Port-Hamiltonian Perspective on Electro-Thermal Magnet Models

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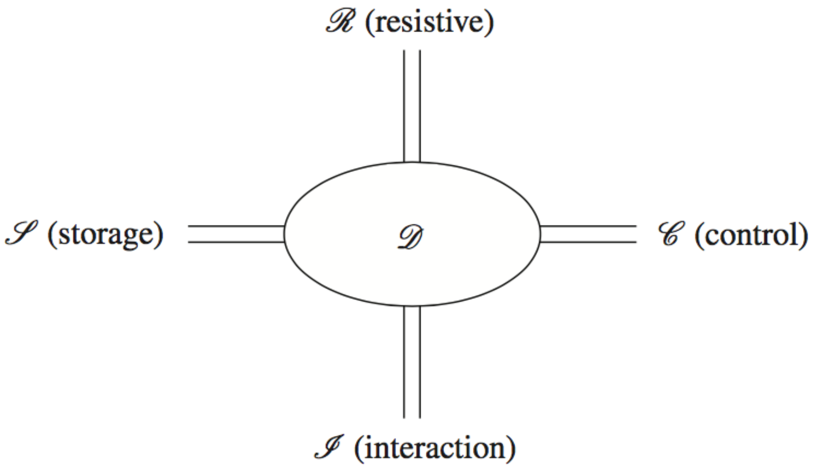
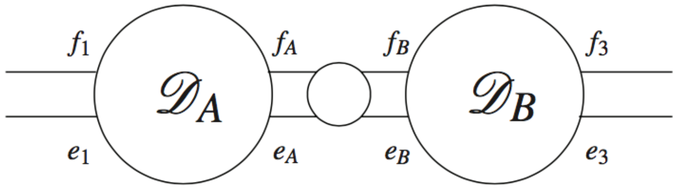
Presentation outline

1. Motivation
2. Electromagnetic Field
3. Field/Circuit Coupling
4. Irreversible Thermodynamics
5. Conclusion

Motivation

- General framework for the multi-physics modelling
- Computational causality
- Analysis of the coupling strength between physical systems
- Distributed and lumped element phenomena

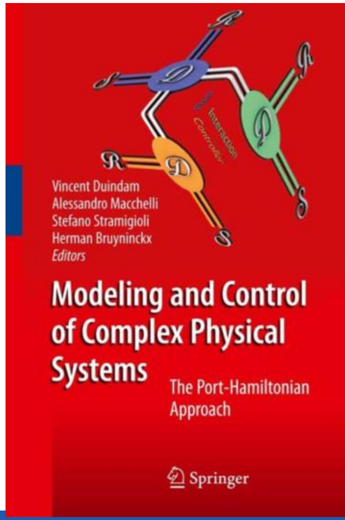




1.12 Future Trends

The following general future trends in bond graphs and port-based modeling can be distinguished:

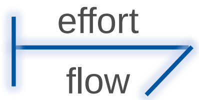
- . . .
- use of the port-based approach for *co-simulation*.



V. Duindam, et al. Modelling and Control of Complex Physical Systems
The Port-Hamiltonian Approach, Springer, 2009.

Modelling with bond graphs

Effort and Flow



effort*flow = Power

Causality assignment

1. Fixed
2. Arbitrary
3. Preferred
4. Restricted

	f flow	e effort	$q = \int f \, \mathrm{d}t$ generalized displacement	$p = \int e \, \mathrm{d}t$ generalized momentum
electro-magnetic	i current	u voltage	$q = \int i \, \mathrm{d}t$ charge	$\lambda = \int u \, \mathrm{d}t$ magnetic flux linkage
mechanical translation	v velocity	F force	$x = \int v \, \mathrm{d}t$ displacement	$p = \int F \, \mathrm{d}t$ momentum
mechanical rotation	ω angular velocity	T torque	$\theta = \int \omega \, \mathrm{d}t$ angular displacement	$b = \int T \, \mathrm{d}t$ angular momentum
hydraulic pneumatic	ϕ volume flow	p pressure	$V = \int \phi \, \mathrm{d}t$ volume	$\Gamma = \int p \, \mathrm{d}t$ momentum of a flow tube
thermal	T temperature	f_S entropy flow	$S = \int f_S \, \mathrm{d}t$ entropy	

Flow and effort

$\mathrm{Sf} \vdash\!\!\rightharpoonup$ $\qquad$ $\mathrm{Se} \rightharpoonup\!\!|$

Resistor

$J \rightharpoonup\!| \; R$ $\qquad$ $J \;|\!\rightharpoonup R$

Inductor and Capacitor

$J \rightharpoonup\!| \; I$ $\qquad$ $J \;|\!\rightharpoonup C$

$$f(t) = \frac{1}{I} \int e(t) \, \mathrm{d}t \qquad e(t) = \frac{1}{C} \int f(t) \, \mathrm{d}t$$

V. Duindam, et al. Modelling and Control of Complex Physical Systems
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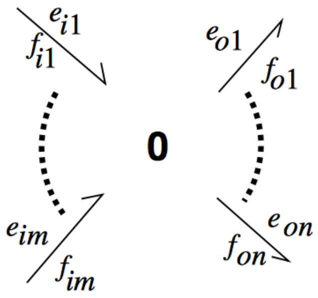
Modelling with bond graphs

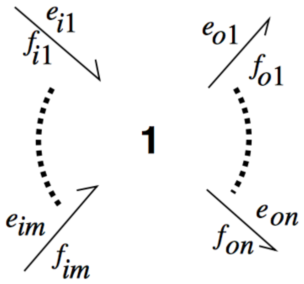
0-junction



$$\sum_{k=1}^{m} f_{ik} = \sum_{k=1}^{n} f_{ok}$$

$$e_{i1} = \cdots = e_{im} = e_{o1} \cdots = e_{on}$$

1-junction



$$\sum_{k=1}^{m} e_{ik} = \sum_{k=1}^{n} e_{ok}$$

$$f_{i1} = \cdots = f_{im} = f_{o1} \cdots = f_{on}$$

Transformer

$$\xrightarrow[f_1]{e_1} \underset{n}{\overset{}{\mathrm{TF}}} \xrightarrow[f_2]{e_2}$$

$$\begin{bmatrix} e_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ n & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ e_2 \end{bmatrix}$$

Gyrator

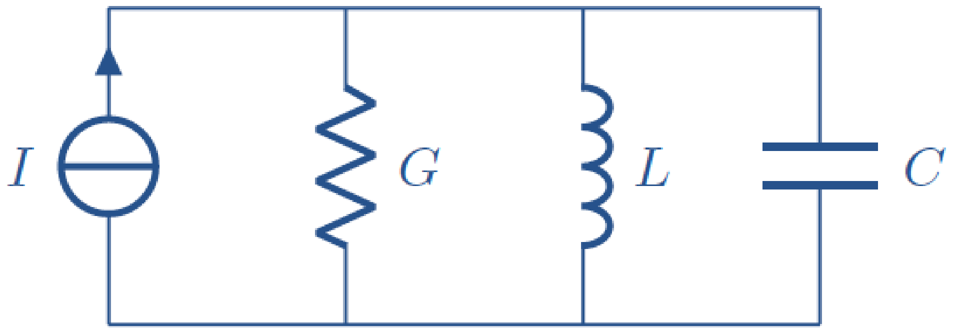
$$\xrightarrow[f_1]{e_1} \underset{n}{\overset{}{\mathrm{TF}}} \xrightarrow[f_2]{e_2}$$

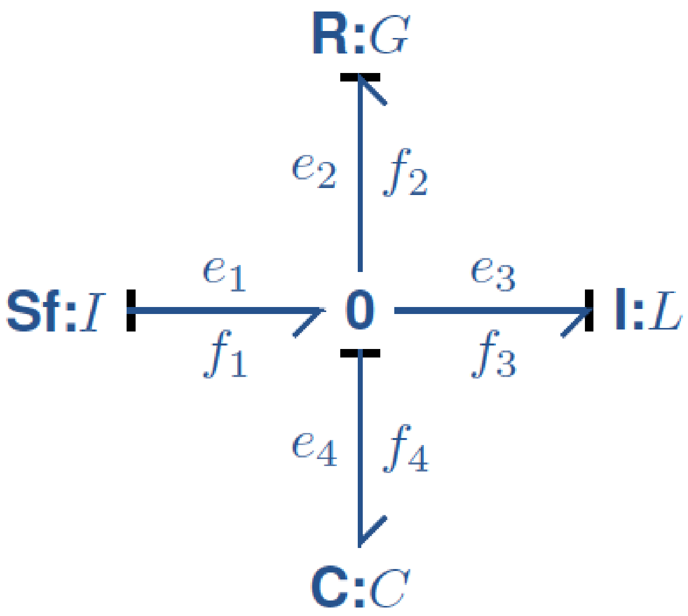
$$\begin{bmatrix} e_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 & n \\ n & 0 \end{bmatrix} \begin{bmatrix} f_1 \\ e_2 \end{bmatrix}$$

Example of a Dirac structure – 0-junction

A Dirac structure is given on a space of flows and efforts such that, the product of elements is equal to 0.

Electrical circuit is composed of a parallel connection of a current source, conductance, inductance, and capacitance is a 0-junction.

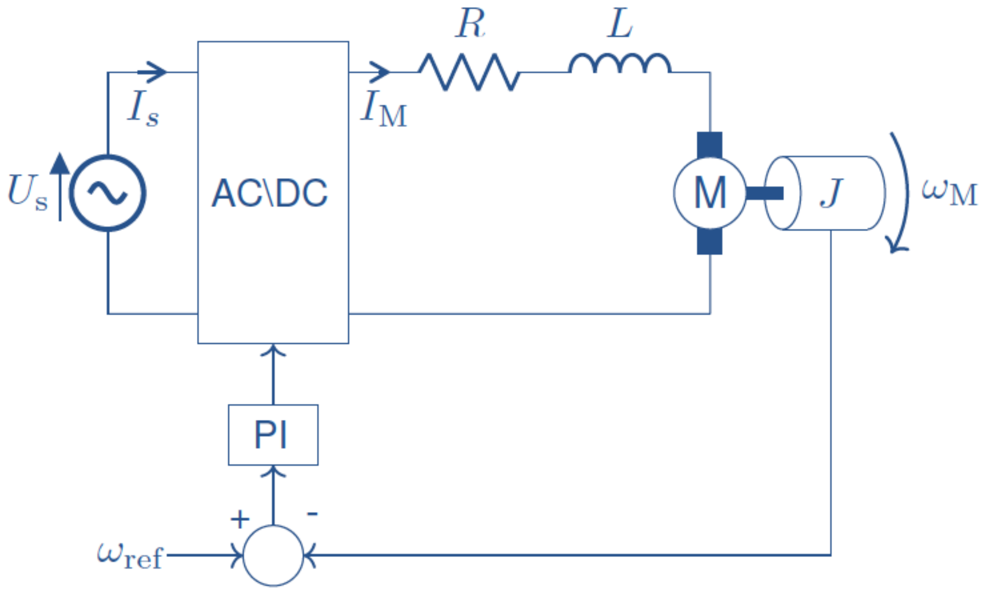


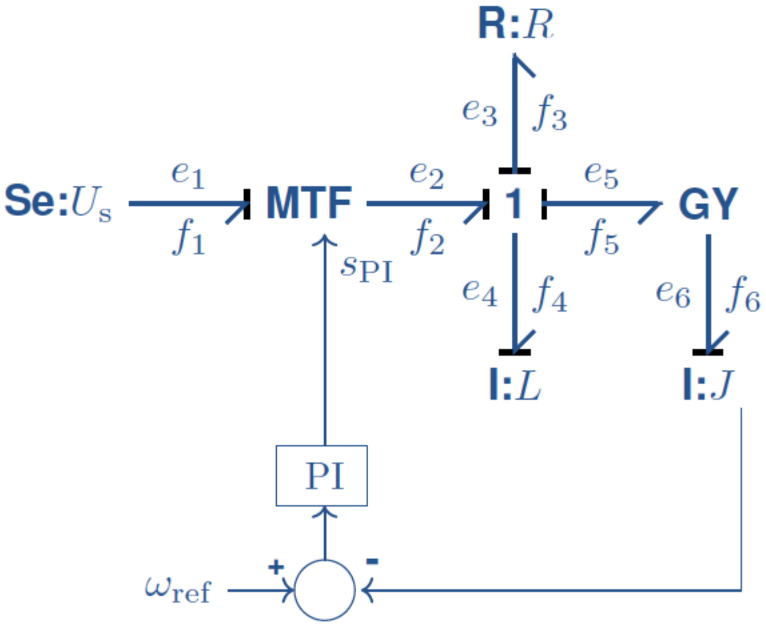


Computational causality indicates that in this case conductance is preferred to represent dissipation.

Interconnection of Dirac structures

Example presents an interconnection of an electrical circuit and a mechanical system.





Interconnection of several Dirac structures is a Dirac structure.

Input-state-output Port Hamiltonian Model

$$\partial_t x = [J(x) - R(x)]\delta_x \mathbb{H}(x) + g(x)u$$
$$y = g^\top(x)\delta_x \mathbb{H}(x) + S(x)u,$$

where $x \in \mathbb{R}^n$ is the state vector,

$J(x) = -J(x)^\top \in \mathbb{R}^n \times \mathbb{R}^n$ is a skew-symmetric interconnection matrix

$R(x) = R(x)^\top \in \mathbb{R}^n \times \mathbb{R}^n$ is a symmetric matrix corresponding to the resistive port.

$g(x) \in \mathbb{R}^n \times \mathbb{R}^m$ is the input vector field

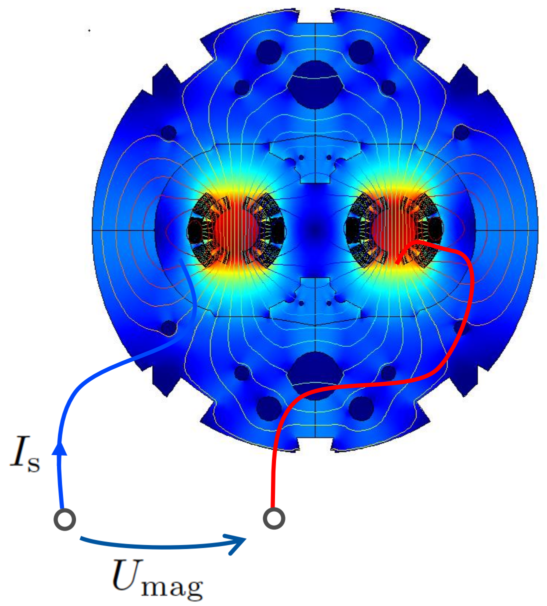
$S(x) \in \mathbb{R}^m \times \mathbb{R}^m$ is a direct feed-through matrix

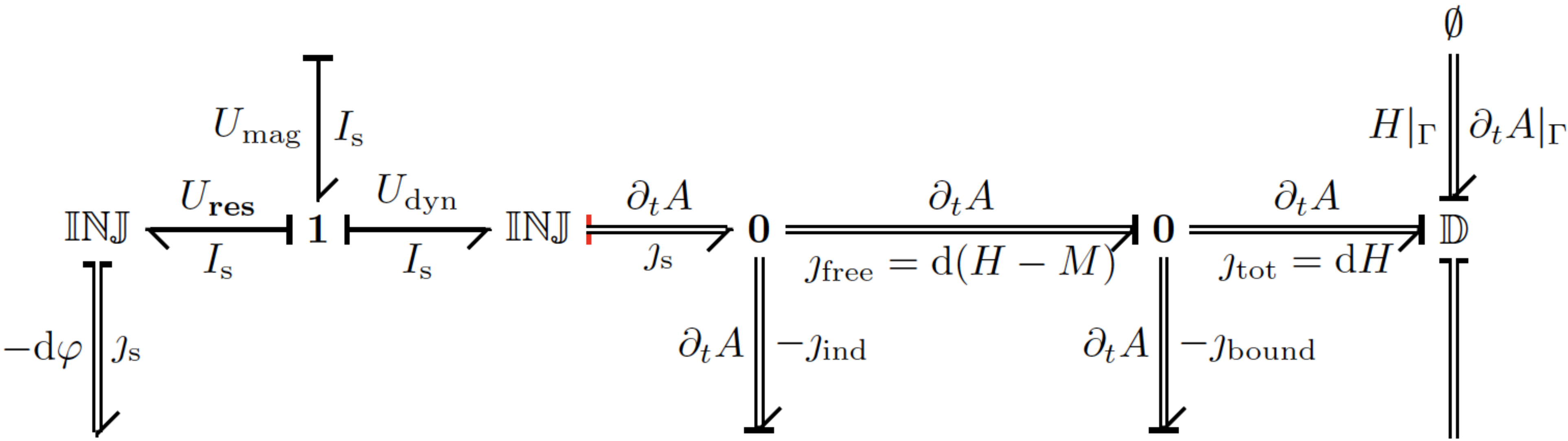
B. Maschke and A. van der Schaft, "From conservation laws to port-hamiltonian representations of distributed-parameter systems," in *16th IFAC World Congress*, Elsevier, 2005.

Modelling of Magnetoquasistatic Domain

For a superconducting magnet model we consider a magnetoquasistatic model including

1. energy stored in the magnetic field,
2. resistive voltage due to a quench,
3. induced voltage influenced by the eddy currents and cable coupling currents.





Electromagnetic Field – port-Hamiltonian Model

Combining the Dirac structures together, the state and output equations are given as

$$
\begin{aligned}
-\partial_t \begin{bmatrix} B \\ 0 \end{bmatrix} &= \left(\begin{bmatrix} 0 & -\mathrm{d} \\ \mathrm{d} & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & *_\kappa + \mathrm{d} *_{\nu\tau} \mathrm{d} \end{bmatrix} \right) \begin{bmatrix} H \\ \partial_t A \end{bmatrix} + \begin{bmatrix} 0 \\ \chi \end{bmatrix} I_\mathrm{s} \\
U_\mathrm{mag} &= \begin{bmatrix} 0 & \chi^\top \end{bmatrix} \begin{bmatrix} H \\ \partial_t A \end{bmatrix} + \begin{bmatrix} 0 & \chi^\top \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & *_\rho \end{bmatrix} \begin{bmatrix} 0 \\ \chi \end{bmatrix} I_\mathrm{s},
\end{aligned}
$$

which forms an input-state-output port-Hamiltonian system

$$
\begin{aligned}
\partial_t x &= \left(J(x) - R(x) \right) \delta_x \mathbb{H}(x) + g u \\
y &= g^\top \delta_x \mathbb{H}(x) + S u,
\end{aligned}
$$

The power of the system is bounded by the product of input and output

$$
\begin{aligned}
\partial_t \mathbb{H}(x) &= \delta_x \mathbb{H}(x)^\top \partial_t x = \delta_x \mathbb{H}(x)^\top \left(\left(J(x) - R(x) \right) \delta_x \mathbb{H}(x) + g u \right) \\
\partial_t \mathbb{H}(x) &= -\delta_x \mathbb{H}(x)^\top R(x) \delta_x \mathbb{H}(x) + y^\top u - u^\top S^\top u \leq y^\top u.
\end{aligned}
$$

Electromagnetic Field – port-Hamiltonian Model

Considering the input-state-output equations representing the magnetoquasistatic mode, the variation of the Hamiltonian function reads

$$
\begin{aligned}
\partial_t \mathbb{H}(x) &= -\delta_x \mathbb{H}(x)^\top R(x) \delta_x \mathbb{H}(x) + y^\top u - u^\top S^\top u \\
&= -\begin{bmatrix} H & \partial_t A \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & *_\kappa + \mathrm{d} *_{\nu\tau} \mathrm{d} \end{bmatrix} \begin{bmatrix} H \\ \partial_t A \end{bmatrix} + \left(\chi^\top \partial_t A + \chi *_\rho \chi^\top I_\mathrm{s}\right) I_\mathrm{s} - I_\mathrm{s} \chi *_\rho \chi^\top I_\mathrm{s} \\
&= \partial_t A \chi I_\mathrm{s} - \partial_t A (*_\kappa + \mathrm{d} *_{\nu\tau} \mathrm{d}) \partial_t A
\end{aligned}
$$

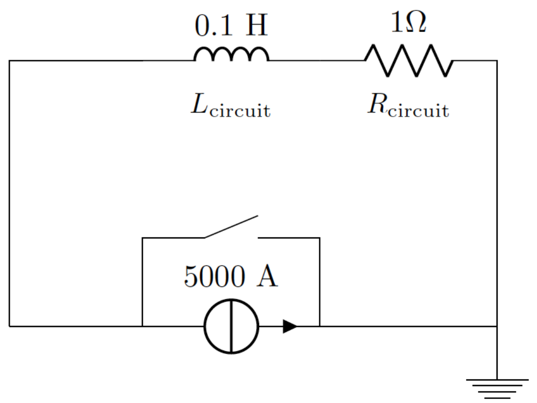
The magnetic energy **decreases only due** to the eddy and coupling current losses and **is not directly** influenced by the Joule losses after a quench.

Field/circuit coupling needed.

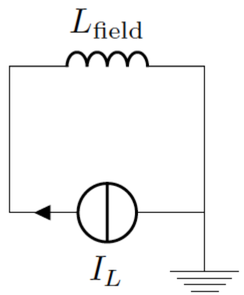
„Field”/circuit coupling – lumped case

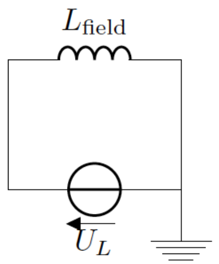
We consider an RL circuit with initial current of 5000 A and a crowbar that closes at t=0s.

Circuit model



„Field” models



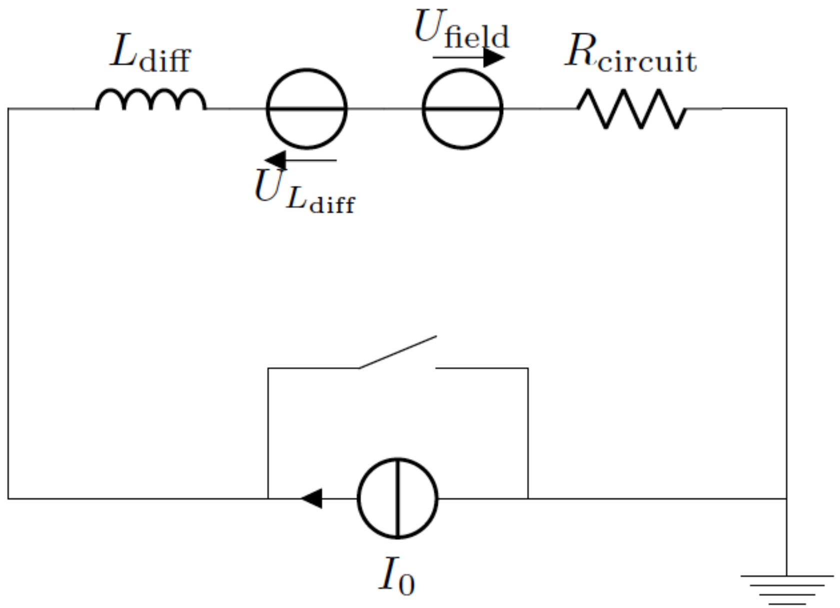


- Field model is represented as a circuit with inductance L_{field} of 0.09 H (R_{field}=0) connected in parallel with either a current (1) or a voltage (2) source.
- Convergence criterion is based on the relative error equal to 1e-6 or the absolute error equal to 1 mA.
- Co-simulation is solved for T=[0, 0.1, 0.2, 0.3, 0.4] s.
- Maximum time step is equal to 100 us.

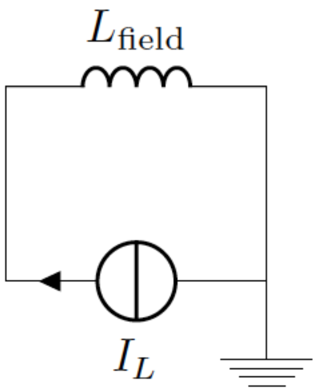
For the converged solution we expect a current discharge on the circuit side with time constant $L_{\text{field}}/R_{\text{circuit}}$.

Computational causality of lumped models (current-driven)

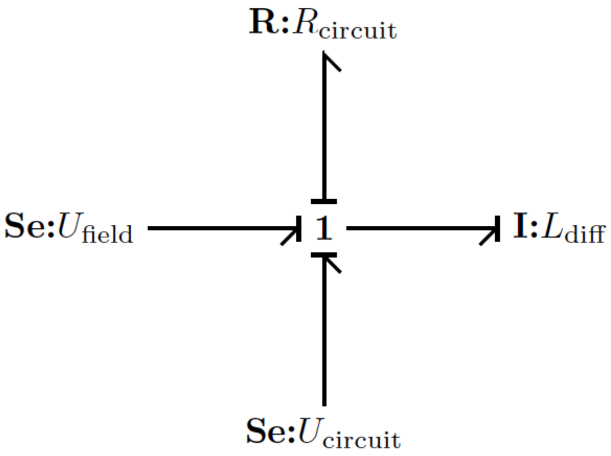
Circuit model



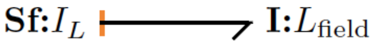
„Field" model



Circuit model



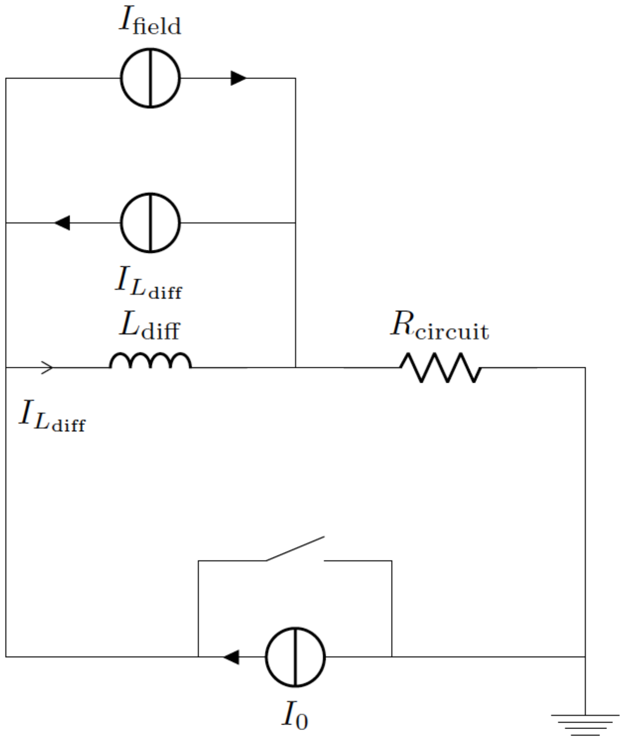
„Field" model



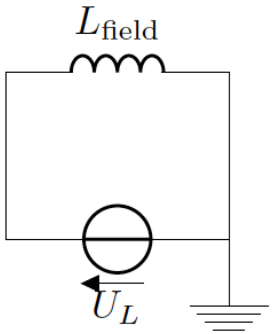
„Field" model has an non-preferred, differential causality

Computational causality of lumped models (voltage-driven)

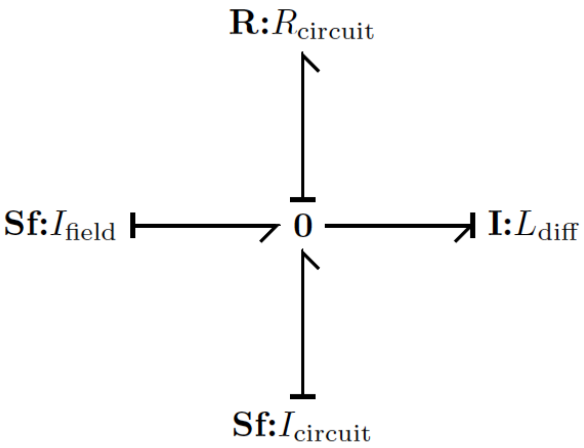
Circuit model



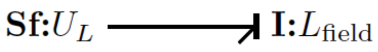
„Field" model



Circuit model



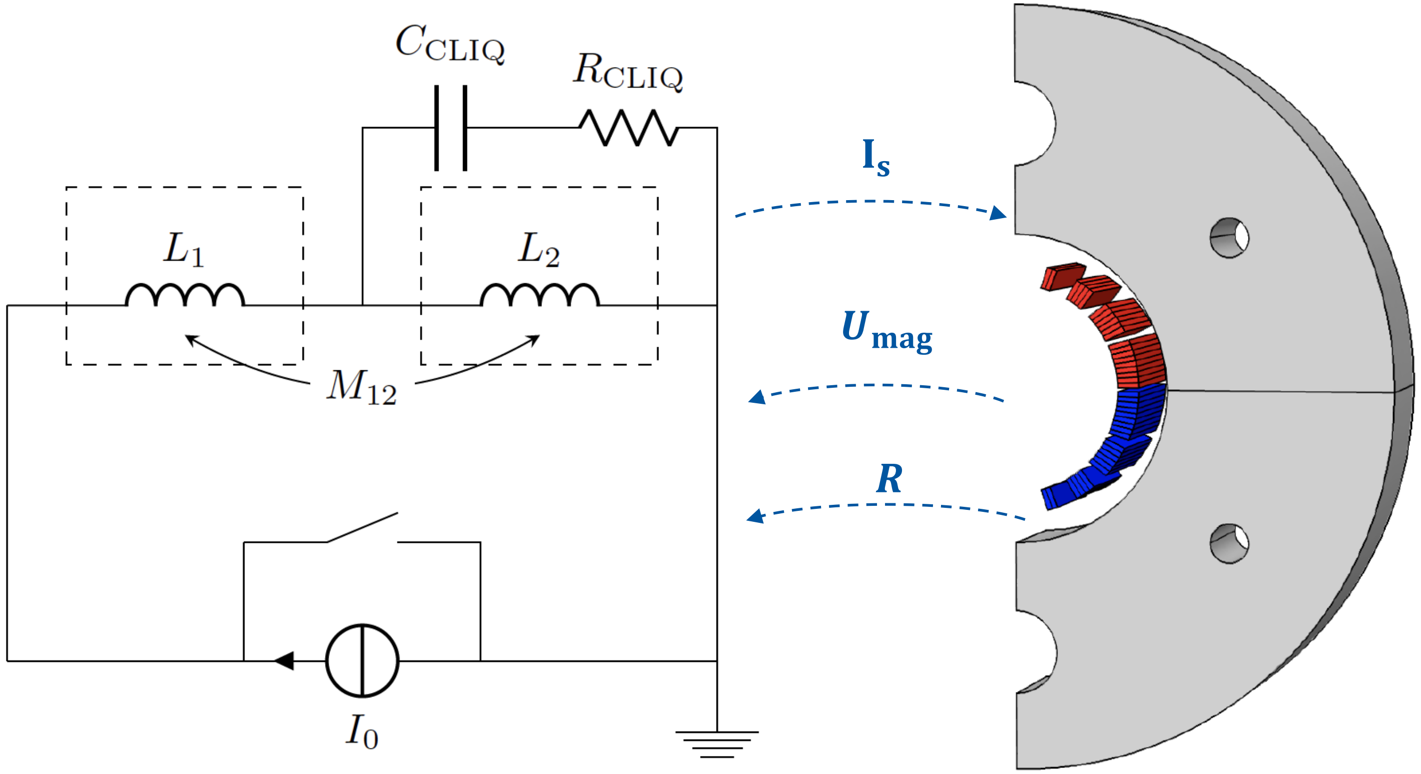
„Field" model



„Field" model has an integral, preferred causality

Field/circuit coupling – distributed field model

- Standalone D1 magnet protected with CLIQ
- Comparison of voltage- and current-driven equivalent circuit models
- Bond-graph computational causality analysis

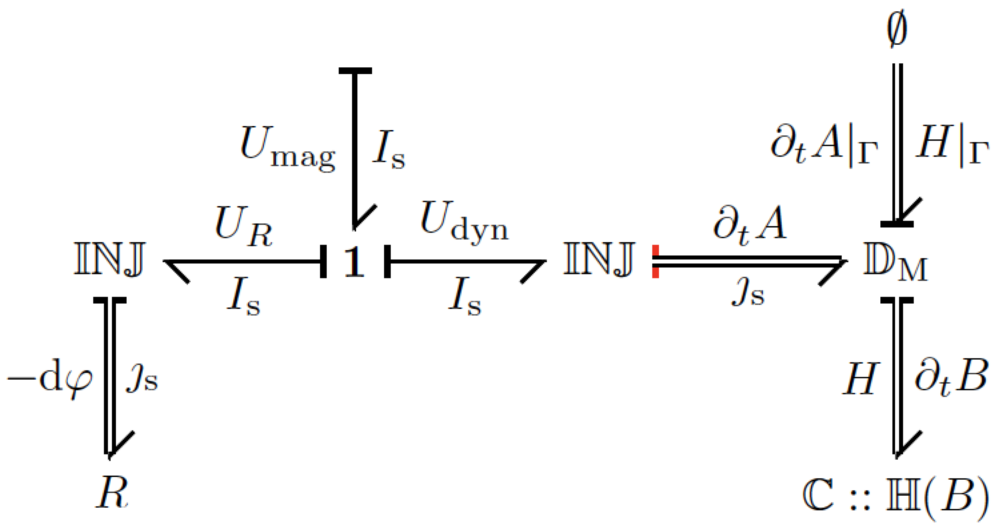


I. Garcia et al, “Optimized Field/Circuit Coupling for the Simulation of Quenches in Superconducting Magnets”, IEEE Journal on Multiscale and Multiphysics Computational Techniques

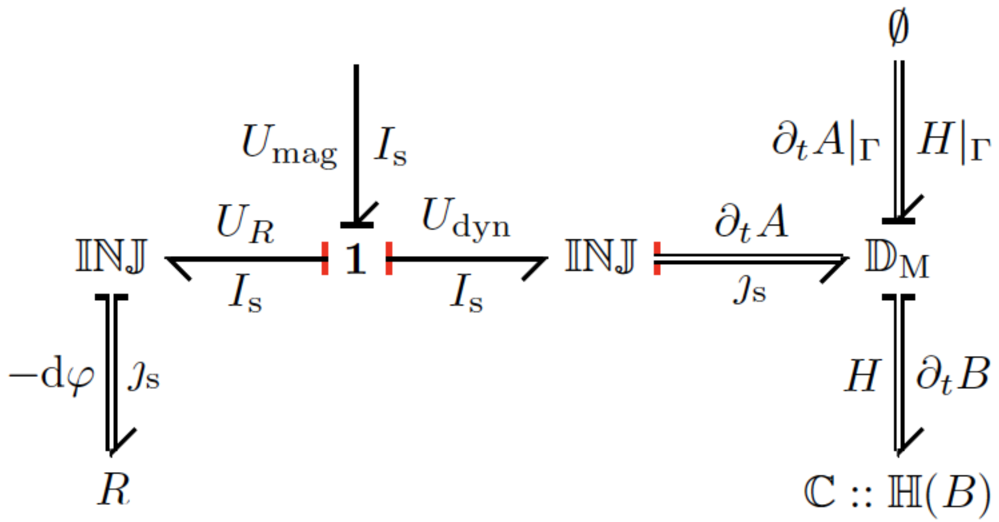
Field/circuit coupling – causality

Without the loss of generality, neglecting the eddy currents in copper wedges and cable yields the following bond graphs

Current-driven mode

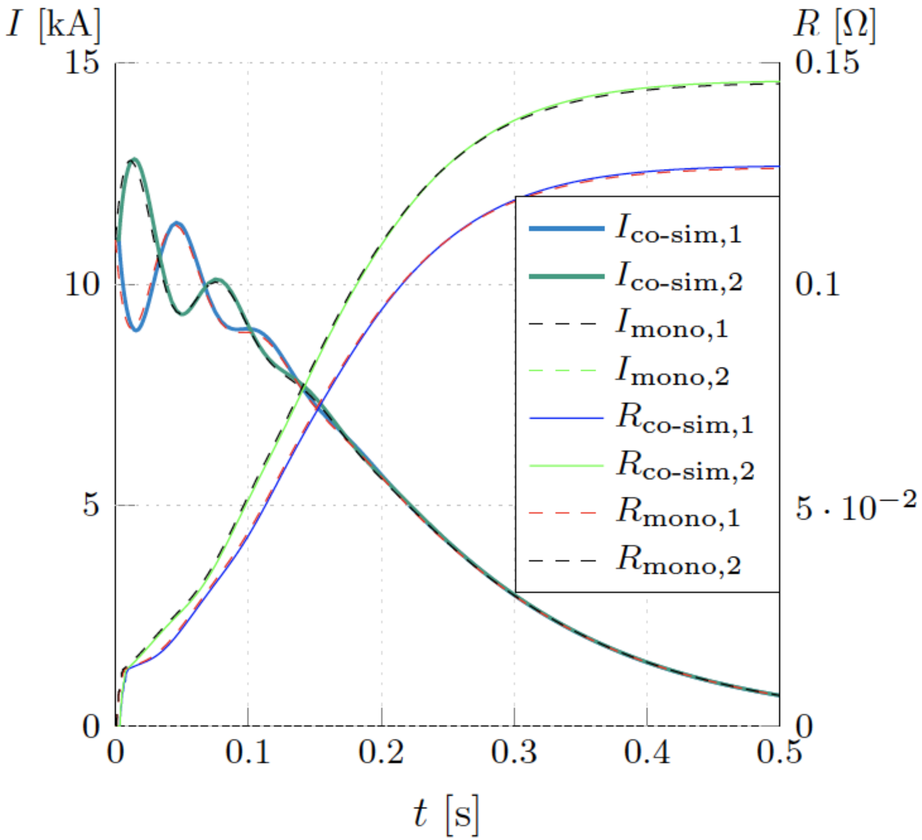


Voltage-driven mode

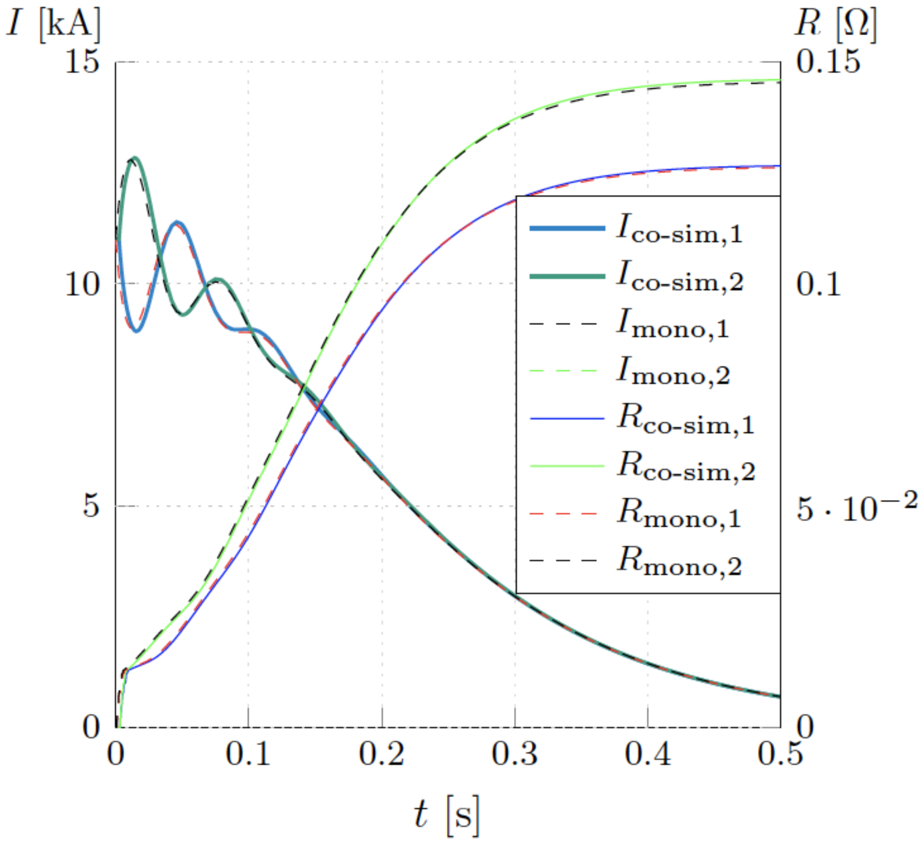


Field/circuit coupling – results

Current-driven mode



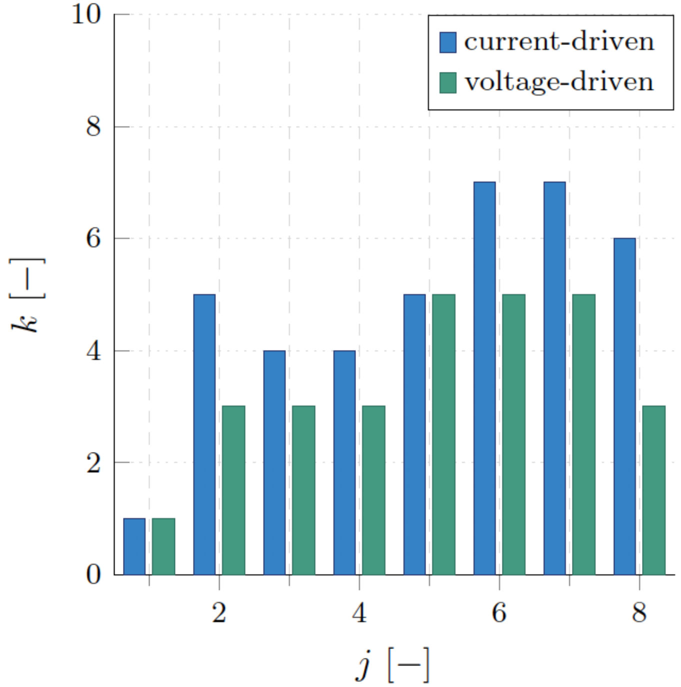
Voltage-driven mode



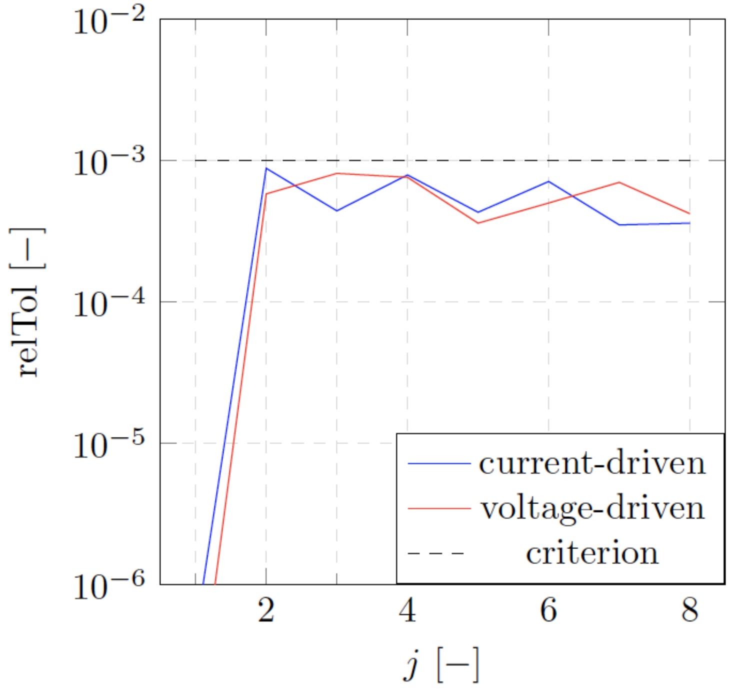
Both modes converged to the monolithic solution with the requested accuracy

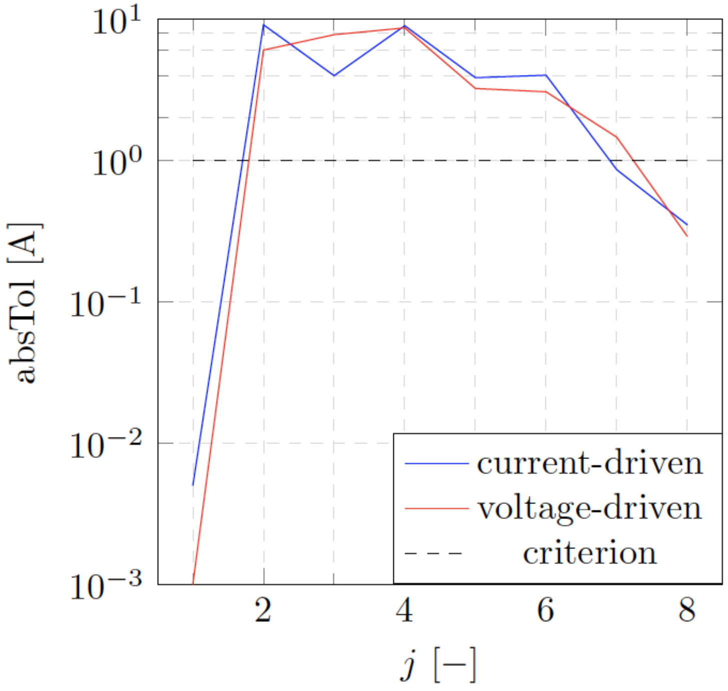
Field/circuit coupling – comparison

Number of iterations



Relative and absolute tolerance



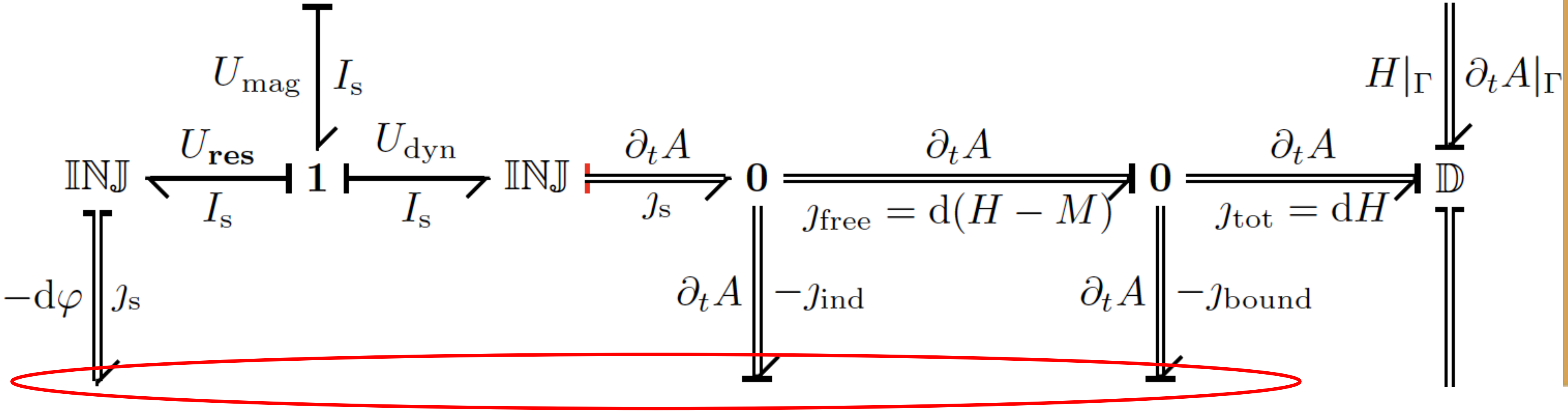


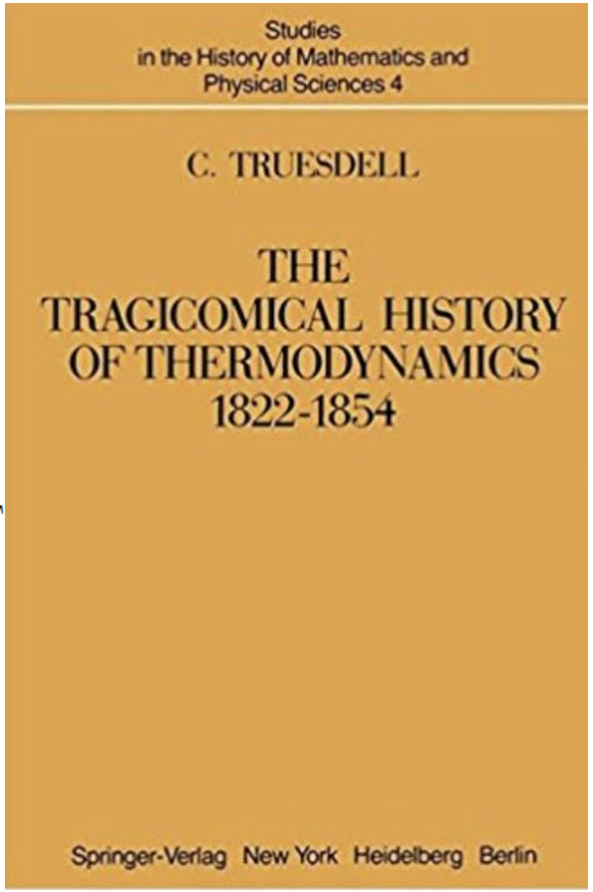
Voltage-drive mode did not outperform the current-driven one

Modelling of Irreversible Thermodynamics

Irreversible thermodynamics modelling considers

1. storage of the thermal energy
2. heat conduction
3. heat generation in the EM domain





Magnetocaloric effect in superconducting magnets

The magnetocaloric effect demonstrates itself in the superconducting state while isentropicly increasing or decreasing the magnetic field, the temperature, respectively decreases or increases.

A reciprocal effect while changing temperature also takes place. Starting with the entropy variation

$$\partial_t s = \partial_t T \partial_T s + \partial_t B \partial_B s$$

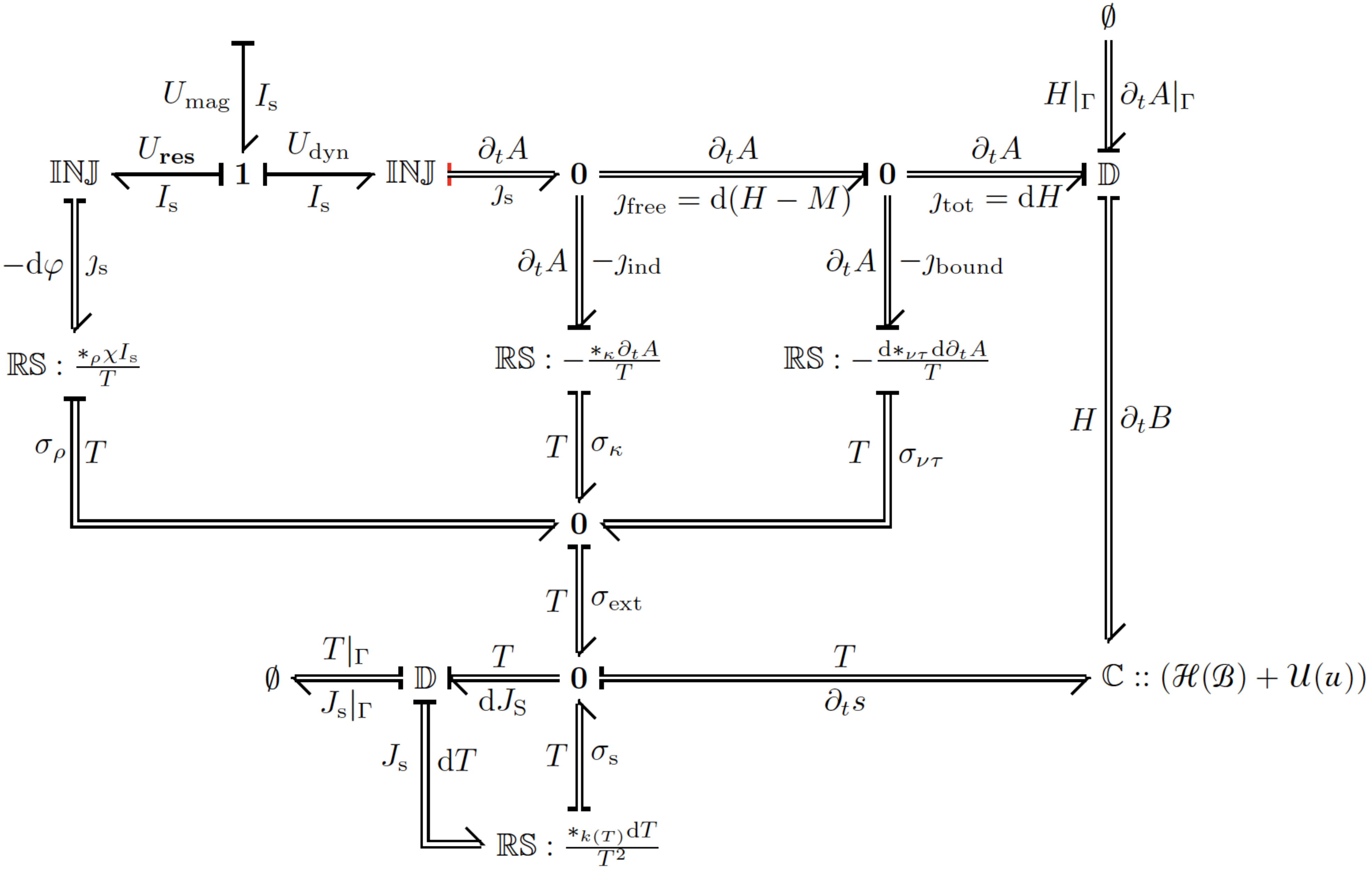
Considering that $\partial_T s = -\partial_T^2 G(T) = C_V / T$, the entropy variation takes the following form

$$\partial_t s = \frac{C_V}{T} \partial_t T + \frac{\lambda_B}{T} \partial_t B,$$

where $\lambda_B = \partial_B s \wedge T$ is the magnetocaloric coefficient. Eventually, taking into account also the heat conduction and external heat sources, the heat balance equation for type-II superconductors reads

$$\frac{C_V}{T} \partial_t T = -\frac{\lambda_B}{T} \partial_t B + \frac{\mathrm{d}(*_{k(T)} \mathrm{d}T)}{T} + \frac{q_0}{T}.$$

Bond graph representation of magneto-thermal magnet model



Summary

1. Port-based modelling allows for a generic representation of multi-physical systems (both, SC magnets, and circuits)
2. Variation of the magnetic and thermal energy is studied
3. Computational causality can be derived from a bond-graph model
4. Tha magnetocaloric effect is captured however has minor influence

- ~~use of the port-based approach for *co-simulation*.~~

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