Finite mass effects in Higgs boson plus jets production

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01.03.2018

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Outline

• Introduction and motivation

• H+1 jets at NLO with full top-quark mass dependence:
  • Process details
  • Numerical computation of 2-loop diagrams

• Phenomenological results:
  • Total cross section
  • Differential distributions

• Conclusions and outlook

Credits: Many thanks to Mattias Kerner and Stephen Jones for sharing material and for clarifications about the technical aspects of the 2-loop computation
The Higgs and the LHC: a success story!!
It was 6 years ago...

... that we were looking at exclusion plots seeing some hints

Consistency of the data with the background-only expectation

- Lowest probability ($p_0 = 1.9 \times 10^{-4}$) for background-only expectation observed for $m_H \sim 126$ GeV
- Local significance of excess: $3.6\sigma$ ($2.8\sigma \rightarrow \gamma \gamma 2.1\sigma H \rightarrow 4\ell, 1.4\sigma H \rightarrow tt \tau \tau$)
- Global significances: $2.5\sigma$ ($p = 0.6\%$) Look-elsewhere-effect over 110 – 146 GeV
  $2.2\sigma$ ($p = 1.4\%$) Look-elsewhere-effect over 110 – 600 GeV

K. Jakobs – Zurich Phenomenology Workshop 2012
in the meanwhile

... many results and steeply increasing statistics

Data collected in 2017

Data collected in 2011 (used for the plots in the previous slide)
New differential results at 13 TeV

• Very recent results by ATLAS:

Data become more and more precise, not only for inclusive quantities, but also for more exclusive and differential observables.
New differential results at 13 TeV

- First results on boosted Higgs by CMS: [CMS, 1709.05543, PRL 120 (2018) 071802]

"Inclusive search for a highly boosted Higgs boson decaying to a bottom quark-antiquark pair"

Search for $H \rightarrow b\bar{b}$ with:
- $p_T > 450$ GeV
- $-2.5 < \eta < 2.5$
- anti-$k_T$ R = 0.8

Look for fat jet:
- Select leading $p_T$ jet
- Apply soft-drop algorithm to groom the jet
- Reconstruct both Z and H

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed signal strength</td>
<td>$2.3^{+1.8}_{-1.6}$</td>
<td>$0.78^{+0.23}_{-0.19}$</td>
</tr>
<tr>
<td>Expected UL signal strength</td>
<td>$&lt; 3.3$</td>
<td>—</td>
</tr>
<tr>
<td>Observed UL signal strength</td>
<td>$&lt; 5.8$</td>
<td>—</td>
</tr>
<tr>
<td>Expected significance</td>
<td>$0.7\sigma$</td>
<td>$5.8\sigma$</td>
</tr>
<tr>
<td>Observed significance</td>
<td>$1.5\sigma$</td>
<td>$5.1\sigma$</td>
</tr>
</tbody>
</table>
Higgs boson production channels

- Gluon-Gluon Fusion
  - 87.73%
  - Largest production mechanism
  - Huge background

- Vector Boson Fusion
  - 7.49%
  - Characteristic signature
  - Coupling to VBs

- Higgs Strahlung
  - 3.77%
  - Clean final state
  - Measure $H \rightarrow bb$

- $ttH$
  - 1.01%
  - Small signal
  - Top-Yukawa coupling

$M_H = 125$ GeV
MSTW2008

[Percentage for 13 TeV]
Higgs boson decay rates in the SM

- Several different decay channels:
  - Experimental analyses tuned to the different decay channels in order to optimize the signal to background ratio
  - The number of accompanying jets also plays an important role

Refined analyses and a very high theoretical accuracy to make the most out of the LHC data!

125 GeV
LHC is a tough environment for precision.

- QCD is omnipresent at LHC:
  - PDF
  - Hard scattering and loop corrections
  - Parton Shower
  - Hadronization
  - Further non perturbative effects

- Master formula:

\[ \sigma_{h_1 h_2 \to X} = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1/a}(x_1, \mu_F^2) f_{h_2/b}(x_2, \mu_F^2) \times \hat{\sigma}_{a,b \to X} \left( x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2} \right) \left[ + \mathcal{O} \left( \frac{1}{Q^2} \right) \right] \]

  PDFs  partonic cross section  power corrections

- All components need to determined with high accuracy!

01/03/2018  - Gionata Luisoni  Seminario, Università Milano Bicocca
Fixed order calculations for H + n Jets

- Partonic cross section:

\[ \hat{\sigma}_{pp\rightarrow H+nJ} = \alpha_s^{2+n} \left[ \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 + \mathcal{O}(\alpha_s^4) \right] \]

<table>
<thead>
<tr>
<th>LO</th>
<th>NLO</th>
<th>NNLO</th>
<th>N^3LO</th>
</tr>
</thead>
</table>

- **LO:** Predicts only the order of magnitude:
  - Scale only rough estimate
  - 1 parton ↔ 1 jet

- **NLO:** First reliable predictions:
  - Scale choices can be made
  - First description of jet substructure

- **NNLO:** Possible to quantify uncertainties:
  - Convergence can be checked
  - Richer jet substructure
State of the art of the theoretical predictions

- **Gluon-gluon fusion**
- **Weak boson fusion**
- **Higgs Strahlung**
- **ttH**

**Latest results:**
- [H]: 1503.06056, 1602.00695, 1610.05497
- [H+1j]: 1504.07922, 1505.03893, 1508.02684, 1607.08817
- [H+3j]: 1307.4737, 1506.01016
- [... and more (see later slide)]

**Latest results:**
- [H+2j]: 1506.02660, 1606.00840

**Latest results:**
- [HW NNLO+PS]: 1603.01620
- [HW NNLO prod+bb-dec]: 1705.10304
- [HW NNLO prod+bb-dec]: 1712.06954

**Latest results:**
- [Off-shell]: 1506.07448, 1612.07138

[I tried not to miss any contribution, my apologies for any omission]
Gluon fusion

• Theoretically gluon fusion has two important key aspects

1. It’s loop induced (Born is a 1-loop process): huge increase in complexity

   However:
   • Since \( m_{\text{top}} > m_H \) we can integrate out top quark and compute in an effective theory where the Born is a tree-level amplitude

2. It suffers from very large perturbative corrections:
   • Huge NLO [\( O(100\%) \)] and NNLO [\( O(20\%) \)] effects!
   • Now known at \( N^3\text{LO} \):

<table>
<thead>
<tr>
<th>( \sigma ) [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
</tr>
<tr>
<td>NLO</td>
</tr>
<tr>
<td>NNLO</td>
</tr>
<tr>
<td>N3LO</td>
</tr>
</tbody>
</table>

   [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger, ’15-’16]
Gluon fusion

- LHC Higgs results allowed to **exclude** a 4th SM-like generation.

- New physics in the Higgs sector could however still hide in the **transverse momentum tail**. How to nail this down?

**Facts to consider:**

- Higher order corrections are particularly **sizable** in Higgs boson production in gluon-gluon fusion (also in association with jets).

- For a precise determination of the most important observables (e.g. the Higgs transverse momentum spectrum) a **good control over higher multiplicities** is relevant.

- LHC Run II is collecting data very fast. This will soon allow for **precise Higgs boson studies** at 13 TeV.

- **What are the dominant effects**: higher order corrections or mass effects?

- **How reliable** are effective field theory results, when do they **break down** and how are the different observables affected by mass corrections?
State of the art of the theoretical predictions

- Gluon fusion calculations in effective and full theory:

Loops: $g_{eff} F_2 \propto q^2$

Jets: $m_{top} \rightarrow \infty$

Latest results:
- [H: 1503.06056, 1602.00695]
- [H+1j: 1504.07922, 1505.03893, 1508.02684, 1607.08817]
- [H+3j: 1307.4737, 1506.01016]

[Latest results: [H+0,1,2j NLO merged + PS: 1410.5806]
[H+0,1,2j approx. NLO merged + PS: 1604.03017]
[H+1,2,3j LO: 1608.01195]
[H+1j approx. NLO: 1609.00367]
[H+1j approx. NNLO for pT: 1607.08817]
[planar 2-loop MI: 1609.06685]
[pp->Hj for nearly massless quarks: 1610.03747, 1702.00426]
[pp->Hj at large pT: 1712.06549, 1801.08226; 1802.02981]
[pp->Hj with full top mass: 1802.00349]

[I tried not to miss any contribution, my apologies for any omission]
H+1j at NLO with full top-quark mass effects
### Process details

- Leading order contributions (1-loop) to $pp \rightarrow H + 1\ jet$:

<table>
<thead>
<tr>
<th>Channel</th>
<th>Diagrams</th>
<th>$\sigma_{\text{LO}}(p_t,j &gt; 30\ GeV)$</th>
<th>$\sigma_{\text{LO}}(p_t,j &gt; 500\ GeV)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow Hg$</td>
<td><img src="image1" alt="Diagrams for gg to Hg" /></td>
<td>6.32 pb (74%)</td>
<td>1.92 x 10^{-3} pb (52%)</td>
</tr>
<tr>
<td>$qg \rightarrow Hq, \bar{q}g \rightarrow H\bar{q}$</td>
<td><img src="image2" alt="Diagrams for qg to Hq and qbar g to Hbar q" /></td>
<td>2.21 pb (26%)</td>
<td>1.72 x 10^{-3} pb (47%)</td>
</tr>
<tr>
<td>$q\bar{q} \rightarrow Hg$</td>
<td><img src="image3" alt="Diagrams for qbar q to Hg" /></td>
<td>0.04 pb (&lt; 1%)</td>
<td>0.04 x 10^{-3} pb (1%)</td>
</tr>
</tbody>
</table>

**Tot:** 8.56 pb 3.68 x 10^{-3} pb

- Computed using analytical implementation of the amplitudes [Baur, Glover 89]
### Process details

- **Real radiation corrections (1-loop):**

<table>
<thead>
<tr>
<th></th>
<th>Diagrams</th>
<th># Diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow Hgg$</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td>98</td>
</tr>
<tr>
<td>$qg \rightarrow Hqg, \bar{q}g \rightarrow H\bar{q}g$</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td>4</td>
</tr>
<tr>
<td>$q\bar{q} \rightarrow Hgg$</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td>20</td>
</tr>
<tr>
<td>$qq \rightarrow Hq'q'$</td>
<td><img src="image4.png" alt="Diagram" /></td>
<td>2</td>
</tr>
<tr>
<td>$qq \rightarrow Hqq$</td>
<td><img src="image5.png" alt="Diagram" /></td>
<td>4</td>
</tr>
</tbody>
</table>

- **Generated using GoSam**
  - Upgraded **GoSam** generates quadruple precision copy of the code to rescue on-the-fly unstable point with **Ninja**

---

[Mastrolia, Mirabella, Peraro ’12] [Peraro ’15]
[v. Deurzen, Mastrolia, Mirabella, Ossola, Peraro, GL, ’14]
### Process details

- **Virtual corrections (2-loops):**

<table>
<thead>
<tr>
<th>Process</th>
<th>Planar</th>
<th>Non-planar</th>
<th>(1-loop)$^2$</th>
<th># Diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>$gg \rightarrow Hg$</td>
<td><img src="image1" alt="Planar Diagram" /></td>
<td><img src="image2" alt="Non-planar Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td>354</td>
</tr>
<tr>
<td>$qg \rightarrow Hq, \bar{q}g \rightarrow H\bar{q}$</td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>$q\bar{q} \rightarrow Hg$</td>
<td><img src="image6" alt="Diagram" /></td>
<td><img src="image7" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Computed using **2-loop extension** of GoSam, REDUZE and SecDec

  [Greiner, Heinrich, Jahn, Jones, Kerner, Zirke][von Manteuffel, Studerus '12]
  [Binoth, Borowka, Carter, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke]

- Analogous method used to compute **HH production at NLO**

  [Borowka, Greiner, Heinrich, Jones, Kerner, Schlenk, Schubert, Zirke]
Analytic results of 2-loop diagrams

- For planar integrals an analytic result exists
  - Reduced to **125** Master Integrals
  - Alphabet with 3 variables, 49 letters which contain 13 square roots
  - Non elliptic master integrals in terms of $\text{Log}$ and $\text{Li}_2$ up to weight 2
  - Weight 3 and 4 expressed in form of 1-fold integrals
  - 2 sectors contain **elliptic functions** computed as iterated integrals over elliptic kernel

- First analytic computation of Feynman integrals for 4-point multiscale amplitudes involving elliptic functions
Numerical computation of 2-loop diagrams

- Decompose amplitude into form factors:
  - **Gluon channel**
    \[ \mathcal{M} = \epsilon_\mu(p_1) \epsilon_\nu(p_2) \epsilon_\rho(p_3) \mathcal{M}^{\mu\nu\rho} \]
    where: \[ \mathcal{M}^{\mu\nu\rho} = F_{212} (s g^{\mu\nu} - 2 p_2^\mu p_1^\nu) (u p_1^\rho - t p_2^\rho) / (2t) \]
    \[ + F_{332} (u g^{\nu\rho} - 2 p_3^\nu p_2^\rho) (t p_2^\mu - s p_3^\mu) / (2s) \]
    \[ + F_{311} (t g^{\rho\mu} - 2 p_1^\rho p_3^\mu) (s p_3^\nu - u p_1^\nu) / (2u) \]
    \[ + F_{312} (g^{\mu\nu} (u p_1^\rho - t p_2^\rho) + g^{\nu\rho} (t p_2^\mu - s p_3^\mu) + g^{\rho\mu} (s p_3^\nu - u p_1^\nu)) \]
  - **Quark channel**
    \[ \mathcal{M} = F_q \left( \bar{u}(p_q) \not{\! p} g v(p_q) p_q \cdot \epsilon - \bar{u}(p_q) \not{\! \epsilon} v(p_q) p_q \cdot p_g \right) \]
    \[ + F_{\bar{q}} \left( \bar{u}(p_\bar{q}) \not{\! p} g v(p_\bar{q}) p_\bar{q} \cdot \epsilon - \bar{u}(p_\bar{q}) \not{\! \epsilon} v(p_\bar{q}) p_\bar{q} \cdot p_g \right) \]

- Full integration-by-part reduction obtained using **REDUZE**
  - In-house modifications:
    - Changed order of solving differential equations (sort by number of unreduced integrals)
    - Allow to specify list of required integrals (consider only equations containing these integrals)
Numerical computation of 2-loop diagrams

- Unreduced amplitude: 3767 integrals
  - up to 3 inverse propagators for 7-propagator integrals
  - up to 4 inverse propagators for factorizing 6-propagator integrals

- Reduced amplitude: 458 integrals
  - up to 6 master integrals per sector

- Choose quasi-finite basis of MI
  - requires integrals in shifted dimension
  - requires reduction of integrals with 2 inverse propagators and 2 dots

- Reduction performed keeping fixed mass ratio:

\[
\frac{m_H^2}{m_T^2} = \frac{12}{23} \iff m_H = 125\text{ GeV} \quad m_T = 173.055\text{ GeV}
\]

- Total size of REDUZE reduction directory: ................................................ 250 GB
- Size of reduced amplitude (symbolic d-dependent coeff.): .................... 780 MB
- Size of C++ code (for coefficients after expansion in \(\varepsilon\) ): ...................... 340 MB
  - Does NOT allow inclusion of massive bottom quark and Higgs width
Numerical computation of 2-loop diagrams

- Loop integrals evaluated numerically using **sector decomposition** and the program **SecDec-3.0**
  - Method to factorize overlapping singularities:
    - Factorization of poles in dim-regulator $\varepsilon$ and expansion in Laurent series
    - Contour deformation (analytic continuation from Euclidian to physical region)

- **Output:**
  - **finite** integrals at each order in $\varepsilon$
  - can be integrated **numerically**

- Python version now also available: **pySecDec**
  - on Github / uses python and FORM
  - creates library of the integrand functions which can be linked to external code

H+1j 2-loop amplitude written in terms of 22675 finite integrals

[Soper '00; Binoth et al. '05; Nagy, Soper '06; Borowka et al. '12]

[Hepp '66; Denner, Roth '96, Binoth, Heinrich '00]
[Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke]
Numerical computation of 2-loop diagrams

- Loop integrals with $r$ propagators and $s$ inverse propagators can be written as:

$$I_{r,s}(s, t, m_{H}^{2}, m_{T}^{2}) = (M^{2})^{-L_{c}}(M^{2})^{2L-r+s}I_{r,s} \left( \frac{s}{M^{2}}, \frac{t}{M^{2}}, \frac{m_{H}^{2}}{M^{2}}, \frac{m_{T}^{2}}{M^{2}} \right)$$

and renormalized form factors as

$$F^{\text{virt}} = a^{3/2} \left( F^{(1)} + a \left( \frac{n_{a}}{2} \delta Z_{A} + \frac{3}{2} \delta Z_{a} \right) F^{(1)} + a \delta m_{T}^{2} F^{\text{ct.}(1)} + a F^{(2)} + O(a^{2}) \right)$$

$$F^{(1)} = \left( \frac{\mu_{R}^{2}}{M^{2}} \right)^{\epsilon} \left[ b_{0}^{(1)} + b_{1}^{(1)} \epsilon + b_{2}^{(1)} \epsilon^{2} + O(\epsilon^{3}) \right]$$

$$F^{\text{ct.}(1)} = \left( \frac{\mu_{R}^{2}}{M^{2}} \right)^{\epsilon} \left[ c_{0}^{(1)} + c_{1}^{(1)} \epsilon + O(\epsilon^{2}) \right]$$

$$F^{(2)} = \left( \frac{\mu_{R}^{2}}{M^{2}} \right)^{2\epsilon} \left[ \frac{b_{2}^{(2)}}{\epsilon^{2}} + \frac{b_{1}^{(2)}}{\epsilon} + b_{0}^{(2)} + O(\epsilon) \right]$$

- Coefficients $b_{i}^{(n)}$, $c_{i}^{(n)}$ do not need to be recomputed for scale variation
- Compute each $b_{i}^{(n)}$ coefficient optimizing the accuracy needed from each integral
Numerical computation of 2-loop diagrams

• Finite integrals evaluated using **Quasi-Monte-Carlo** integration
  • After sector decomposition and expansion in $\varepsilon$ amplitude is written in terms of **22’675 finite integrals**

• **QMC rank-1 lattice rule:**

\[ I = \int d\vec{x} f(\vec{x}) \approx I_k = \frac{1}{n} \sum_{i=1}^{n} f(\vec{x}_{i,k}) \]

\[ \vec{x}_{i,k} = \left\{ \frac{i \cdot \vec{g}}{n} + \vec{\Delta}_k \right\} \]

\[ \{ \ldots \} = \text{fractional part} \]

\[ \vec{g} = \text{generating vector} \]

\[ \vec{\Delta}_k = \text{randomized shift} \]

[See Dick, Kuo and Sloan for a review]

[Pictures by S.Jones, 1608.03846]

• Compute $m$ different estimates $I_1 \ldots I_m$ to estimate the error

• Error scales as $O(n^{-1})$

• Generating vector constructed component-by-component [Nuyens ‘07]

• Dynamically set $n$ for each integral, minimizing

\[ T = \sum_{\text{integral } i} t_i + \lambda \left( \sigma^2 - \sum_{i} \sigma_i^2 \right) \quad \sigma_i = c_i \cdot t_i^{-\varepsilon} \]

$\sigma_i$ = error estimate (including coefficients in amplitude)

$\lambda$ = Lagrange multiplier \hspace{1cm} $\sigma$ = precision goal

Computed on GPU [Li, Wang, Yan, Zhao ‘15]
Numerical computation of 2-loop diagrams

- Comparison between $H+1j$ and $HH$ virtual 2-loop amplitudes

<table>
<thead>
<tr>
<th></th>
<th>HJ production</th>
<th>HH production</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>#Form factors</strong></td>
<td>4+2</td>
<td>2</td>
</tr>
<tr>
<td><strong>Full reduction</strong></td>
<td>✓</td>
<td>only planar</td>
</tr>
<tr>
<td><strong>(quasi-) finite basis</strong></td>
<td>✓</td>
<td>only planar</td>
</tr>
<tr>
<td><strong>#Master integrals</strong></td>
<td>458</td>
<td>327</td>
</tr>
<tr>
<td>including crossings</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>#Master integrals</strong></td>
<td>120</td>
<td>215</td>
</tr>
<tr>
<td>neglecting crossings</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>#Integrals</strong></td>
<td>22675</td>
<td>11244</td>
</tr>
<tr>
<td>after sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>decomposition and expansion in $\epsilon$</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Code size</strong></td>
<td>~340 MB</td>
<td>~80 MB</td>
</tr>
<tr>
<td>coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Code size</strong></td>
<td>~330 MB</td>
<td>~580 MB</td>
</tr>
<tr>
<td>integrals</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Compile time</strong></td>
<td>~2 weeks</td>
<td>few days</td>
</tr>
<tr>
<td>coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Compile time</strong></td>
<td>~4 hours</td>
<td>~1-2 days</td>
</tr>
<tr>
<td>integrals</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Time</strong></td>
<td>~3-4 days</td>
<td>few hours</td>
</tr>
<tr>
<td>for linking the program</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[M.Kerner, RADCOR2017]
NLO calculation

- Separate integration over **phase space** for **B+I+RS** and **V**:

  - **B+I+RS**:
    - 1-loop Born and real radiation matrix elements implemented in the POWHEG-BOX-V2
    - easy to interface
    - easy to be made public
    - straightforward matching to parton-shower
    - further phenomenological developments (MiNLO, NNLOPS, ...)
      
      [Alioli, Nason, Oleari, Re ‘10]
      
      [Frixione, Nason, Oleari ‘07]
      
  - **V**:
    - generated unweighted events based on differential LO cross section
    - included additional $p_T$-dependent reweighting factor to sample sufficiently also at large transverse momenta

- Contributions combined at the level of the differential histograms
Phenomenological results
Results

• Consider the following setup:

LHC @ 13 TeV:

scale: \( \frac{H^T}{2} = \frac{1}{2} \left( \sqrt{m_H^2 + p_{T,H}^2} + \sum_i |p_{T,i}| \right) \), uncertainty with 7-pt variation

jets: anti-kt with \( R = 0.4, \quad p_{T,j} > 30 \text{ GeV} \)

PDFs: PDF4LHC15_nlo_30_pdfas

• We compare three different computations
  
  • Higgs Effective Field Theory (HEFT):
    \[
    d\sigma_{NLO}^{HEFT} = \int dP S_2 \left( d\sigma_B^{HEFT} + d\sigma_V^{HEFT} \right) + \int dP S_3 d\sigma_R^{HEFT}
    \]

  • Full Theory approximated (FT\text{approx}):
    \[
    d\sigma_{NLO}^{FT\text{approx}} = \int dP S_2 \left( d\sigma_B^{Full} + \frac{d\sigma_B^{Full}}{d\sigma_B^{HEFT}} d\sigma_V^{HEFT} \right) + \int dP S_3 d\sigma_R^{Full}
    \]

  • Full theory (Full)
    \[
    d\sigma_{NLO}^{Full} = \int dP S_2 \left( d\sigma_B^{Full} + d\sigma_V^{Full} \right) + \int dP S_3 d\sigma_R^{Full}
    \]
Results: total cross section

<table>
<thead>
<tr>
<th>THEORY</th>
<th>LO [pb]</th>
<th>NLO [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEFT:</td>
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- Top-quark mass effects: **LO**: +4.3%  **NLO**: +9%  (+6% compared with FT$_{approx}$)

- However for inclusive cross section non-negligible top-bottom interference for H+1 jet production:
  - At LO:
    - $\sigma_{LO, m_{t,b}}$: top- and bottom-quark loops
    - $\sigma_{LO, m_t}$: top-quark loops only

\[ \sigma_{LO, m_{t,b}} = |M_t|^2 + |M_b|^2 + 2\Re(M_t M_b) \]

**positive definite**  **potentially negative**
Results: total cross section

<table>
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- Top-quark mass effects: **LO**: + 4.3%  **NLO**: + 9%  (+ 6% compared with FT$_{\text{approx}}$)

- However for inclusive cross section non-negligible **top-bottom** interference for H+1 jet production:

  At NLO:

  [Lindert, Melnikov, Tancredi, Wever ‘17]

\[
R_{\text{int}}[\mathcal{O}] = \frac{\int d\sigma_{tb} \delta(\mathcal{O} - \mathcal{O}(\bar{x}))}{\int d\sigma_{tt} \delta(\mathcal{O} - \mathcal{O}(\bar{x}))}
\]
• Full theory and HEFT start **deviating** substantially for $p_T > 200$ GeV
• Above 150 GeV **stable** K-factor (peculiar to this scale choice?)
• Scale uncertainty slightly **reduced** compared to $\text{FT}_{\text{approx}}$
• Full virtual gives **+8%** correction w.r.t. HEFT virtual

[Broad agreement with observations of Lindert, Kudashkin, Melnikov, Wever ‘18]
Higgs transverse momentum spectrum at 13 TeV

- Importance of $H+2j$ and $H+3j$ contributions in Higgs $p_T$ spectrum:

![Graph showing the spectrum](image-url)
Higgs transverse momentum spectrum at 13 TeV

- Importance of H+2j and H+3j contributions in Higgs $p_T$ spectrum:

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01/03/2018 - Gionata Luisoni

Seminario, Università Milano Bicocca
Ratios of successive differential cross sections:

\[ R_n(O) = \frac{\frac{d\sigma}{dO}(H+n\text{ jets})}{\frac{d\sigma}{dO}(H+(n-1)\text{ jets})} \]

- suggests that the different transverse momentum scaling of effective and full theory also holds for higher multiplicities

- relative importance of higher multiplicities remains stable under mass corrections
Transverse momentum scaling for large $p_T$
**Interludio: Effective vs. Full theory scaling**

- Breakdown of effective theory can be understood comparing the high energy limit of a pointlike $ggH$ interaction with that of a loop-mediated one:

  - Consider the **transverse momentum** behaviour of the $g^*g^* \rightarrow H$ amplitude (i.e. when gluons are **off shell**)

    - Transverse momenta can reach kinematic limit given by CM energy
    - Contribution from large transverse momenta suppressed by massive quark loop

    \[
    \hat{\sigma} \sim \begin{cases} 
    \sum_{k=1}^{\infty} \alpha_s^k \ln^{2k-1} \left( \frac{s}{m_H^2} \right) & \text{pointlike: } m_t \rightarrow \infty \\
    \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1} \left( \frac{s}{m_H^2} \right) & \text{resolved: finite } m_t
    \end{cases}
    \]

    Corresponding scaling in Higgs $p_T$ computed recently:

    - as $p_T, H \rightarrow \infty$ differential cross section (in $p_T^2$):
      - drops like $(p_T^2, H)^{-1}$
      - drops like $(p_T^2, H)^{-2}$
Effective theory starts to break down at about $p_T, H \approx 200$ GeV and NLO corrections start to become subdominant compared to mass effects.

- Define $R_{m_{t,b}}(O) \equiv \left. \frac{d\sigma}{dO} \right|_{m_{t,b}}$, then the rough scaling behavior from plots is given by

$$\frac{R_{m_{t,b}}(p_T, H = 1.0 \text{ TeV})}{R_{m_{t,b}}(p_T, H = 0.4 \text{ TeV})} \approx \frac{10\%}{60\%} = \frac{1}{6} = 0.167$$

while the high energy limit prediction is

$$\left( \frac{400 \text{ GeV}}{1000 \text{ GeV}} \right)^2 = \frac{4}{25} = 0.16$$

- Very similar behavior for the three different multiplicities
New update at NLO:

- Check on double logarithmic scale:

- Consider points at 100 GeV and 1 TeV:
  \[
  \left( \frac{100 \text{ GeV}}{1000 \text{ GeV}} \right)^2 = \frac{1}{100}
  \]

So at 100 GeV Full should be a factor of 10 larger, as confirmed from the plot.

At NLO same scaling!
Conclusions and Outlook

- Presented new NLO QCD results on top-quark mass effects in H+1 jet production
  - NLO cross section is enhanced compared to HEFT (beware of bottom-quark effects)
  - Stable K-factor for transverse momentum distribution
  - Slight increase compared to NLO in FT_{approx}

- This opens the possibility for many further computations:
  - H+1 jet with full top- and bottom-quark mass effects
  - Matching to parton shower / MiNLO / NNLOPS (POWHEG BOX, Geneva, ... )
  - Matching to analytical resummation
  - Inclusive Higgs production with full mass dependence at NNLO in QCD