AGT: the duality between 4d SYM and 2d CFT

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Plan

- N=2 SYM and SW prepotential
- Moduli space of instantons: ADHM construction
- Induced action
- Ω-bacground: Generalized partition function
- Liouville theory
- 4 point conformal block and AGT relation
- Conclusions

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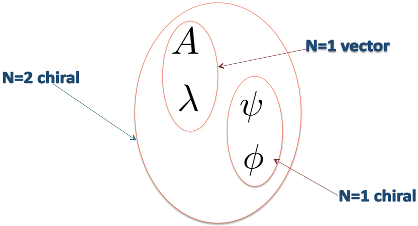
The field content and action



$$S = \int d^4xd^4\theta\,\Im\tau\mathrm{tr}\Psi^2$$

Scalar potential: $V \sim \mathrm{tr}[\phi, \phi^\dagger]^2$

Low energy effective action

Below Ψ includes only massless fields (i.e. those frome the Cartan of the gauge group)

$$S_{eff} = \int d^4 x d^4 \theta \, \Im \mathcal{F}(\Psi)$$

$\mathcal{F}$ - the Seiberg Witten prepotential

In the case of $SU(2)$

$$\mathcal{F}(\Psi) = \frac{i}{2\pi} \Psi^2 \log \frac{2\Psi^2}{e^3 \Lambda^2} - \frac{i}{\pi} \sum_{k=1}^{\infty} \mathcal{F}_k \left(\frac{\Lambda}{\Psi} \right)^{4k} \Psi^2$$

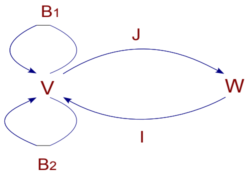
$\mathcal{F}_1 = \frac{1}{2}, \mathcal{F}_2 = \frac{5}{16}, \mathcal{F}_3 = \frac{3}{4}, \mathcal{F}_4 = \frac{1469}{512}, \ldots$

Moduli space of instantons, ADHM

gauge group: $U(N)$; instanton number: k; $V=\mathbb{C}^k$; $W=\mathbb{C}^N$
ADHM equations:

$$[B_1,B_2]+IJ=0;\quad \left[B_1,B_1^\dagger\right]+\left[B_2,B_2^\dagger\right]+II^\dagger-J^\dagger J=\zeta$$

Equivalence relation: $(B_i,I,J)\sim(\phi B_i\phi^{-1},\phi I,J\phi^{-1})$, $\phi\in U(k)$
Global gauge trans. : $(B_i,I,J)\to(B_i,Ig,g^{-1}J)$, $g\in U(N)$
Rotations of Euclidean space time: $(z_1,z_2)\to(e^{i\epsilon_1}z_1,e^{i\epsilon_1}z_2)$
$(B_i,I,J)\to(e^{i\epsilon_i}B_i,I,e^{i\epsilon_1+i\epsilon_2}J)$,



The induced action

R.Flume, R.P., H.Storch 'arXiv:hep-th/0110240

$$\mathcal{F}_k \simeq \int_{\mathcal{M}'_k} e^{-d_x \omega},$$

$d_x \equiv d + i_x$ is an equivariant exterior derivative, i_x denotes contraction with the vector field x which generates the $U(1)$ subgroup of global gauge transformations selected by the choice of "Higgs" expectation values $\langle \phi \rangle_{cl} = diag(a_1, \ldots, a_N)$.
ω is the differential one-form

$$\omega = G(x, \bullet)$$

$G(\bullet, \bullet)$ is the natural induced metric on moduli space.

Localization to the zero locus of the vector field x

The coefficient $\mathcal{F}_k$ may be deformed into

$$\mathcal{F}_k(t) \equiv \int_{\mathcal{M}'_k} e^{-\frac{1}{t} d_x \omega}$$

Compute

$$\frac{d}{dt}\mathcal{F}_k(t) = -\frac{1}{t^2}\int_{\mathcal{M}'_k} d_x \left(\omega e^{-\frac{1}{t} d_x \omega}\right) = -\frac{1}{t^2}\int_{\mathcal{M}'_k} d \left(\omega e^{-\frac{1}{t} d_x \omega}\right).$$

The saddle point approximation is exact! There are contributions only from the points where $x = 0$. Unfortunately they are too many: in fact union of sub-manifolds of dimensions $2Nk - 4$ (c.f. $\dim \mathcal{M}'_k = 4Nk - 4$)

Incorporating space-time rotations

A wonderful way out: modify the vector field x incorporating (Euclidean) space-time rotations (parametrized by ϵ_1, ϵ_2) with the global gauge transformations (parametrized by the expectations values $a_1, \ldots, a_N$)

$$Z_k\left(a_u, \epsilon_1, \epsilon_2\right) \equiv \int_{\mathcal{M}_k} e^{-d_{\tilde{x}}\tilde{\omega}},$$

$\tilde{x}$ is the modified vector field and

$$\tilde{\omega} = G(\tilde{x}, \bullet)$$

Now we are lucky: the vector field $\tilde{x}$ has finitely many zeros!

Generalized partition function

complete localization!

$$Z_k\left(a_u, \epsilon_1, \epsilon_2\right) = \sum_{i \in fixed\ points} \frac{1}{\det \mathcal{L}_{\tilde{x}}}\Bigg|_i .$$

How this is related to SW prepotential? Introduce the partition function Nekrasov 'arXiv:hep-th/0206161

$$Z(a_u, \epsilon_1, \epsilon_2, q) \equiv 1 + \sum_{k=1}^{\infty} Z_k(a, \epsilon_1, \epsilon_2) q^k = e^{\frac{1}{\epsilon_1 \epsilon_2}\mathcal{F}(a_u, \epsilon_1, \epsilon_2, q)}$$

$\frac{1}{\epsilon_1 \epsilon_2}$ is the "voume factor" and $\mathcal{F}(a_u, 0, 0, q)$ coincides with the instanton part of SW prepotential.

$\mathcal{N} = 2$ SYM in Ω bacground

From the point of view of the initial theory above modification boils down to the consideration of the $\mathcal{N} = 2$ SYM in a specific presently commonly known as Ω- background. The two parameters ϵ_1, ϵ_2 specifying the general Ω- background are introduced in [Moor,Nekrasov,Shatashvili 'arXiv:hep-th/9712241], Losev,Nekrasov,Shatashvili 'arXiv:hep-th/9801061 to regularize the integrals over moduli space of instantons.

- In Nekrasov 'arXiv:hep-th/0206161 is shown how the partition function in this background is related to the Seiberg-Witten prepotential.
- In the same paper Nekrasov performed explicit calculation of the prepotential up to 5 instantons choosing $h = \epsilon_1 = -\epsilon_2$ and demonstrated that at vanishing h one exactly recovers the results extracted from the Seiberg-Witten curve.

Partition function with generic ϵ_1, ϵ_2

- In Flume, R.P. 'arXiv:hep-th/0208176 a closed combinatorial formula which allows to calculate the Nekrasov partition function for generic ϵ_1, ϵ_2 was found. The partition function is represented as a sum over arrays of Young tableau with total number of boxes equal to the number of instantons.
- The partition function with generic ϵ_1, ϵ_2 is essential also from the point of view of the AGT duality Alday,Gaiotto,Tachikawa ' arXiv:0906.3219 relating this partition function to the conformal blocks in 2d Conformal Field Theory.

Partition function with generic ϵ_1 and $\epsilon_2 = 0$

- In a parallel very interesting development Nekrasov and Shatashvily in 'arXiv:0908.4052 show that when $\epsilon_2 = 0$ the prepotential is related to the quantum integrable many body systems.
  It is established inR.P. 'arXiv:1006.4822 that in this limit we are led to the notion of "quantum" Seiberg-Witten curve.
- Note one more point which to my opinion makes the investigation of $\epsilon_2 = 0$ case even more interesting: namely, due to above mentioned AGT relation this is related to the quasi-classical $(c \to \infty)$ limit of conformal blocks and hence to the (semi-) classical Liouville field theory.

Classification of the fixed points

As we saw it is useful to generalize the Seiberg-Witten prepotential including into the game besides unbroken global gauge transformations also the space time rotations which allowed to localize instanton contributions around finite number of fixed points.

For the gauge group $U(N)$ the fixed points are in 1-1 correspondence with the arrays of Young tableau $\vec{Y} = (Y_1, ..., Y_N)$ with total number of boxes $|\vec{Y}|$ being equal to the instanton charge k.

The character

At a fixed point the tangent space of the moduli space of instantons decomposes into sum of (complex) one dimensional irreducible representations of the Cartan subgroup of $U(N) \times O(4)$

[R.Flume, R.P. '02]

$$\chi = \sum_{\alpha,\beta=1}^{N} \frac{e_\alpha}{e_\beta} \left\{ \sum_{s \in Y_\alpha} T_1^{-l_{Y_\beta}(s)} T_2^{a_{Y_\alpha}(s)+1} + \sum_{s \in Y_\beta} T_1^{l_{Y_\alpha}(s)+1} T_2^{-a_{Y_\beta}(s)} \right\}$$

where $(e_1, ..., e_N) = (e^{ia_1}, \ldots, e^{ia_N}) \in U(1)^N \subset U(N)$ and $(T_1, T_2) = (e^{i\epsilon_1}, e^{i\epsilon_2}) \in U(1)^2 \subset O(4)$, $l_Y(s)$ $(a_Y(s))$ is the distance of the right (top) edge of the box s from the limiting polygonal curve of the Young tableaux Y in horizontal (vertical) direction taken with plus sign if $s \in Y$ and with minus sign otherwise.

Demonstration: arm and leg lengths

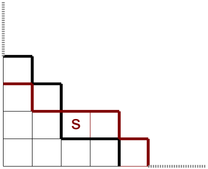
Let Y_α be the black tableaux and Y_β the red one, (the box $s \in Y_\beta$)

$a_{Y_\alpha}(s) = -1$

$a_{Y_\beta}(s) = 0$

$l_{Y_\alpha}(s) = -1$

$l_{Y_\beta}(s) = 1$



Determinant of $\tilde{x}$

One-dimensional subgroups of the $N+2$ dimensional torus are parametrized by $a_1, \ldots, a_N$ and ϵ_1, ϵ_2. From the physical point of view a_α are the vacuum expectation values of the complex scalar of the $\mathcal{N}=2$ gauge multiplet. The parameters of the Ω-background ϵ_1, ϵ_2 as we saw are related to space time rotations. The contribution of a fixed point to the Nekrasov partition function in the basic $\mathcal{N}=2$ case without extra hypermultiplets is simply the inverse determinant of the vector field action on the tangent space. All the eigenvalues of this vector field can be directly read off from the character formula.

Contribution of the gauge multiplet

As a result [Flume, R.P. 'arXiv:hep-th/0208176]

$$P_{gauge}(\vec{Y}) = \prod_{\alpha,\beta=1}^{N} \prod_{s \in Y_\alpha} \frac{1}{E_{\alpha,\beta}(s)(\epsilon - E_{\alpha,\beta}(s))},$$

where

$$E_{\alpha,\beta} = a_\alpha - a_\beta - \epsilon_1 l_{Y_\beta}(s) + \epsilon_2 (a_{Y_\alpha}(s) + 1)$$

In general the theory may include "matter" hypermultiplets in various representations of the gauge group. In that case one should multiply the gauge multiplet contribution by another factor P_{matter}.

The "matter" part

The respective matter factors read [Bruzzo, Fucito, Morales, Tanzini '03]

$$P_{antifund}(\vec{Y}) = \prod_{\ell=1}^{f} \prod_{\alpha=1}^{N} \prod_{s_\alpha \in Y_\alpha} (\phi_{\alpha,s_\alpha} + m_\ell)$$

$$P_{adj}(\vec{Y}) = \prod_{\alpha,\beta=1}^{N} \prod_{s \in Y_\alpha} (E_{\alpha,\beta}(s) - M)(\epsilon - E_{\alpha,\beta}(s) - M),$$

where m_ℓ, M are the masses of the hypermultiplets, $\epsilon = \epsilon_1 + \epsilon_2$,

$$\phi_{\alpha,s_\alpha} = a_\alpha + (i_{s_\alpha} - 1)\epsilon_1 + (j_{s_\alpha} - 1)\epsilon_2$$

and i_{s_α}, j_{s_α} are the numbers of the column and the row of the tableaux Y_α where the box s_α is located.

The partition function of the theory with matter hyper-multiplets

Finally the instanton part of generalized partition function is:

$$Z_{inst} = \sum_{\vec{Y}} q^{|\vec{Y}|} P_{gauge}(\vec{Y}) P_{matter}(\vec{Y})$$

$q = e^{2\pi i \tau_g}$, with τ_g the usual gauge theory coupling.

Example: 1-instanton computation for pure $SU(2)$

1 instanton: two fixed points

- $$\begin{aligned}\chi(\square,\bullet) &= T_1 + T_2 + \frac{e_2}{e_1} T_1 T_2 + \frac{e_1}{e_2} \\ \det(\square,\bullet) &= \epsilon_1\epsilon_2(a_2 - a_1 + \epsilon_1 + \epsilon_2)(a_1 - a_2)\end{aligned}$$

- $$\begin{aligned}\chi(\bullet,\square) &= \frac{e_2}{e_1} + \frac{e_1}{e_2} T_1 T_2 + T_1 + T_2 \\ \det(\bullet,\square) &= (a_2 - a_1)(a_1 - a_2 + \epsilon_1 + \epsilon_2)\epsilon_1\epsilon_2\end{aligned}$$

Hence

$$Z_1 = \frac{1}{\det(\square,\bullet)} + \frac{1}{\det(\bullet,\square)} = \frac{2}{\epsilon_1\epsilon_2((a_1 - a_2)^2 - (\epsilon_1 + \epsilon_2)^2)}$$

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Example: 2-instantons in pure $SU(2)$

5 fixed points $(\square\square,\bullet)$, $(\begin{smallmatrix}\square\\ \square\end{smallmatrix},\bullet)$, $(\square,\square)$, $(\bullet,\square\square)$; $(\bullet,\begin{smallmatrix}\square\\ \square\end{smallmatrix})$

- $$\chi_1=\frac{T_2}{T_1}+T_2+T_1^2+T_1+\frac{e_2}{e_1}(T_1T_2+T_1^2T_2)+\frac{e_1}{e_2}(1+T_1^{-1})$$
- $$\chi_3=T_1+T_2+\frac{e_2}{e_1}(T_1+T_2)+\frac{e_1}{e_2}(T_1+T_2)+T_1+T_2$$

Other characters by symmetry. Hence

$$Z_2=\sum_{i=1}^{5}\frac{1}{\det_i}$$

$$=\frac{2a_{12}^2-8\epsilon_1^2-8\epsilon_2^2-17\epsilon_1\epsilon_2}{\epsilon_1^2\epsilon_2^2(a_{12}^2-(\epsilon_1+\epsilon_2)^2)(a_{12}^2-(2\epsilon_1+\epsilon_2)^2)(a_{12}^2-(\epsilon_1+2\epsilon_2)^2)}$$

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- $$\chi_1 = \frac{T_2}{T_1} + T_2 + T_1^2 + T_1 + \frac{e_2}{e_1}(T_1 T_2 + T_1^2 T_2) + \frac{e_1}{e_2}(1 + T_1^{-1})$$

- $$\chi_3 = T_1 + T_2 + \frac{e_2}{e_1}(T_1 + T_2) + \frac{e_1}{e_2}(T_1 + T_2) + T_1 + T_2$$

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$$= \frac{2a_{12}^2 - 8\epsilon_1^2 - 8\epsilon_2^2 - 17\epsilon_1\epsilon_2}{\epsilon_1^2\epsilon_2^2(a_{12}^2 - (\epsilon_1+\epsilon_2)^2)(a_{12}^2 - (2\epsilon_1+\epsilon_2)^2)(a_{12}^2 - (\epsilon_1+2\epsilon_2)^2)}$$

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Example: two instanton generalized prepotential in pure $SU(2)$

$$\mathcal{F}(a_1, a_2, \epsilon_1, \epsilon_2) = \epsilon_1 \epsilon_2 \log Z(a_1, a_2, \epsilon_1, \epsilon_2, q) = \frac{2q}{(a_{12}^2 - (\epsilon_1 + \epsilon_2)^2)}$$

$$+\frac{q^2(5a_{12}^2 + 7\epsilon_1^2 + 7\epsilon_2^2 + 16\epsilon_1\epsilon_2)}{(a_{12}^2 - (\epsilon_1 + \epsilon_2)^2)(a_{12}^2 - (2\epsilon_1 + \epsilon_2)^2)(a_{12}^2 - (\epsilon_1 + 2\epsilon_2)^2)} + O(q^3)$$

Liouville theory

Action

$$S = \frac{1}{4\pi} \int (\partial_a \varphi \partial^a \varphi + 4\pi \mu e^{2b\varphi}) d^2 x$$

This is a CFT endowed with holomorphic $T(z)$

$$T(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

L_n-s are the Virasoro generators satisfying

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{n+m,0}$$

The central charge $c = 1 + 6Q^2$, where $Q = (b + 1/b)$. Primary fields are the exponentials $V_\alpha = \exp 2\alpha\varphi$ with dimension $\Delta_\alpha = \alpha(Q - \alpha)$.

4-point correlation functions and conformal blocks

The main object of our interest are the 4-point correlation functions

$$\langle V_{\alpha_4}(\infty) V_{\alpha_3}(1) V_{\alpha_2}(q) V_{\alpha_1}(0) \rangle$$

Denote its holomorphic building block with fixed s-channel intermediate dimension Δ_α by $G(\alpha, \alpha_i; q)$.
AGT correspondence states:

$$Z(m_\ell, \epsilon_2, \epsilon_2; q) = q^{\Delta - \Delta_3 - \Delta_4} \, (1-q)^{2\alpha_2 \, (Q-\alpha_3)} \, G(\alpha, \alpha_i; q)$$

with α_i and m_i related

AGT dictionary

$$\begin{aligned} \alpha_1 &= \tfrac{\epsilon}{2}+\tfrac{1}{2}(m_1-m_2)\,; \qquad \alpha_2=-\tfrac{1}{2}(m_1+m_2)\,; \\ \alpha_3 &= \epsilon-\tfrac{1}{2}(m_1+m_2)\,; \qquad \alpha_4=\tfrac{\epsilon}{2}+\tfrac{1}{2}(m_1-m_2)\,; \\ \alpha &= \tfrac{\epsilon}{2}+a\,; \qquad \epsilon=\epsilon_1+\epsilon_2=Q\,; \\ \epsilon_1 &= b\,; \qquad \epsilon_2=b^{-1}\,; \end{aligned}$$

Why this is true?
Two "explanations"

- Physical: Through M-theory engineering of a 6d theory on $R^4 \times S_{2,4}$
- Algebraic geometry: There is a natural action of the Virasoro (W) algebra on the moduli space of instantons

THANKS