Multi-Particle Effects in Particle Accelerators (I)

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Overview

2 hours to go through the basics of multi-particle effects, the reasons why they are object of study and show a few study cases

- 1 hour
  - → General concepts
  - → Space charge
  - → Some examples of instabilities, bunch oscillation modes
- 1 hour
  - → Numerical modeling of multi-particle effects
  - → Examples of experimental observations
  - → Cures

General definition of *multi-particle processes* in an accelerator or storage ring

Class of phenomena in which the evolution of the particle beam cannot be studied as if the beam was a single particle (as is done in beam optics), but depends on the combination of external fields and interaction between particles. Particles can interact between them through

- Self generated fields:

  → Direct space charge fields

  → Electromagnetic interaction of the beam with the surrounding environment through the beam's own images and the wake fields (impedances)

  → Interaction with the beam's own synchrotron radiation

- Long- and short-range Coulomb collisions, associated to intra-beam scattering and Touschek effect, respectively

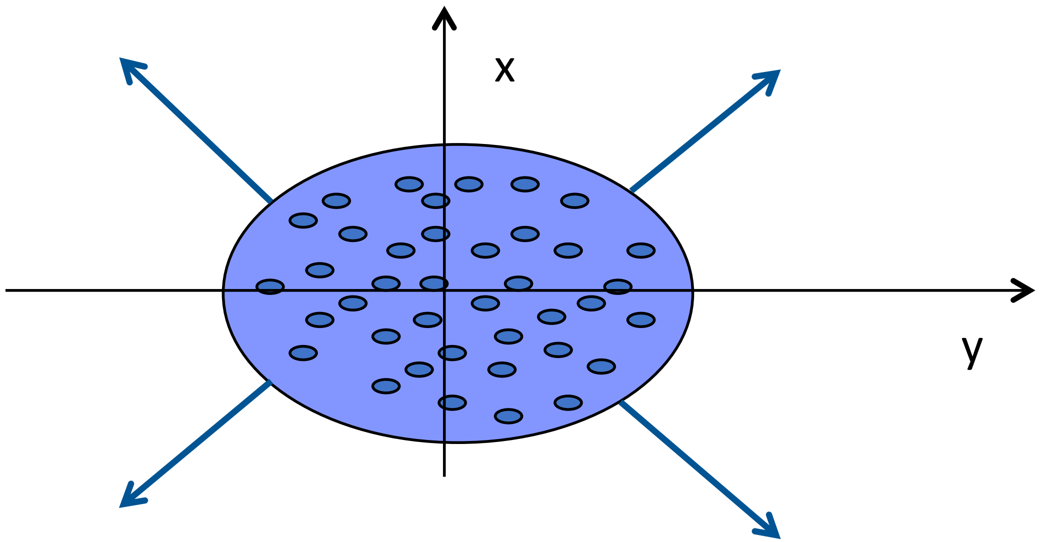
- Interaction of electron beams with trapped ions, proton/positron/ion beams with electron clouds, beam-beam in a collider ring, electron cooling for ions

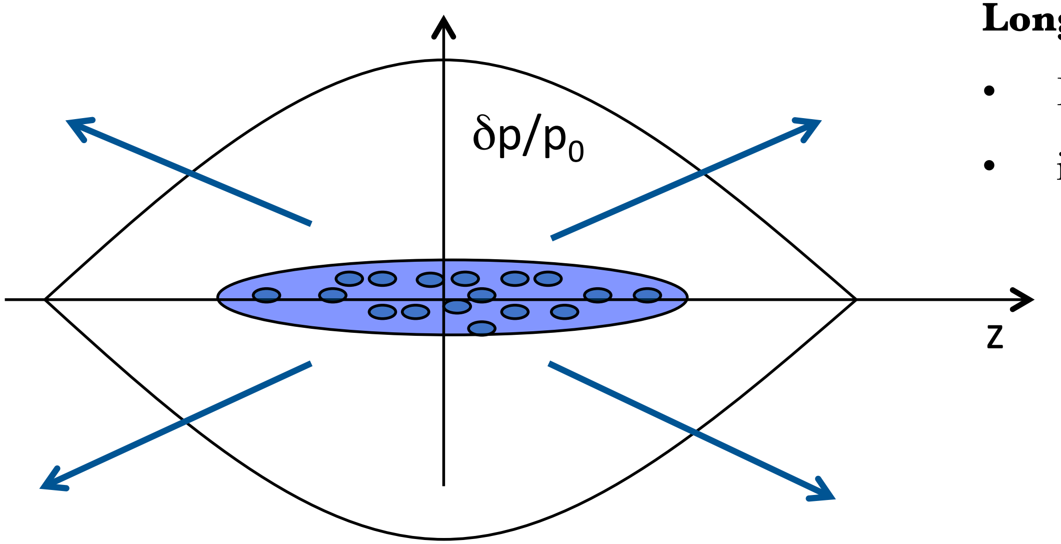
MULTI-PARTICLE PROCESSES ARE DETRIMENTAL FOR THE BEAM (DEGRADATION AND LOSS, SEE NEXT SLIDES)

Direct space charge forces

Transverse space charge

- Force decays like $1/\gamma^2\beta$
- It is always repulsive

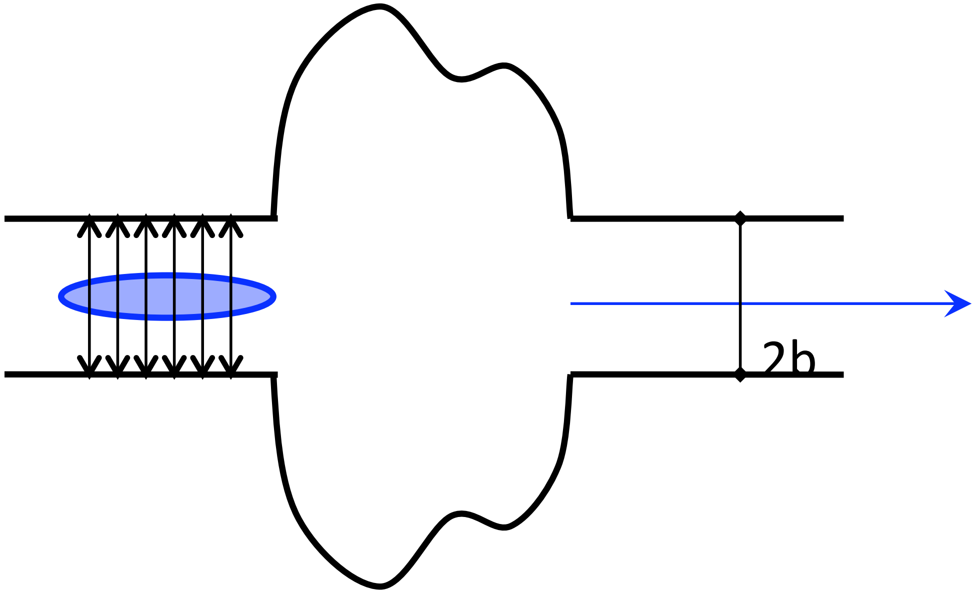


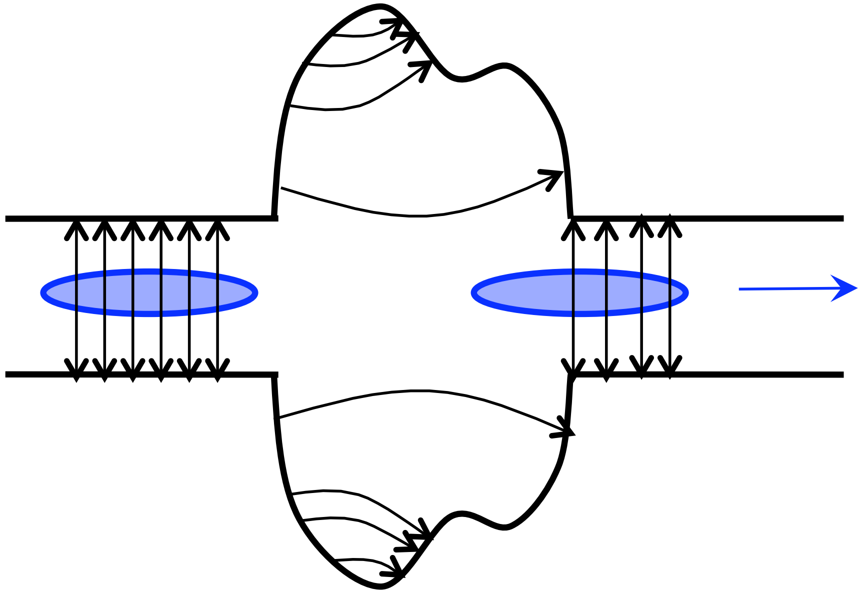


Longitudinal space charge

- Force decays like $1/\gamma^2$
- it can be attractive above transition

Wake fields (impedances)

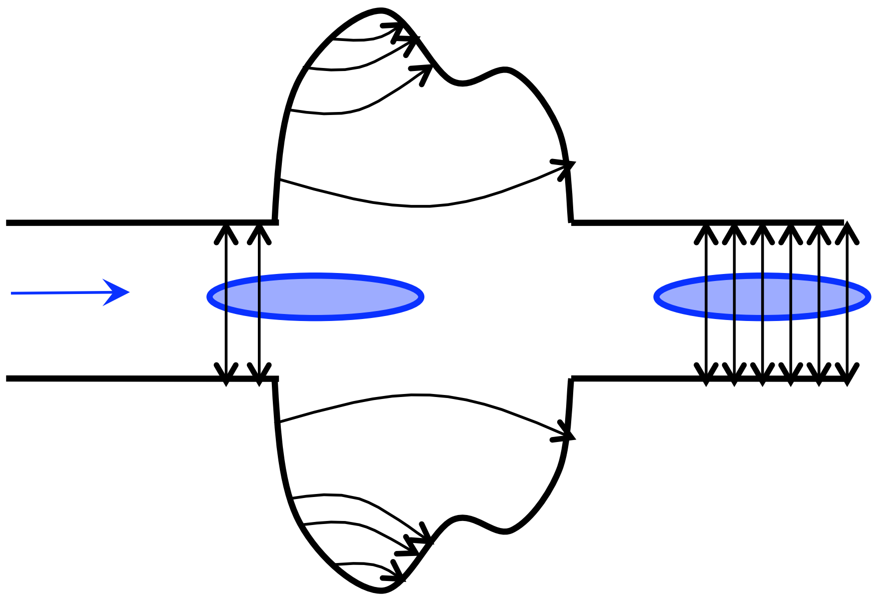




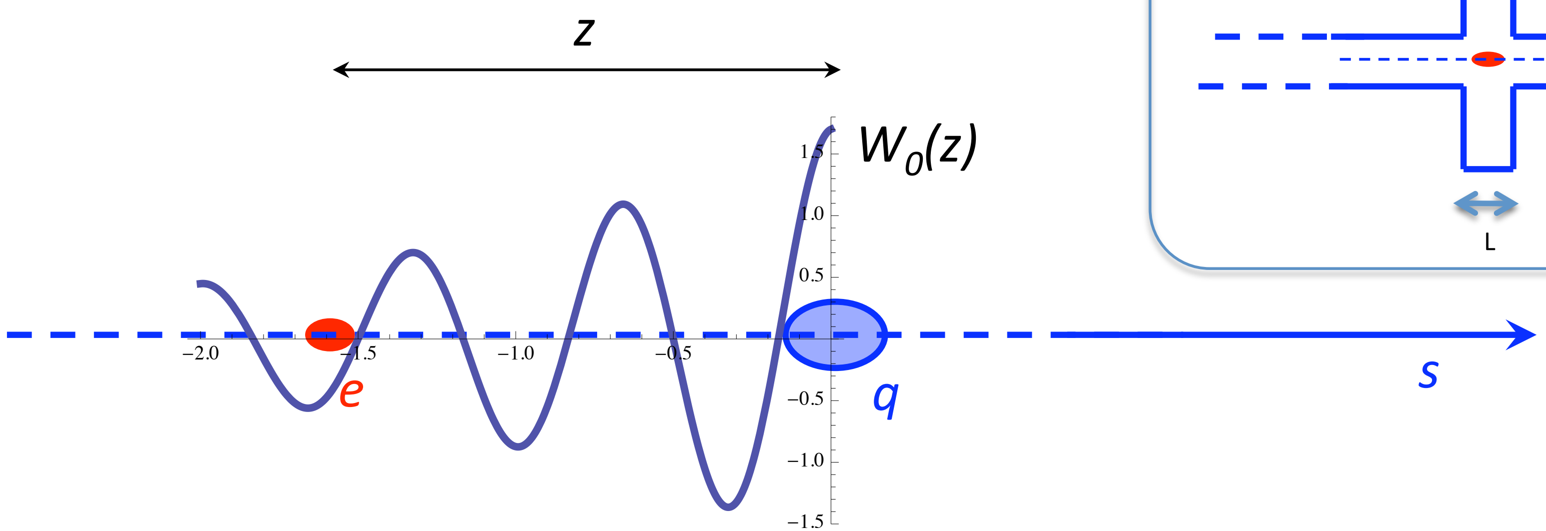
When the beam goes through a discontinuity, it induces an e-m field which keeps ringing after the beam has passed:

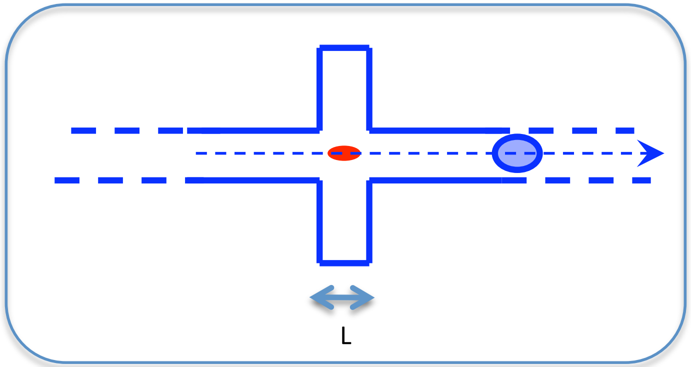
→ Energy loss

→ Effect intra-bunch and on following bunches



Wake fields (impedances)





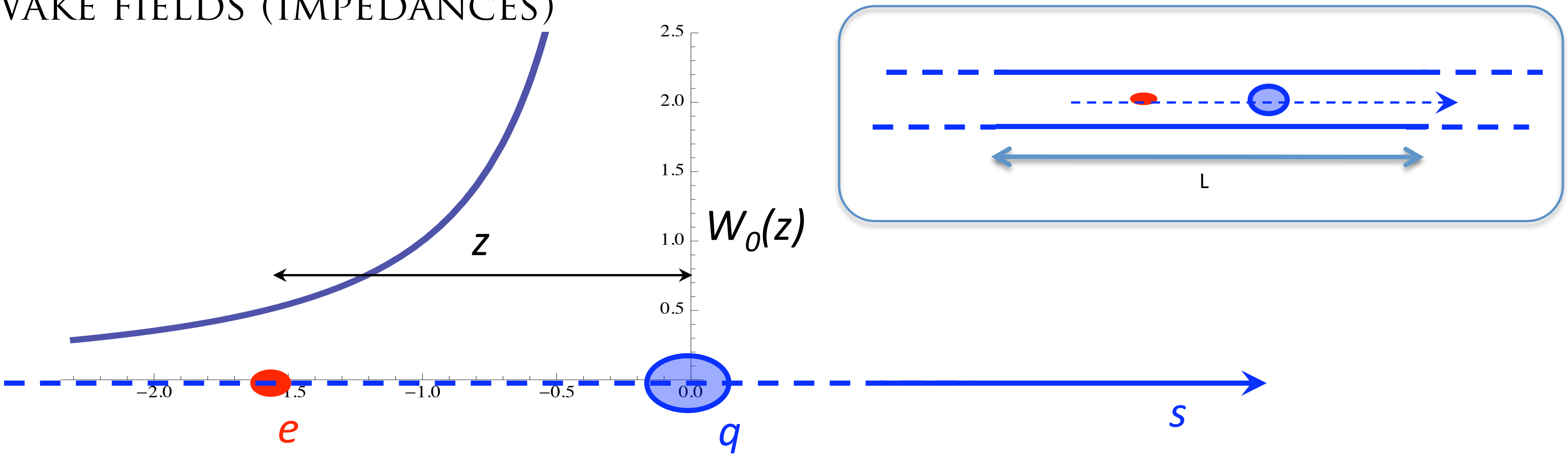
Model:

A particle ***q*** going through a device of length L, ***s (0,L),*** leaves behind an oscillating field and a probe charge ***e*** at distance ***z*** feels a force as a result. The integral of this force over the device defines the ***wake field*** and its Fourier transform is called the ***impedance of the device of length L***.

$$\int_0^L F_{||}(s,z)ds = -eqW_{||}(z) \qquad \int_0^L F_{\perp}(s,z)ds = -eqxW_{\perp}(z)$$

$$Z_{||\,(\perp)}(\omega) \equiv \frac{1}{c}\int_{-\infty}^{\infty} dz\, e^{-i\omega z/c} W_{||\,(\perp)}$$

Wake fields (impedances)



Model (cont's):

The device of length L can also be a segment of accelerator (defined by the simple beam pipe) and the wake is generated by the finite conductivity of the pipe material. In this case the wake field and the impedance are said to be of **resistive wall** type and the integration can be done over L=2π R

$$\int_0^L F_{||}(s,z)ds = -eqW_{||}(z) \qquad \int_0^L F_{\perp}(s,z)ds = -eqxW_{\perp}(z)$$

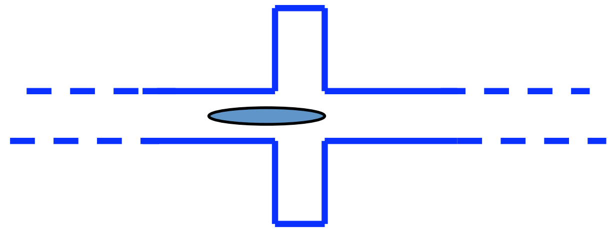
$$Z_{||\,(\perp)}(\omega) \equiv \frac{1}{c}\int_{-\infty}^{\infty} dz\, e^{-i\omega z/c} W_{||\,(\perp)}$$

Wake fields (impedances)

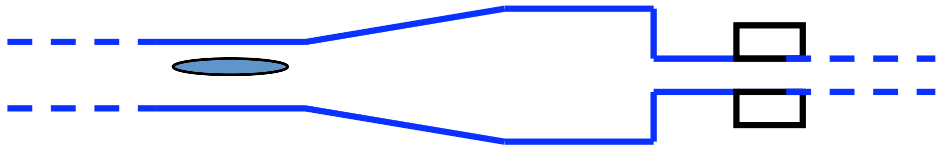
The full ring is usually modeled with a so called total impedance made of three main components:

- Resistive wall impedance

- Several narrow-band resonators at lower frequencies than the pipe cutoff frequency c/b (b beam pipe radius)



- One broad band resonator at $\omega_r \sim c/b$ modeling the rest of the ring (pipe discontinuities, tapers, other non-resonant structures like pick-ups, kickers bellows, etc.)



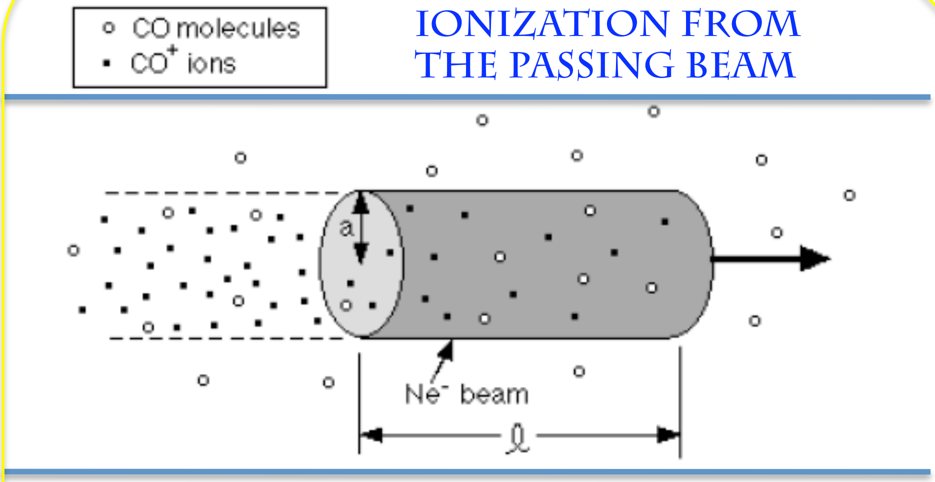
⇒ The total impedance is **allocated to the single ring elements** by means of off-line calculation prior to construction/installation

⇒ **Total impedance** designed such that the nominal intensity is stable

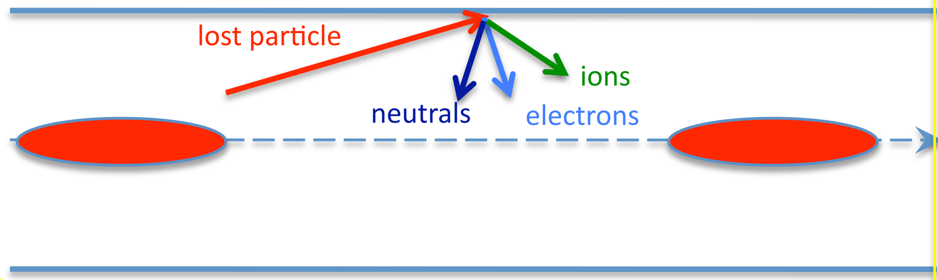
Electron cloud

Primary generation of electrons

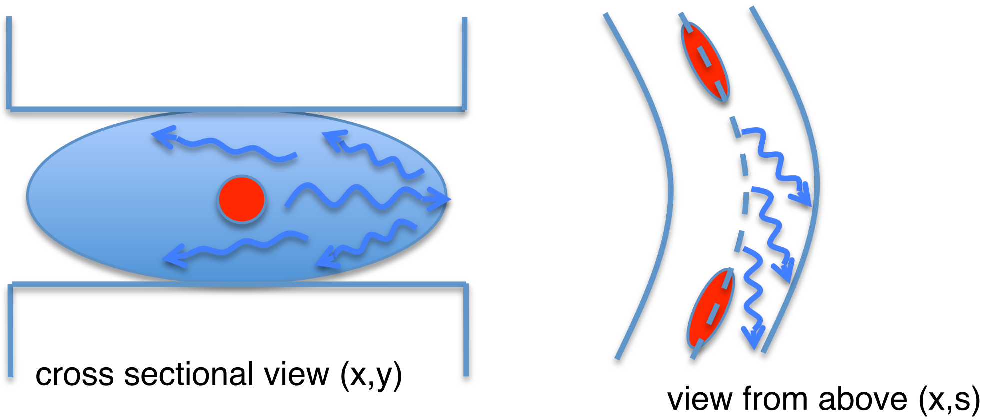
Residual gas ionization from the passing beam



Desorption from beam particle loss



Photoemission

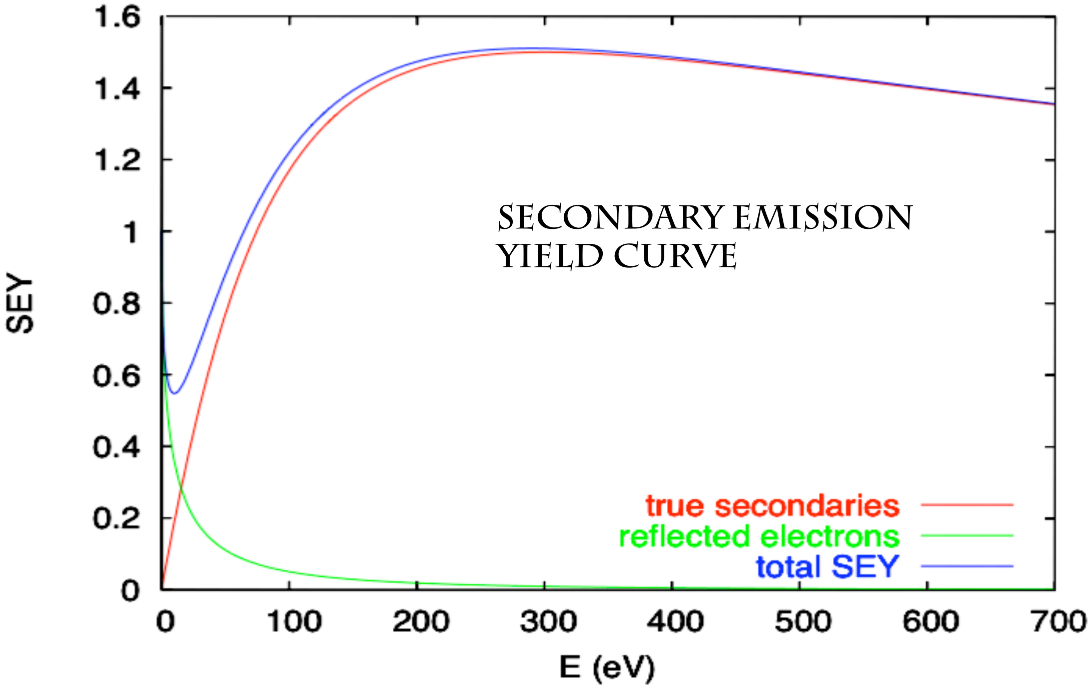


All these electrons could be in principle harmless (except the photoelectrons, which could be already generated in large amounts), but unfortunately they get accelerated in the beam transverse field, and usually hit the beam pipe before the next bunch comes

Electron cloud

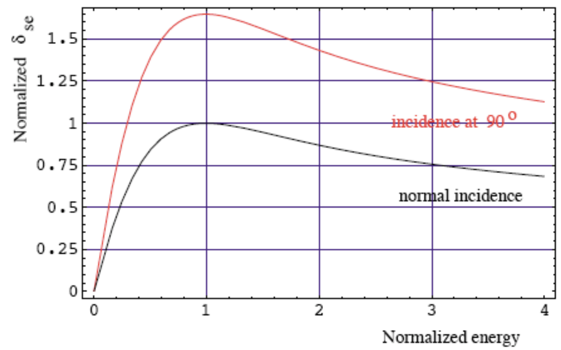
What happens when the primary electrons hit the beam pipe?

→ Secondary production of electrons



$$\delta_{se}(E_p,\theta) = \delta_{max} 1.11 x^{-0.35}[1 - \exp(-2.3x^{1.35})] \times \exp\left(\frac{1-\cos\theta}{2}\right).$$

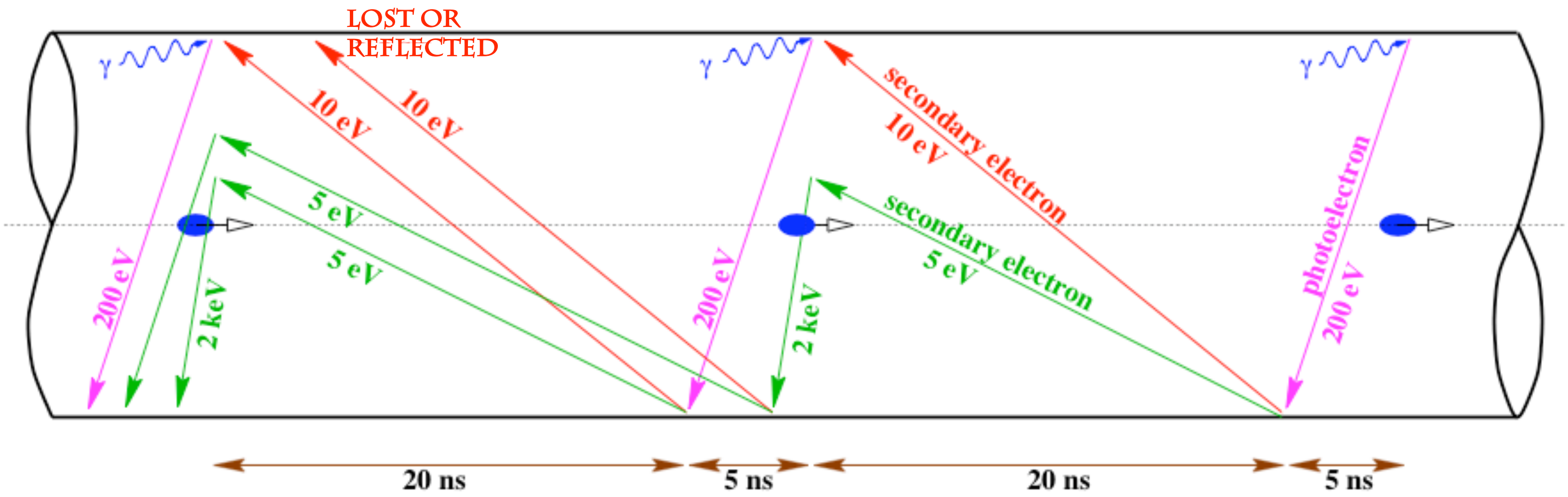
$$x = E_p/\epsilon_{max}$$



- Electrons hitting the beam pipe with high energies (typically above in the hundreds of eV range) can produce more electrons
- Electrons hitting the pipe with low energies have a high probability to be elastically reflected
- Electrons hitting the beam pipe with grazing incidence can produce even higher numbers of secondaries….

Electron cloud

Example of LHC

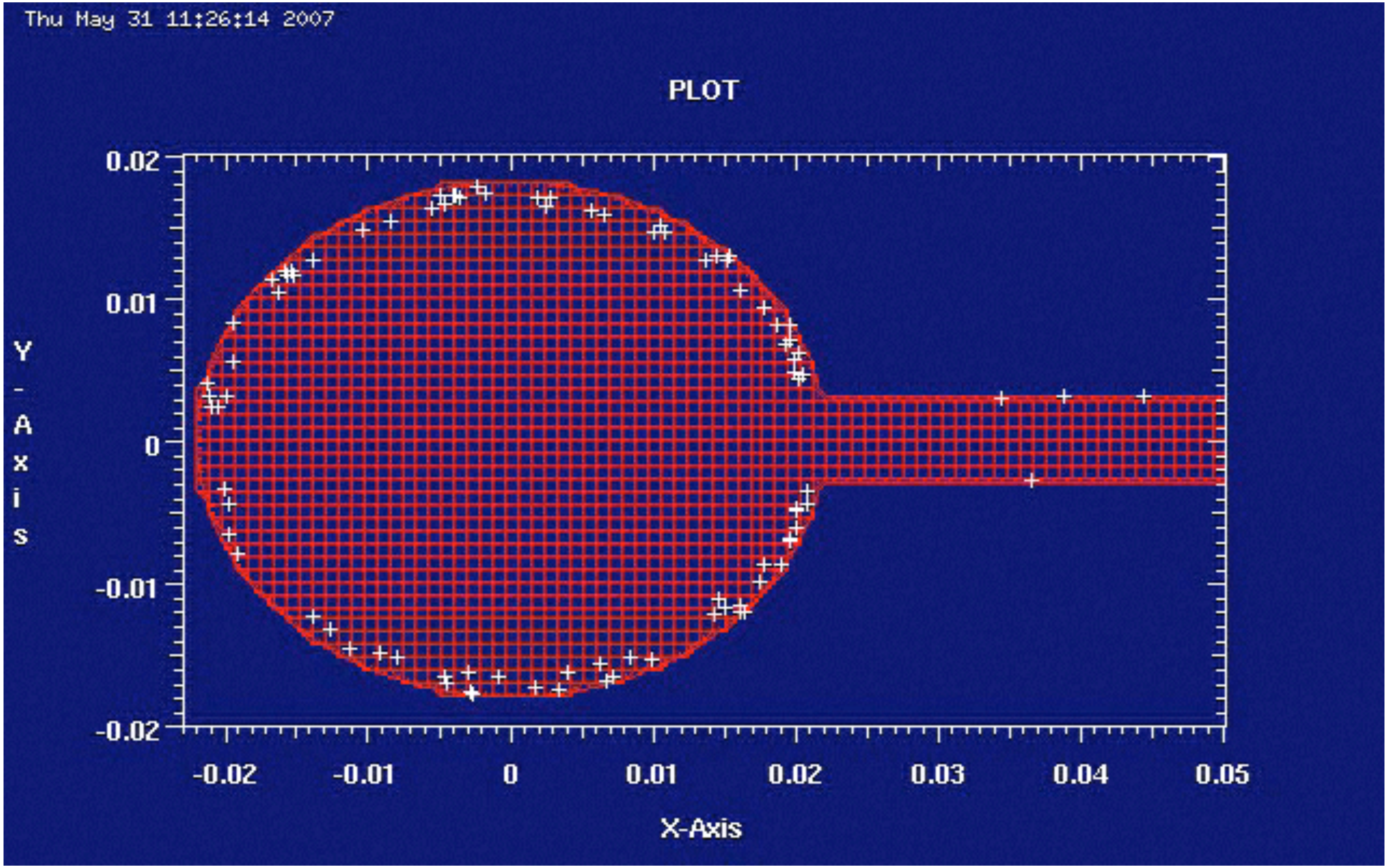


The LHC case provides a simple illustration of the interplay between the „ingredients“ that lead to electron cloud formation:

- ✓ Primary electron generation due to photoemission
- ✓ Electron acceleration in the field of the passing bunch
- ✓ Some of the secondary electrons perfectly synchronized with the incoming bunch (ideal multipacting condition, it is not a very narrow resonance anyway)
- ✓ Secondary emission with efficiency larger than one, when the electrons hit the inner pipe walls with high enough energy

Electron cloud

Photoelectrons produced by the synchrotron radiation are accelerated by the bunches and quickly accumulate in the vacuum chamber (example, wiggler of a CLIC damping ring)



Several names to describe these effects...

‚Multi-particle‘ is the most generic attribute. ***‚High-current‘, ‚high-intensity‘, ‚high brightness‘*** are also used because these effects are important when the beam has a high density in phase space (many particles crammed in a little volume)

Multi-particle effects are also usually referred to as COLLECTIVE EFFECTS, which is actually a subclass that further separates into two types

- **Collective** effects (general**)**:

  → The beam exhibits a visible response to a self-induced electromagnetic excitation

  → **Coherent collective** effects:

    ✓ Produced by the electromagnetic fields generated by the beam distribution, they affect the lower order momenta of the beam distribution itself (i.e. charge distribution, centroid)

    ✓ Are **fast** and experimentally observable in the beam **charge distribution** or **offset motion** (coherent tune shift, instabilities)

  → **Incoherent collective** effects:

    ✓ In some cases, the self-excitation moves along with the beam and it therefore cannot affect the lower order momenta

    ✓ Lead to particle diffusion in phase space and **slow** emittance growth

Several names to describe these effects...

Other effects are also **multi-particle** as they involve more than one particle, and become significant when the beam is very dense in the 6D phase space

- **Collisional** effects (incoherent):

  → Isolated two-particle encounters (short and long range Coulomb scattering) have a global effect on the beam dynamics (diffusion and emittance growth, lifetime)

  → they do not strictly depend on the beam distribution, but they result into a final effect on the beam distribution (halo, tails → higher order momenta)

- **Two-stream** phenomena (coherent or incoherent):

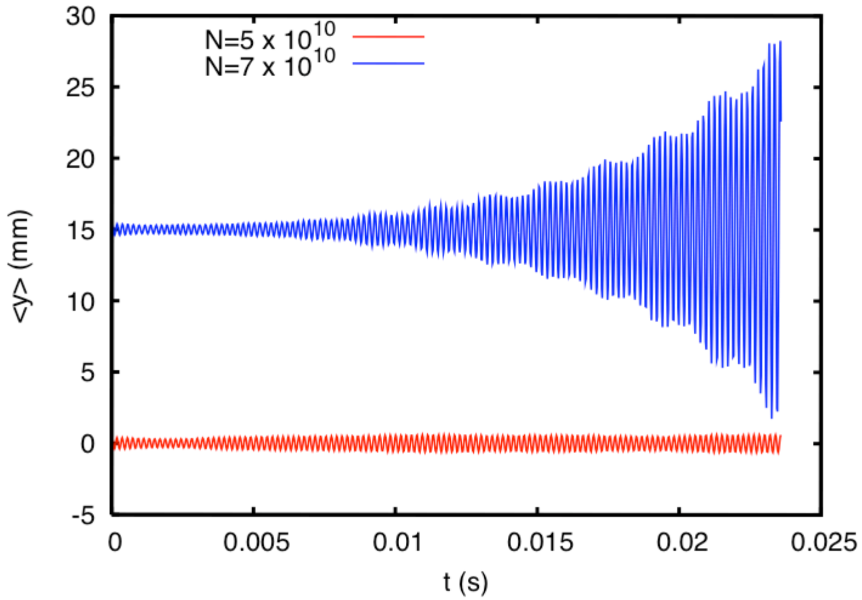
  → They need the presence of a second particle distribution, besides the beam (similar to a two component plasma). Examples:

  - ✓ beam-beam in colliders
  - ✓ hadron/positron beam with an electron cloud or an electron cooler
  - ✓ electron beam with trapped ions

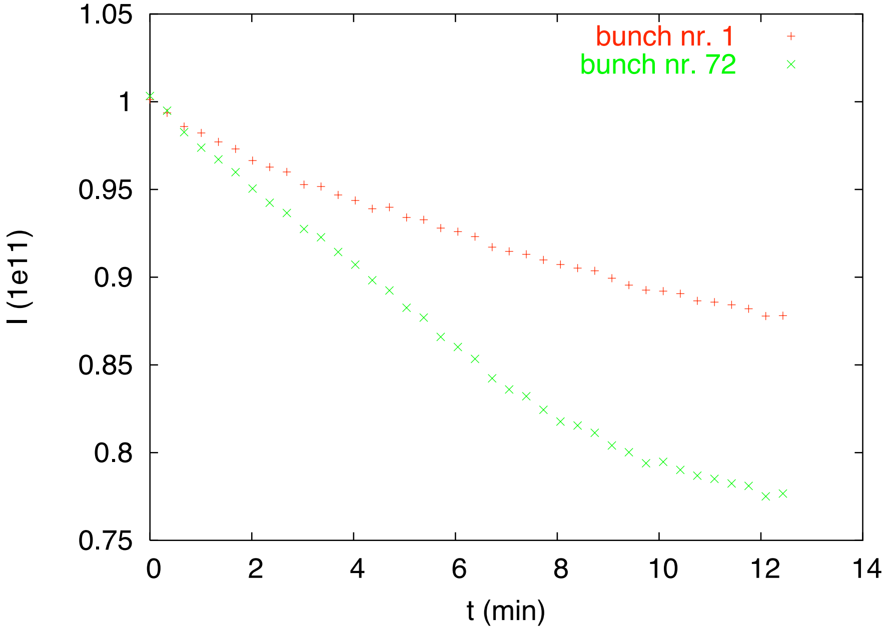
  → The beam exhibits a visible reaction to an electromagnetic excitation caused by another „beam“ (coherent or incoherent, like in the collective effects)

  → The beam suffers the effect of two particle encounters with particles of the second „beam“ (incoherent, like in the collisional effects)

Some examples....



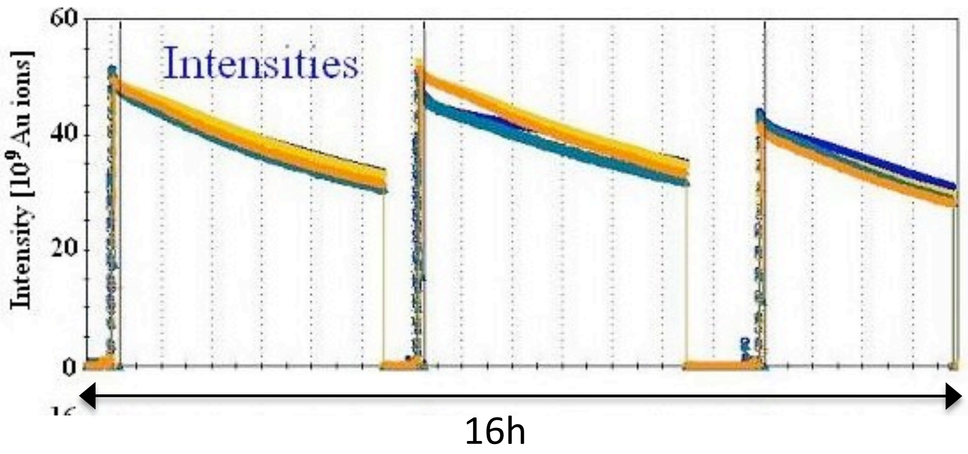
Coherent effect:
When the bunch current exceeds a certain limit (current threshold), the centroid of the beam, e.g. as seen by a BPM, exhibits an exponential growth and the beam is lost within few milliseconds (simulation of an SPS bunch)



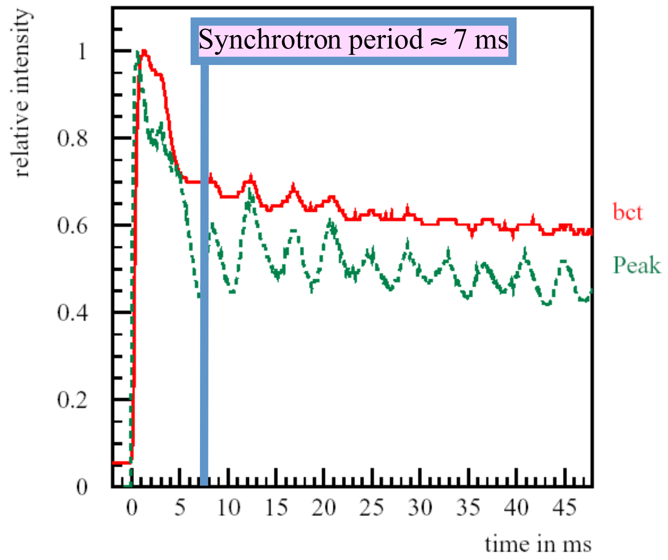
Incoherent effect:
Slow beam loss. First and last bunch of an SPS train of 72 bunches gradually lose their particles (10% and 25%, respectively) over several minutes (data from SPS, 2004)

More examples… (plots show measured beam intensity vs time)

BNL-RHIC, Au-Au operation, Run-4 (2004)



CERN-SPS (II) TMC Instability (2003)



NOTE THE DIFFERENT TIME SCALE ON THESE PLOTS!

Why are multi-particle effects important ?

The performance of an accelerator is usually limited by a multi-particle effect!!

➔ When the beam current in a machine is pushed above a certain limit (intensity threshold), **intolerable losses or beam quality degradation** appear due to these phenomena

➔ How these effects can affect the beam:

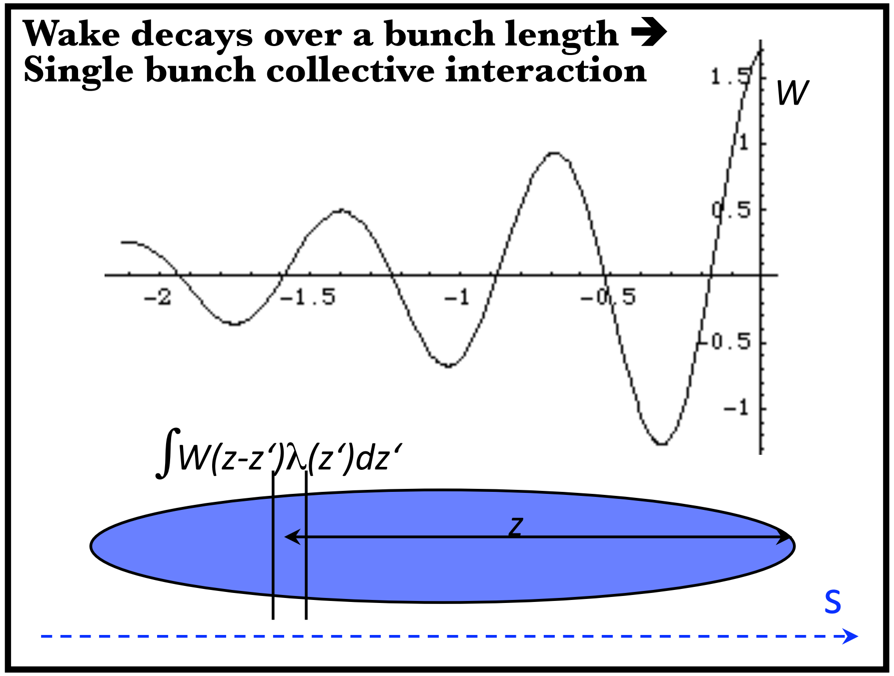
- Coherent effects (collective or two-stream) can be:
  - → Transverse
    - ✓ coherent tune shift
    - ✓ fast beam loss due to instability when the excitation is resonant
  - → Longitudinal
    - ✓ energy loss, bunch lengthening, synchrotron tune shift
    - ✓ beam quality degradation because of an instability
- Incoherent effects (collective, collisional, two-stream) can be:
  - → Longitudinal or transverse
    - ✓ single particle tune shifts resulting into a tune spread
    - ✓ emittance growth, bad lifetime

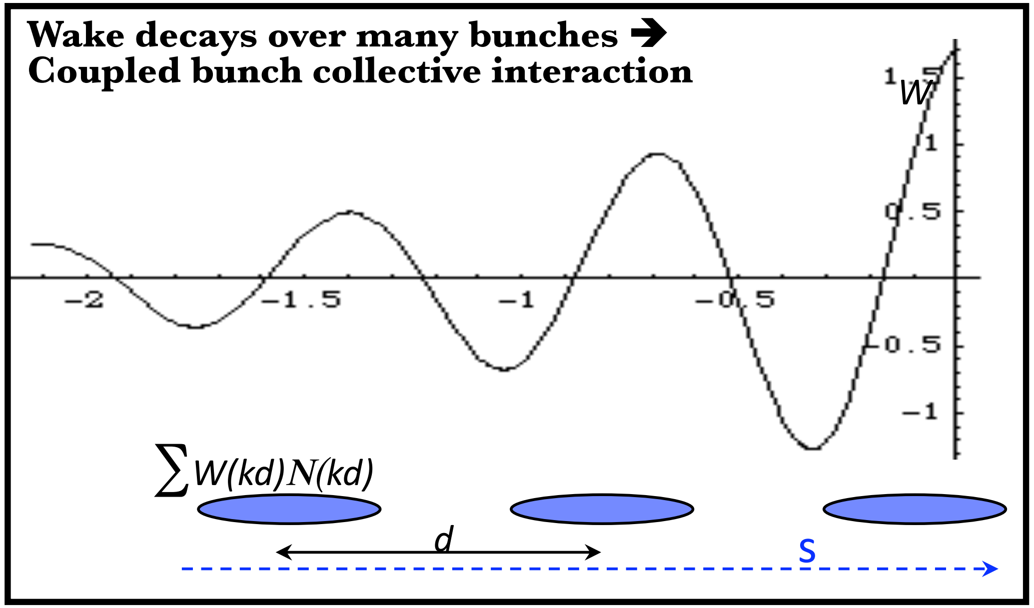
Types of coherent effects (I)

• Single or multi-bunch behaviour is determined by the range of action of the driving force (e.g. a wake field, another beam, an electron cloud, etc.)

→ Single bunch effects are caused by short lived excitations (e.g. broad-band impedances, beam-beam, electron cloud pinch)

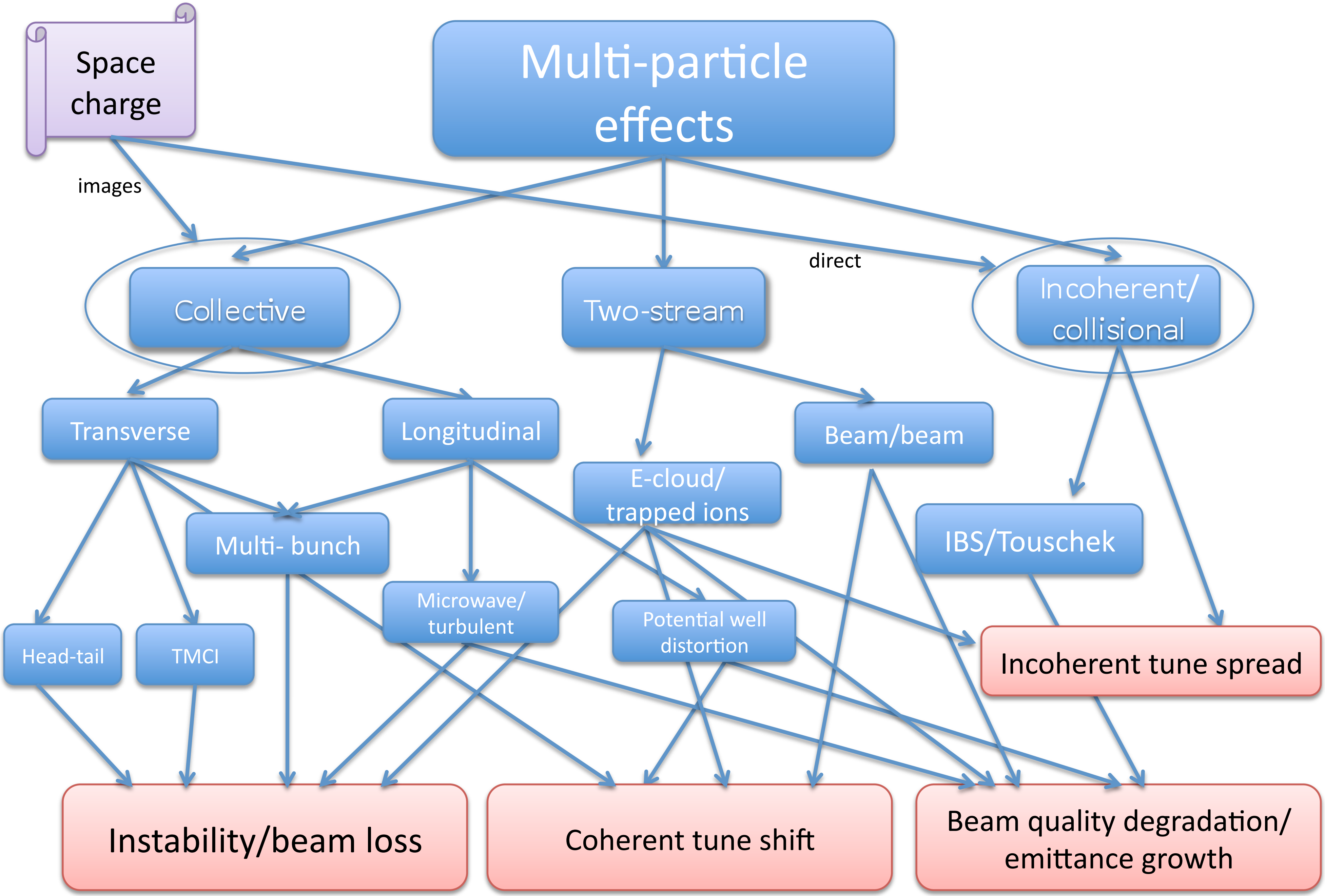
→ Multi bunch or multi-turn effects are necessarily associated to long lived excitations (e.g. resistive wall, narrow-band impedances, bunch-to-bunch electron clouds, ions)





Types of collective effects (II)

- Transverse:
  - → Single bunch
    - ✓ Rigid bunch instability, head-tail instability
    - ✓ Transverse Mode Coupling Instability (TMCI), also referred to as beam break-up or strong (fast) head-tail instability
    - ✓ Electron Cloud Instability (ECI)
  - → Coupled bunch instability
    - ✓ Coupling between subsequent bunches through long range wake fields or electron cloud, Ion Instability, Fast-Ion Instability
- Longitudinal:
  - → Single bunch
    - ✓ Potential well distortion, energy loss
    - ✓ Dipolar, quadrupolar, higher order oscillations
    - ✓ Microwave instability, also referred to as turbulent bunch lengthening
  - → Coupled bunch instability


Space charge
Multi-particle effects
images
direct
Collective
Two-stream
Incoherent/ collisional
Transverse
Longitudinal
Beam/beam
E-cloud/ trapped ions
IBS/Touschek
Multi- bunch
Microwave/ turbulent
Potential well distortion
Head-tail
TMCI
Incoherent tune spread
Instability/beam loss
Coherent tune shift
Beam quality degradation/ emittance growth

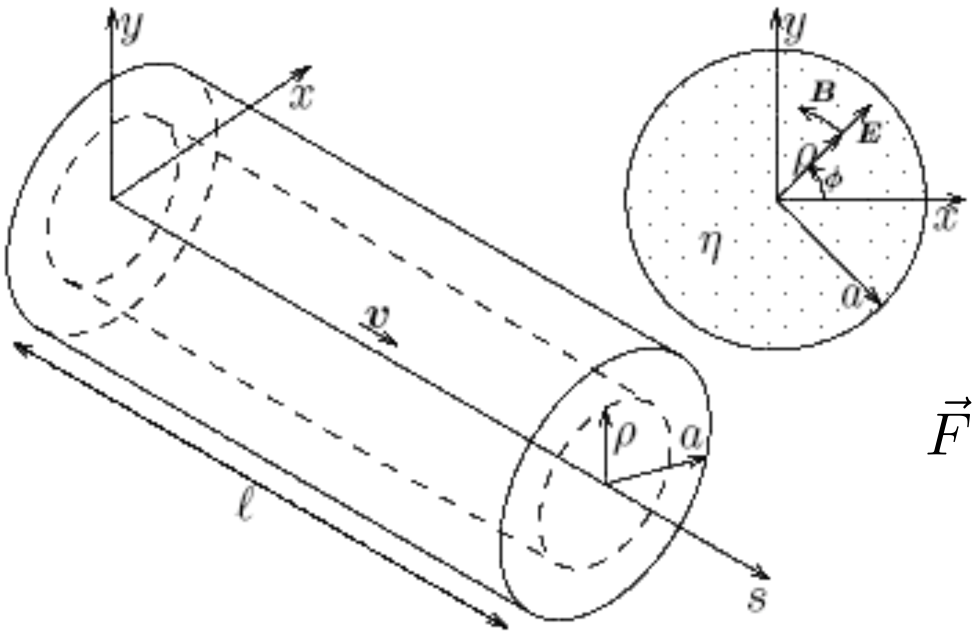
Direct space charge effect (transverse)

- To quantify the direct space charge effect

→ Consider a continuous beam with circular cross section

→ Calculate the electromagnetic force from the beam acting on a generic particle inside the beam itself

→ Electric and magnetic components have opposite signs and scale between them with β^2. There is perfect compensation only when $\beta=1$.



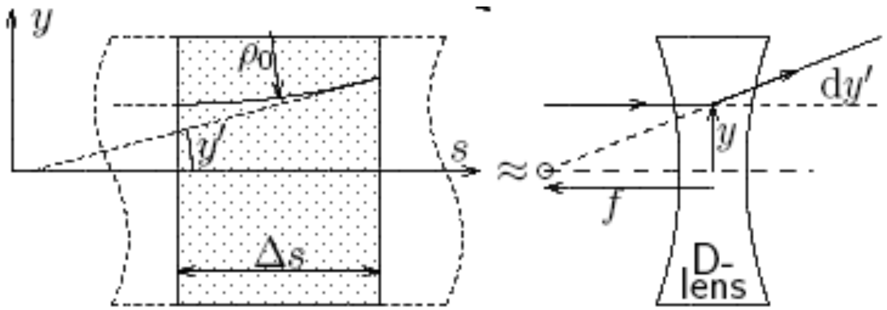
$$\vec{F} = \vec{F}_E + \vec{F}_B = e\left(\vec{E} + \vec{v} \times \vec{B}\right) =$$

$$= \frac{e\lambda\vec{\rho}}{2\epsilon_0 \pi a^2}(1-\beta^2) = \frac{e\lambda\vec{\rho}}{2\pi\epsilon_0\gamma^2 a^2}$$

Defocusing space charge....

- Looking now only at the vertical plane
  - → Additional force causes locally a defocusing deflection *dy'*
  - → This translates into a local contribution to the tune depression $dQ_y(s)$
  - → Tune shift ΔQ_y is found integrating all over the machine circumference

$$\begin{cases} F_y = m_0\gamma \dfrac{d^2y}{dt^2} = m_0\gamma\beta^2c^2\dfrac{dy'}{ds} \\ \dfrac{dy'}{y} = \dfrac{2r_0\lambda ds}{e\beta^2\gamma^3a^2} \\ dQ_y(s) = -\dfrac{\beta_y(s)}{4\pi}\dfrac{dy'}{y} = -\dfrac{r_0\lambda\beta_y(s)ds}{2\pi e\beta^2\gamma^3a^2(s)} \end{cases}$$



$$\Delta Q_y = \oint dQ_y(s) = -\frac{r_0\lambda}{2\pi e\beta^2\gamma^3}\oint\frac{\beta_y(s)ds}{a^2(s)} = -\frac{r_0\lambda R}{e\beta\gamma^2\epsilon_{yn}}$$

Defocusing space charge....

- The direct space charge tune shift of a continous beam with circular cross section

  → Is negative, because space charge transversely always defocuses

  → Is proportional to the number of particles inside the beam N

  → decreases with the beam energy ($\beta^{-1}\gamma^{-2}$)

  → is inversely proportional to the normalized emittance and does not depend on the local beta functions or beam sizes.

$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi e \beta \gamma^2 \epsilon_{xn,yn}}$$

$$r_0 = \frac{e^2}{4\pi\epsilon_0 m_0 c^2} = \begin{array}{l} 1.54 \cdot 10^{-18}\,\mathrm{m\ protons} \\ 2.82 \cdot 10^{-15}\,\mathrm{m\ electrons} \end{array}$$

Some important extensions (I)

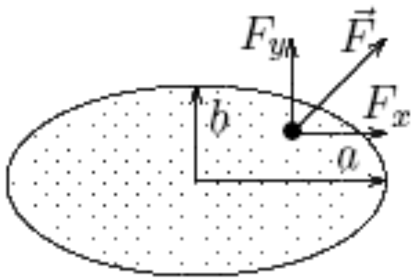
- If the beam has an **elliptical cross section**:

  → forces are still linear and can be calculated

  → defocusing contribution in both transverse directions

$$\Delta Q_x = -\frac{r_0 \lambda}{\pi e \beta \gamma^2 \epsilon_{xn}} \oint \frac{a}{a+b} ds$$

$$\Delta Q_y = -\frac{r_0 \lambda}{\pi e \beta \gamma^2 \epsilon_{yn}} \oint \frac{b}{a+b} ds$$



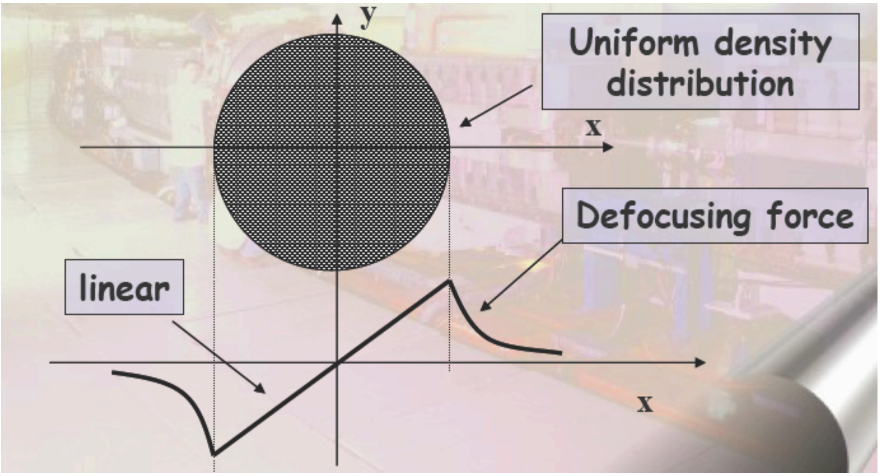
Some important extensions (II)

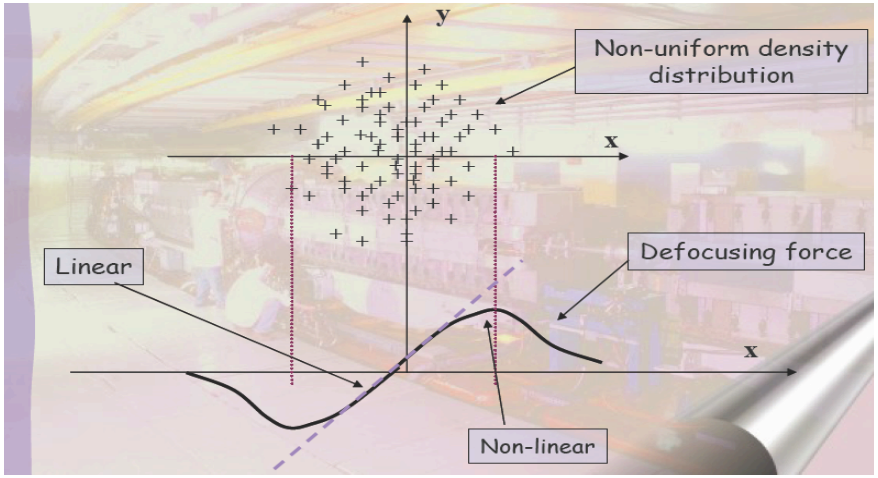
• If the beam has a **more general cross section** (neither circular nor elliptical)

→ Beam field over the beam cross section is nonlinear

→ Particles will see different tune shifts according to their betatron amplitudes. This causes a tune spread over the beam particles

→ The previous formulae will still hold for particles with small amplitudes (in the linear region of the beam force) assuming rms emittances





Some important extensions (III)

- If the beam is **bunched** and has an **s-dependent line density**

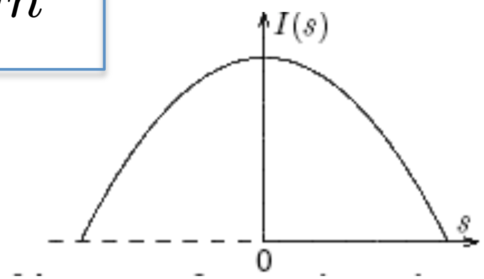
→ Relativistic field has longitudinally an opening angle of $1/\gamma$

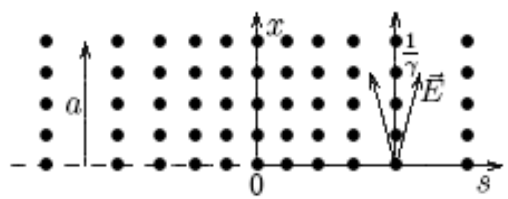
→ We can look at the bunched beam as locally continuous if the density $\lambda(s)$ changes smoothly (i.e. does not change much over $\Delta s=a/\gamma$)

→ The tune shift will depend on the position of the particle along the bunch

→ This translates into tune modulation with the synchrotron period, because particles execute synchrotron oscillations longitudinally

$$\Delta Q_{x,y} = -\frac{r_0 \lambda(s) R}{e\beta\gamma^2 \epsilon_{xn,yn}}$$





The effect of the images (incoherent)

- The **presence of a beam pipe** also contributes to the single particle tune shift

  → Assume two parallel plates at a distance h from the beam and calculate the tune shift of a particle having vertical position *y* within the beam

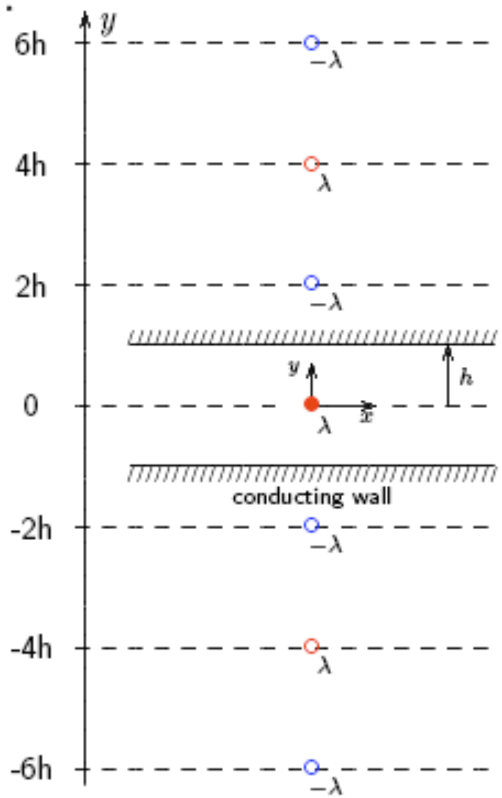
  → There are infinite images placed at ±2nh, which contribute to E_i

$$E_{iny} = \frac{(-1)^n \lambda}{2\pi\epsilon_0}\left(\frac{1}{2nh+y} - \frac{1}{2nh-y}\right) \approx -\frac{\lambda y}{4\pi\epsilon_0 h^2}\frac{(-1)^n}{n^2}$$

$$E_{iy} = \sum_1^{\infty} E_{iny} = \frac{\lambda y}{4\pi\epsilon_0 h^2}\frac{\pi^2}{12}, \quad \text{div}\vec{E}_i = 0 \rightarrow E_{ix} = -\frac{\lambda x}{4\pi\epsilon_0 h^2}\frac{\pi^2}{12}$$

$$F_{ix} = -\frac{e\lambda}{\pi\epsilon_0}\frac{\pi^2}{48h^2}x \qquad F_{iy} = \frac{e\lambda}{\pi\epsilon_0}\frac{\pi^2}{48h^2}y$$

$$\Delta Q_{x,y} = \pm\frac{2r_0\lambda R\langle\beta_{x,y}\rangle}{e\beta^2\gamma}\frac{\pi^2}{48h^2}$$



The effect of the images (coherent)

• The **presence of a beam pipe** also causes a net electric force on the beam, when it is rigidly offset as a whole

→ Direct space charge cannot because it moves along with the beam

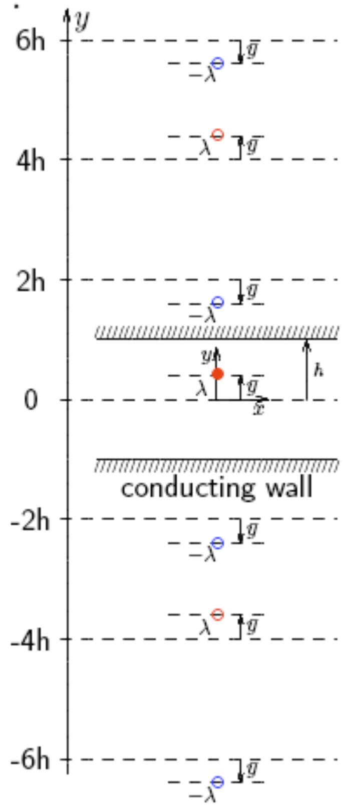
→ This force F_c affects the beam collective motion, therefore it is responsible for a coherent tune shift

$$E_{cny} = -\frac{(-1)^n \lambda \bar{y}}{4\pi\epsilon_0 h^2}\left(\frac{1}{n^2} - \frac{(-1)^n}{n^2}\right)$$

$$E_{cy} = \sum_1^\infty E_{cny} = \frac{\lambda \bar{y}}{4\pi\epsilon_0 h^2}\left(\frac{\pi^2}{12} + \frac{\pi^2}{6}\right)$$

$$F_{cy} = \frac{e\lambda}{\pi\epsilon_0}\frac{\pi^2}{16h^2}\bar{y}$$

$$\Delta Q_{cy} = -\frac{2r_0\lambda R\langle\beta_y\rangle}{e\beta^2\gamma}\frac{\pi^2}{16h^2}$$



Summary of the space charge tune shifts

- All the **space charge tune shifts** of a continuous beam with circular cross section between two parallel perfectly conducting plates

	Incoherent (direct)	Incoherent (indirect)	Coherent (indirect)
Horizontal	$\Delta Q_{dx} = -\frac{r_0 N}{2\pi e \beta \gamma^2 \epsilon_{xn}}$	$\Delta Q_{ix} = \frac{2 r_0 \lambda R \langle \beta_x \rangle}{e \beta^2 \gamma} \frac{\pi^2}{48 h^2}$	$\Delta Q_{cx} = 0$
Vertical	$\Delta Q_{dy} = -\frac{r_0 N}{2\pi e \beta \gamma^2 \epsilon_{yn}}$	$\Delta Q_{iy} = -\frac{2 r_0 \lambda R \langle \beta_y \rangle}{e \beta^2 \gamma} \frac{\pi^2}{48 h^2}$	$\Delta Q_{cy} = -\frac{2 r_0 \lambda R \langle \beta_y \rangle}{e \beta^2 \gamma} \frac{\pi^2}{16 h^2}$

$$Q_{xcoh} - Q_{xinc} = -(\Delta Q_{dx} + \Delta Q_{ix}) = \frac{r_0 \lambda R}{e \beta \gamma} \left(\frac{1}{\gamma \epsilon_{xn}} - \frac{\langle \beta_x \rangle \pi^2}{24 \beta h^2} \right)$$

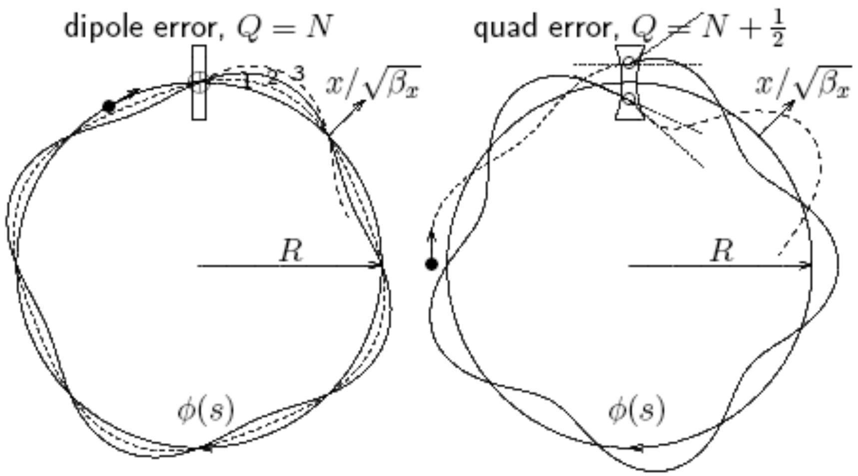
$$Q_{ycoh} - Q_{yinc} = \Delta Q_{cy} - (\Delta Q_{dy} + \Delta Q_{iy}) = \frac{r_0 \lambda R}{e \beta \gamma} \left(- \frac{2 \langle \beta_y \rangle \pi^2}{16 \beta h^2} + \frac{1}{\gamma \epsilon_{yn}} + \frac{2 \langle \beta_y \rangle \pi^2}{48 \beta h^2} \right) =$$

$$= \frac{r_0 \lambda R}{e \beta \gamma} \left(\frac{1}{\gamma \epsilon_{yn}} - \frac{\langle \beta_y \rangle \pi^2}{12 \beta h^2} \right)$$

What space charge tune shifts can cause

Problems caused by space-charge in rings

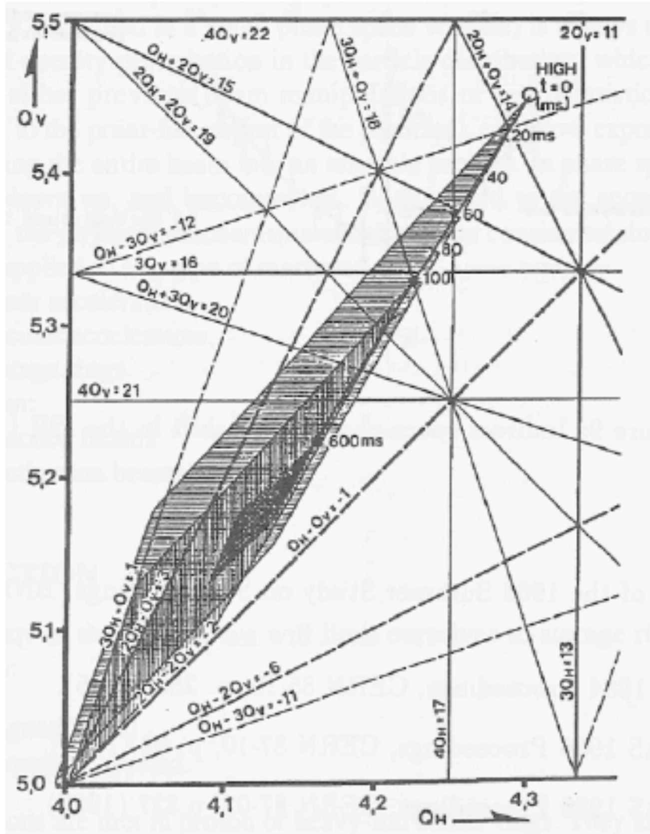
In rings space-charge can shift tunes into resonances where $Q = N/M$ is a simple rational fraction. Dipole imperfection deflects particle each turn in phase if Q =integer and for a quadrupole error this happens if Q =half integer. Since space-charge shifts coherent and incoherent tunes differently and produces spread it may be difficult to avoid all resonances.



Related effects

Beam-beam effect: electric and magnetic forces ad.

Ions and electron clouds: don't move, no B-force.



Direct space-charge tune shift and spread at different γ during acceleration (shaded areas), CERN booster, E. Brouzet, K.H. Schindl.

Instabilities

- Coherent transverse or longitudinal motion of a bunch can result into beam instability

A. **Single rigid bunch instability**: this is an instability affecting a bunch over multiple passages through the same impedance source

   ✓ Long-range wake field, must extend over more than one turn

   ✓ Typical source: narrow-band resonators, cavities, HOM in cavities

B. **Coupled rigid bunch instability**: this is an instability caused by the effect on a bunch of the sum of the wake fields left behind by the previous bunches

   ✓ Long-range wake field, must extend over several interbunch gaps

   ✓ Typical source: resistive wall, narrow-band resonators

C. **Single non-rigid bunch instability**: this is an instability in which the motion of the tail of a bunch is coupled to that of the head through a wake field

   ✓ Short-range wake field, usually damped over one full bunch length

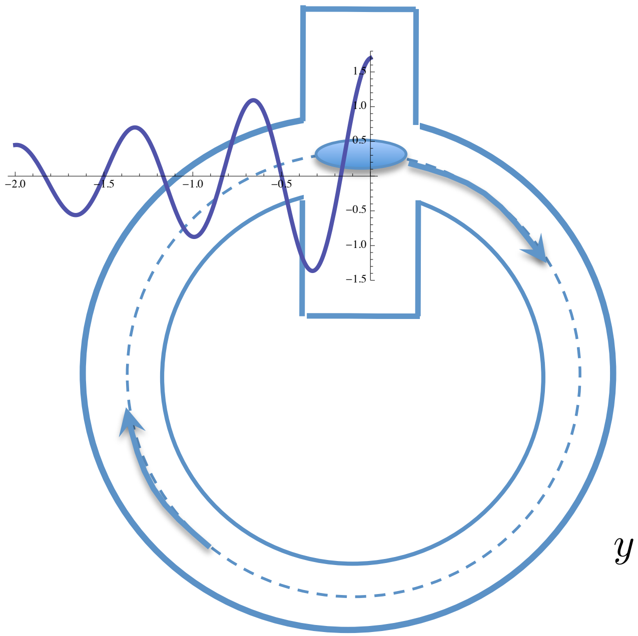
   ✓ Typical source: broad-band impedance

A. Single rigid bunch instability (transverse)

- A bunch can become unstable passing several times through an impedance source (over successive turns, ring having circumference C=2πR)

  → Force felt by a probe charge at distance d from the bunch is proportional to the wake field $W_1(-d)$ and to the bunch offset y

  → This force F_c affects the bunch itself on successive turns and **causes tune shift**



$$F_y = -\frac{Ne^2}{C} y(s) W_1(-d)$$

$$\frac{dy'}{ds} + \left(\frac{\omega_\beta}{c}\right)^2 y(s) = -\frac{Ne^2}{m_0 \gamma c^2 C} \sum_{k=1}^{\infty} y(s-kC) W_1(-kC)$$

$$y \propto \exp\left(\frac{-i\Omega s}{c}\right) \Longrightarrow$$

$$\Omega^2 - \omega_\beta^2 = \frac{Ne^2}{m_0 \gamma C} \sum_{k=1}^{\infty} \exp(ik\Omega T_0) W_1(-kC)$$

$$= -i \frac{Ne^2}{m_0 \gamma C T_0} \sum_{p=-\infty}^{\infty} Z_1^{\perp}(p\omega_0 + \Omega)$$

A. Single rigid bunch instability (II)

- Therefore, the interaction with the impedance over successive turns gives rise to a **complex tune shift,** to be interpreted in the following way

  → Real part gives the coherent tune shift, i.e. the shift with respect to the nominal tune of the beam centroid oscillation frequency

  → Imaginary part gives growth or damping time of this coherent oscillation, depending on its sign

$$\frac{\mathrm{Re}[\Omega-\omega_\beta]}{\omega_0} = \Delta Q_{cx,y} \approx \frac{Ne^2\bar{\beta}_{x,y}}{4\pi m_0\gamma c C}\sum_{p=-\infty}^{\infty}\mathrm{Im}\left[Z_1^\perp(p\omega_0+\omega_\beta)\right]$$

$$\mathrm{Im}(\Omega-\omega_\beta) = \tau^{-1} \approx -\frac{Ne^2\bar{\beta}_{x,y}}{2m_0\gamma C^2}\sum_{p=-\infty}^{\infty}\mathrm{Re}\left[Z_1^\perp(p\omega_0+\omega_\beta)\right]$$

A. Single rigid bunch instability (III)

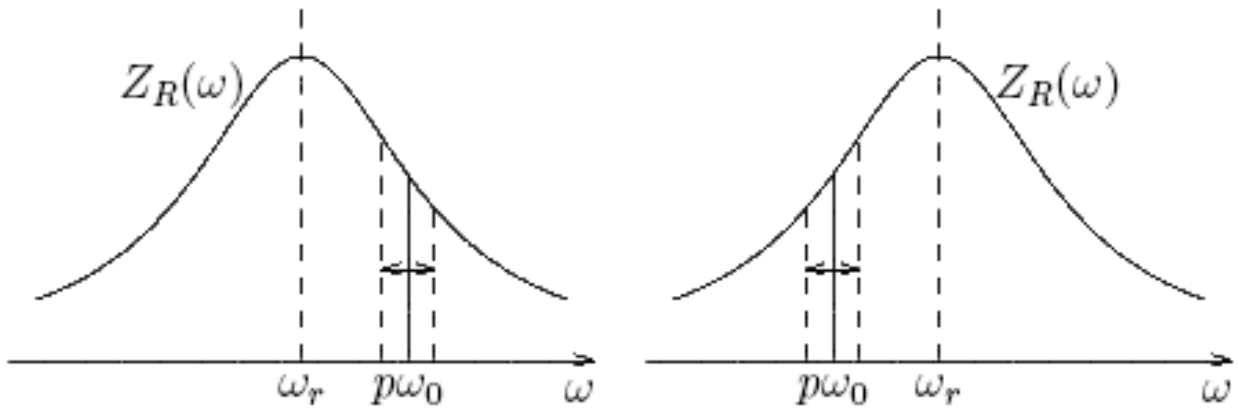
- An interesting case is when there is only one narrow-band resonator acting on the beam

→ Its resonant frequency ω_r is very close to a multiple of ω_o ($p\omega_o$)

→ Then stability depends on the tuning of the betatron frequency with respect to the resonator frequency

$$\tau^{-1} \approx -\frac{Ne^2\bar{\beta}_{x,y}}{2m_0\gamma C^2}\left(\mathrm{Re}\left[Z_1^{\perp}(p\omega_0+\Delta_\beta\omega_0)\right]-\mathrm{Re}\left[Z_1^{\perp}(p\omega_0-\Delta_\beta\omega_0)\right]\right)$$

If $\Delta_\beta > 0$ (tune above the integer), beam is stable when $p\omega_0 < \omega_r$
If $\Delta_\beta < 0$ (tune below the integer), beam is stable when $p\omega_0 > \omega_r$



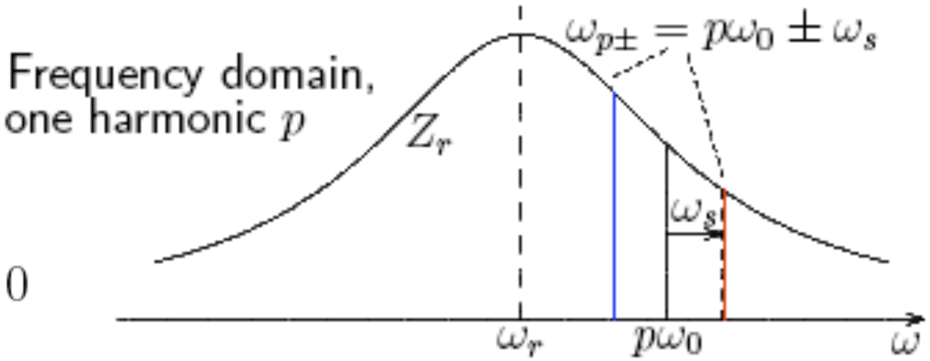
B. Single rigid bunch instability (longitudinal)

- There is an equivalent longitudinal of this type of instability, which is called **Robinson instability**. It has an intuitive explanation

  → When a bunch above transition is executing longitudinal dipole oscillations, it will have an excess of energy when $\omega<\omega_0$

  → If the cavity is tuned on $\omega_r<p\omega_0$ the beam is stable because it will see a higher impedance (and therefore will lose more) when it is sitting on the lower side-band, i.e. when it has an excess of energy.

  → Below transition the situation is reversed.



$$\epsilon = \hat{\epsilon} e^{-\alpha_s t} \sin(\omega_s t), \text{ damping if } \alpha_s > 0$$

$$\alpha_s = \frac{\omega_{s0} p I_p^2 (Z_r(\omega_{p+}) - Z_r(\omega_{p-}))}{2 I_0 h \hat{V} \cos\phi_s}$$

$$\gamma > \gamma_T, \cos\phi_s < 0, \text{ stable } Z_r(\omega_{p-}) > Z_r(\omega_{p+})$$

C. Single non-rigid bunch modes (Head-tail modes)

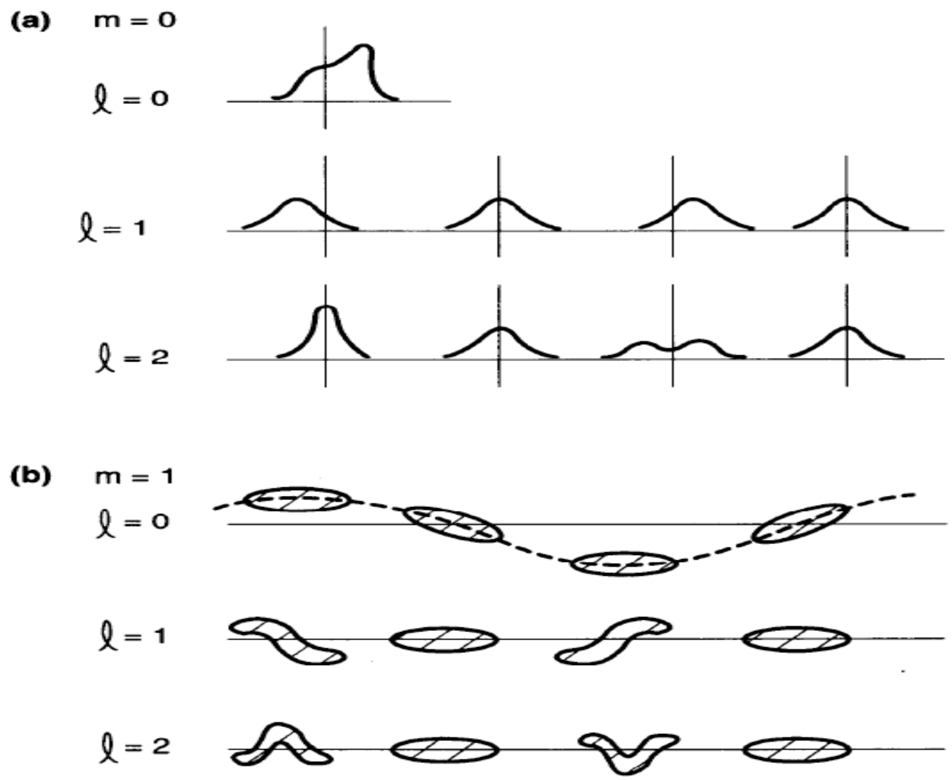
- A bunch can also become unstable because one or more of its intrinsic modes of oscillation are driven unstable by an external wake field/impedance

  → Longitudinal modes are described by a longitudinal mode number, *l*

  → Transverse modes are described by a transverse mode number, *m*

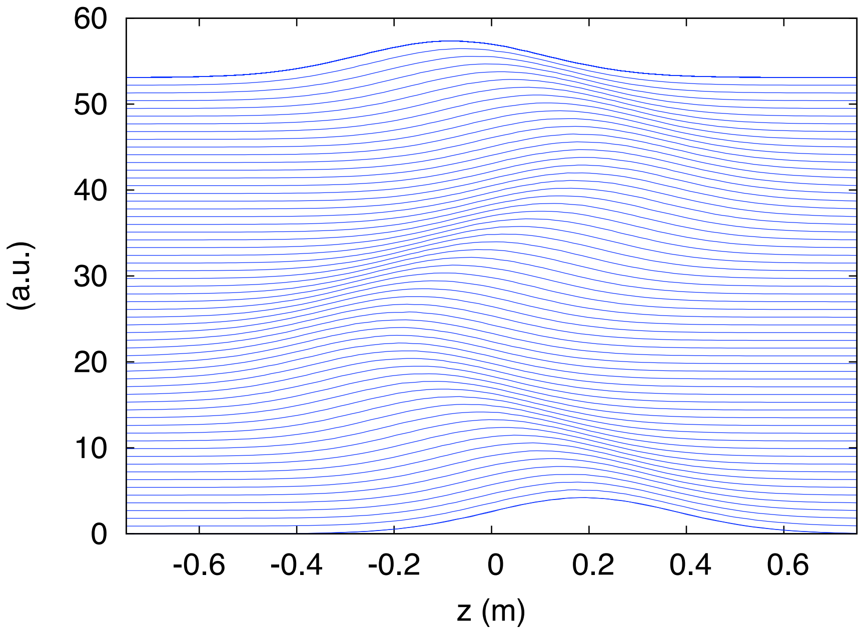
Examples:

- ***m*=0, *l*=1** dipole oscillation inside a bucket
- ***m*=1, *l*=0** rigid dipole oscillation
- ***m*=1, *l*=1** transverse dipole oscillation with head and tail of the bunch out of phase by π
- ….

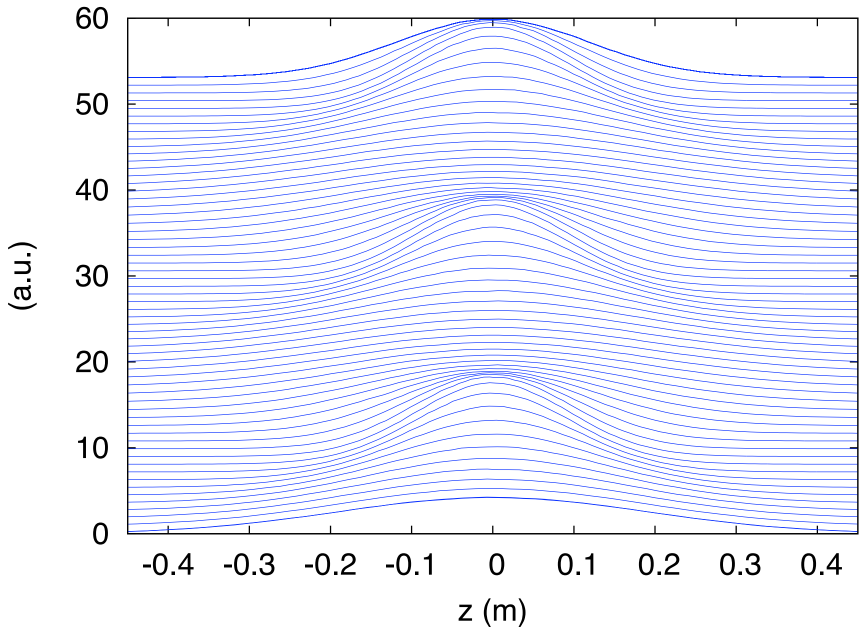


C. Head-tail modes seen by a Wall Current Monitor

1. (m=0 and l=1)



2. (m=0 and l=2)

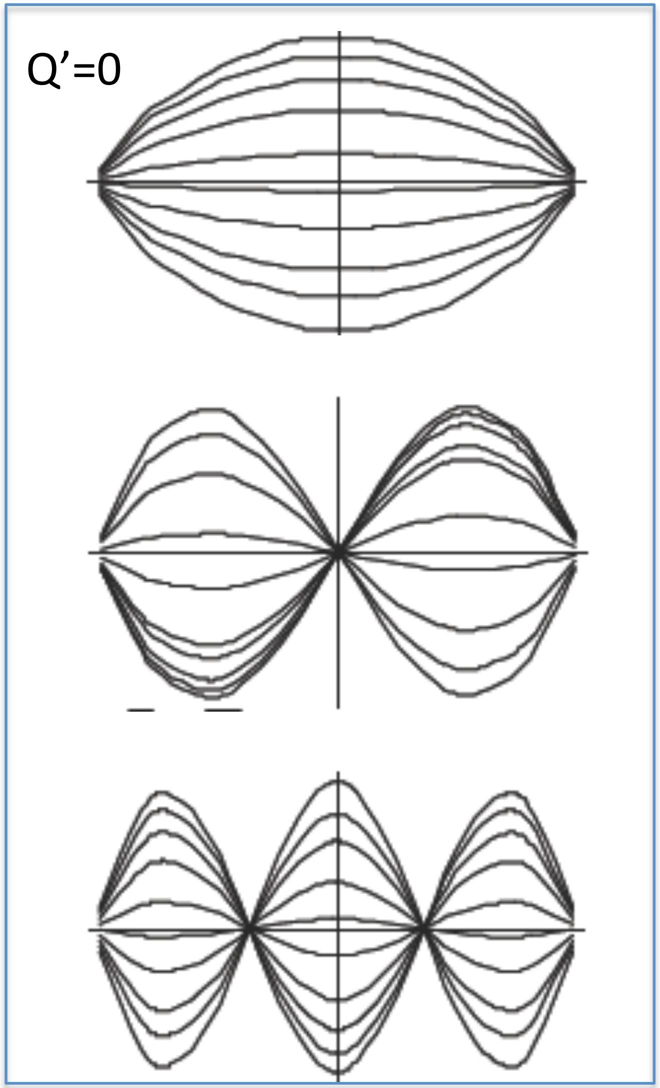


These are mountain range plots showing the evolution in time of the bunch shape

1. The bunch is executing a rigid dipole oscillation in the longitudinal phase space at the synchrotron frequency
2. The bunch is mismatched in the bucket and it executes a quadrupole oscillation in the longitudinal phase space at twice the synchrotron frequency

C. Head-tail modes seen at a wide-band pick-up (BPM)

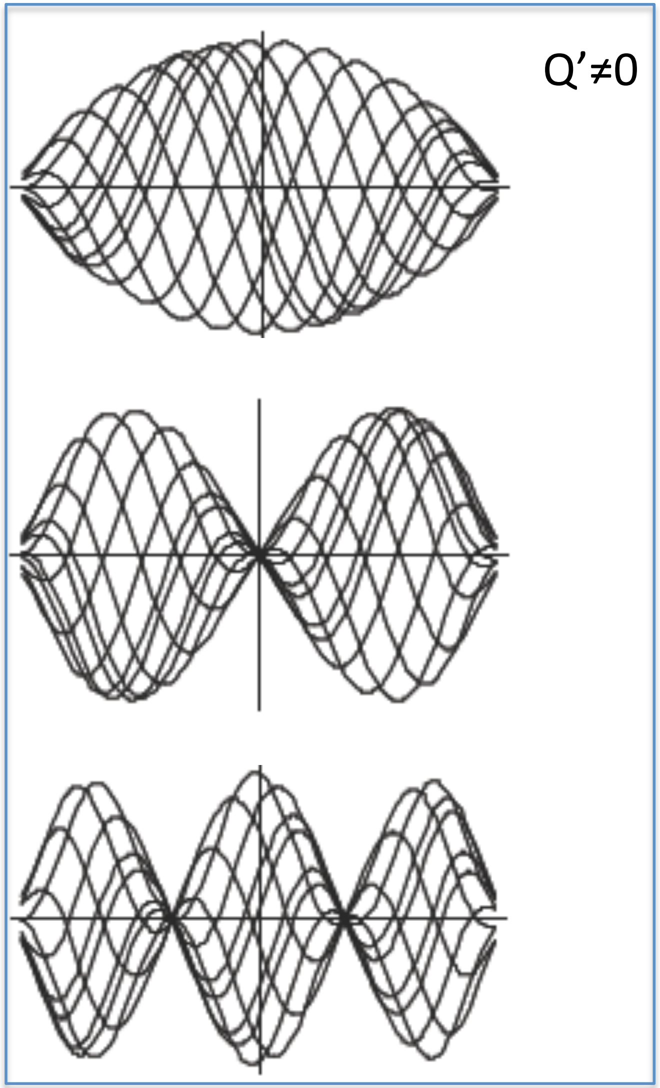
- The patterns of the head-tail modes depend on chromaticity



← *m=1* and *l=0* →

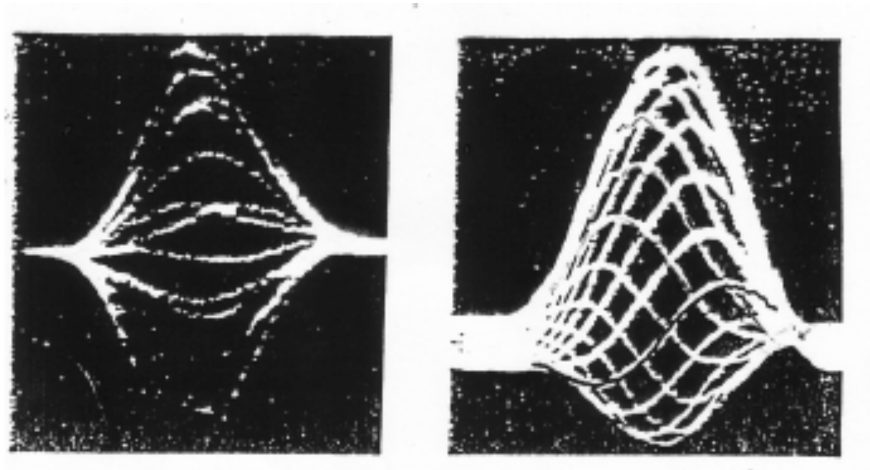
← *m=1* and *l=1* →

← *m=1* and *l=2* →

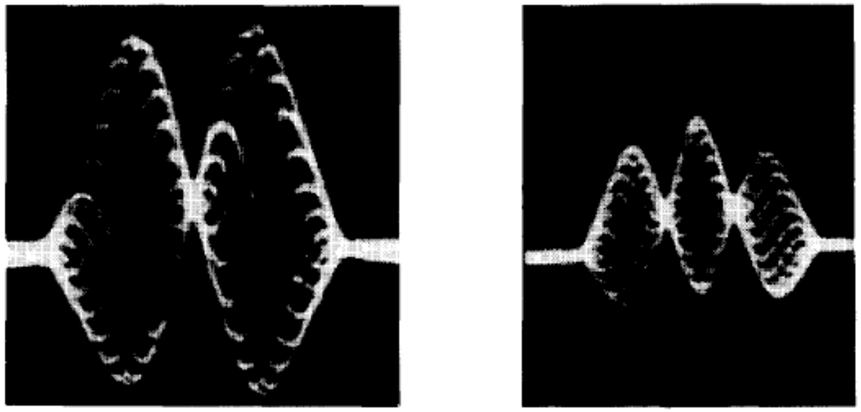


C. Head-tail modes seen at a wide-band pick-up (BPM)

m=1 and *l=0*



$Q' = 0$ $Q' > 0$



$\ell = 1$ $\ell = 2$

Figure 6.32. Transverse beam oscillation modes observed at the CERN PS Booster. The head-tail phase χ is properly defined for the observed bunch shape. (Courtesy Jacques Gareyte, 1992).

C. Head-tail modes

- All the head-tail modes are affected by the interaction of the bunch with an external impedance and therefore oscillate at frequencies dependent on the bunch intensity

→ At very low intensity, where the influence of the impedance is small, these frequencies are multiples of the synchrotron frequency (m=0), or they cluster around the betatron frequency spaced by multiples of the synchrotron frequency (m=1)

✓ $m\omega_{\beta}+l\omega_{s}$

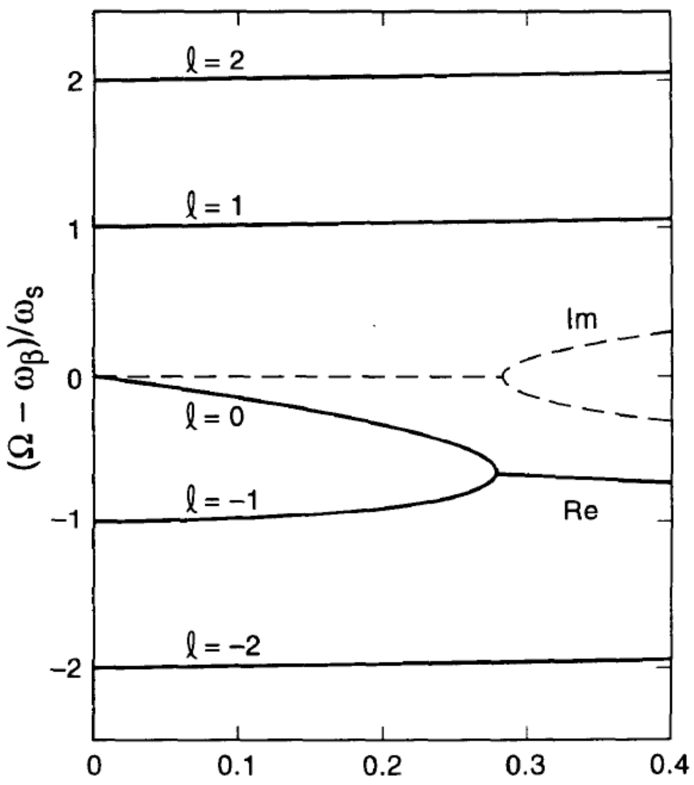
→ At „reasonably" low intensities, the transverse modes (m=1) are:

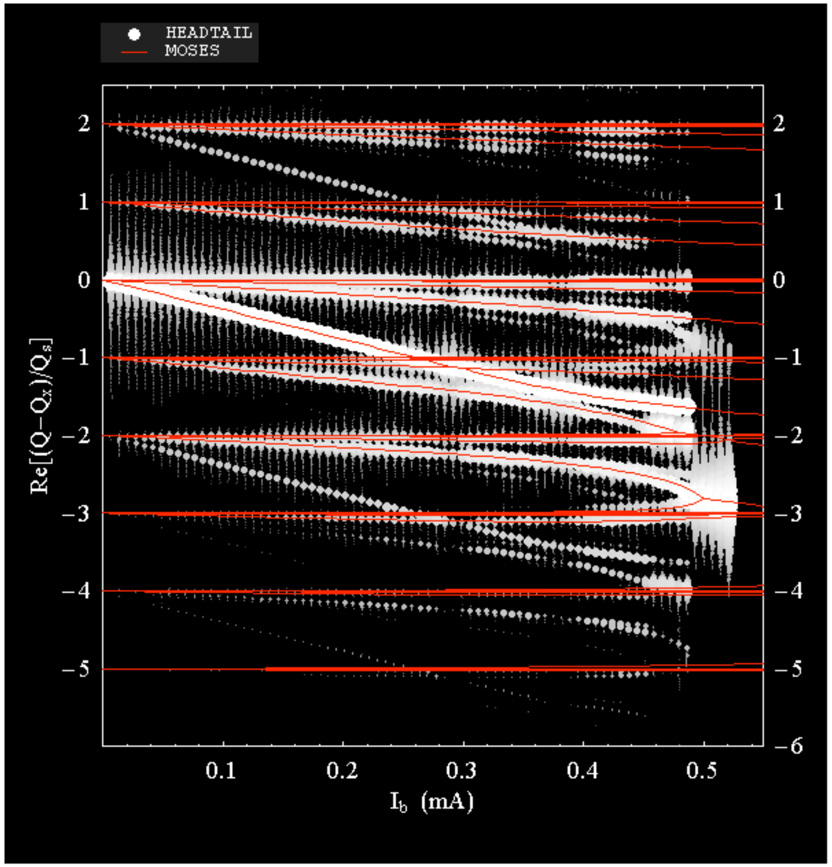
- Zero chromaticity (ξ=0): all stable (neither growing nor damped)
- Positive chromaticity (ξ>0):
  - ⇒ l=0 unstable and |l|≥1 stable below transition (η<0)
  - ⇒ l=0 stable and |l|≥1 unstable above transition (η>0)
- Negative chromaticity (ξ<0):
  - ⇒ l=0 stable and |l|≥1 unstable below transition (η<0)
  - ⇒ l=0 unstable and |l|≥1 stable above transition (η>0)

→ As intensity increases, modes *(m=1, l)* shift due to the impedance, and there is an intensity threshold at which two of them (l≤0) couple and become unstable (**Transverse Mode Coupling Instability**)

C. Head-tail modes

- Typical pictures of modes shifting and coupling with intensity (m=1)
  - ⇒ Left picture: text book case, short bunch (calculation from Vlasov equation)
  - ⇒ Right picture: SPS case, long bunch (calculation + macroparticle simulation)



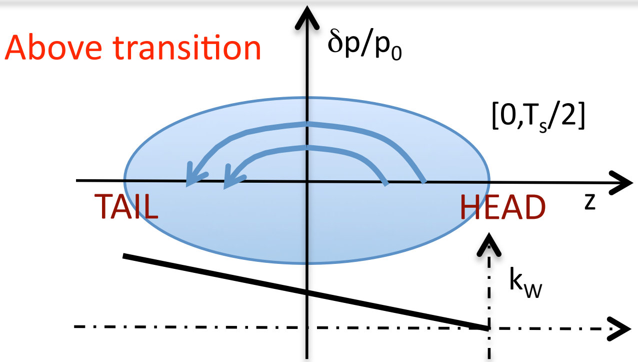


C. Head-tail modes

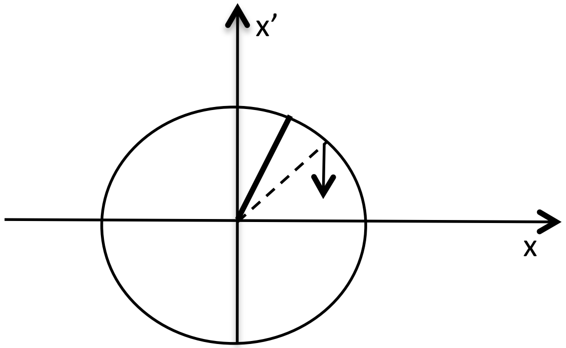
- The stability criterion of the mode *m=1, l=0* can be understood qualitatively

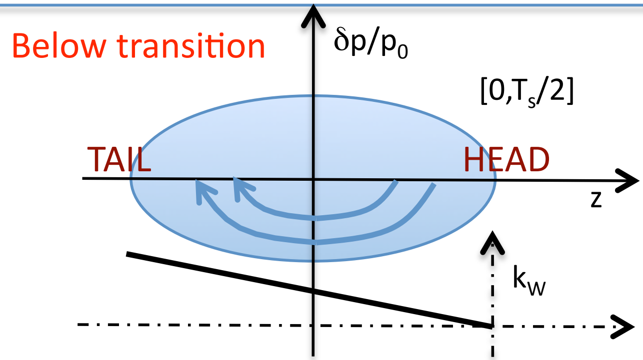
→ Above transition particles rotate counter-clockwise in longitudinal phase space and, with negative chromaticity, they lose betatron phase going from head to tail ($\Delta p/p_0>0$ and $Q'<0$, thus $\Delta Q<0$)

→ The opposite happens below transition

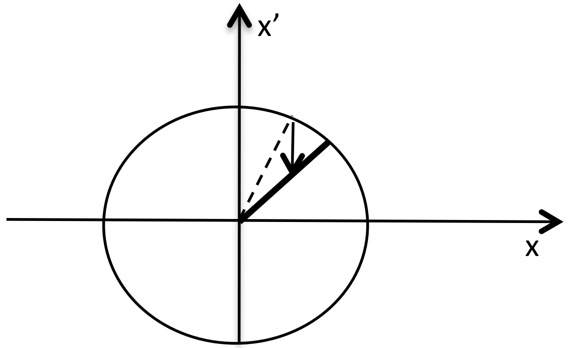


If Q'<0, particles from head to tail accumulate betatron phase delay. The further defocusing kick from the wake makes the beam unstable over several synchrotron periods -> UNSTABLE





If Q'<0, particles from head to tail accumulate extra betatron phase advance. The defocusing kick from the wake partially compensates over several synchrotron periods -> STABLE



C. Head-tail modes

- For intensities well below the mode coupling threshold, the **complex tune shift** of the m=1 modes is given by an analytical formula

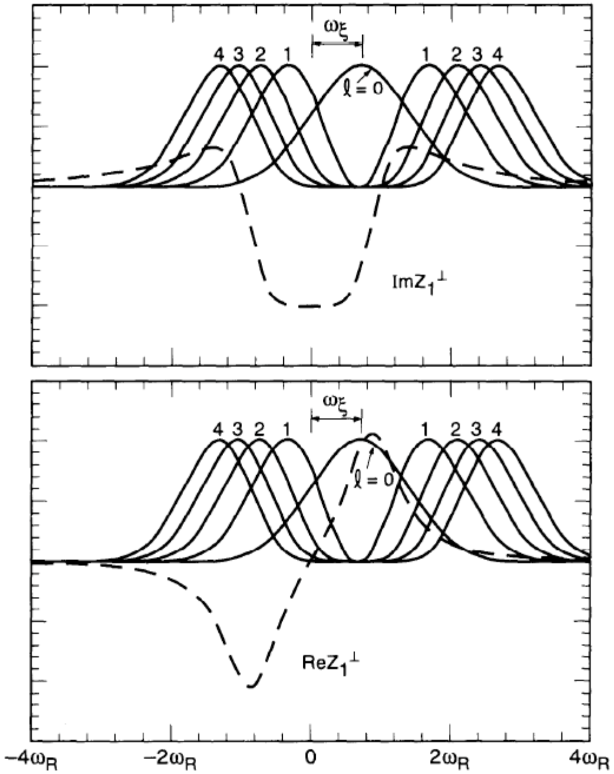
  → Valid for bunches with longitudinal Gaussian distribution

  → Real part for l=0 gives the coherent tune shift and imaginary part gives growth/damping rates of the different modes

$$\Omega^{(l)} - \omega_\beta - l\omega_s \approx -\frac{i}{4\pi}\frac{\Gamma(l+\frac{1}{2})}{2^l l!}\frac{Ne^2\bar{\beta}_{x,y}}{m_0\gamma C\sigma_z}\frac{\sum_{p=-\infty}^{\infty} Z_1^\perp(\omega')h_l(\omega'-\omega_\xi)}{\sum_{p=-\infty}^{\infty} h_l(\omega'-\omega_\xi)}$$

$$\omega' = p\omega_0 + \omega_{\beta x,y} + l\omega_s$$

$$\omega_\xi = \frac{\xi_{x,y}\omega_{\beta x,y}}{\eta}$$



Spectra of $h_l(\omega-\omega_\xi)$ and real and imaginary part of a broad-band impedance