From saturation to high p_t jets:

toward a unified picture of parton production at all transverse momenta

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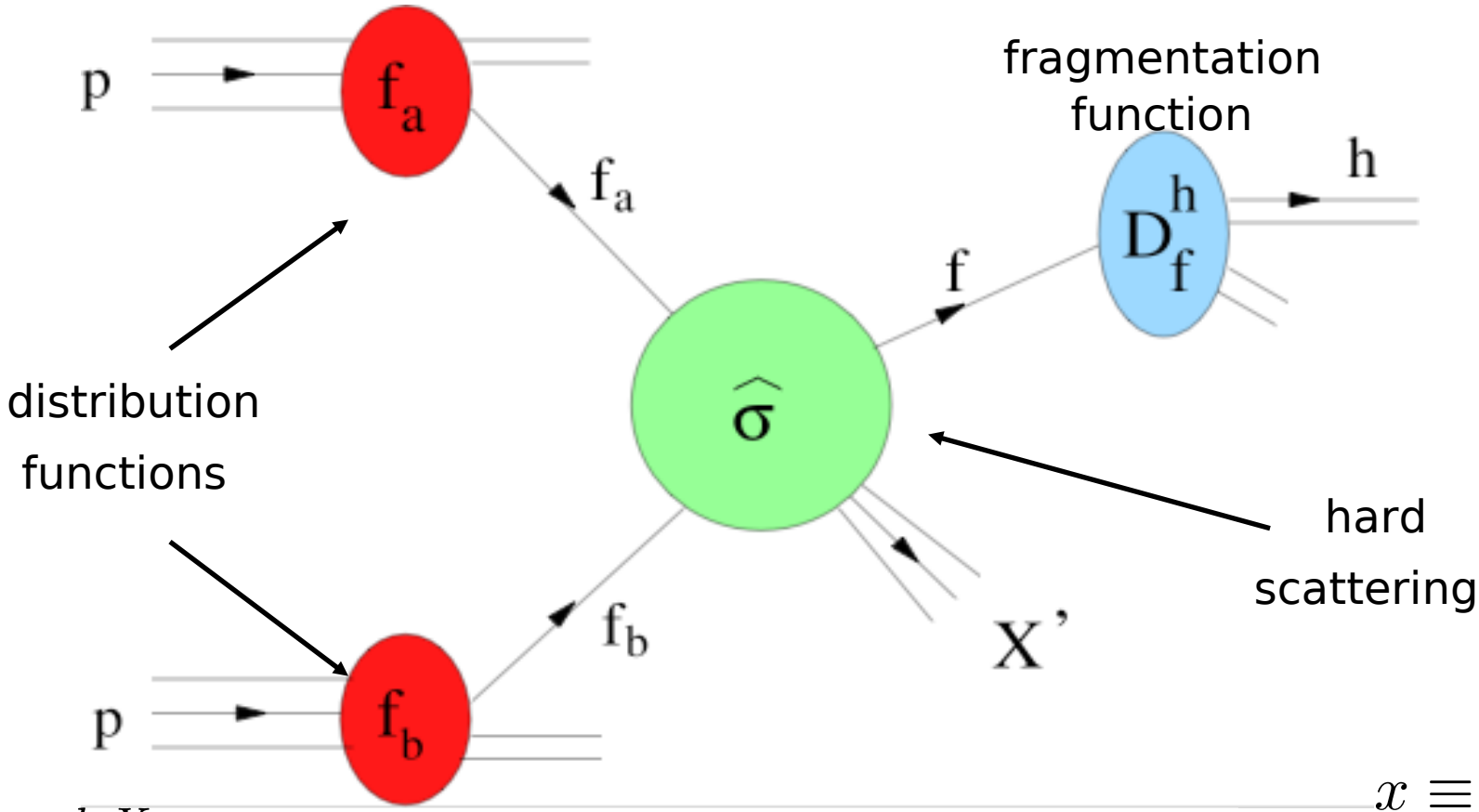
High p_t particle production: pp collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



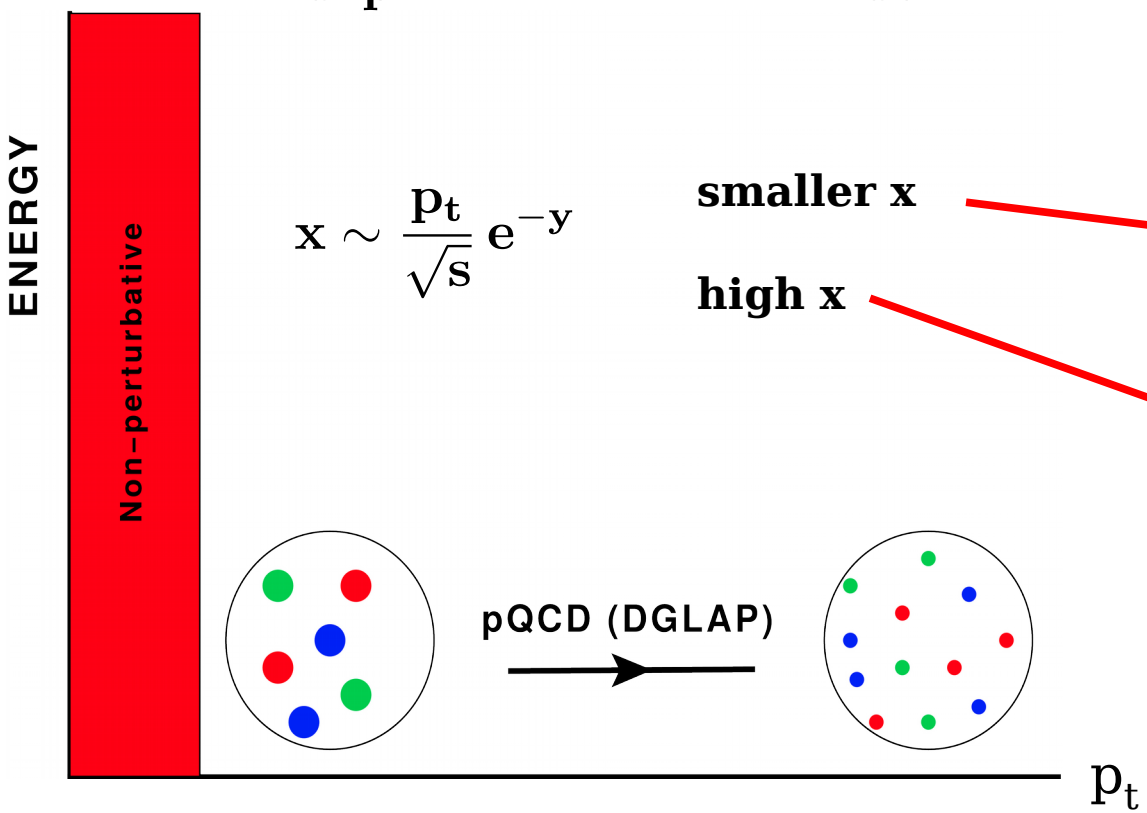
$$x \equiv \frac{p^+}{P^+}$$

$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections

pQCD: the standard paradigm

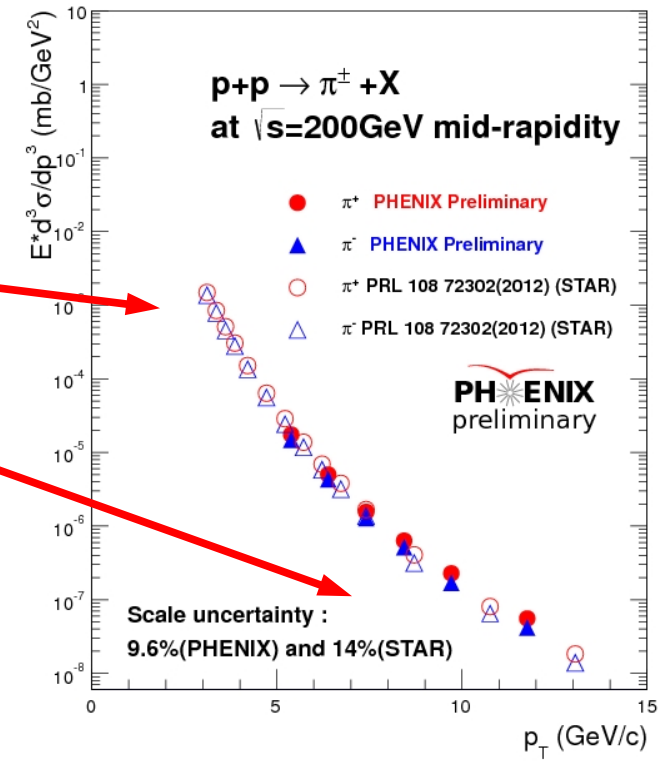
$$E\frac{d\sigma}{d^3p} \sim f_1(x) \otimes f_2(x) \otimes \frac{d\sigma}{dt} \otimes D(z)$$



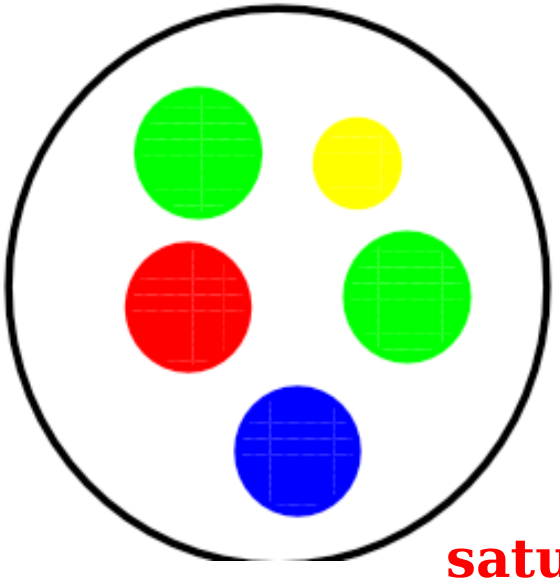
$$x \sim \frac{p_t}{\sqrt{s}} e^{-y}$$

smaller x

high x



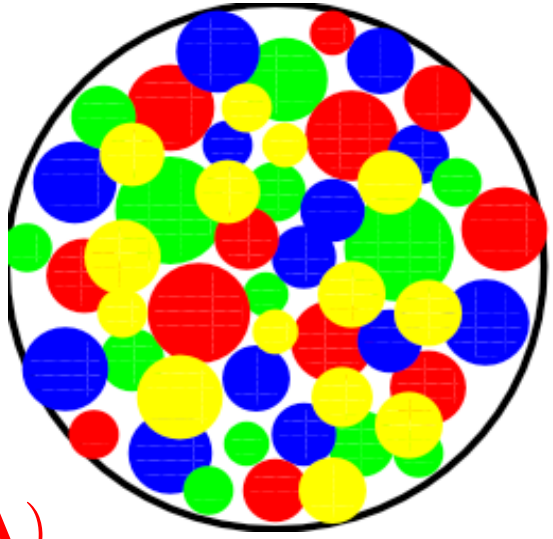
bulk of QCD phenomena happens at low p_t (small x)



energy ~ 1/x

gluon radiation $\mathbf{P_{gg}(x) \sim \frac{1}{x}}$

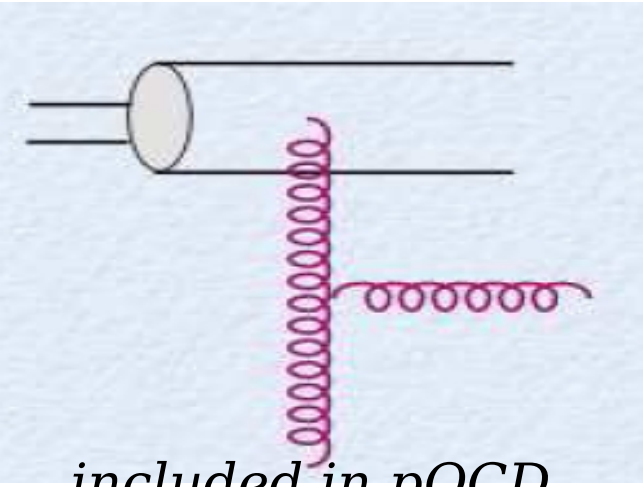
$$\frac{\alpha_s}{Q^2}\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$



saturation scale $\mathbf{Q_s^2(x, b_t, A)}$

collinear factorization breaks down at small x

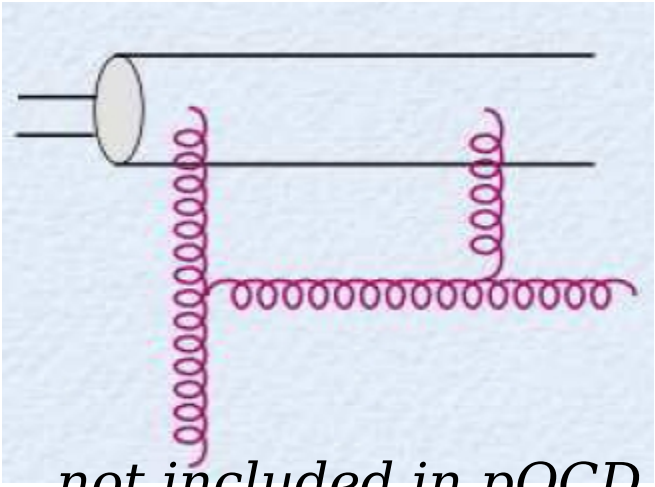
"attractive" bremsstrahlung vs. "repulsive" recombination



included in pQCD

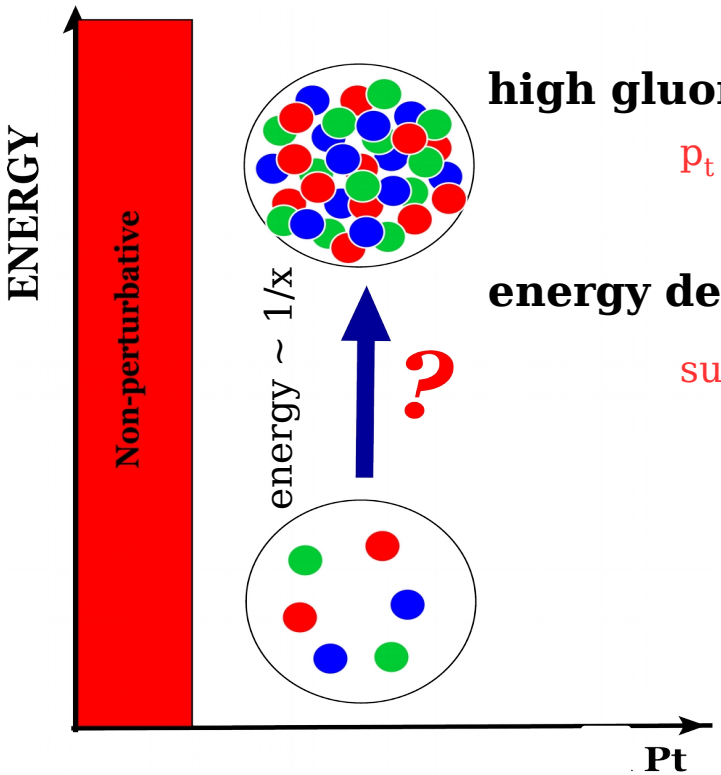
$$S \to \infty, \quad Q^2 \ fixed$$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$



not included in pQCD

A hadron/nucleus at high energy: gluon saturation



high gluon density: multiple scattering via Wilson lines

p_t broadening

energy dependence: x-evolution via JIMWLK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

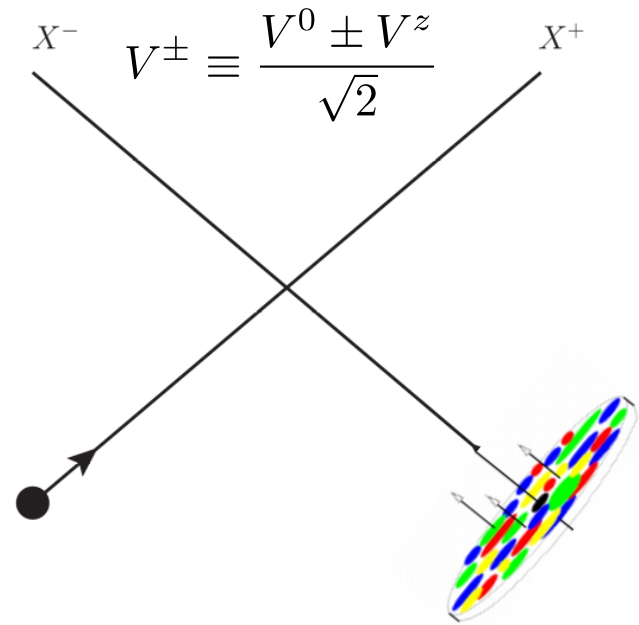
$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Initial conditions for hydro
Thermalization ?
Long range rapidity correlations
Azimuthal angular correlations
Nuclear modification factor

eliminate/minimize medium effects (*proton-nucleus*)



Eikonal approximation

$$J_a^\mu \simeq \delta^{\mu -} \rho_a$$

$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0$$ (in $A^+ = 0$ gauge)

does not depend on x^-

solution to classical EOM:

$$A_a^-(x^+, x_t) \equiv n^- S_a(x^+, x_t)$$

with

$$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$

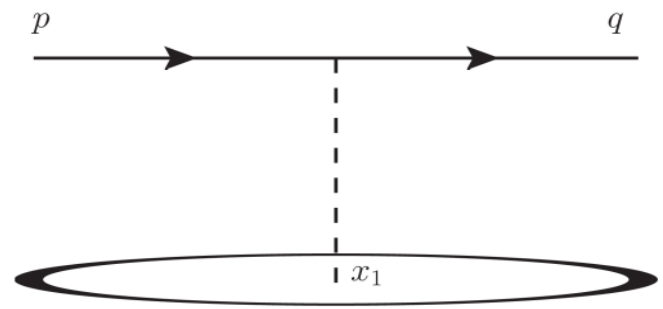
$$n^2 = 2 n^+ n^- - n_\perp^2 = 0$$

recall (eikonal limit): $\bar{u}(q)\gamma^\mu u(p) \to \bar{u}(p)\gamma^\mu u(p) \sim p^\mu$

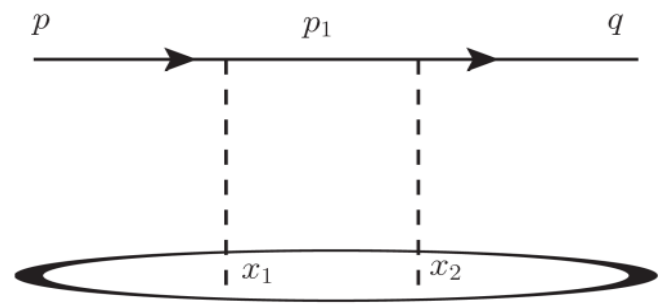
$$\bar{u}(q)\not{A} u(p) \to p \cdot A \sim p^+ A^-$$

multiple scattering of a quark from background color field $A_a^-(x^+, x_t)$

$$
\begin{aligned}
i\mathcal{M}_1 &= (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p) \\
&= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}} \\
&\quad \bar{u}(q)\, \left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)
\end{aligned}
$$



$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2 \int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(q-p_1)x_2} \\
&\quad \bar{u}(q)\, \left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right]\, u(p)
\end{aligned}
$$

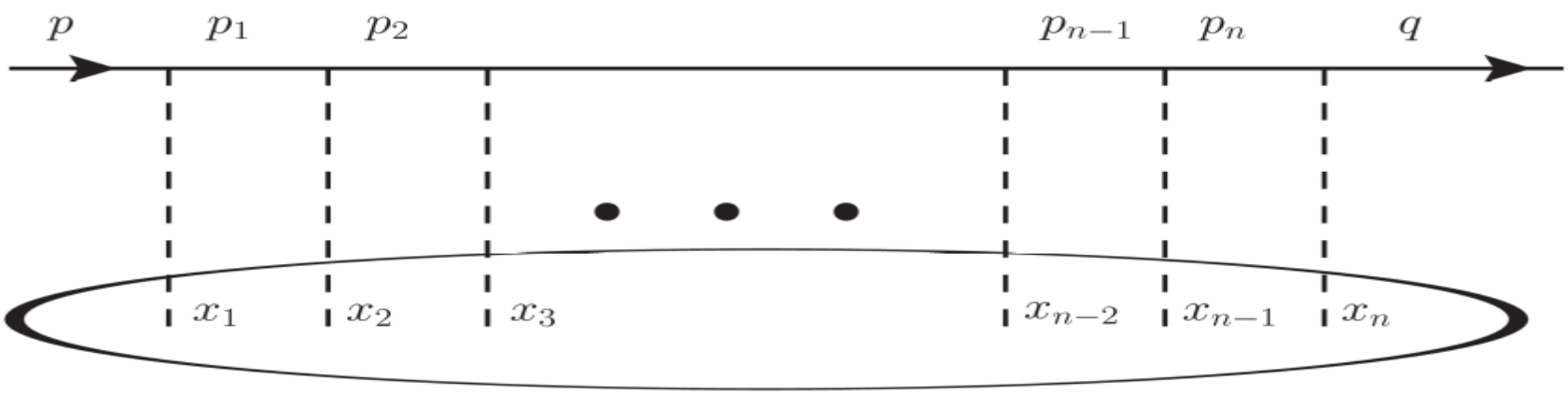


$$
\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\, \theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}
$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\, \frac{\not{p}_1}{2n\cdot p}\, \not{n} = \not{n}$

$$
\begin{aligned}
i\mathcal{M}_2 &= (ig)^2\, (-i)(i)\, 2\pi\delta(p^+ - q^+) \int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+) \int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}} \\
&\quad \bar{u}(q)\, \left[S(x_2^+, x_{1t})\, \not{n}\, S(x_1^+, x_{1t})\right]\, u(p)
\end{aligned}
$$



$$i\mathcal{M}_n = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}$$
$$\Big\{(ig)^n\,(-i)^n(i)^n\int dx_1^+\,dx_2^+\,\cdots dx_n^+\,\theta(x_n^+ - x_{n-1}^+)\,\cdots\theta(x_2^+ - x_1^+)$$
$$\big[S(x_n^+,x_t)\,S(x_{n-1}^+,x_t)\,\cdots S(x_2^+,x_t)S(x_1^+,x_t)\big]\Big\}\,u(p)$$

sum over all scatterings $\quad i\mathcal{M} = \sum_n i\,\mathcal{M}_n$

$$i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\;[V(x_t)-1]\;u(p)$$

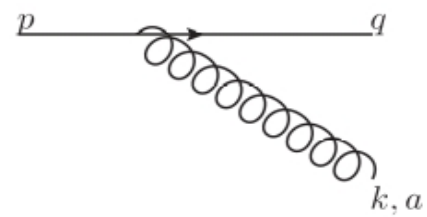
with $V(x_t) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{+\infty} dx^+\,n^-\,S_a(x^+,x_t)\,t_a\right\}$

$$\frac{d\,\sigma^{q\,T\to q\,X}}{d^2p_t\,dy} \sim |i\mathcal{M}|^2 \sim \;F.T.\; < Tr\,V(x_t)\,V^\dagger(y_t) >$$

a dipole

1-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)



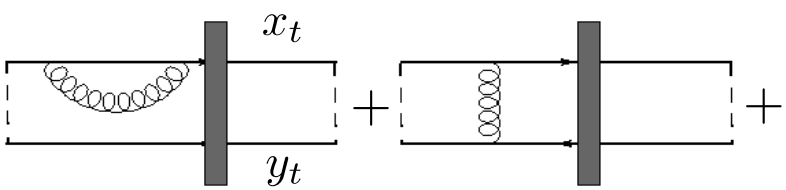
$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^\mu_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$

coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2} e^{ik_t\cdot(x_t-z_t)} 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$

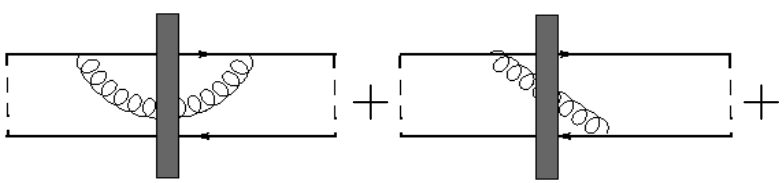
x_t, z_t are transverse coordinates of the quark and gluon

virtual corrections:



$$\longrightarrow Tr\,V(x_t)\,V^\dagger(y_t)$$

real corrections:



$$\longrightarrow Tr\,V(x_t)\,V^\dagger(z_t)\,Tr\,V(z_t)\,V^\dagger(y_t)$$

$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

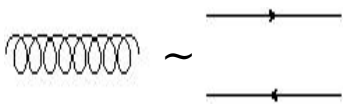
the S matrix

$$S(x_t,y_t) \equiv \frac{1}{N_c} Tr\,V(x_t)\,V^\dagger(y_t)$$

1-loop correction: BK eq.

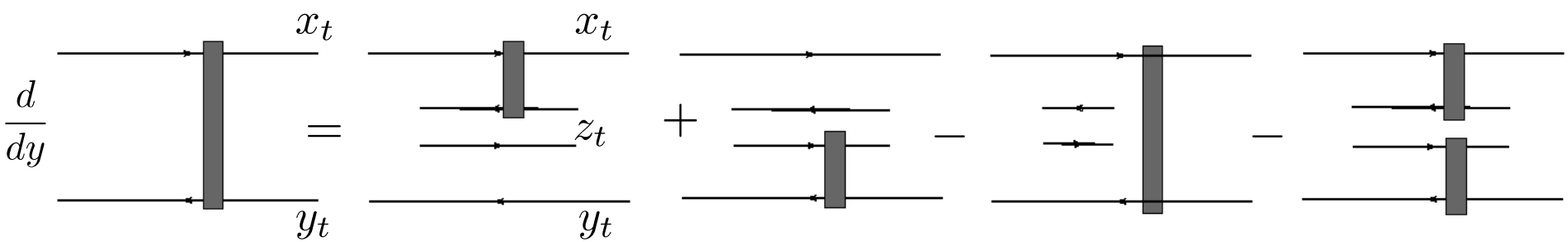
at large N_c

$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$



$$\frac{d}{dy}T(x_t,y_t)=\frac{N_c\alpha_s}{2\pi^2}\int d^2z_t\,\frac{(x_t-y_t)^2}{(x_t-z_t)^2(y_t-z_t)^2}\left[T(x_t,z_t)+T(z_t,y_t)-T(x_t,y_t)-T(x_t,z_t)T(z_t,y_t)\right]$$

$$T \equiv 1 - S$$



$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log\left[\frac{Q_s^2}{p_t^2}\right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^{\gamma} \qquad Q_s^2 < p_t^2$$

nuclear modification factor

$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2p_t\,dy}}{A^{1/3}\,\frac{d\sigma^{pp}}{d^2p_t\,dy}}$$

suppression of p_t spectra
nuclear shadowing
centrality dependence
.....

Particle production in high energy collisions

pQCD and collinear factorization at high p_t

breaks down at low p_t *(small x)*

CGC at low p_t

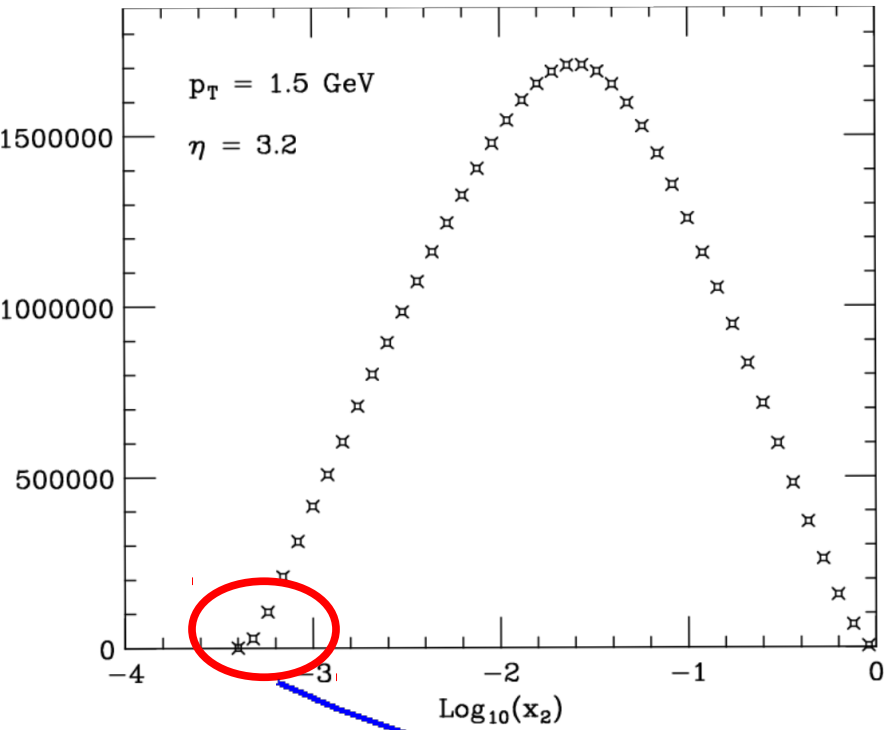
breaks down at large x (high p_t*)*

need a unified formalism

Particle production at RHIC: kinematics

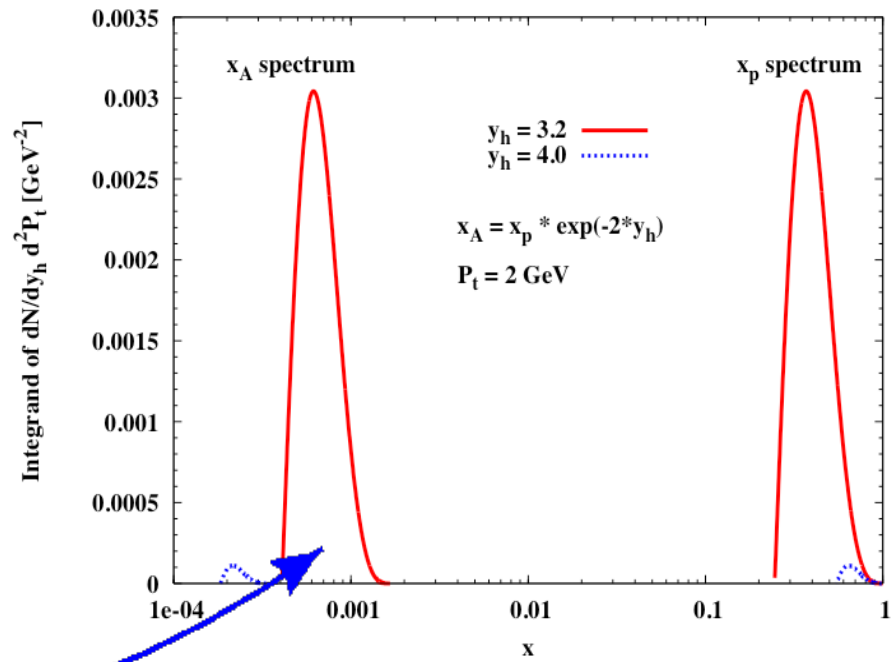
collinear factorization

GSV, PLB603 (2004) 173-183



CGC

DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x,Q^2)\cdots\cdots \longrightarrow x_{min}G(x_{min},Q^2)\cdots$$

this is an extreme approximation with potentially severe consequences!

how to tackle this problem?

what should be the *starting point/expression/operator*?

pQCD: quark and gluon operators

$$\overline{\Psi}(y^-, 0_t)\gamma^+\Psi(0^-, 0_t)$$

renormalization lead to DGLAP evolution eq.

CGC: correlators of Wilson lines (DIS, Hybrid,....)

$$F_2 \sim Tr\, V(x_t)\, V^\dagger(y_t)$$

renormalization leads to JIMWLK/BK evolution eq.

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$
\begin{aligned}
p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\
S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t)
\end{aligned}
$$

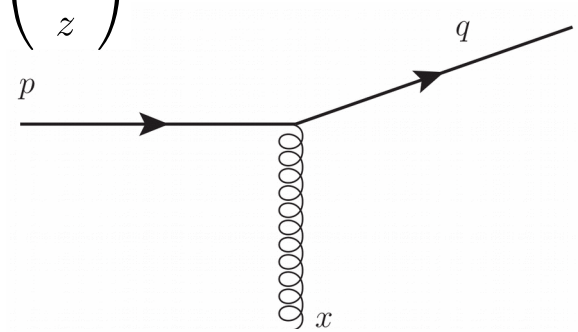
allow hard scattering by including one all x field $A_a^\mu(x^+, x^-, x_t)$ during which there is large momenta exchanged and quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering

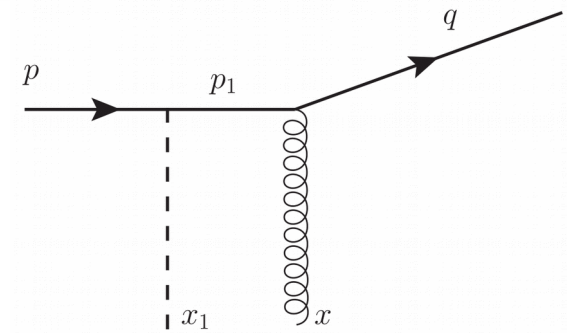
hard scattering: large deflection
scattered quark travels in the new "z" direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

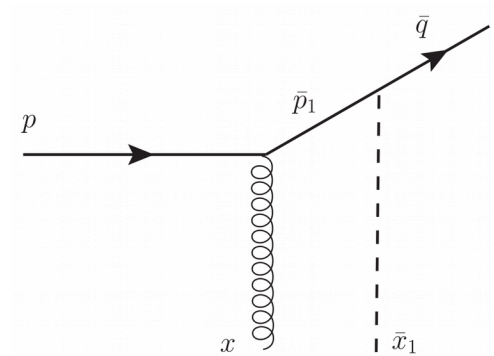
$$i\mathcal{M}_1 = (ig) \int d^4x \, e^{i(\bar{q}-p)x} \, \bar{u}(\bar{q}) \, \left[\not{A}(x)\right] \, u(p)$$



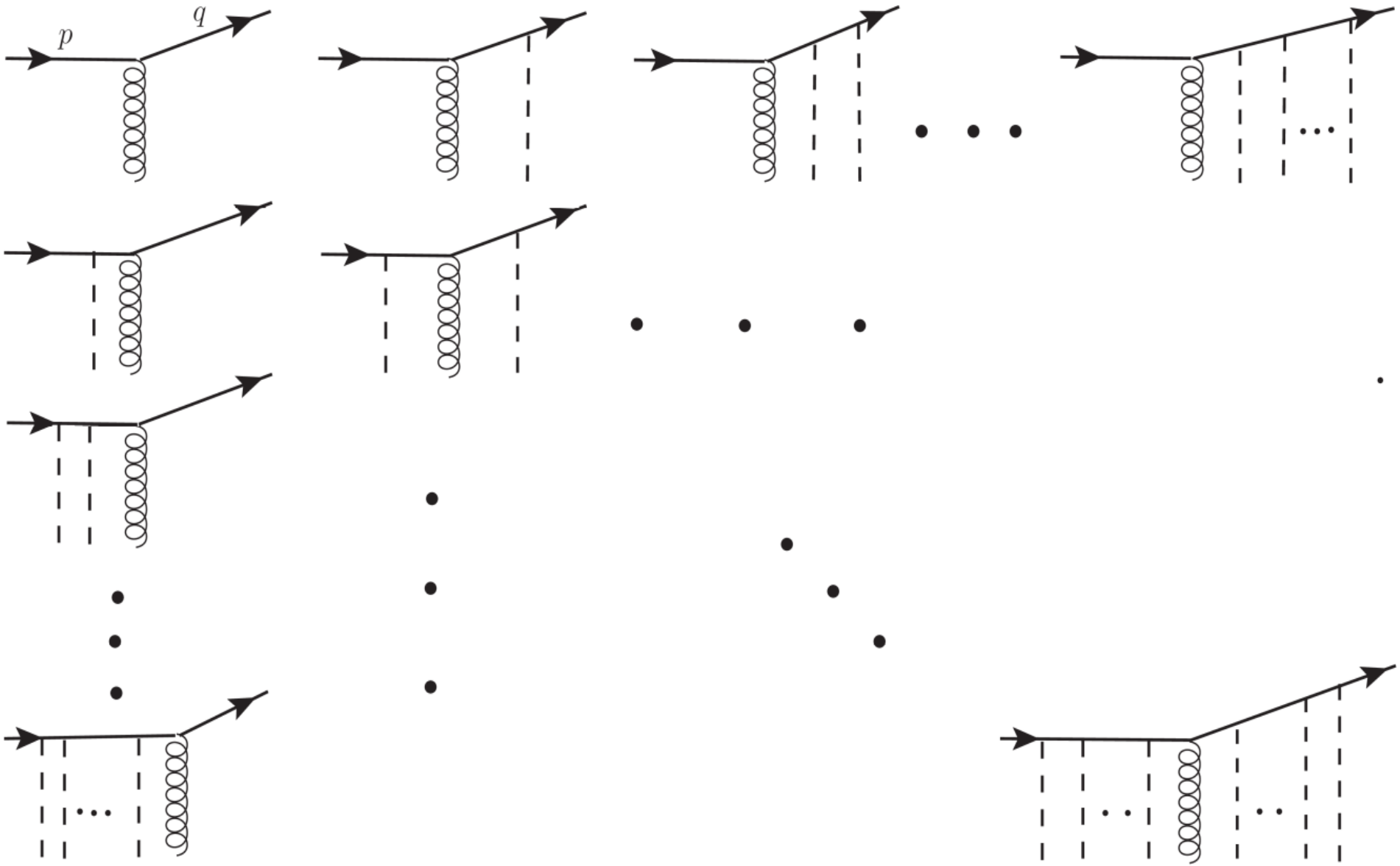
$$\begin{aligned} i\mathcal{M}_2 \quad = \quad & (ig)^2 \int d^4x \, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(\bar{q}-p_1)x} \\ & \bar{u}(\bar{q}) \left[\not{A}(x) \, \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1)\right] \, u(p) \end{aligned}$$



$$\begin{aligned} i\mathcal{M}_2 \quad = \quad & (ig)^2 \int d^4x \, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} \, e^{i(\bar{p}_1-p)x} \, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \\ & \bar{u}(\bar{q}) \left[\not{\bar{n}} \, \bar{S}(\bar{x}_1) \, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2 + i\epsilon} \not{A}(x)\right] \, u(p) \end{aligned}$$



with $\bar{v}^\mu = \Lambda^\mu_\nu \, v^\nu$

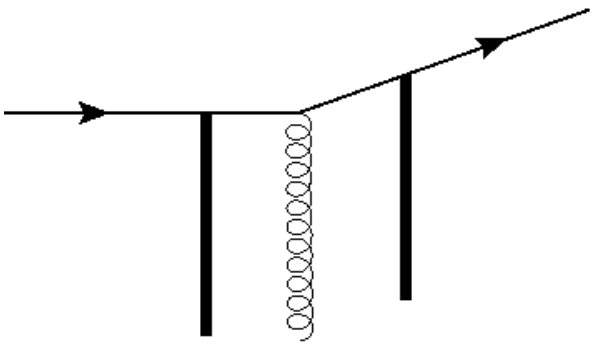

p
q

summing all the terms gives:

$$i\mathcal{M}_1 = \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\, \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t}$$

$$\bar{u}(\bar{q})\left[\overline{V}_{AP}(x^+,\bar{z}_t)\,\not{n}\,\frac{\not{\bar{k}}}{2\bar{k}^+}\,\left[ig\not{A}(x)\right]\,\frac{\not{k}}{2k^+}\,\not{n}\,V_{AP}(z_t,x^+)\right]u(p)$$

with



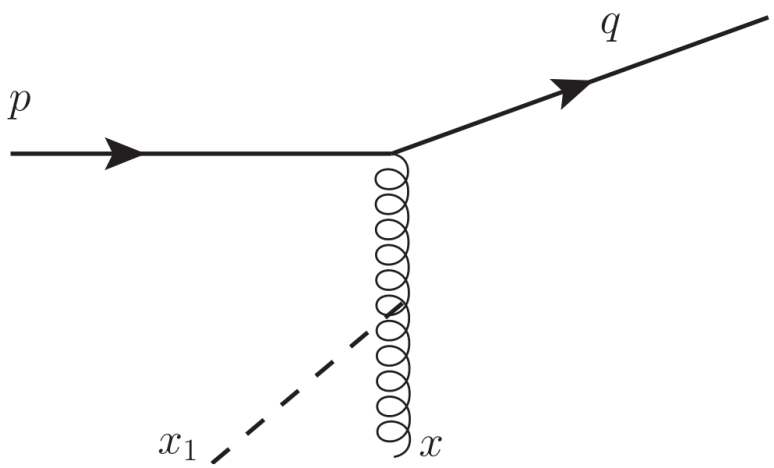
$$\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\,\bar{S}_a^-(\bar{z}_t,\bar{z}^+)\,t_a\right\}$$

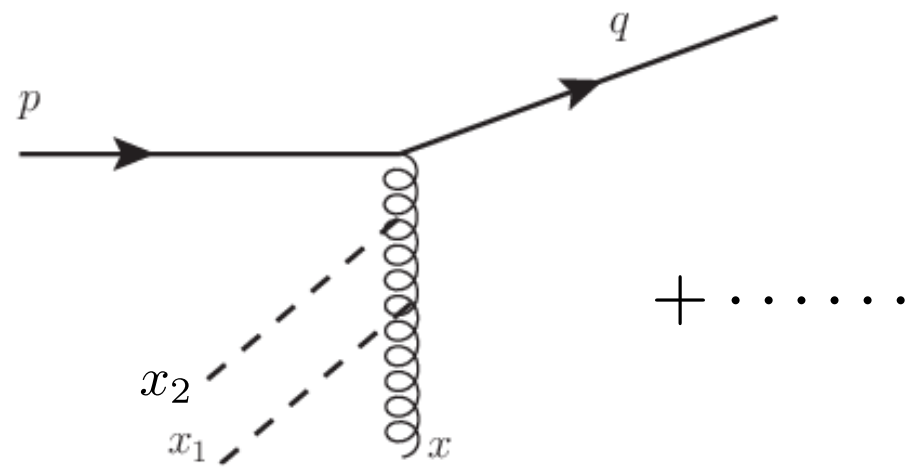
$$V_{AP}(z_t,x^+) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{x^+} dz^+\,S_a^-(z_t,z^+)\,t_a\right\}$$

can extract the effective quark propagator $\quad i\mathcal{M}(p,\bar{q}) = \bar{u}(\bar{q})\,\tau_F\,u(p)$

this quark propagator is the building block for DIS structure functions, single inclusive particle production in pA,....

but there is more to do: interactions of large and small x modes



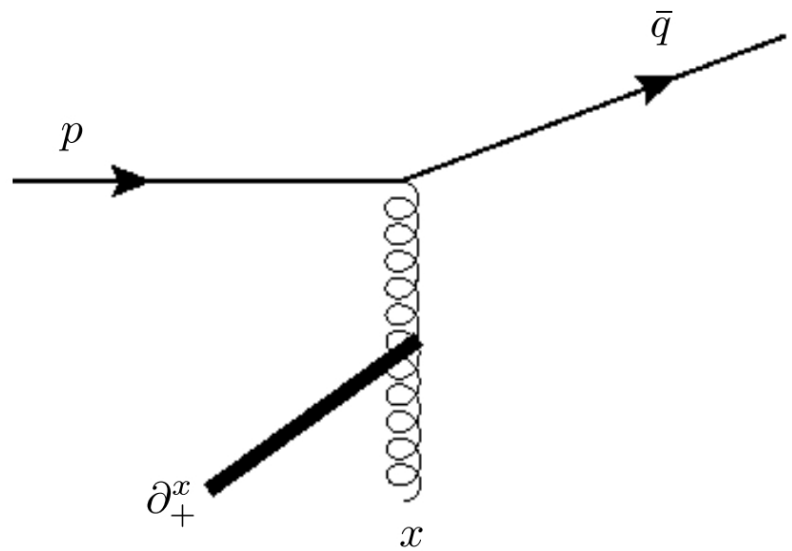


these re-sum to

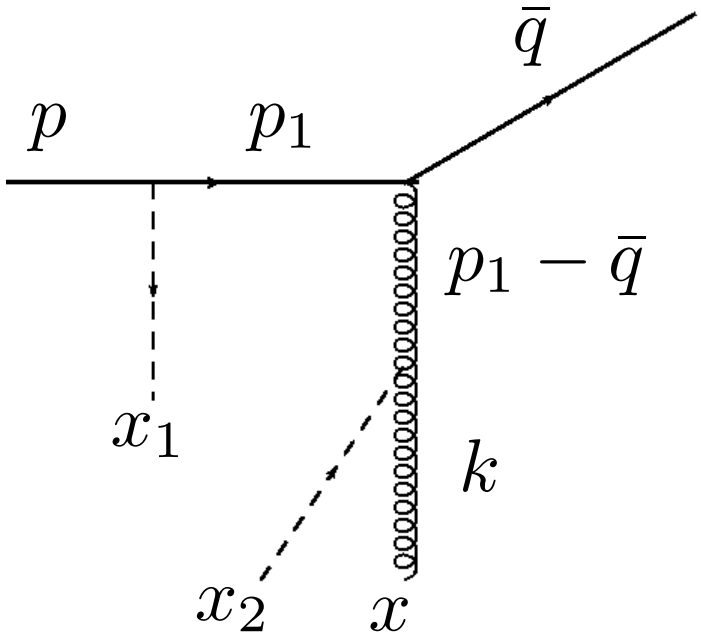
$$
\begin{aligned}
i\mathcal{M}_2 \quad = \quad & \frac{2\,i}{(p-\bar{q})^2} \int d^4x\, e^{i(\bar{q}-p)x}\, \bar{u}(\bar{q}) \Big[(ig\, t^a)\, \Big[\partial_{x^+}\, U^\dagger_{AP}(x_t, x^+)\Big]^{ab} \\
& \Big[n\cdot(p-\bar{q})\not{A}_b(x) - (p-\bar{q})\cdot A_b(x)\,\not{n} \Big] \Big]\, u(p)
\end{aligned}
$$

with

$$
U_{AP}(x_t, x^+) \equiv \hat{P}\, exp\left\{ ig \int_{-\infty}^{x^+} dz^+\, S^-_a(z^+, x_t)\, T_a \right\}
$$



but there is more!



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\,[(p_1 - \bar{q})^2 + i\epsilon]}$$

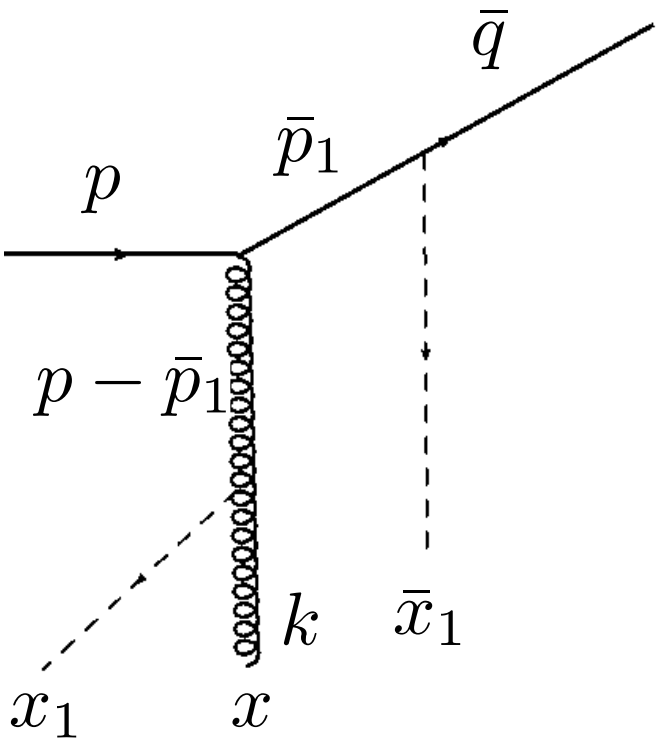
both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t}-\bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

how about the final state quark interactions?



integration over $\bar{p}_1^-$

$$\int \frac{d\bar{p}_1^-}{2\pi} \frac{e^{i\bar{p}_1^-(\bar{x}_1^+ - x^+)}}{[\bar{p}_1^2 + i\epsilon]\,[(p_1 - \bar{p}_1)^2 + i\epsilon]}$$

now the poles are on the opposite side of the real axis, we get both ordering

$$\theta(x^+ - \bar{x}_1^+) \text{ and } \theta(\bar{x}_1^+ - x^+)$$

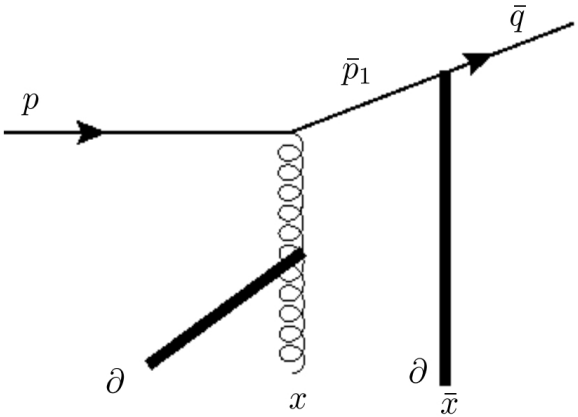
ignoring the phases the contribution of the two poles add!

path ordering is lost!

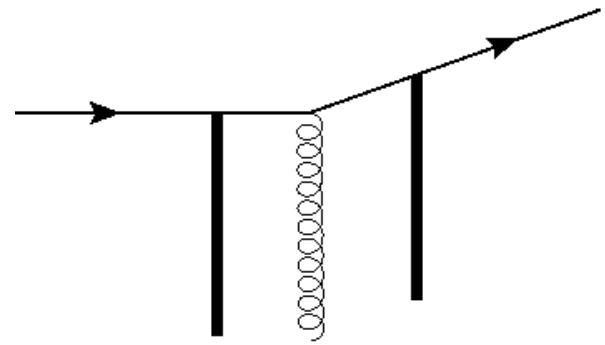
however further rescatterings are still path-ordered before/after $\mathrm{x}_1^+, \bar{\mathrm{x}}_1^+$

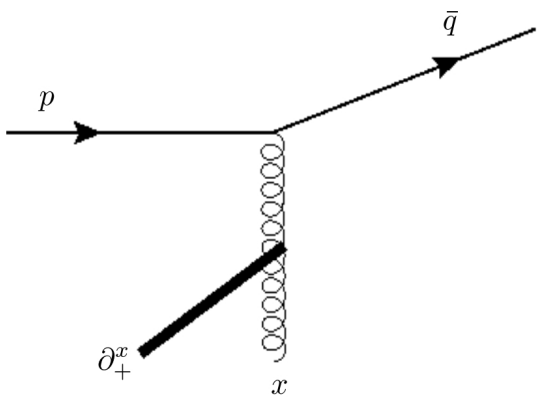
these contributions resum to

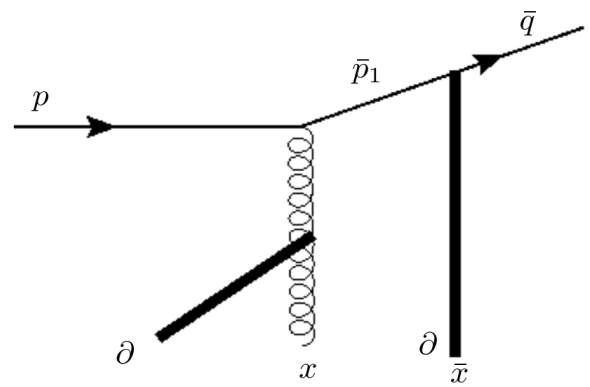
$$
\begin{aligned}
i\mathcal{M}_3 \;=\; & -2\,i \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+\, \frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+ - p^+)x^-}\; e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\; e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot \bar{x}_t} \\
& \bar{u}(\bar{q}) \Bigg[\left[\partial_{\bar{x}^+} \overline{V}_{AP}(\bar{x}^+, \bar{x}_t)\right] \not{n}\, \not{p}_1\, (igt^a)\, \left[\partial_{x^+} U^\dagger_{AP}(x_t, x^+)\right]^{ab} \\
& \frac{\left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\, 2n\cdot(p-\bar{q})\, p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]} \Bigg]\, u(p)
\end{aligned}
$$



full amplitude: $\mathbf{i\mathcal{M} = i\mathcal{M}_{eik} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3}$







soft (eikonal) limit: $A^\mu(x) \to n^- \, S(x^+, x_t) \qquad n \cdot \bar{q} \to n \cdot p$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

Need to clarify, further steps to be taken

The limit of intermediate/large x ?

ignore S^- ?

Gauge invariance ?

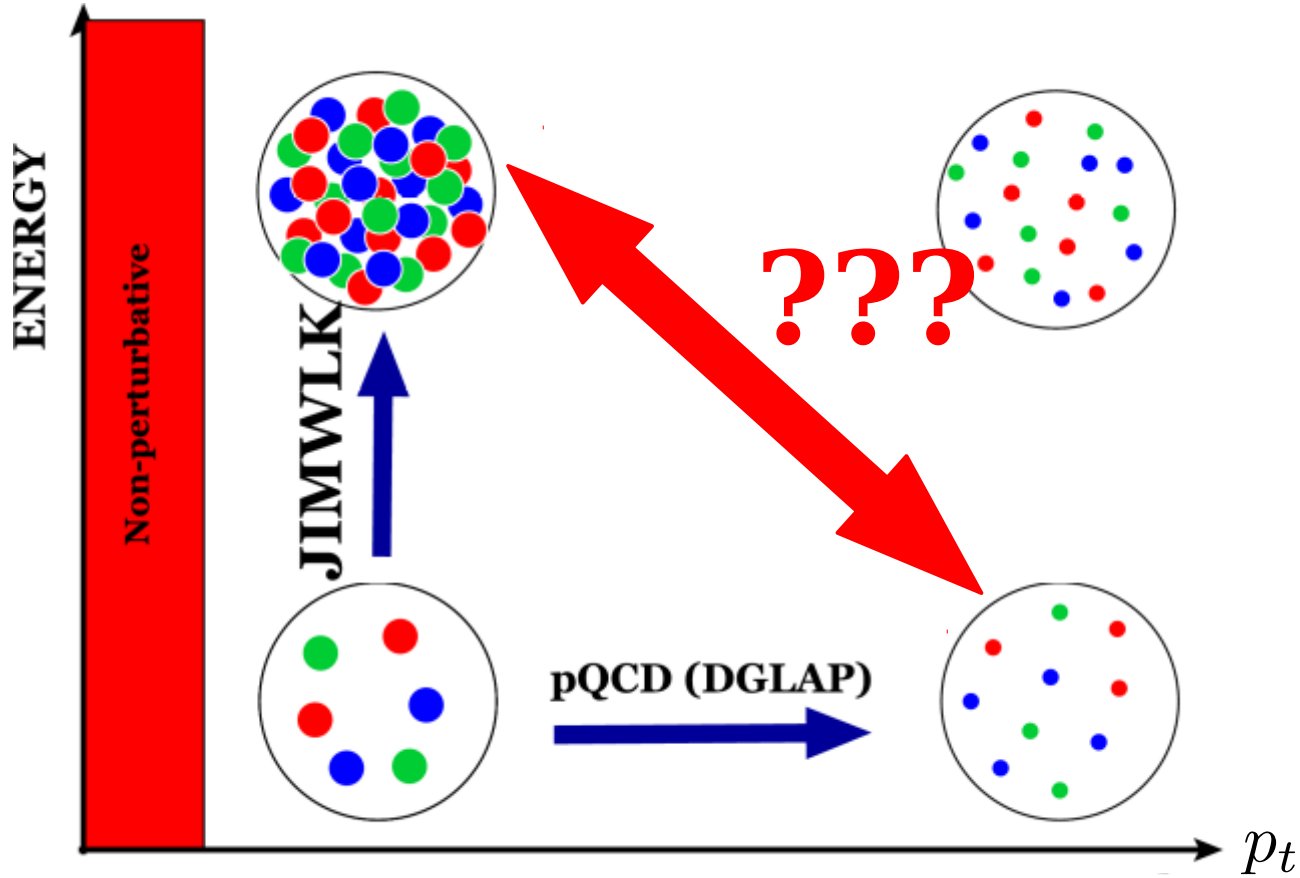
Matching between small and large x ?

brute force?

Gluon scattering and radiation

One-loop correction: a new evolution equation

QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

kinematics of saturation: *where is saturation applicable?*

jet physics, *high p_t (polar and azimuthal) angular correlations*

cold matter energy loss, spin physics?, ...

single inclusive pion production

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