Designing a linearized partonic transport model and application to open charm

Weiyao Ke, Yingru Xu, Wenkai Fan, and Steffen Bass

13th International Workshop on High-p_T Physics in the RHIC/LHC era
University of Tennessee, Knoxville

Mar 21, 2019

This work is supported by US DOE Grant no. DE-FG02-05ER41367, and NSF Grant no. ACI-1550225.

Based on PRC 97 014907, PRC 98 064901, ArXiv:1810.08177, and work in progress.

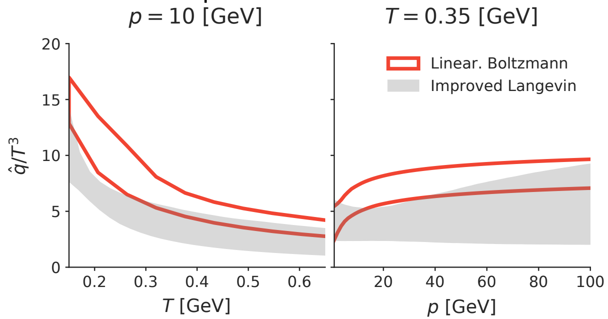
Transport model approach to hard probes in QGP

- Advantages:
  - Particle-based simulation.
  - Easy to be coupled to a hydrodynamic background.
- Challenges:
  - Testing the underlying assumptions.
  - Range of validity of semi-classical approach.

Assumption of probe-medium interaction:

- Relatively dilute & perturbative scattering centers?
- Many soft scatterings $\rightarrow$ diffusion.
- Non-perturbative contribution, are they modeled by diffusion?

The assumption affects the extraction of the transport parameter from experiments



Large model uncertainty in extracting $\hat{q}$!

- Linearized Boltzmann: perturbative el & inel scatterings.
- Improved Langevin: diffusive propagation & radiation.

How to include the LPM effect in a transport simulation?

- Interference between multiple scatters changes the branching spectrum.
- Different transport models implement the LPM effect quite differently.
- How much uncertainty does it introduce? Can we calibrate it to theoretical calculations in certain limits?

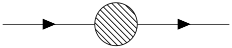
Goals

- Let model take more flexible form of probe-medium interaction
  $\rightarrow$ a combination of diffusion at small angle and large angle scatterings.

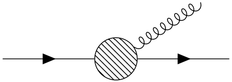
- Revisit the medium-induced radiation implementation
  $\rightarrow$ modifying the semi-classical particle-based transport scheme.

Probe-medium interaction: a separation between hard and soft modes

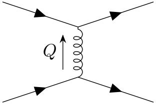
Small-Q diffusion $\mathcal{D}[f]$



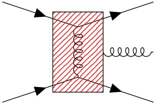
Diffusion-induced radiation $\mathcal{C}^{1\leftrightarrow 2}[f]$



Large-Q elastic collision $\mathcal{C}^{2\leftrightarrow 2}[f]$



Large-Q inelastic collision $\mathcal{C}^{2\leftrightarrow 3}[f]$



$$\frac{df}{dt} = \mathcal{D}[f] + \mathcal{C}^{1\leftrightarrow 2}[f] + \mathcal{C}^{2\leftrightarrow 2}[f] + \mathcal{C}^{2\leftrightarrow 3}[f]$$

LIDO provides a particle based Monte-Carlo solver of the transport equation (Q, q, g):

$$\mathcal{D}: \quad \begin{cases} \Delta\vec{x}/\Delta t = \vec{p}/E \\ \Delta\vec{p}/\Delta t = -\eta_D\vec{p} + \vec{\xi}(t) \end{cases}, \langle\vec{\xi}(t)\vec{\xi}(0)\rangle = \delta(t)(\hat{P}_L\hat{q}_{S,L} + \hat{P}_T\hat{q}_S/2)$$

$$\mathcal{C}: \quad \text{Sample } \vec{p}_1, \vec{p}_2, \cdots \text{ according to } \frac{\Delta t \cdot dR}{dp_1^3 dp_2^3 \cdots}$$

Probe-medium interaction: a separation between hard and soft modes

Small-Q by diffusion $\mathcal{D}(Q < Q_{\rm cut})$

- Do not resolve details of medium.
- Transport coefficients from weakly-coupled theory (J Ghiglieri et al JHEP 03 095)

$$\hat{q}_S = \int_0^{Q^2_{\rm cut}} \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{C_R T g^2 m_D^2}{q_\perp^2 + m_D^2}, \hat{q}_{S,L} = \cdots$$

- Can also model certain non-perturbative effect by parametric $\Delta\hat{q}$.

Large-Q by scatterings $\mathcal{C}(Q > Q_{\rm cut})$

- Medium participates as quasi-particles.
- Few body scattering rates:

$$R = \frac{1}{2E_1} \int_{|t|>Q^2_{\rm cut}} d[\mathrm{PS}] f_0(p_2) |M|^2_{a,b\to c,d,\cdots}$$

Approximating the LPM effect in a semi-classical approach

- The medium-induced gluon radiation: coherent over $t_1 - t_2 \sim \tau_f$, $\tilde{\lambda} = m_D^2/\hat{q}$

$$\frac{dP}{dx} = g^2 \frac{P(x)}{\pi} \int_0^\infty dt_1 \int_{t_1}^\infty dt_2 \underbrace{\mathfrak{Re} \int_{\vec{q}_\perp, \vec{p}_\perp} \frac{i\vec{q}_\perp \cdot \vec{p}_\perp}{\delta E(\vec{q}_\perp)} \mathcal{C}(t_2) K(t_2, \vec{q}_\perp; t_1, \vec{p}_\perp)}_{F(t_2;t_1),\ \text{path-dependent}}$$

S Caron-Huot and C Gale, PRC 82 064902

Approximating the LPM effect in a semi-classical approach

- The medium-induced gluon radiation: coherent over $t_1 - t_2 \sim \tau_f$, $\tilde{\lambda} = m_D^2/\hat{q}$

$$\frac{dP}{dx} = \int_0^\infty dt_1 \underbrace{g^2 \frac{P(x)}{\tilde{\lambda}\pi}}_{\text{Incoherent rate}} \tilde{\lambda} \int_{t_1}^\infty dt_2 F(t_2; t_1)$$

- Boltzmann/rate Eq: $df/dt = R(t)$, single time variable (needs fundamental change).

Approximating the LPM effect in a semi-classical approach

- The medium-induced gluon radiation: coherent over $t_1 - t_2 \sim \tau_f$, $\tilde{\lambda} = m_D^2/\hat{q}$

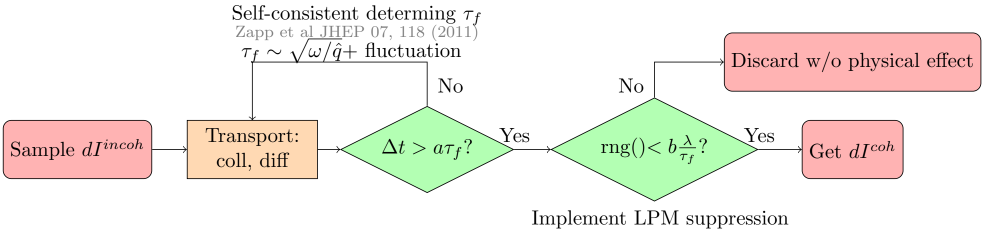
$$\frac{dP}{dx} = \int_0^\infty dt_1 \underbrace{g^2 \frac{P(x)}{\tilde{\lambda}\pi}}_{\text{Incoherent rate}} \tilde{\lambda} \int_{t_1}^\infty dt_2 \underbrace{F(t_2; t_1)}_{\text{Proxy ansatz } \sum_{\tau_f} \frac{b}{\tau_f} P(\tau_f)\delta(t_2 - t_1 - \tau_f)}$$

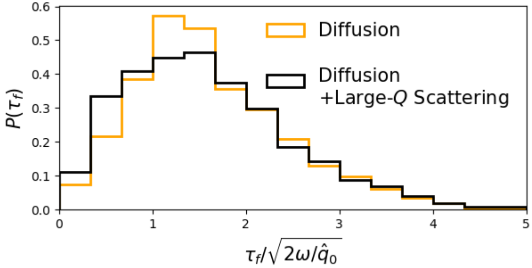
- Boltzmann/rate Eq: $df/dt = R(t)$, single time variable (needs fundamental change).
- An ansatz for test particles:
  - (a) stochastic process samples from the distribution $P(\tau_f)$ for a splitting.
  - (b) Meanwhile, system is propagated to $t + \tau_f$. Incoherently generated processes rejected by,

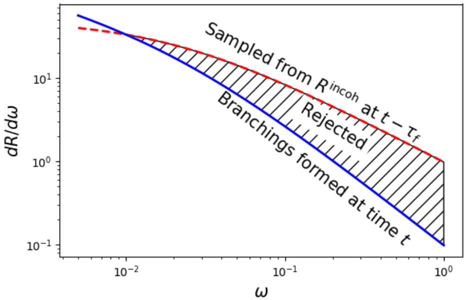
$$\frac{dP}{dx} \approx \int_0^\infty dt_1 \frac{dR^{\text{incoh}}}{d\omega}(t_1) \times \frac{b\tilde{\lambda}}{\tau_f} \longrightarrow \text{Probabilistic rejection at } t_2 = t_1 + \tau_f$$

Ke, Xu, and Bass, arXiv:1810.08177

Approximating the LPM effect in a semi-classical approach







Ke, Xu, and Bass, arXiv:1810.08177

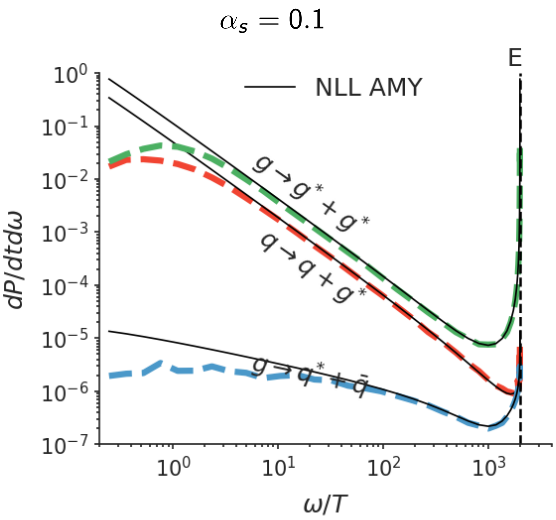
“Calibrate” this approach in the deep-LPM region

- The determined formation time has an logarithmic-Q_0 ambiguity

$$\langle\tau_f\rangle^{-1} \propto \sqrt{\hat{q}/\omega} \propto \sqrt{\frac{g^4 T^3}{\omega} \ln\left(1+\frac{Q_0^2}{m_D^2}\right)}, \quad q_\perp^2 |M|^2 dq_\perp^2 \propto dq_\perp^2/q_\perp^2$$

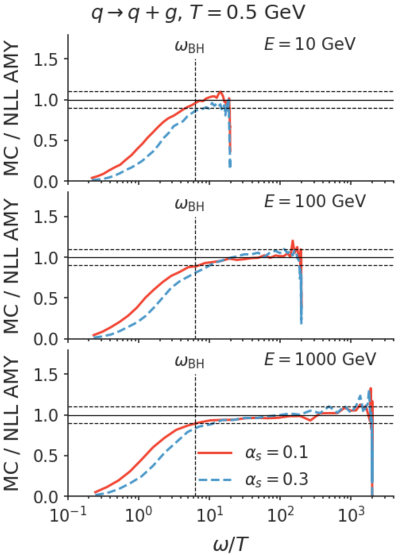
- For incoherent case, $q_\perp$ limited by its maximum $q_{\max} < \sqrt{s}$. On average $Q_0^2 = s \sim 6ET$.
- Analysis at NLL order Arnold and Dogan, PRD 78 065008 and recently by Mehtar-Tani, arXiv:1903.00506 suggest that, $Q_0^2 \propto \sqrt{\hat{q}\omega} \propto \sqrt{g^4 T^3 \omega \ln(Q_0^2/m_D^2)}$.
- This mismatch in Q_0 is fixed by a scale dependent rejection probability $b\tilde{\lambda}/\tau_f$ where $b \propto \sqrt{\frac{\ln(1+\tau_f/\tilde{\lambda})}{\ln(1+6ET/m_D^2)}}$ $(\sqrt{\hat{q}\omega}/m_D^2 \approx \tau_f \hat{q}/m_D^2 = \tau_f/\tilde{\lambda})$.

Compare simulation to the theory in the deep-LPM region



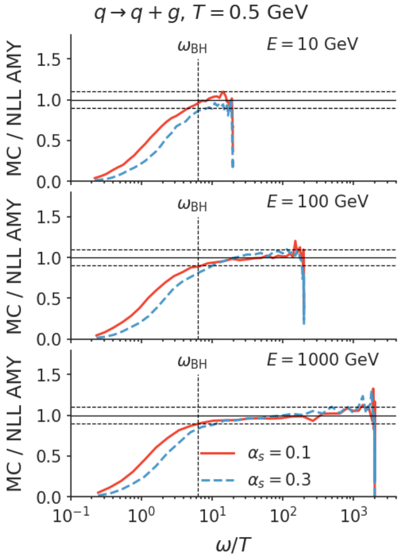
- Comparing the modified transport approach to NLL solutions of AMY equation Arnold and Dogan, PRD 78 065008.
- $\omega \lesssim T \ll E$, incoherent radiation by Gunion-Bertsch cross-section.
- $\omega \gg T$, consistent with theory calculation.
- For heavy quark, the dead-cone effect further suppress the radiation spectrum $1/k_\perp^2 \to 1/(k_\perp^2 + x^2 M^2), \cdots$

Compare simulation to the theory in the deep-LPM region



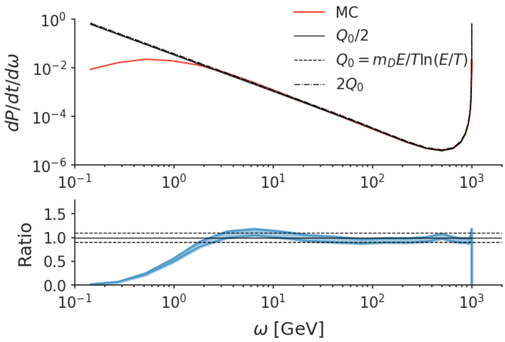
- Agrees when $\omega/T \gg 1$ varying E and α_s.

Compare simulation to the theory in the deep-LPM region



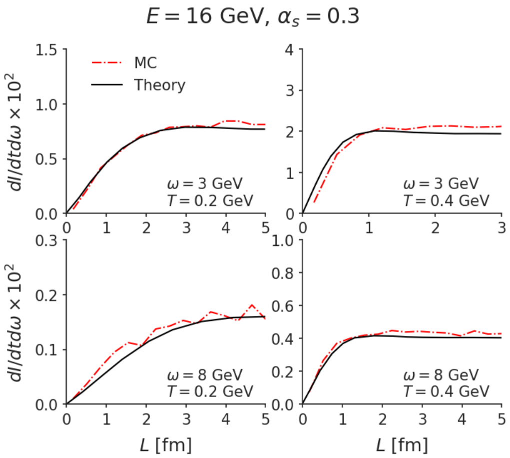
- Agrees when $\omega/T \gg 1$ varying E and α_s.
- Running coupling is also implemented,
  $\alpha_s(Q^2 = q_\perp^2)$ for elastic vertices
  $\alpha_s(Q^2 = k_\perp^2)$ for splitting vertices.

Use running α_s in theory and simulation



In a finite medium, complex interference pattern

- In principle, this transport approach is better applied to a large / dense medium. And one should apply, for instance, opacity expansion (Wiedemann, Gyulassy, Levai, Vitev) when $L \lesssim \tau_f$.

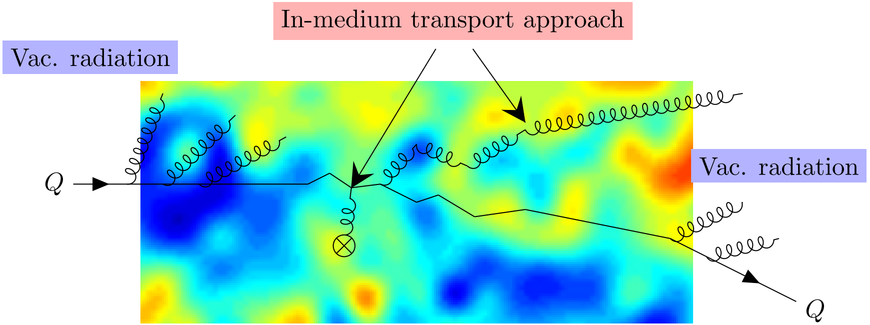


- But the currently implementation does demonstrate certain finite-size effect.
- Qualitatively the right transition to the large-L limit.
- Systematic deviations at small L. Theory curves from S Caron-Huot and C Gale, PRC 82 064902

Simulation framework for open charm

- Medium evolution: event-by-event 2+1D viscous hydrodynamics
- **Production, high-virtuality**: Pythia8 T. Sjöstrand et al JHEP 05 026, CPC 178 852 .
- **HQ transport model at low-virtuality**: this talk.
- Heavy flavor hadronization: Recombination + fragmentation S. Cao et al. PRC 88, 044907.
- Hadronic phase: Ultra-relativistic Quantum Molecular Dynamics S Bass et al. PPNP 41 225-370, M Bleicher et al. JPG 25 1859 with D-π, D-ρ cross-sections Z Lin et al. NPA 689, 965

Where to apply the transport model?



- Interface high/low-virtuality models at scale $\Delta k_\perp^2 \sim \sqrt{\hat{q}\omega}$ (P. Caucal et al. PRL 120, 232001).
- Surviving vacuum radiations are either $k_\perp^2 > \sqrt{\hat{q}\omega}$ or $\tau_f > \tau_{\mathrm{QGP}}$.
  τ_{QGP}: the time when the parton leaves the QGP.

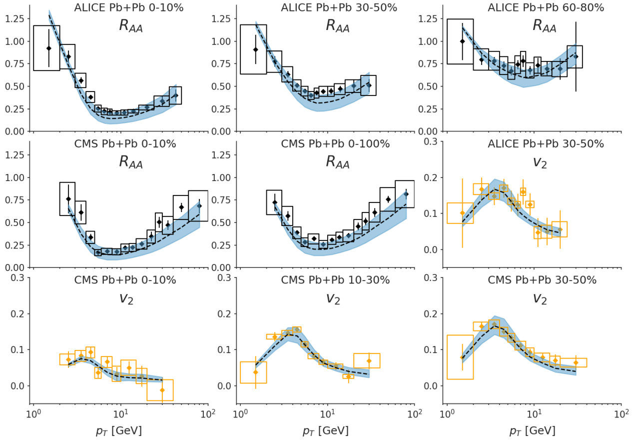
Bayesian Analysis (Preliminary): Identifying Tunable Parameter

- Effective coupling: $\alpha_s(Q) = \frac{2\pi}{9}\left(\ln\frac{\max\{Q,\mu\pi T\}}{\Lambda_{\rm QCD}}\right)^{-1}$.
- Medium pre-equilibrium time τ_i.
- Matching between vacuum-like and medium-induced radiation $R_v\Delta k_\perp^2$.
- Hard / soft-mode separation scale $Q_{\rm cut}^2 = c m_D^2$.
- Soft contribution to $\hat{q}$: $\hat{q}^S = \hat{q}^S_{\rm pert.} + \underbrace{\Delta\hat{q}}_{\text{additional part}}$
- $\Delta\hat{q}$ takes a parametric form, and is allowed to be anisotropic,

$$\Delta\hat{q} = \frac{K T^3}{\left[1+\left(a\frac{T}{T_c}\right)^p\right]\left[1+\left(b\frac{E}{T}\right)^q\right]},$$

$$\Delta\hat{q}_L = \frac{\Delta\hat{q}}{2}\left(\frac{E}{M}\right)^\gamma.$$

Global parameter calibration



- Currently compared to LHC open-charm measurements.
- Calculation at RHIC energy in progress.

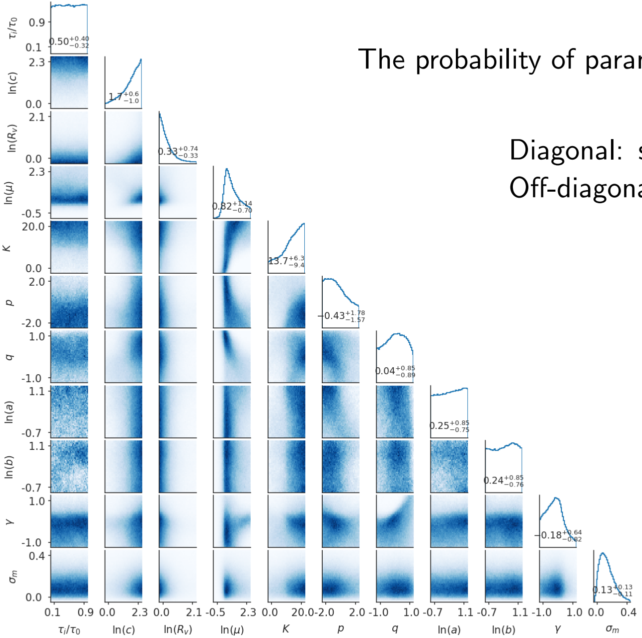
ALICE: v_2 PRL 120, 102301

ALICE: R_{AA} JHEP 10 (2018) 174

CMS: v_n PRL 120, 202301

CMS R_{AA} PLB 782, 474

The probability of parameters given model & data

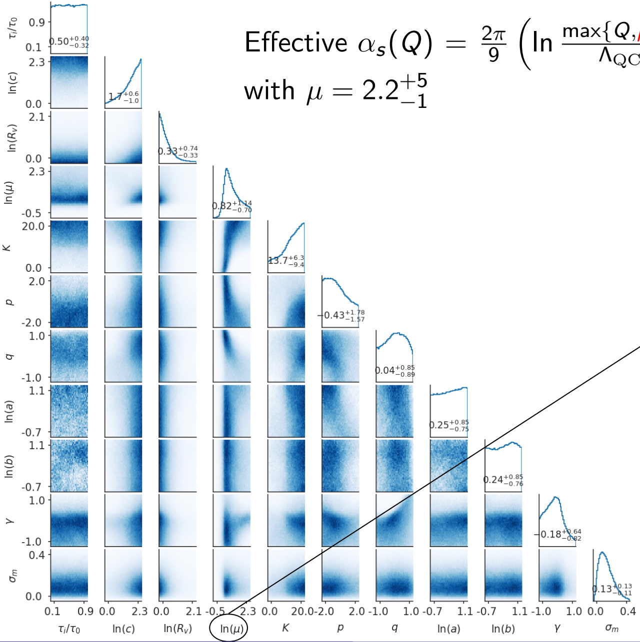


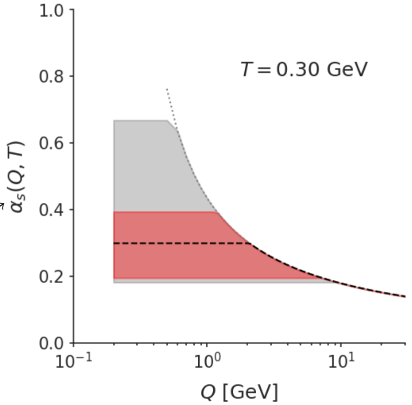
Diagonal: single parameter distribution.

Off-diagonal: two-parameter joint distribution.

Effective $\alpha_s(Q) = \frac{2\pi}{9}\left(\ln\frac{\max\{Q,\mu\pi T\}}{\Lambda_{\rm QCD}}\right)^{-1}$,

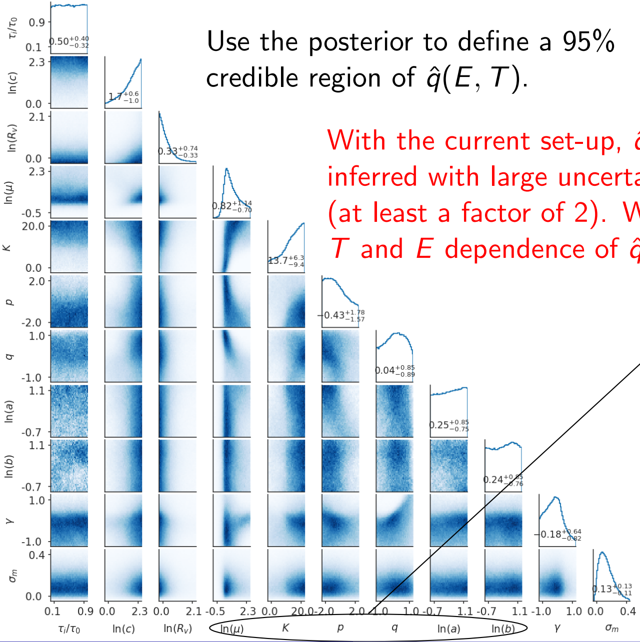
with $\mu = 2.2^{+5}_{-1}$



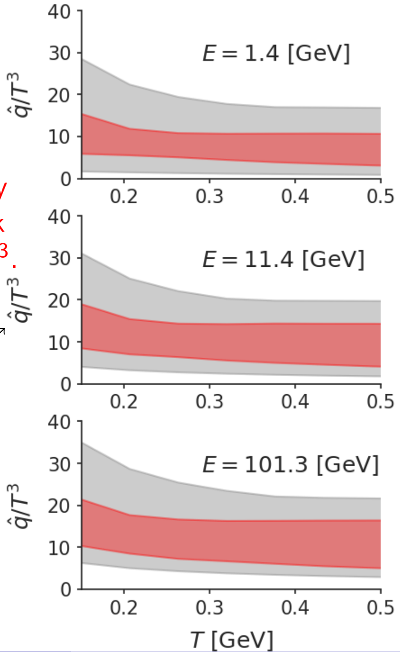


Effectively $\alpha_s = 0.2$ - 0.4 ($g = 1.5$ - 2.2) at $T = 0.3$ GeV.

Use the posterior to define a 95% credible region of $\hat{q}(E, T)$.

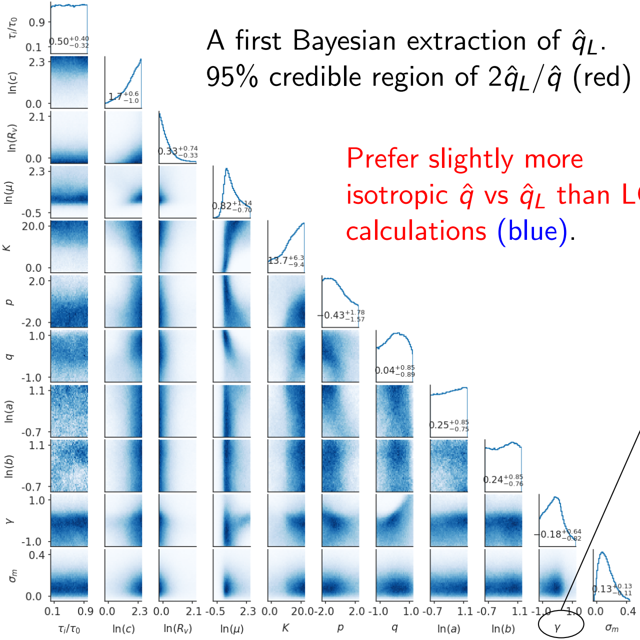


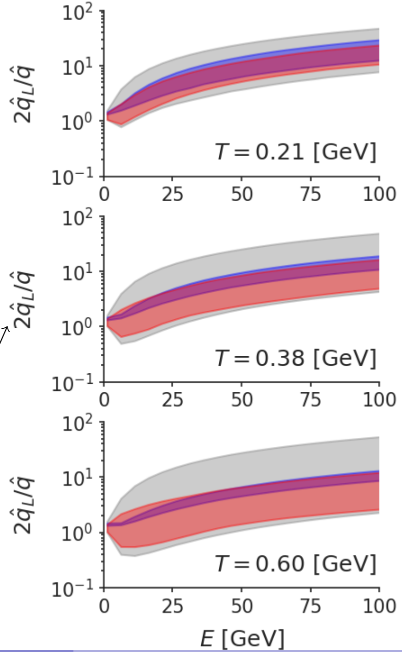
With the current set-up, $\hat{q}$ is inferred with large uncertainty (at least a factor of 2). Weak T and E dependence of $\hat{q}/T^3$.



A first Bayesian extraction of $\hat{q}_L$.
95% credible region of $2\hat{q}_L/\hat{q}$ (red)

Prefer slightly more isotropic $\hat{q}$ vs $\hat{q}_L$ than LO calculations (blue).

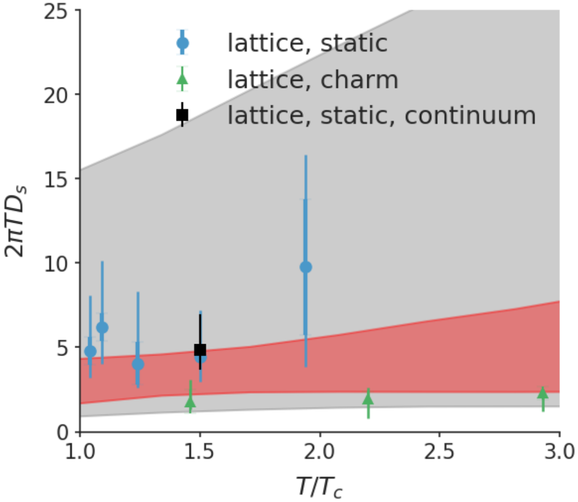




Summary

- Recent efforts in LIDO transport model:
  - Allow for an interpolating description between diffusion and large angle scattering.
  - Improving the LPM treatment in the Boltzmann transport approach.
- Application to open charm: preliminary Bayesian analysis.
  - A large effective coupling $\alpha_s(2.2^{+5}_{-1}\pi T)$.
  - Large uncertainty in $\hat{q}$ given present model and data.
  - A first extraction of the longitudinal transport coefficient.
- Future:
  - Better small-L treatment.
  - Jet study within the the modular jet simulation framework JETSCAPE (NPA 982 615-618)

Extrapolate the calibrated $\hat{q}$ to zero momentum



Spatial diffusion coefficient D_S

$$2\pi T D_s = \frac{8\pi T^3}{\hat{q}(p \to 0)}$$

Remark for reading this plot:

- Data helps to constrain the large / finite momentum part of $\hat{q}$.
- But the extrapolation in obtaining D_s can be sensitive to how the parametrization behaves when $p \to 0$.
- The red band may be underestimating the uncertainty. How to gain sensitivity to D_s?

Back-up: improving the matrix-elements

An improved version of the Gunion-Bertsch cross-section (J Gunion and G Bertsch PRD 25 746, O Fochler et al. PRD 88 014018 collinear, and $xq_\perp \ll k_\perp$, $y > 0$),

$$M_{23}^2 \propto M_{22}^2 C_A (1-x)^2 \left(\frac{\vec{k}_\perp}{k_\perp^2} - \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} \right)^2$$

Relaxing the condition $xq_\perp \ll k_\perp$,

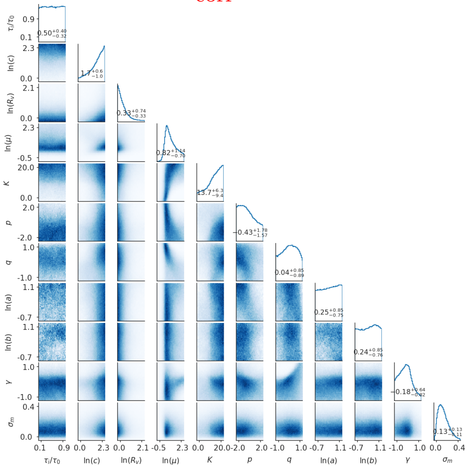
$$\begin{aligned} M_{23}^2 &\propto M_{22}^2 (1-x) P(x) \left(C_F \vec{A}^2 + C_F \vec{B}^2 - (2C_F - C_A) \vec{A} \cdot \vec{B} \right), \\ \vec{A} &= \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{k}_\perp - x\vec{q}_\perp}{(\vec{k}_\perp - x\vec{q}_\perp)^2} \\ \vec{B} &= \frac{\vec{k}_\perp - \vec{q}_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{k}_\perp}{\vec{k}^2} \end{aligned}$$

Bayesian Analysis: Bayes' rule and Posterior

$$\text{Posterior} \propto L(\text{Exp}|p, \text{Model}) \times \text{Prior}(p)$$

Experimental inputs on L_{corr} will be very helpful!

$L_{\text{corr}} = 1$



$L_{\text{corr}} = 0.5$