Width Difference in $B_s - \overline{B}_s$ System at $O(\alpha_s^2 N_f)$

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Motivation

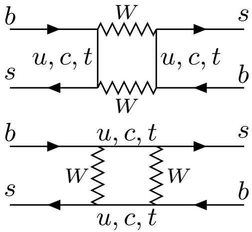
Why $\Delta\Gamma_s$?

1. Nice test to understand the non-perturbative effects in QCD
2. One of the few unambiguous theoretical predictions that are relatively easy to test experimentally
3. Theoretical uncertainty can be estimated order by order: precision studies

$B_s - \overline{B}_s$ mixing basics

- Time evolution of a decaying particle $B_s(t)$:

$$\mathrm{i}\frac{d}{dt}\begin{pmatrix} |B_s(t)\rangle \\ |\overline{B}_s(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2}\hat{\Gamma}\right)\begin{pmatrix} |B_s(t)\rangle \\ |\overline{B}_s(t)\rangle \end{pmatrix}$$



- $B_s - \overline{B}_s$ oscillation is due to weak interactions

$$\hat{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} \quad \hat{\Gamma} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix}$$

- The diagonalization of $\hat{M}$ and $\hat{\Gamma}$ gives the physical eigenstates B_L and B_H with the masses M_L, M_H and the decay widths Γ_L, Γ_H.

Two mass eigenstates:

Lighter eigenstate (CP-even): $|B_L\rangle = p|B_s^0\rangle + q|\overline{B}_s^0\rangle$
Heavier eigenstate (CP-odd): $|B_H\rangle = p|B_s^0\rangle - q|\overline{B}_s^0\rangle$, with $|p|^2 + |q|^2 = 1$.

- 3 physical quantities in $B_s - \overline{B}_s$ mixing:

$$|M_{12}|, \quad |\Gamma_{12}|, \quad \phi = arg(-M_{12}/\Gamma_{12})$$

Physical Observables in $B_s - \overline{B}_s$ mixing

$|M_{12}|, |\Gamma_{12}|$ and $\phi = arg(-M_{12}/\Gamma_{12})$ related to three observables:

- Mass difference: $\Delta M = M_H - M_L \simeq 2|\mathrm{M}_{12}|$

ΔM simply equals to the frequency of the $B_s - \overline{B}_s$ oscillations.

$|\mathrm{M}_{12}|$: dispersive (real) part of box, only internal t is relevant in SM $\rightarrow$ is very sensitive to virtual effects of new heavy particles.

- Decay rate difference: $\Delta\Gamma = \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}|cos\phi$

$|\Gamma_{12}|$: absorptive (imaginary) part of box (tree-level decays), only internal u, c contribute $\rightarrow$ can be barely affected from new physics.

- Flavor-specific CP asymmetry: measures CP violation in mixing.

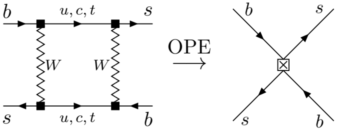
$$\mathrm{a}_{fs} = \frac{\Gamma(\overline{B}_s(t)\rightarrow f) - \Gamma(B_s(t)\rightarrow \bar{f})}{\Gamma(\overline{B}_s(t)\rightarrow f) + \Gamma(B_s(t)\rightarrow \bar{f})} = \mathrm{Im}\left(\frac{\Gamma_{12}}{\mathrm{M}_{12}}\right) = \left|\frac{\Gamma_{12}}{\mathrm{M}_{12}}\right| sin\phi$$

The standard way to measure a_{fs} uses $B_s \rightarrow X_s \ell^+ \nu_\ell$ decay, so that it is often called a *semileptonic CP asymmetry*.

$\phi \sim arg(-M_{12}/\Gamma_{12})$: can be enhanced by new physics if GIM suppression is lifted.

Operator Product Expansion

- Calculation of the Box diagram with an internal top-quark gives:



$$M_{12} = \frac{G_F^2 M_W^2 \hat{\eta}_B}{2(4\pi)^2} \left(V_{tb} V_{ts}^*\right)^2 S_0(x_t) \langle B_s | Q | \overline{B}_s \rangle, \quad (\mathit{Inami},\ \mathit{Lim}\ (1981))$$

- Perturbative QCD corrections: $\hat{\eta}_B$ (Buras, Jamin, Weisz (1990))
- Local four-quark $\Delta B = 2$ operator:

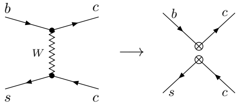
$$Q = (\bar{s}_i \gamma_L^\mu b_i) \otimes (\bar{s}_j \gamma_{\mu L} b_j)$$

- Hadronic matrix element: $\langle B_s | Q | \overline{B}_s \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B$

$\Delta B = 1$ Effective Hamiltonian

- $m_b << M_W$: Large logarithms arise $\alpha_s(m_b)\ln\left(\frac{m_b^2}{M_W^2}\right) \approx 6\alpha_s(m_b)$
- OPE: integrate out heavy particles (e.g. W-boson):

$$\frac{1}{k^2 - M_W^2} \to -\frac{1}{M_W^2} + \mathcal{O}\left(\frac{k^2}{M_W^4}\right)$$



$$H_{eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb}\left[\sum_{i=1}^{6} C_i O_i + C_8 O_8\right] + h.c. \ldots$$

$$O_1 = (\bar{s}_i c_j)_{V-A}(\bar{c}_j b_i)_{V-A}, \quad O_2 = (\bar{s}_i c_i)_{V-A}(\bar{c}_j b_j)_{V-A},$$
$$O_3 = (\bar{s}_i b_i)_{V-A}(\bar{q}_j q_j)_{V-A}, \quad O_4 = (\bar{s}_i b_j)_{V-A}(\bar{q}_j q_i)_{V-A},$$
$$O_5 = (\bar{s}_i b_i)_{V-A}(\bar{q}_j q_j)_{V+A}, \quad O_6 = (\bar{s}_i b_j)_{V-A}(\bar{q}_j q_i)_{V+A},$$
$$O_8 = \frac{g_s}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu}(1-\gamma_5) T_{ij}^a b_j G_{\mu\nu}^a.$$

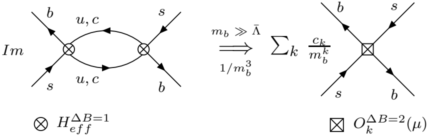
The i, j are colour indices, q = u, d, s, c, b and $V \pm A$ refers to $\gamma_\mu(1 \pm \gamma_5)$.

At this step our width difference is represented by matrix element of bilocal operator:

$$\Gamma_{12} = Abs\langle B_s | i \int d^4x T\left(H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0)\right)|\overline{B}_s\rangle.$$

Heavy Quark Expansion

- Large energy release $\quad m_b \gg \overline{\Lambda} = M_b - m_b \sim \Lambda_{QCD}$



- Systematic expansion of Γ_{12} in powers of Λ_{QCD}/m_b

$$\Gamma_{12} = \sum_k \frac{\vec{c}_k(\mu)}{m_b^k} \langle B_s | \vec{Q}_k^{\Delta B=2}(\mu) | \overline{B}_s \rangle \qquad \text{Spectator effects}$$

[Khoze, Shifman, Uraltsev and Voloshin, 1987]

$$\Gamma_{12} = -\frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left(c_1(\mu) \langle B_s | Q | \overline{B}_s \rangle + c_2(\mu) \langle B_s | Q_S | \overline{B}_s \rangle + \delta_{1/m_b} \right)$$

- Local four-quark $\Delta B = 2$ op.-s: $\; Q = (\bar{s}_i \gamma_L^\mu b_i) \otimes (\bar{s}_j \gamma_{\mu L} b_j), \; Q_S = (\bar{s}_i L b_i) \otimes (\bar{s}_j L b_j)$

Separation of the scales

- $c_k(\mu)$: short distance (perturbative)
- $\langle B_s | Q_k^{\Delta B=2}(\mu) | \overline{B}_s \rangle$: long distance (non perturbative)

State of the Art for Γ_{12}

$$\Gamma_{12} = \left(\frac{\Lambda}{m_b}\right)^3 \left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma_3^{(2)} + \ldots\right) + \left(\frac{\Lambda}{m_b}\right)^4 \left(\Gamma_4^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_4^{(1)} + \ldots\right) + \ldots$$

$$= \left(\frac{\Lambda}{m_b}\right)^3 \Gamma_3^{(0)} \Bigg(1 + \underset{\sim 35\%}{O(\alpha_s)} + \underset{\%?}{O(\alpha_s^2)} + \underset{\sim 20\%}{O\left(\frac{\Lambda}{m_b}\right)} + \underset{?\%}{O\left(\frac{\alpha_s \Lambda}{m_b}\right)} + \underset{< 1\%}{O\left(\frac{\Lambda^2}{m_b^2}\right)} + \ldots\Bigg)$$

- $\Gamma_3^{(0)}$: many collaborations; (1981...)
- $\Gamma_3^{(1)}$: Beneke, Buchalla, Greub, Lenz, Nierste (1998,2003)
  Ciuchini, Franco, Lubicz, Mescia, Tarantino (2003)
  Lenz, Nierste (2006)
- $\Gamma_3^{(2,N_f)}$: Asatrian, A. H., Nierste, Yeghiazaryan (2017)
- $\Gamma_4^{(0)}$: Beneke, Buchalla, Dunietz (1996)
- $\Gamma_4^{(1)}$: A. H., Nierste, in progress
- $\Gamma_5^{(0)}$: Badin, Gabbiani, Petrov, Onishchenko (2003,2004,2007)
- $\langle||\rangle$: HPQCD, JLQCD, Becirevic et al.; Gimenez, Reyes; ... (1999...)

New Basis

- In the calculation of Γ_{12} at LO in HQE 4 operators arise

$$Q = (\bar{s}_i b_i)_{V-A}(\bar{s}_j b_j)_{V-A} \ , \quad \tilde{Q} = (\bar{s}_i b_j)_{V-A}(\bar{s}_j b_i)_{V-A}$$
$$Q_S = (\bar{s}_i b_i)_{S-P}(\bar{s}_j b_j)_{S-P} \ , \quad \tilde{Q}_S = (\bar{s}_i b_j)_{S-P}(\bar{s}_j b_i)_{S-P}$$

- They are not independent: $(\alpha_i = 1 + \mathcal{O}(\alpha_s))$

$$\tilde{Q} = Q \quad \text{and} \quad R_0 = Q_S + \alpha_1 \tilde{Q}_S + \alpha_2 Q/2 = \mathcal{O}\left(1/m_b\right)$$

$$\langle B|Q_S|\overline{B}_s\rangle = -\frac{5}{3} M_{B_s}^2 f_{B_s}^2 \frac{M_{B_s}^2}{(\overline{m}_b + \overline{m}_s)^2} B_S \ , \quad \langle B_s|\tilde{Q}_S|\overline{B}_s\rangle = \frac{1}{3} M_{B_s}^2 f_{B_s}^2 \frac{M_{B_s}^2}{(\overline{m}_b + \overline{m}_s)^2} \tilde{B}_S$$

- Old Basis: $\{Q, Q_s\}$ $\quad K_1 \sim 1/N_c, \ K_2 \sim N_c^0$

$$\Gamma_{12} \sim \underbrace{F}_{small}\langle Q\rangle + \underbrace{(K_1 - K_2)F_S}_{big}\langle Q_S\rangle + \underbrace{K_2 F_S \langle R_0\rangle}_{negative \sim 30\%}$$

Bad News!

- New Basis: $\{Q, \tilde{Q}_s\}$ Good News! [Lenz, Nierste (2006)]

$$\Gamma_{12} \sim \underbrace{(F - (K_1 - K_2)F_S/2)\langle Q\rangle}_{big} - \underbrace{(K_1 - K_2)F_S\langle \tilde{Q}_S\rangle}_{small} + \underbrace{K_1 F_S \langle R_0\rangle}_{reduced\ by\ a\ factor\ 3!}$$

Why do we need NNLO?

To make a rigorous comparison between experiment and theory a precise theoretical predictions is needed

$$\Delta\Gamma^{\mathrm{exp}} = (0.089 \pm 0.006)\, ps^{-1}$$

[an average of measurements by LHCb, ATLAS, CMS, and CDF]

$$\Delta\Gamma^{NLO} = \left(0.091 \pm 0.020_{\mathrm{scale}} \pm 0.006_{B,\overline{B}_S} \pm 0.017_{\Lambda_{QCD}/m_b}\right)\, ps^{-1} \qquad \text{(pole)}$$

$$\Delta\Gamma^{NLO} = \left(0.104 \pm 0.008_{\mathrm{scale}} \pm 0.007_{B,\overline{B}_S} \pm 0.015_{\Lambda_{QCD}/m_b}\right)\, ps^{-1} \qquad (\overline{\mathrm{MS}})$$

[Lenz, Nierste, 2006]

- Reduction of renormalization scale μ-dependence
- Reduction of renormalization scheme dependence
- Reduction of m_b scheme dependence, i.e. definition of b-quark mass $m_b^{pole} \leftrightarrow \overline{m}_b$
  $\Delta\Gamma \sim m_b^2$
- Test the perturbative expansion

Wilson Coefficients at order $\alpha_s^2 N_f$

- Optical theorem $\Gamma_{12} = \langle B_s | \mathcal{T} | \overline{B}_s \rangle$

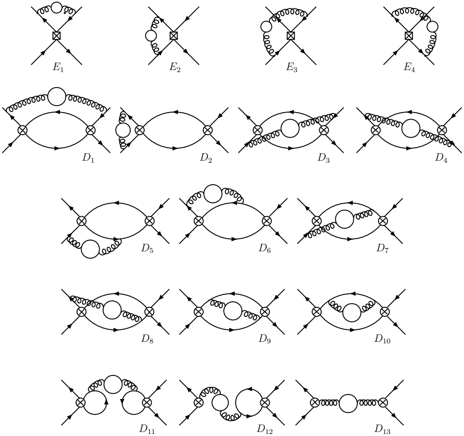
- Transition operator

$$\mathcal{T} = \mathrm{Abs}\ i \int d^4x T \left[\mathcal{H}_{eff}^{\Delta B=1}(x) \mathcal{H}_{eff}^{\Delta B=1}(0) \right]$$

- Matching between full and effective theory at the NNLO in α_s and LO in HQE

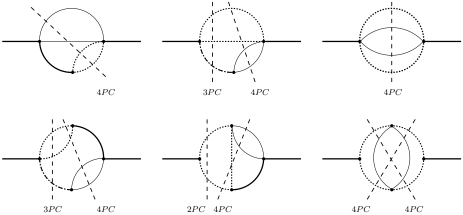
$$\langle B_s | \mathcal{T} | \overline{B}_s \rangle = \vec{c}(\mu) \cdot \langle B_s | \vec{O}^{\Delta B=2}(\mu) | \overline{B}_s \rangle$$

$\alpha_s^2 N_f$ order diagrams



Evaluation of Im part of MI

- Tensorial Integrals $\rightarrow$ Scalar Integrals
- Reduction to Master Integrals (MI): FIRE (A. Smirnov)
- Examples of Master Integrals: three types of cuts



Cutkosky Rule

$$\frac{i}{q_j^2 - m_j^2 - i\epsilon} \rightarrow 2\pi\delta^+(q_j^2 - m_j^2)$$

Angular integration

b-quark rest frame $\rightarrow$ (p_1 , p_3) rest frame

- In MI-s the infrared singularities appear as $\log^n(m_g)$, n = 1, 2.
- Results are presented as analytic functions of m_c/m_b.

Example of MI

$$\int_0^1 dx \int_0^1 dy \left[\left(\frac{1}{x(1-x)y(1-y)} + \frac{1-y}{y^2} \right) \log\left(x(1-x)(1-y)^2 + y \right) + \frac{\log\left(x(1-x)\right)}{y} + \frac{1}{y} - \left(\frac{1}{x(1-x)y(1-y)} + \frac{1}{y^2} \right) \log\left(x(1-x)(1-y) + y \right) \right].$$

The exact value of this integral is $-1 - 2\zeta(2) = -3.404114$.

	N	10^4	10^5	10^6	10^7	10^8	10^9
MT,	*Time*, *c*	0.0021	0.020	0.19	1.8	18	181
Plain	*Result*	-3.413520	-3.411583	-3.403035	-3.407948	-3.403809	-3.403920
MX,	*Time*, *c*	0.0016	0.011	0.095	0.93	8.7	87
Plain	*Result*	-3.384531	-3.449768	-3.398291	-3.404808	-3.403767	-3.404495
MT,	*Time*, *c*	0.0020	0.019	0.19	1.9	19	187
MISER	*Result*	-3.413520	-3.411583	-3.403035	-3.407948	-3.403809	-3.403920
MX,	*Time*, *c*	0.0014	0.012	0.10	1.0	10	101
MISER	*Result*	-3.418790	-3.409007	-3.403363	-3.403912	-3.404168	-3.404087

Table : MT is Mersenne-Twister RNG and MX is MixMax RNG.

Example of MI

$$\int_0^1 d\lambda \int_{4z}^1 ds \frac{(\sqrt{s}-1)^3 \sqrt{1-\frac{4z}{s}}\sqrt{\lambda\left(2\sqrt{s}(2-\lambda)+(s+1)\lambda\right)}}{\left((2-\sqrt{s})\sqrt{s}(1-\lambda)+\lambda\right)}$$
$$\left(1-\epsilon\cdot\left(\log(s-4z)+\log\left(\lambda\left(2\sqrt{s}(2-\lambda)+(s+1)\lambda\right)\right)+4\log\left(1-\sqrt{s}\right)+\log(1-\lambda)\right)\right)$$
$$= \frac{1}{4}\left[\frac{2z^7(35\log z+611)}{1225}+\frac{1}{200}z^6(20\log z+151)+z^5\left(\frac{\log z}{5}+\frac{559}{900}\right)+z^4\left(\frac{\log z}{2}+\frac{7}{24}\right)\right.$$
$$+z^3\left(2\log z-\frac{11}{3}\right)+z^2\left(\log^2 z-7\log z+\frac{27}{2}\right)-\frac{2}{3}z\left(6\log z+\pi^2+6\right)+\frac{\pi^2}{3}-\frac{7}{2}$$
$$+\epsilon\cdot\left(z^7\left(-0.0792168\log z-1.3829\right)+z^6\left(-0.138629\log z-1.04665\right)\right.$$
$$+z^5\left(-0.277259\log z-0.861043\right)+z^4\left(4.3\log z-8.65202\right)+z^3\left(-1.33333\log z-14.4059\right)$$
$$+z^2\left(-\log^3 z+4.5\log^2 z-11.6595\log z+33.472\right)+z\left(2\log^2 z-32\log z-102.311\right)$$
$$\left.\left.-3.00788z^{7/2}-21.0552z^{5/2}+105.276z^{3/2}-2.50034\right)\right]+\mathcal{O}\left(z^8,\epsilon^2\right),$$

where $z = m_c^2/m_b^2$.

Example of MI

For $z = 0.048$ the value of this integral is $-0.0173512 - 0.223989\epsilon$
For $z = 0.048$ the numerical integration on Mathematica: $-0.0173512 - 0.223992\epsilon$

ϵ^0	N	10^4	10^5	10^6	10^7	10^8	10^9
MT, Plain	$T.$, c	0.0012	0.009	0.09	0.9	8.7	91.7
	Res.	-0.0172745	-0.0172893	-0.0173722	-0.0173497	-0.0173485	-0.0173504
MX, Plain	$T.$, c	0.0007	0.007	0.07	0.6	6.5	70.4
	Res.	-0.0175178	-0.0174568	-0.0173638	-0.0173509	-0.0173523	-0.0173517
MT, MISER	$T.$, c	0.0012	0.01	0.1	1.0	10	100.5
	Res.	-0.0173973	-0.0173422	-0.0173511	-0.0173515	-0.0173512	-0.0173512
MX, MISER	$T.$, c	0.0008	0.075	0.08	0.8	8.0	80.5
	Res.	-0.0174198	-0.0173519	-0.0173508	-0.0173512	-0.0173512	-0.0173512
ϵ^1							
MT, Plain	$T.$, c	0.0018	0.016	0.15	1.5	15.2	163.3
	Res.	-0.222966	-0.222942	-0.224253	-0.223959	-0.223956	-0.223980
MX, Plain	$T.$, c	0.0012	0.011	0.11	1.1	10.7	115.7
	Res.	-0.226169	-0.225194	-0.224148	-0.223990	-0.224004	-0.223999
MT, MISER	$T.$, c	0.0018	0.016	0.16	1.6	16.3	168.3
	Res.	-0.224368	-0.223902	-0.224004	-0.223988	-0.223992	-0.223991
MX, MISER	$T.$, c	0.0012	0.011	0.12	1.2	12.3	133.2
	Res.	-0.224448	-0.224267	-0.224001	-0.223989	-0.223992	-0.223992

Table : MT is Mersenne-Twister RNG and MX is MixMax RNG.

Renormalization and infrared regularization

- we renormalize the operators in the naive dimensional regularization (NDR) scheme
- evanescent operators ($D = 4 - 2\epsilon$):

$$[\gamma^\mu\gamma^\nu(1-\gamma_5)]_{ij}[\gamma_\nu\gamma_\mu(1-\gamma_5)]_{kl} \to (8-8\epsilon)[1-\gamma_5]_{il}[1-\gamma_5]_{kj} + 4\epsilon^2[1-\gamma_5]_{ij}[1-\gamma_5]_{kl},$$

$$[\gamma^\mu\gamma^\alpha\gamma^\nu(1-\gamma_5)]_{ij}[\gamma_\nu\gamma_\alpha\gamma_\mu(1-\gamma_5)]_{kl} \to (4-8\epsilon+4\epsilon^2)[\gamma^\mu(1-\gamma_5)]_{ij}[\gamma_\mu(1-\gamma_5)]_{kl}.$$

- gluon propagator (similar to the W boson propagator in an R_ξ gauge with $\xi = 0$)

$$\frac{-i\delta_{ab}}{k^2 - m_g^2 + i\epsilon}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right).$$

- the NLO renormalization constants of the gluon mass and g_s in $\overline{\mathrm{MS}}$ scheme

$$\delta Z_x^{(1),N_f} = -\frac{\alpha_s}{2\pi\epsilon}N_f, \qquad \delta Z_{g_s}^{(1),N_f} = \frac{\alpha_s}{6\pi\epsilon}N_f T_R \qquad \text{with} \quad T_R = \frac{1}{2}.$$

- for the external quark lines in the 't Hooft-Feynman gauge is

$$\delta Z_q^{(2),N_f} = \frac{\alpha_s^2}{(4\pi)^2}\frac{4}{3\epsilon}N_f, \quad q = b, s.$$

Renormalization and infrared regularization

- We now turn to the counterterms for the $\Delta B = 1$ operators.

$$H_{eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_j^6 \left[C_j O_j\right]^{\text{bare}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_j^6 \left[C_j O_j\right]^{\text{ren}}$$
$$= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_{j,k}^6 C_j^{\text{bare}} Z_{jk} O_k^{\text{ren}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{cb} \sum_{j,k}^6 C_j^{\text{ren}} Z_{jk} O_k^{\text{bare}}.$$

The last lines illustrates that one can view Z_{jk} as either renormalising the operator O_k or the Wilson coefficient C_j. Traditionally the renormalization is attributed to the operator, but we adopt the latter viewpoint, with $C_j \equiv C_j^{\text{ren}}$ and $O_k \equiv O_k^{\text{bare}}$.

Writing $Z_{jk} = \delta_{jk} + \delta Z_{jk}$ and expanding $\delta Z_{jk} = \frac{\alpha_s}{4\pi}\delta Z_{jk}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \delta Z_{jk}^{(2)} + \mathcal{O}(\alpha_s^3)$ we find the following counterterms at order $\alpha_s^2 N_f$:

$$\delta Z_{11}^{(2),N_f} = \delta Z_{22}^{(2),N_f} = -\frac{1}{3}\delta Z_{12}^{(2),N_f} = -\left(\frac{1}{3\epsilon^2} + \frac{1}{18\epsilon}\right) N_f .$$

Renormalization Penguins

• For the penguin-diagram contributions we need the counterterms δZ_{2k} related to the mixing of O_2 into the four-fermion operators O_{3-6}.

$$\delta Z_{42}^{(1)} = \delta Z_{62}^{(1)} = \frac{1}{3\epsilon},$$
$$\delta Z_{32}^{(2),N_f} = \delta Z_{52}^{(2),N_f} = -\frac{2}{27\epsilon^2} N_f,$$
$$\delta Z_{42}^{(2),N_f} = \delta Z_{62}^{(2),N_f} = \frac{2}{9\epsilon^2} N_f.$$

• The second type of counterterms involves the mixing of the penguin operators O_{3-6} among themselves. Together with $\delta Z_{42}^{(1)}$ and $\delta Z_{62}^{(1)}$ written above, the additional non-zero contributions, $i, j = 3, \ldots, 6$, are:

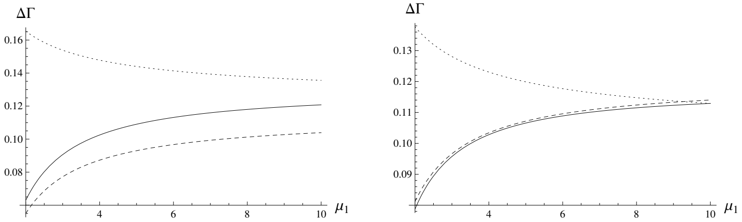
$$\delta Z_{32}^{(1)} = \delta Z_{52}^{(1)} = -\frac{1}{9\epsilon}.$$

• Finally the $\mathcal{O}(\alpha_s)$ counterterms needed to renormalize the penguin diagram D_{12}. Here the counterterms are $\delta Z_{42}^{(1)}$ and $\delta Z_{62}^{(1)}$.

Results

$$\Delta\Gamma^{NLO} = (0.091 \pm 0.020_{\rm scale})\ ps^{-1}, \qquad \Delta\Gamma^{NNLO} = (0.108 \pm 0.021_{\rm scale})\ ps^{-1} \quad \text{(pole)}$$
$$\Delta\Gamma^{NLO} = (0.104 \pm 0.015_{\rm scale})\ ps^{-1}, \qquad \Delta\Gamma^{NNLO} = (0.103 \pm 0.015_{\rm scale})\ ps^{-1} \quad (\overline{\rm MS})$$

where $\mu_1 \in [m_b/2, 2m_b]$ and for the central values of $\Delta\Gamma$ we took $\mu_1 = m_b^{\rm pole}$ and $\mu_1 = \overline{m}_b$ for the pole and $\overline{\rm MS}$ schemes, respectively ($\overline{m}_b = 4.18$ GeV, $m_b^{\rm pole,NLO} = 4.58$ GeV, $m_b^{\rm pole,NNLO} = 4.85$ GeV).



Renormalization scale dependence for $\Delta\Gamma$ at LO (dotted), NLO (dashed), and NNLO (solid) results for the pole scheme (left) and the $\overline{\rm MS}$ scheme (right).

$$\Delta\Gamma^{\rm exp} = (0.089 \pm 0.006)\, ps^{-1}$$

Results NNA

We have discussed the naive non-abelianization approach (NNA) as well . If we trade N_f for β_0 in G, G_S and the relation between $\overline{m}_b = 4.18\,\mathrm{GeV}$ and m_b^{pole}, we find $m_b^{\mathrm{pole}} = 4.87\,\mathrm{GeV}$, which is close to the full two-loop result, and

$$\Delta\Gamma^{NNA} = (0.071 \pm 0.020_{\mathrm{scale}})\; ps^{-1} \qquad (\text{pole})$$
$$\Delta\Gamma^{NNA} = (0.099 \pm 0.012_{\mathrm{scale}})\; ps^{-1} \qquad (\overline{\mathrm{MS}}).$$

We find that the $\overline{\mathrm{MS}}$ result is quite stable, if we change the literal $\alpha_s^2 N_f$ result to the NNA one, while the pole-scheme result is not.

Until a full NNLO calculation is available, we recommend to use the $\overline{\mathrm{MS}}$ NLO value:

$$\Delta\Gamma = \; (1.86 \pm 0.17)\, f_{B_s}^2 B \;+\; (0.42 \pm 0.03) f_{B_s}^2 \widetilde{B}_S' \;+\; (-0.55 \pm 0.29)\, f_{B_s}^2 .$$
$$\Delta\Gamma = \left(0.104 \pm 0.015_{\mathrm{scale}} \pm 0.007_{B,\widetilde{B}_S} \pm 0.015_{\Lambda_{QCD}/m_b}\right)\; ps^{-1} \qquad (\overline{\mathrm{MS}}).$$

Summary

• On the examples of integrals, which originate from contributing to $\Delta\Gamma$ in the $B_s - \overline{B}_s$ system three loop Feynman diagrams, we tested the MIXMAX RNG. In particular, we calculated these integrals applying the Monte-Carlo Plain and the Monte-Carlo MISER methods using the MIXMAX and the Mersenne-Twister RNG-s.

• We found, that the calculation of these integrals with the MixMax RNG is faster than with the Mersenne-Twister RNG.

• Also, the results of the integrals obtained by using the MIXMAX RNG more often have better accuracy and less variance then when one is using the Mersenne-Twister RNG.