamoto, PRD 87 (2013)]

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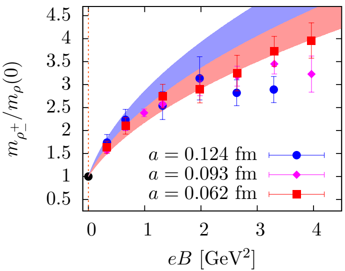

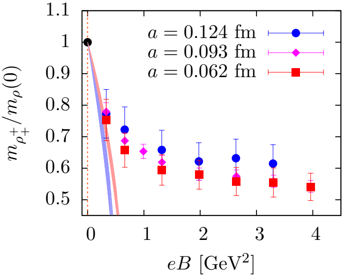

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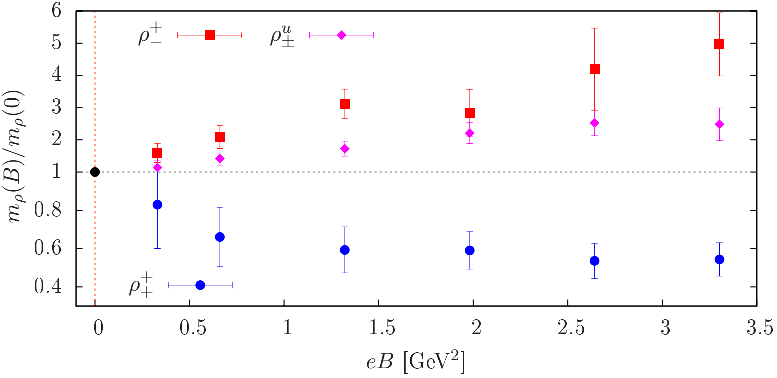
$\Rightarrow$ supported by our data

Results for the spectrum – ρ-mesons continuum

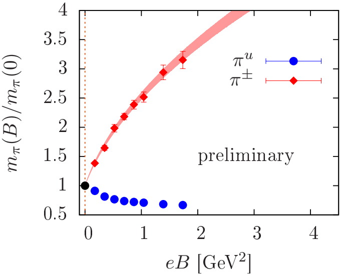


$\rho^{0,\pm}_{s_z=0}$-mesons: magnetic field enables mixing $\rho_0^{0,\pm} \longleftrightarrow \pi^{\pm,0}$

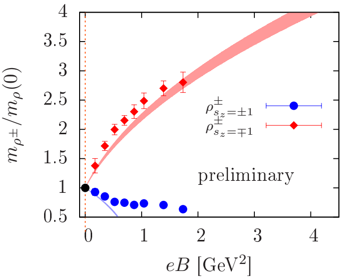
(extraction of mass eigenstates via correlation matrices)

Results for the spectrum – $N_f = 2 + 1$

results for pions:



results for $\rho^\pm$ mesons:



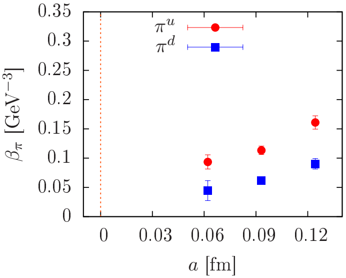
$\Rightarrow$ qualitative agreement with quenched results

Polarisabilities – quenched setup

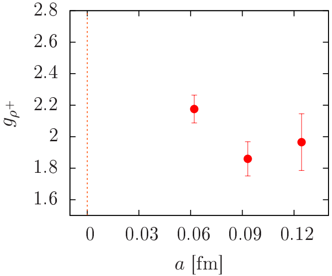
for extraction of polarisabilities: fit the spectrum to the form

$$E^2_{H;n} = m^2 + (1+2n)|qB| - g_H s_z qB - 4\pi m \beta_H |eB|^2 + \dots \qquad n \in \mathbb{Z}_0^+$$

polarisability of $\pi^{u/d}$:



magnetic moment of ρ^+:



(unfortunately: volumes rather small $\Rightarrow$ comparably large B)

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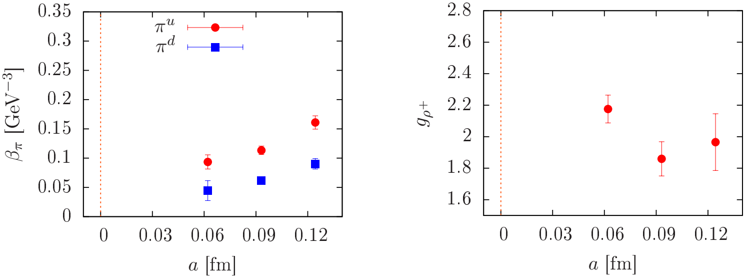
polarisability of $\pi^{u/d}$: magnetic moment of ρ^+:



(unfortunately: volumes rather small $\Rightarrow$ comparably large B)

for g_ρ: good agreement with χPT (≈ 2)

[Djukanovic *et al*, PLB 730 (2014)]

Polarisabilities – effect of improvement

polarisability defined by: $\beta_H = \frac{1}{8\pi m} \frac{\partial^2 E_{H;0}^2}{\partial |eB|^2}\Big|_{B=0}$

background field method: compute full derivative

$$\frac{dE_{H;0}^2}{d|eB|} = \frac{\partial E_{H;0}^2}{\partial |eB|} - \sum_f \frac{\partial E_{H;0}^2}{\partial m_f} \frac{\partial m_{c;f}}{\partial |eB|}$$

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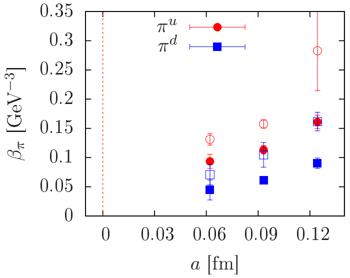

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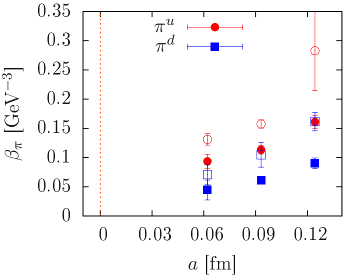
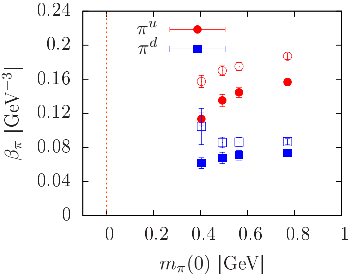


more important for small m_f



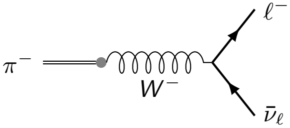
4. Pion decay constants and weak pion decay

[PRL 121 (2018), arXiv:1805.10971]

Weak pion decay

external magnetic fields: also affects the decay rates of particles!

decay of π^- into leptons:



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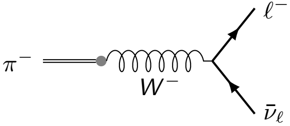
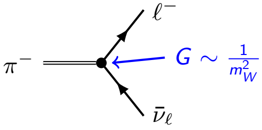


$\Longrightarrow$

effective four-fermi theory:



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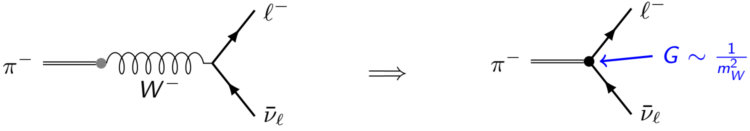

full decay rate: $\quad \Gamma = \int d\Phi \sum_{<out>} |\mathcal{M}|^2$

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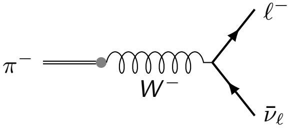

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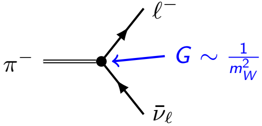

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amplitude: $$\mathcal{M} = \frac{G\,\cos(\theta_c)}{\sqrt{2}} \underbrace{\bar{u}_\ell \gamma^\mu (1-\gamma_5) v_\nu}_{\equiv L^\mu \text{ leptonic}} \underbrace{\langle 0 | \bar{d}(x) \gamma_\mu (1-\gamma_5) u(x) | \pi^-(\vec{p}=0) \rangle}_{\equiv H_\mu \text{ QCD}}$$

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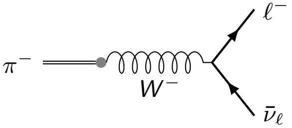
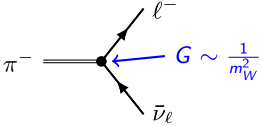


$\Longrightarrow$



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$\Rightarrow$ need to compute the QCD matrix element non-perturbatively

QCD matrix element – general form

focus on QCD matrix element:

$$H_\mu = \langle 0 | \bar{d}(x)\gamma_\mu u(x) | \pi^-(p) \rangle - \langle 0 | \bar{d}(x)\gamma_\mu\gamma_5 u(x) | \pi^-(p) \rangle$$

parameterised in terms of Lorentz (axial-)vectors (at $T = 0$)

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parameterised in terms of Lorentz (axial-)vectors (at $T = 0$)

- at $B = 0$: (available: p_μ)

  $\Rightarrow$ definition of pion decay constant f_π

$$\langle 0 | \bar{d}(x)\gamma_\mu u(x) | \pi^-(p) \rangle = 0$$

$$\langle 0 | \bar{d}(x)\gamma_\mu\gamma_5 u(x) | \pi^-(p) \rangle = i e^{ipx} f_\pi p_\mu$$

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  $\Rightarrow$ definition of pion decay constant f_π

- at $B > 0$: (available: p_μ and tensor $F_{\mu\nu}$)

  $\Rightarrow$ 3 decay constants (has been unknown before)

$$\langle 0 | \bar{d}(x)\gamma_\mu u(x) | \pi^-(p) \rangle = ie^{ipx} \left[i \frac{f'_\pi}{2} \epsilon_{\mu\nu\rho\sigma} eF^{\nu\rho} p^\sigma \right]$$

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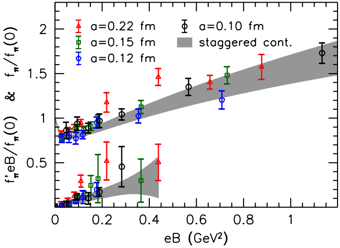
$$\Rightarrow \qquad H_\mu(B) = -ie^{im_\pi(B)x_0} m_\pi(B) \left[f_\pi(B)\delta_{\mu 0} + if'_\pi(B) eB\delta_{\mu 3} \right]$$

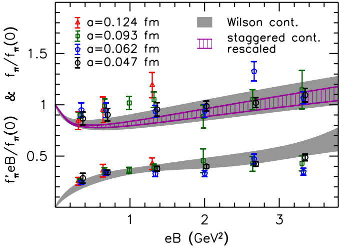
(π in lowest Landau level)

QCD Amplitude – decay constants for $B > 0$

dynamical staggered quarks:
pion mass: $m_\pi = 135$ MeV
(setup: [Bali, Endrődi, JHEP 1202 (2012)])



quenched Wilson quarks:
pion mass: $m_\pi = 417$ MeV
(setup as before)



fit functions:

$$\frac{f_\pi(B)}{f_\pi(0)} = \left[1 + c_1|eB|\right] \frac{m_\pi(0)}{m_\pi(B)}$$

$$\frac{f'_\pi(B)}{f_\pi(0)} = \left[d_0 + d_1|eB| + d_2|eB|^2\right] \frac{m_\pi(0)}{m_\pi(B)}$$

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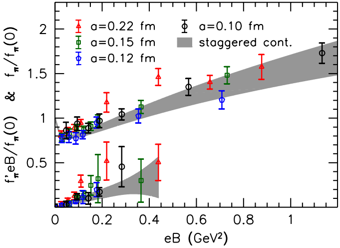

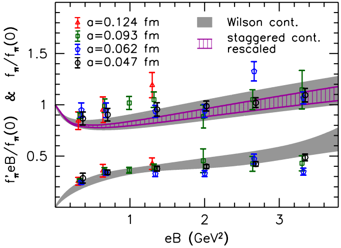

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for small magnetic fields: $f'_\pi = 0.10(2)\ \mathrm{GeV}^{-1} + O(B)$

Full decay rate [PRL 121 (2018), arXiv:1805.10971]

$$\frac{\Gamma(B)}{\Gamma(0)} = \frac{f_\pi^2(B) + [f_\pi'(B)\, eB]^2}{f_\pi^2(0)} \cdot \left[1 - \frac{m_\ell^2}{m_\pi^2(0)}\right]^{-2} \cdot \frac{2\,|eB|}{m_\pi(0) m_\pi(B)}$$

(valid: $eB \gg m_\ell^2$; LLL approximation exact: $eB > m_\pi^2(0) - m_\ell^2(0)$)

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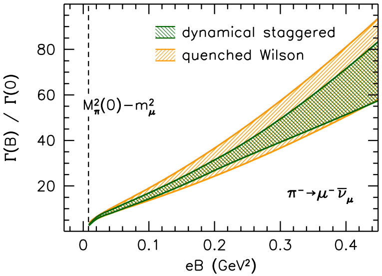
dominant π^--decay channel:

$\pi^- \to \mu^- + \bar{\nu}_\mu$
(decay fraction 99.98%)

enhanced by factor ≈ 50
at $eB \approx 0.3$ GeV^2

(Wilson: rescaled to $m_\pi = 135$ MeV)



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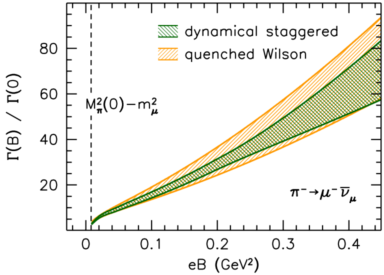

typical $B > 0$ lifetime: $\tau_\pi = 5 \cdot 10^{-10}$ sec at $B \approx 0.3$ GeV^2/e$= 5 \cdot 10^{15}$ T

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(valid: $eB \gg m_\ell^2$; LLL approximation exact: $eB > m_\pi^2(0) - m_\ell^2(0)$)

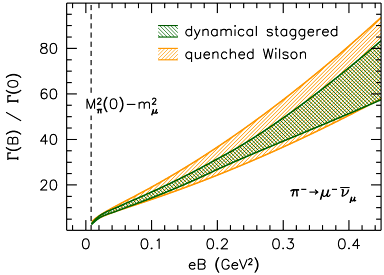
subdominant π^--decay channel:

$\pi^- \to e^- + \bar{\nu}_e$

(decay fraction 0.012%)

enhanced by factor ≈ 10
at $eB \approx 0.3$ GeV^2

change of mass m_ℓ



Summary

- Spectrum and decay rates for $B > 0$: important quantities (particularly interesting for astrophysical applications)
- Quenched lattice results in the continuum:
  - $\pi^{\pm}$ and $\rho^{\pm}_{\mp}$ masses increase as for point particles
  - π^0 masses decrease down to 60% of $m_\pi(0)$
  - $\rho^{\pm}_{\pm}$ masses decrease but remains > 0 $\Rightarrow$ no ρ-meson condensation!?
- key ingredient: tuning of m_q with B for Wilson fermions
- new decay constants appear for $B > 0$
  (here: example of $\pi^{\pm}$; same happens for π^0, $K^{\pm,0}$, ...)

  we have measured the new decay constants on the lattice
- weak $\pi^{\pm}$ decay: drastically enhanced (much smaller lifetimes)
- Future prospects:
  - look at other decays (Urca processes, ...)
  - compute polarisabilities

Thank you for your attention!

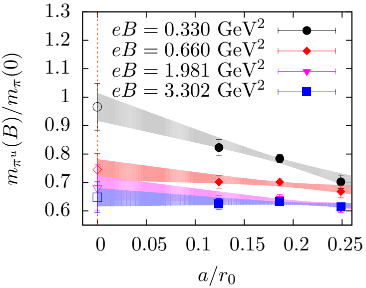
Results for the spectrum – continuum extrapolations

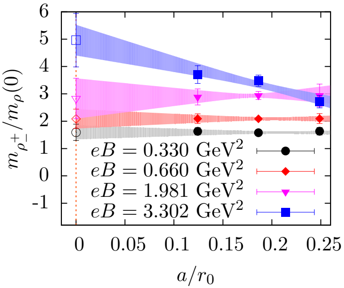
unimproved Wilson fermions $\Rightarrow$ lattice artefacts of $O(a)$

here: perform linear continuum extrapolations in a

check for relevant higher order terms:
use only two smallest lattice spacings for extrapolation

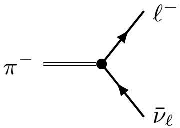
$\Rightarrow$ included in systematic error





Changes to perturbative computation

[PRL 121 (2018), arXiv:1805.10971]



- external states for π^- and ℓ^- at finite B: so called Landau levels

  energies lepton: $E_{n,k_z,s_z} = \sqrt{(2n+1+2s_z)eB + k_z^2 + m_\ell^2}$

  lowest Landau level (LLL): $n = 0, \quad s_z = -1/2$

  energy conservation: $eB > m_\pi(0)^2 - m_\ell^2$ only LLL contributes for lepton

  multiplicities of Landau-Levels $\sim \Phi = eBL^2$
  $\Rightarrow$ regularisation in a finite volume $V = L^3$ needed

- outgoing ν_ℓ state has $s_z = 1/2$

  $\Rightarrow$ no spin sum for outgoing states

  $\Rightarrow$ neutrino momentum $\parallel$ **B**

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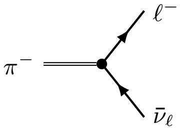

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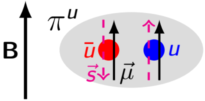
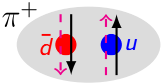
Free case

Alternative: Look at the energy levels of free "pions"!

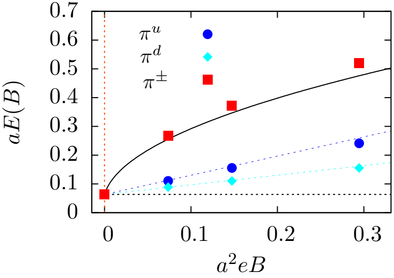
(in practice: two quarks in a box with imposed π-quantum numbers)





$\Rightarrow \quad E_{\pi^u/\pi^d} = 2m_{u/d}$

$\Rightarrow \quad E_{\pi^\pm} = m_u + \sqrt{m_d + 2|q_d B|}$



$\Rightarrow$ Quark mass changes with magnetic field!

Free case – adjusted κ

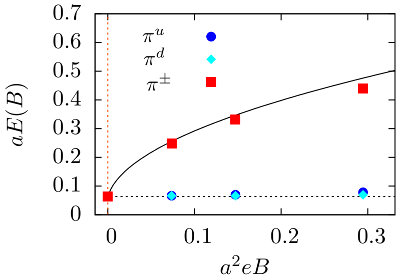
change in the quark masses $\Rightarrow$ change in m_c!

$\Rightarrow$ recompute m_c with condition

$$E_{\pi^u/\pi^d}(\mathbf{B}) \sim m - m_{c,u/d}(\mathbf{B}) = \text{const}$$

in the free case: $am_{c,f}(aB) - am_{c,f}(0) \approx a^2|q_f B|/2$

with retuned masses:



Results for the spectrum – ρ-mesons $s_z = 0$

magnetic field enables mixings: $\rho_0^{0,\pm} \longleftrightarrow \pi^{\pm,0}$

mass eigenstates:

$$\big|(\pi')^{\pm,0}\big\rangle = \cos(\theta)\big|\pi^{\pm,0}\big\rangle + \sin(\theta)\big|\rho_0^{\pm,0}\big\rangle$$
$$\big|(\rho')_0^{\pm,0}\big\rangle = -\sin(\theta)\big|\pi^{\pm,0}\big\rangle + \cos(\theta)\big|\rho_0^{\pm,0}\big\rangle$$

$\Rightarrow$ need a correlation matrix to extract the $(\rho')_0^{\pm,0}$ mass

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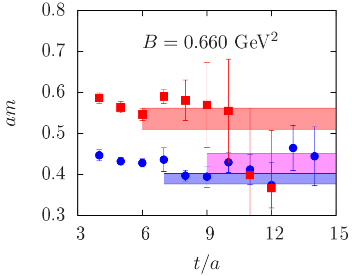
plateaus after diagonalisation:



results for $(\rho')_0^{\pm}$ masses: