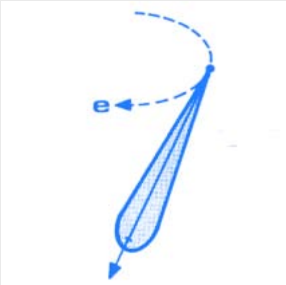


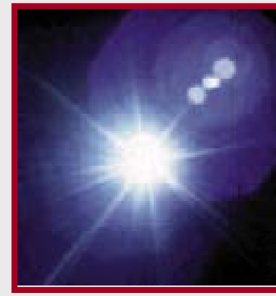

# SYNCHROTRON RADIATION AND ELECTRON DYNAMICS

*Lenny Rivkin*

*Ecole Polythechnique Federale de Lausanne (EPFL)*

*and Paul Scherrer Institute (PSI), Switzerland*

***CERN Accelerator School: Introduction to Accelerator Physics***

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# Radiation effects in electron storage rings

## Average radiated power restored by RF

$$U_0 \cong 10^{-3} \text{ of } E_0$$

- Electron loses energy each turn
- RF cavities provide voltage to accelerate electrons back to the nominal energy

$$V_{RF} > U_0$$

## Radiation damping

- Average rate of energy loss produces DAMPING of electron oscillations in all three degrees of freedom (if properly arranged!)

## Quantum fluctuations

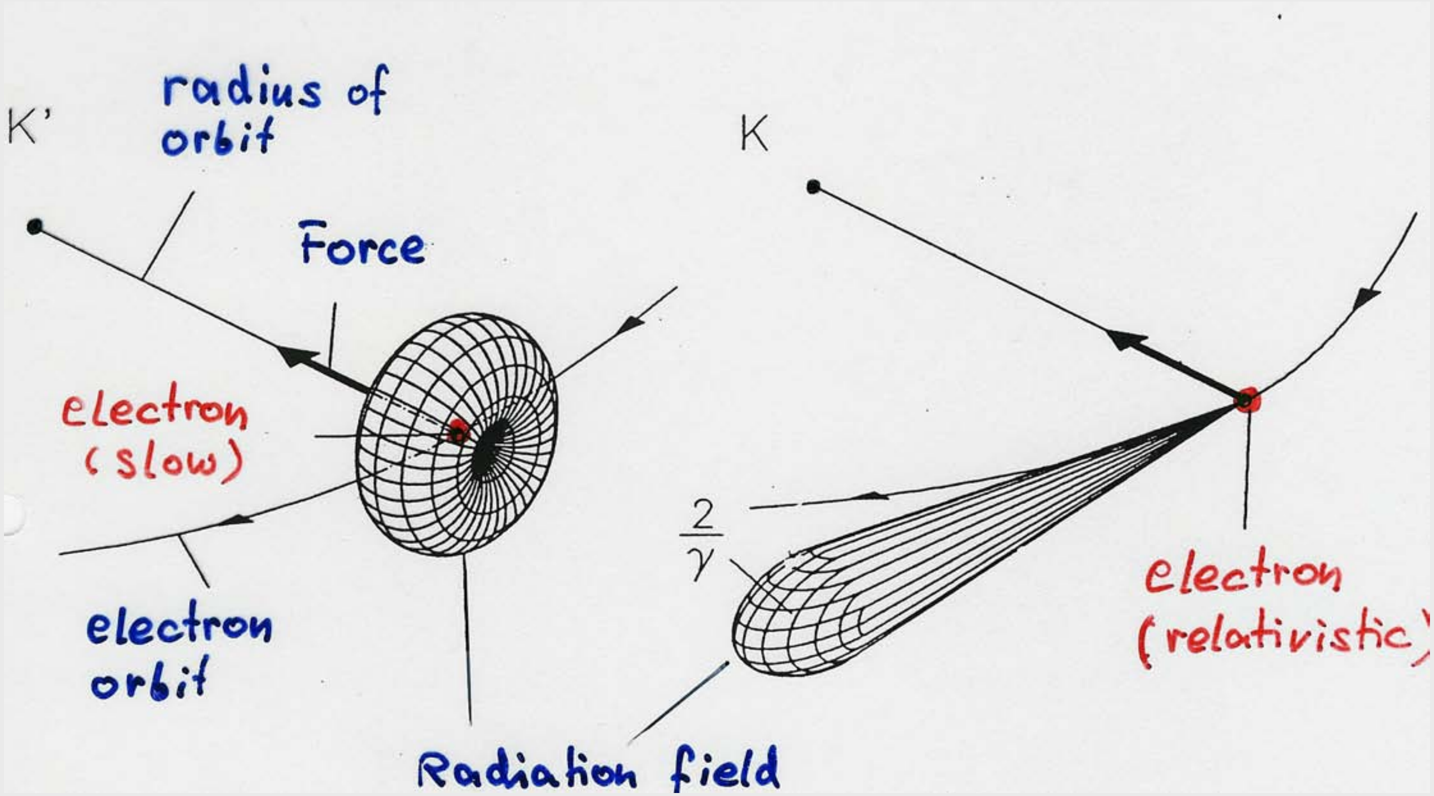
- Statistical fluctuations in energy loss (from quantised emission of radiation) produce RANDOM EXCITATION of these oscillations

## Equilibrium distributions

- The balance between the damping and the excitation of the electron oscillations determines the equilibrium distribution of particles in the beam

# Radiation is emitted into a narrow cone

$$\boxed{\theta = \frac{1}{\gamma} \cdot \theta_e}$$




**v << c**

**v ≈ c**

# Synchrotron radiation power

Power emitted is proportional to:

$$P \propto E^2 B^2$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3} \alpha \hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197 \text{ Mev} \cdot \text{fm}$$

Energy loss per turn:

$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

$$U_0 = \frac{4\pi}{3} \alpha \hbar c \frac{\gamma^4}{\rho}$$

# RADIATION DAMPING

# TRANSVERSE OSCILLATIONS

# Average energy loss and gain per turn

- Every turn electron radiates small amount of energy

$$E_1 = E_0 - U_0 = E_0\left(1 - \frac{U_0}{E_0}\right)$$

- only the amplitude of the momentum changes

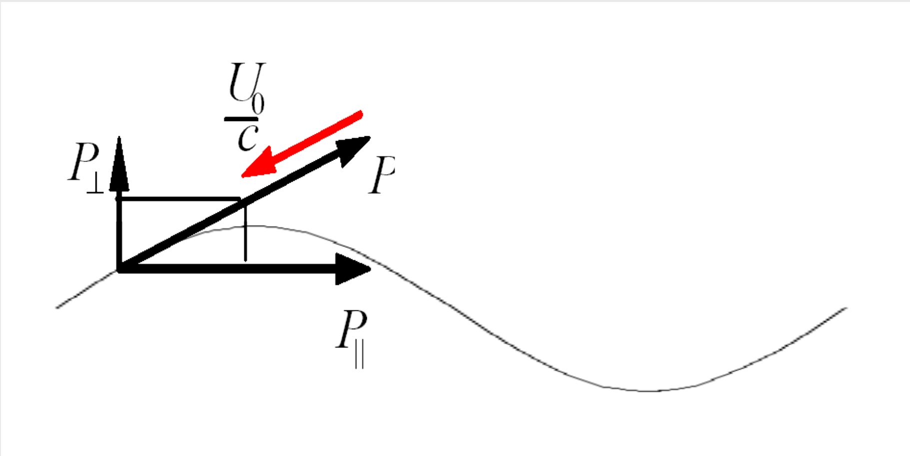
$$P_1 = P_0 - \frac{U_0}{c} = P_0\left(1 - \frac{U_0}{E_0}\right)$$

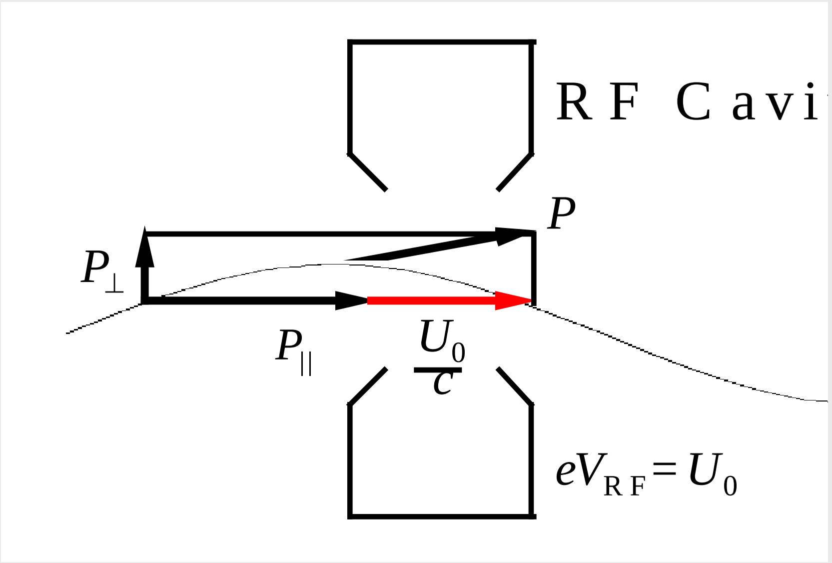
- Only the longitudinal component of the momentum is increased in the RF cavity

- Energy of betatron oscillation

$$\boxed{E_\beta \propto A^2}$$

$$A_1^2 = A_0^2\left(1 - \frac{U_0}{E_0}\right) \quad \text{or} \quad A_1 \cong A_0\left(1 - \frac{U_0}{2E_0}\right)$$



# Damping of vertical oscillations

- But this is just the exponential decay law!

$$\frac{\Delta A}{A} = -\frac{U_0}{2E}$$

$$\boxed{A = A_0 \cdot e^{-t/\tau}}$$

- The oscillations are exponentially **damped** with the **damping time (milliseconds!)**

$$\tau = \frac{2E\,T_0}{U_0}$$

the time it would take particle to 'lose all of its energy'

- In terms of radiation power

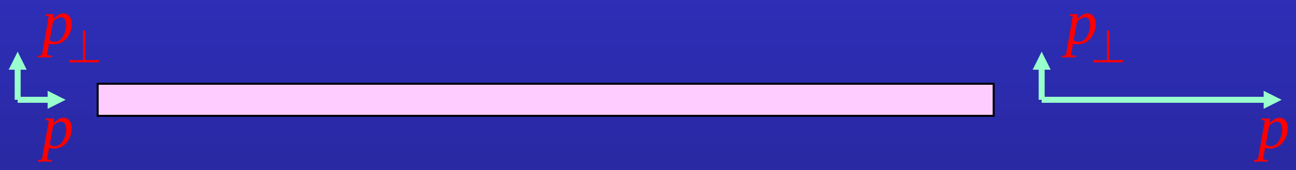
$$\tau = \frac{2E}{P_\gamma}$$ and since $$P_\gamma \propto E^4$$

$$\boxed{\tau \propto \frac{1}{E^3}}$$

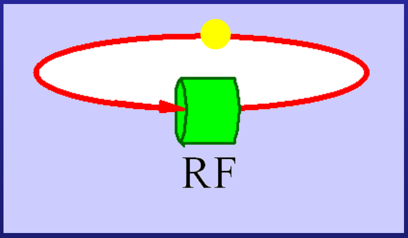
# Adiabatic damping in linear accelerators

In a **linear accelerator**:

$$x' = \frac{p_\perp}{p} \quad \text{decreases} \quad \propto \frac{1}{E}$$




In a **storage ring** beam passes many times through same RF cavity

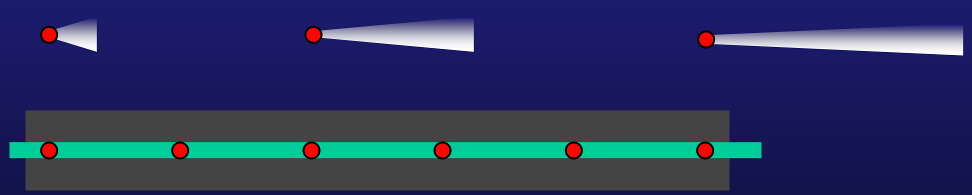



- **Clean loss of energy every turn (no change in x')**
- **Every turn is re-accelerated by RF (x' is reduced)**
- **Particle energy on average remains constant**

# Emittance damping in linacs:

$\varepsilon \qquad \frac{\varepsilon}{2} \qquad \frac{\varepsilon}{4}$

$\Omega \qquad \frac{\Omega}{2} \qquad \frac{\Omega}{4}$



$\gamma \qquad 2\gamma \qquad 4\gamma$

$$\varepsilon \propto \frac{1}{\gamma}$$

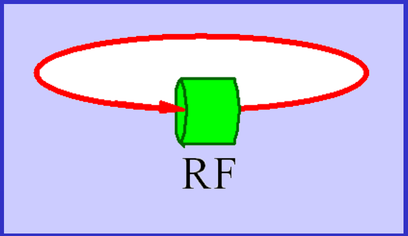
or

$$\gamma\varepsilon = \text{const.}$$

# RADIATION DAMPING

# LONGITUDINAL OSCILLATIONS

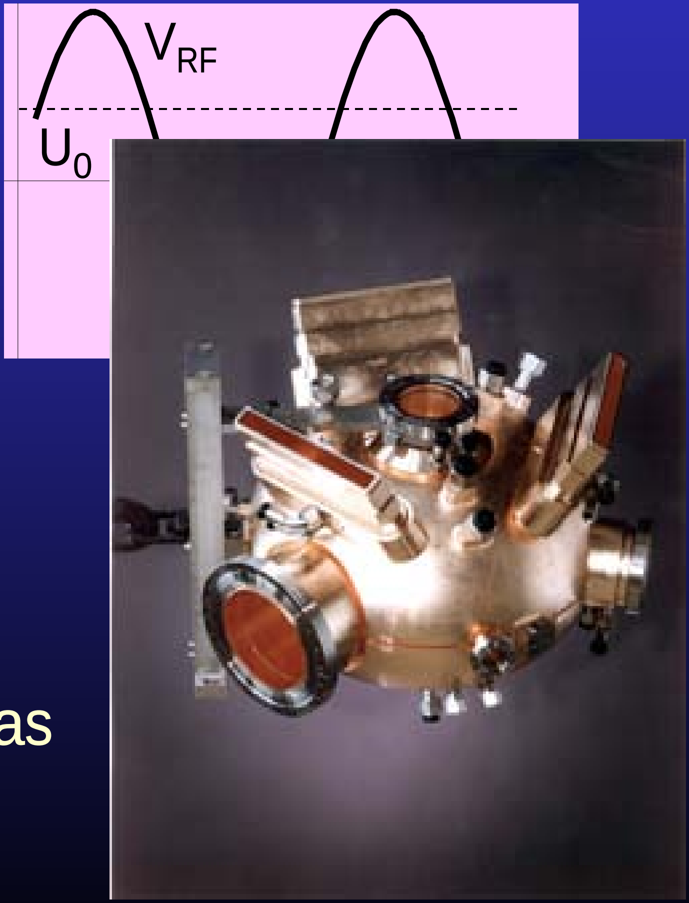
# Longitudinal motion: compensating radiation loss $U_0$




- RF cavity provides accelerating field with frequency

$$f_{RF} = h \cdot f_0$$

  - h – harmonic number

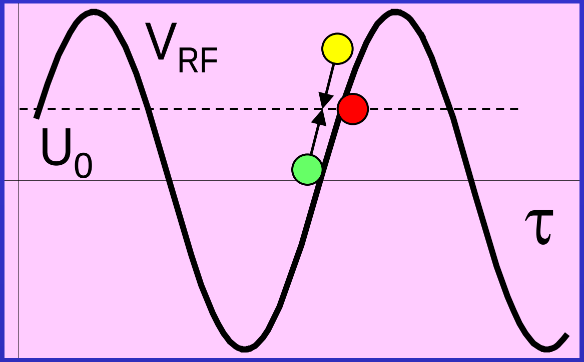



- The energy gain:

$$U_{RF} = eV_{RF}(\tau)$$

- Synchronous particle:
  - has design energy
  - gains from the RF on the average as much as it loses per turn $U_0$

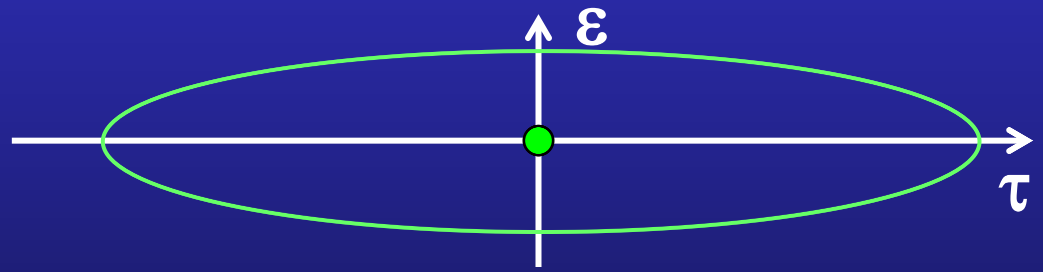
# Longitudinal motion: phase stability




- Particle ahead of synchronous one
  - gets too much energy from the RF
  - goes on a longer orbit (not enough B)
    >> takes longer to go around
  - comes back to the RF cavity closer to synchronous part.
- Particle behind the synchronous one
  - gets too little energy from the RF
  - goes on a shorter orbit (too much B)
  - catches-up with the synchronous particle

# Longitudinal motion: energy-time oscillations

energy deviation from the design energy, or the energy of the synchronous particle



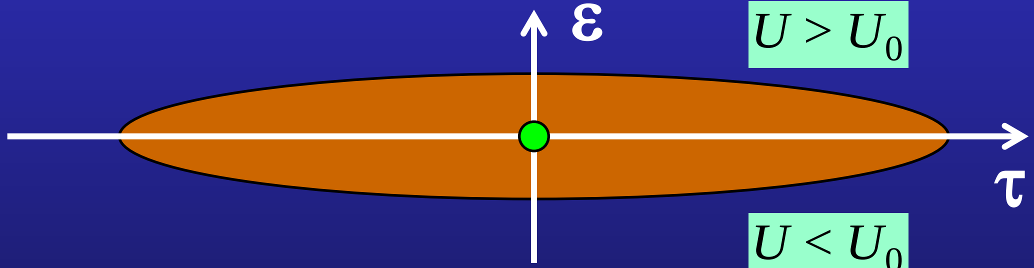

longitudinal coordinate measured from the position of the synchronous electron

# Longitudinal motion: damping of synchrotron oscillations

$$P_\gamma \propto \boldsymbol{E}^2 B^2$$

During one period of synchrotron oscillation:

- when the particle is in the upper half-plane, it loses more energy per turn, its energy gradually reduces




$U > U_0$

$U < U_0$

- when the particle is in the lower half-plane, it loses less energy per turn, but receives $U_0$ on the average, so its energy deviation gradually reduces

The synchrotron motion is damped

- the phase space trajectory is spiraling towards the origin

# Robinson theorem: Damping partition numbers

- Transverse betatron oscillations are damped with

$$\boxed{\tau_x = \tau_z = \frac{2ET_0}{U_0}}$$

- Synchrotron oscillations are damped twice as fast

$$\boxed{\tau_\varepsilon = \frac{ET_0}{U_0}}$$

- The total amount of damping (Robinson theorem) depends only on energy and loss per turn

$$\boxed{\frac{1}{\tau_x} + \frac{1}{\tau_y} + \frac{1}{\tau_\varepsilon} = \frac{2U_0}{ET_0} = \frac{U_0}{2ET_0}\left(J_x + J_y + J_\varepsilon\right)}$$

the sum of the partition numbers $\boxed{J_x + J_z + J_\varepsilon = 4}$

# Radiation loss

$$P_\gamma \propto E^2 B^2$$

Displaced off the design orbit particle sees fields that are different from design values

- **energy deviation ε**
  - different energy: $P_\gamma \propto E^2$
  - different magnetic field **B**
    particle moves on a different orbit, defined by the **off-energy** or **dispersion** function $D_x$

**both contribute to linear term in** $P_\gamma(\varepsilon)$

- **betatron oscillations**: zero on *average*

# Radiation loss

$$\mathbf{P_\gamma \propto E^2 B^2}$$

To first order in $\varepsilon$

$$\mathbf{U_{rad} = U_0 + U' \cdot \varepsilon}$$

*electron energy changes slowly, at any instant it is moving on an orbit defined by* $\mathbf{D_x}$

$$\mathbf{U' \equiv \frac{dU_{rad}}{dE}\bigg|_{E_0}}$$

*after some algebra one can write*

$$U' = \frac{U_0}{E_0}\left(2 + \mathcal{D}\right)$$

$$\mathcal{D} \neq 0 \quad \textit{only when} \quad \frac{k}{\rho} \neq 0$$

# Damping partition numbers

$$\boxed{J_x + J_z + J_\varepsilon = 4}$$

- Typically we build rings with no vertical dispersion

$$\boxed{J_z = 1} \qquad \boxed{J_x + J_\varepsilon = 3}$$

- Horizontal and energy partition numbers can be modified via $\mathcal{D}$ :

$$\boxed{J_x = 1 - \mathcal{D}} \qquad \boxed{J_\varepsilon = 2 + \mathcal{D}}$$

  - Use of combined function magnets
  - Shift the equilibrium orbit in quads with RF frequency

# EQUILIBRIUM BEAM SIZES

# Quantum nature of synchrotron radiation

## Damping only

- If damping was the whole story, the beam emittance (size) would shrink to microscopic dimensions!
- Lots of problems! (e.g. coherent radiation)

## Quantum fluctuations

- Because the radiation is emitted in quanta, radiation itself takes care of the problem!
- It is sufficient to use quasi-classical picture:
  - » *Emission time is very short*
  - » *Emission times are statistically independent (each emission - only a small change in electron energy)*

**Purely stochastic (Poisson) process**

# Visible quantum effects

*I have always been somewhat amazed that a purely quantum effect can have gross macroscopic effects in large machines;*

*and, even more,*

*that Planck's constant has just the right magnitude needed to* <u>*make practical*</u> *the construction of large electron storage rings.*

*A significantly larger or smaller value of* $\hbar$

*would have posed serious -- perhaps insurmountable -- problems for the realization of large rings.*

*Mathew Sands*

# Quantum excitation of energy oscillations

Photons are emitted with typical energy $u_{ph} \approx \hbar\omega_{typ} = \hbar c \dfrac{\gamma^3}{\rho}$

at the rate (photons/second) $\mathcal{N} = \dfrac{P_\gamma}{u_{ph}}$

**Fluctuations in this rate excite oscillations**

During a small interval $\Delta t$ electron emits photons $N = \mathcal{N} \cdot \Delta t$

losing energy of $N \cdot u_{ph}$

Actually, because of fluctuations, the number is $N \pm \sqrt{N}$

resulting in **spread in energy loss** $\pm\sqrt{N} \cdot u_{ph}$

For large time intervals RF compensates the energy loss, providing damping towards the design energy $E_0$

**Steady state**: typical deviations from $E_0$
≈ typical fluctuations in energy during a damping time $\tau_\varepsilon$

# Equilibrium energy spread: rough estimate

We then expect the rms energy spread to be $\sigma_\varepsilon \approx \sqrt{N \cdot \tau_\varepsilon} \cdot u_{ph}$

and since $\boxed{\tau_\varepsilon \approx \frac{E_0}{P_\gamma}}$ and $P_\gamma = N \cdot u_{ph}$

$\sigma_\varepsilon \approx \sqrt{E_0 \cdot u_{ph}}$ geometric mean of the electron and photon energies!

Relative energy spread can be written then as:

$$\frac{\sigma_\varepsilon}{E_0} \approx \gamma \sqrt{\frac{\lambda\!\!\!^{-}_e}{\rho}}$$

$$\boxed{\lambda\!\!\!^{-}_e = \frac{\hbar}{m_e c} \simeq 4 \cdot 10^{-13} m}$$

it is roughly constant for all rings

- typically $\rho \propto E^2$

$$\boxed{\frac{\sigma_\varepsilon}{E_0} \sim const \sim 10^{-3}}$$

# Equilibrium energy spread

More detailed calculations give

- for the case of an 'isomagnetic' lattice $\rho(s) = \begin{cases} \rho_0 & \text{in dipoles} \\ \infty & \text{elsewhere} \end{cases}$

$$\boxed{\left(\frac{\sigma_\varepsilon}{E}\right)^2 = \frac{C_q E^2}{J_\varepsilon \rho_0}}$$

with $$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{(m_e c^2)^3} = 1.468 \cdot 10^{-6} \left[\frac{\mathrm{m}}{\mathrm{GeV}^2}\right]$$

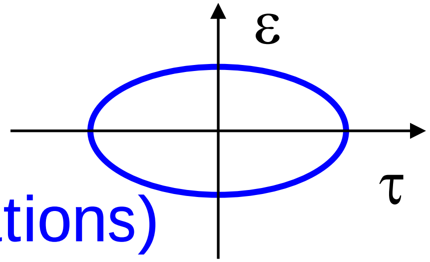
It is difficult to obtain energy spread < 0.1%

- limit on undulator brightness!

# Equilibrium bunch length

Bunch length is related to the energy spread

- Energy deviation and time of arrival (or position along the bunch) are conjugate variables (synchrotron oscillations)




$$\hat{\tau} = \frac{\alpha}{\Omega_s}\left(\frac{\hat{\varepsilon}}{E}\right)$$

$$\boxed{\sigma_\tau = \frac{\alpha}{\Omega_s}\left(\frac{\sigma_\varepsilon}{E}\right)}$$

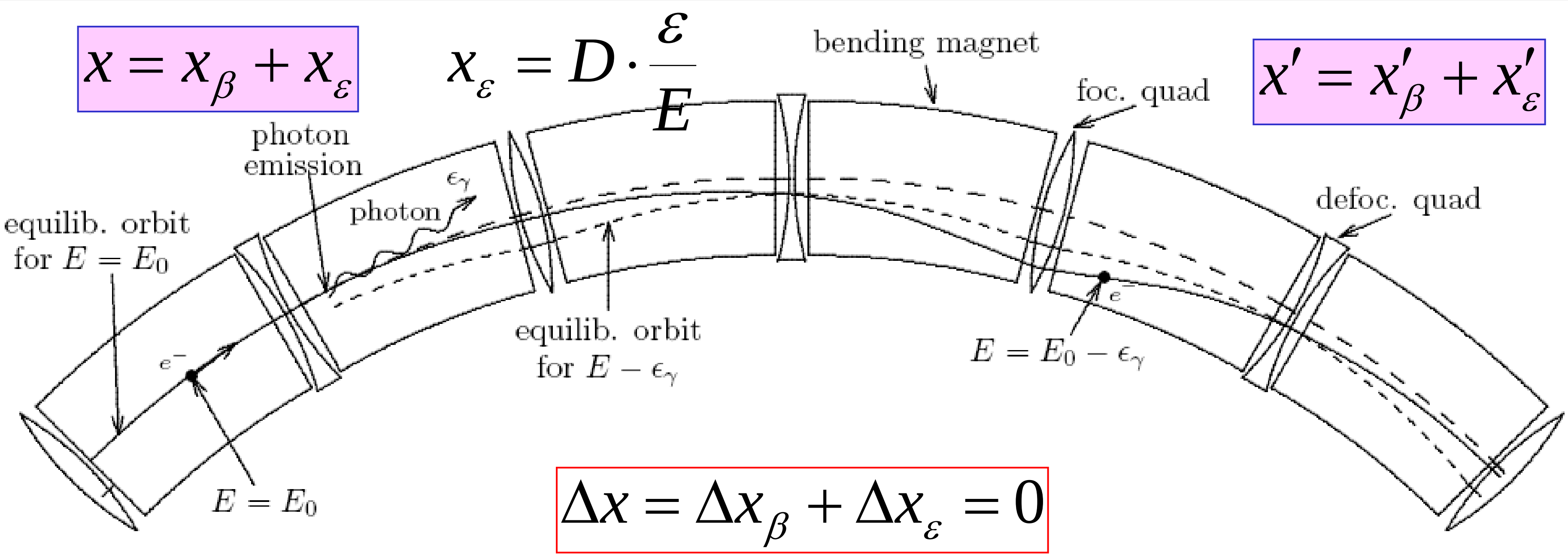
- recall that $\boxed{\Omega_s \propto \sqrt{V_{RF}}}$

Two ways to obtain short bunches:

- RF voltage (power!) $\boxed{\sigma_\tau \propto 1/\sqrt{V_{RF}}}$
- Momentum compaction factor in the limit of $\alpha = 0$
  isochronous ring: particle position along the bunch is frozen

$$\boxed{\sigma_\tau \propto \alpha}$$

# Excitation of betatron oscillations




$$x = x_\beta + x_\varepsilon$$

$$x_\varepsilon = D \cdot \frac{\varepsilon}{E}$$

$$x' = x'_\beta + x'_\varepsilon$$

$$\Delta x = \Delta x_\beta + \Delta x_\varepsilon = 0$$

$$\Delta x_\beta = -D \cdot \frac{\varepsilon_\gamma}{E}$$

Courant Snyder invariant

$$\Delta x'_\beta = -D' \cdot \frac{\varepsilon_\gamma}{E}$$

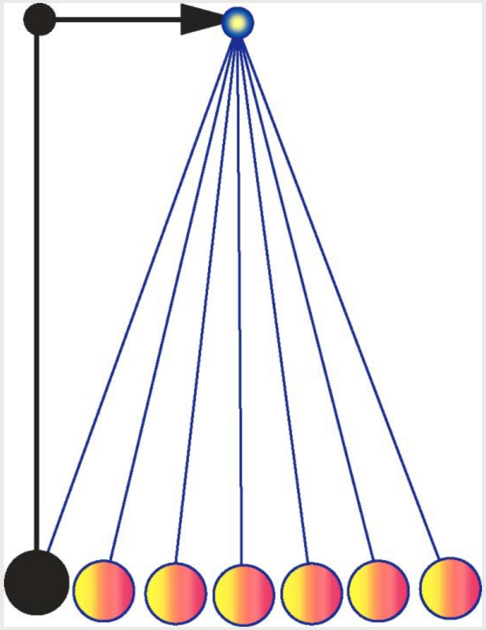
$$\Delta \varepsilon = \gamma \Delta x_\beta^2 + 2\alpha \Delta x_\beta \Delta x'_\beta + \beta \Delta x'^2_\beta = \left[\gamma D^2 + 2\alpha D D' + \beta D'^2\right] \cdot \left(\frac{\varepsilon_\gamma}{E}\right)^2$$

# Excitation of betatron oscillations

Electron emitting a photon

- at a place with non-zero dispersion
- starts a betatron oscillation around a new reference orbit

$$\boxed{x_\beta \approx D \cdot \frac{\varepsilon_\gamma}{E}}$$



# Horizontal oscillations: equilibrium

Emission of photons is a random process

- Again we have random walk, now in **x**. How far particle will wander away is limited by the radiation damping
- The balance is achieved on the time scale of the damping time $\tau_x = 2\ \tau_\varepsilon$

$$\sigma_{x\beta} \approx \sqrt{\mathcal{N} \cdot \tau_x} \cdot D \cdot \frac{\varepsilon_\gamma}{E} = \sqrt{2} \cdot D \cdot \frac{\sigma_\varepsilon}{E}$$

- Typical horizontal beam size ~ 1 mm

**Quantum effect visible to the naked eye!**

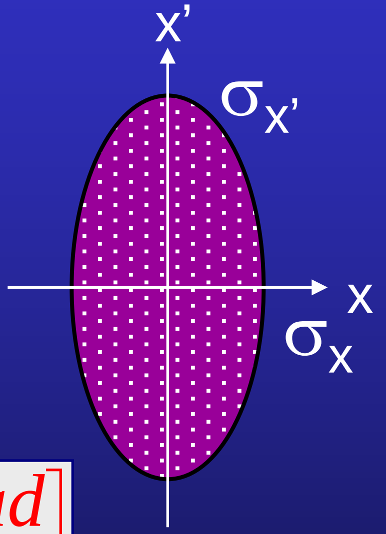
- Vertical size - determined by coupling

# Beam emittance

$$\text{Area} = \pi \cdot \varepsilon$$

## Betatron oscillations

- Particles in the beam execute betatron oscillations with different amplitudes.




## Transverse beam distribution

- Gaussian (electrons)
- "Typical" particle: 1 - $\sigma$ ellipse
  (in a place where $\alpha = \beta' = 0$)

Units of $\varepsilon$ $[m \cdot rad]$

$$\text{Emittance} \equiv \frac{\sigma_x^2}{\beta}$$

$$\varepsilon = \sigma_x \cdot \sigma_{x'}$$

$$\sigma_x = \sqrt{\varepsilon\,\beta}$$

$$\sigma_{x'} = \sqrt{\varepsilon / \beta}$$

$$\beta = \frac{\sigma_x}{\sigma_{x'}}$$

# Equilibrium horizontal emittance

Detailed calculations for isomagnetic lattice

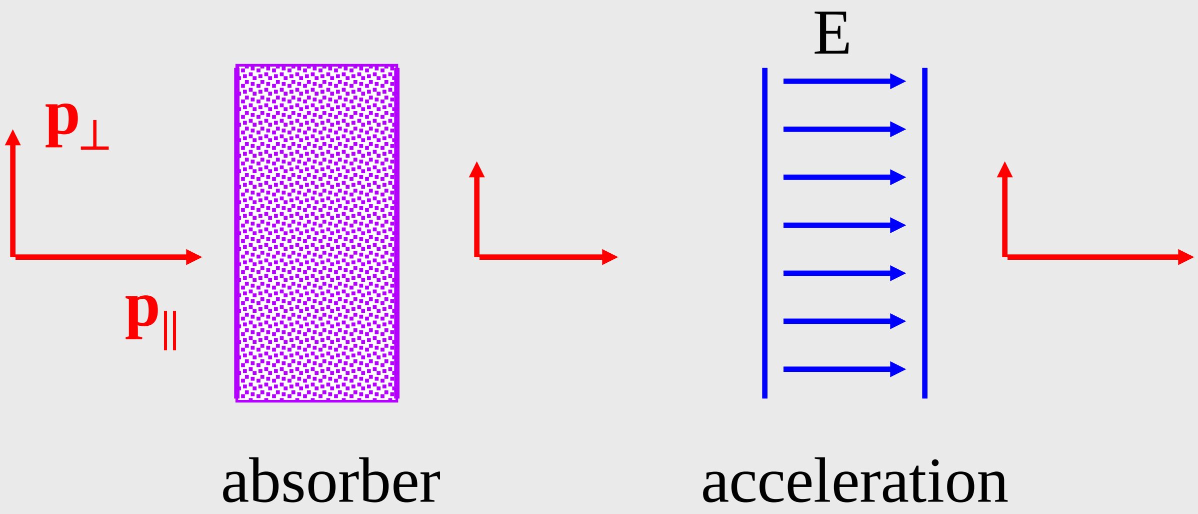
$$\boxed{\varepsilon_{x0} \equiv \frac{\sigma_{x\beta}^2}{\beta} = \frac{C_q E^2}{J_x} \cdot \frac{\langle \mathcal{H} \rangle_{mag}}{\rho}}$$

where

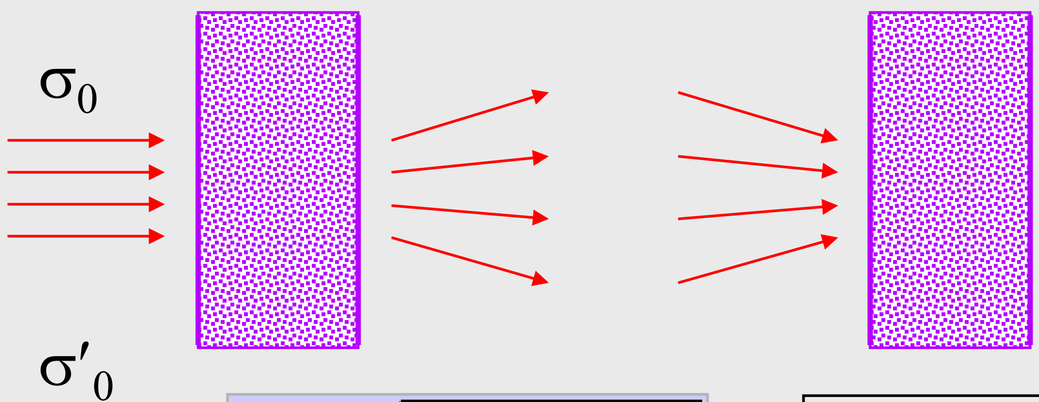
$$\begin{aligned} \mathcal{H} &= \gamma D^2 + 2\alpha D D' + \beta D'^2 \\ &= \frac{1}{\beta}\left[D^2 + (\beta D' + \alpha D)^2\right] \end{aligned}$$

and $\langle \mathcal{H} \rangle_{mag}$ is average value in the bending magnets

# Ionization cooling




similar to radiation damping, but there is multiple scattering in the absorber that blows up the emittance




$$\boxed{\sigma' = \sqrt{\sigma'^2_0 + \sigma'^2_{MS}}}$$

$$\boxed{\sigma'_0 >> \sigma'_{MS}}$$

to minimize the blow up due to multiple scattering in the absorber we can **focus** the beam

# Summary of radiation integrals

Momentum compaction factor

$$\boxed{\alpha = \frac{I_1}{2\pi R}}$$

Energy loss per turn

$$\boxed{U_0 = \frac{1}{2\pi} C_\gamma E^4 \cdot I_2}$$

$$I_1 = \oint \frac{D}{\rho}\, ds$$

$$I_2 = \oint \frac{ds}{\rho^2}$$

$$I_3 = \oint \frac{ds}{|\rho^3|}$$

$$I_4 = \oint \frac{D}{\rho}\left(2k + \frac{1}{\rho^2}\right) ds$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho^3|}\, ds$$

$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{\left(m_e c^2\right)^3} = 8.858 \cdot 10^{-5} \left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

# Summary of radiation integrals (2)

Damping parameter

$$\boxed{\mathcal{D} = \frac{I_4}{I_2}}$$

Damping times, partition numbers

$$\boxed{J_\varepsilon = 2 + \mathcal{D}\,, \quad J_x = 1 - \mathcal{D}\,, \quad J_y = 1}$$

$$\boxed{\tau_i = \frac{\tau_0}{J_i}} \qquad \boxed{\tau_0 = \frac{2ET_0}{U_0}}$$

Equilibrium energy spread

$$\boxed{\left(\frac{\sigma_\varepsilon}{E}\right)^2 = \frac{C_q E^2}{J_\varepsilon} \cdot \frac{I_3}{I_2}}$$

Equilibrium emittance

$$\boxed{\varepsilon_{x0} = \frac{\sigma_{x\beta}^2}{\beta} = \frac{C_q E^2}{J_x} \cdot \frac{I_5}{I_2}}$$

$$I_1 = \oint \frac{D}{\rho}\, ds$$

$$I_2 = \oint \frac{ds}{\rho^2}$$

$$I_3 = \oint \frac{ds}{|\rho^3|}$$

$$I_4 = \oint \frac{D}{\rho}\left(2k + \frac{1}{\rho^2}\right) ds$$

$$I_5 = \oint \frac{\mathcal{H}}{|\rho^3|}\, ds$$

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{(m_e c^2)^3} = 1.468 \cdot 10^{-6} \left[\frac{\mathrm{m}}{\mathrm{GeV}^2}\right]$$

$$\mathcal{H} = \gamma D^2 + 2\alpha D D' + \beta D'^2$$

# Damping wigglers

Increase the radiation loss per turn $U_0$ with WIGGLERS

- reduce damping time

$$\boxed{\tau = \frac{E}{P_\gamma + P_{wig}}}$$

- emittance control

wigglers at high dispersion: blow-up emittance

e.g. storage ring colliders for high energy physics

wigglers at zero dispersion: decrease emittance

e.g. damping rings for linear colliders
e.g. synchrotron light sources (PETRAIII, 1 nm.rad)

END