Beam dynamics for LHC upgrades

T. Pugnat[1]

B. Dalena[1], O. Napoly[1]

L. Bonavantura[2], A. Simona[2]

R. De Maria[3], M. Giovannozzi[3], E. Maclean[3], J. Molson[3], S. Roussenschuck[3], E. Todesco[3], R. Tomás[3]

[1]CEA - DRF/Irfu/DACM/LEDA

[2]MOX, Politecnico di Milano, Milano, Italy

[3]CERN

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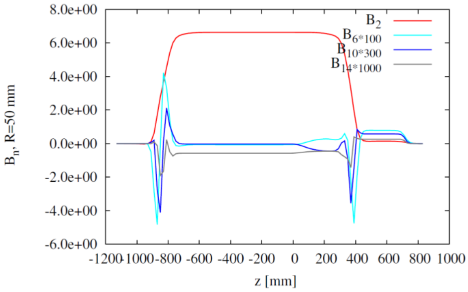
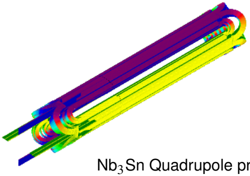
Thesis project

In order to be able to improve the design and performance of future colliders, models of the magnetic fields non-linearities needs deeper understanding. These non-linearities mainly come from magnet fringe fields and ends connections.





Nb_3Sn Quadrupole prototype for HL-LHC
Simulated with ROXIE
Courtesy of CERN magnet group

Goals:

- Develop a "realistic" non-linear transfer map for tracking studies.
- Use calculated or measured magnetic field map given by the magnet designers.
- Define observables sensitive to the longitudinal field description.

Contents

Hamiltonian and Vector potential representation

E. Forest (Ref. [1]):

8 D equivalent Hamiltonian of a quadrupole ($a(x,y,z) = q\dfrac{A(x,y,z)}{P_0 c}$ ***):***

$$H\left[x, p_x, y, p_y, s, \delta, z, p_z; \sigma\right] = -\sqrt{(1+\delta)^2 - (p_x - a_x)^2 - (p_y - a_y)^2} + p_z - a_z$$

$$\Downarrow$$

$$K\left[x, p_x, y, p_y, s, \delta, z, p_z; \sigma\right] = p_z - a_z - \delta + \frac{(p_x - a_x)^2}{2(1+\delta)} + \frac{(p_y - a_y)^2}{2(1+\delta)}$$

A. Simona (Ref. [2]), M. Venturini (Ref. [3]) and A.J. Dragt (Ref. [4]):

Generalized Gradient: $C^{[n]}_{m,*}(z) = \dfrac{i^n}{2^m m!}\dfrac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\dfrac{k^{m+n+1}}{I'_m(Rk)}\widehat{B}_{m,*}(R,k)e^{ikz}dk$

Vector potential representation: $A(x,y,z) = \sum_{i,j} x^i y^j c_{ij}(z)$

Gauge:

- **AF:** $A_\phi \equiv 0$
- **HFC:** $\mathbf{A} = \mathbf{A}' + \nabla\lambda$ such that $A_x \equiv 0$ with $\nabla \cdot \mathbf{A} = 0$

Errors in the gradient reconstruction for $R \geqslant R_{analysis}$

2^{nd} order Lie Integrator

For the position $\mathbf{q} = (x, y, \ldots)$ and the momentum $\mathbf{p} = (p_x, p_y, \ldots)$:

$$
\begin{array}{lcll}
\mathbf{q}^{i+1/13} & = & \mathbf{q}^{i} & + \left(0, \quad 0, \quad \ldots \right) \\
\mathbf{q}^{i+2/13} & = & \mathbf{q}^{i+1/13} & + \left(0, \quad 0, \quad \ldots \right) \\
\mathbf{q}^{i+3/13} & = & \mathbf{q}^{i+2/13} & + \left(0, \quad 0, \quad \ldots \right) \\
\mathbf{q}^{i+4/13} & = & \mathbf{q}^{i+3/13} & + \left(\frac{\Delta\sigma}{2}\frac{p_x^{i+3/13}}{(1+\delta)}, \quad 0, \quad \ldots \right) \\
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$$
\begin{array}{lcll}
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\mathbf{p}^{i+1} & = & \mathbf{p}^{i+12/13} & + \left(0, \quad 0, \quad \ldots \right)
\end{array}
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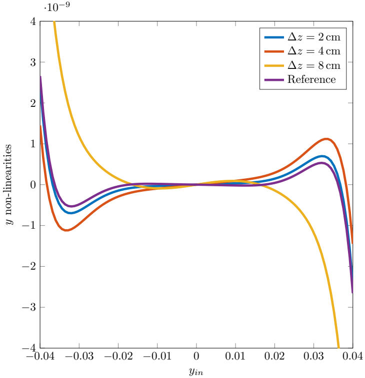
\mathbf{p}^{i+1} &= \mathbf{p}^{i+12/13} + \left(0,\quad 0,\quad \dots\right)
\end{aligned}
$$

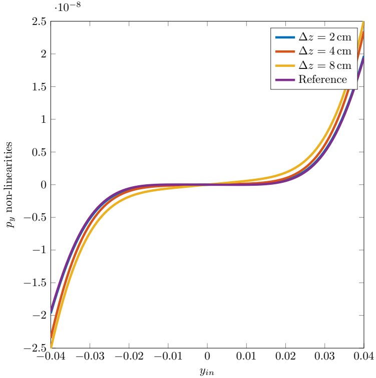
In the Hard Edge case ($A_x = A_y = 0$).

Step size in z

Procedure:

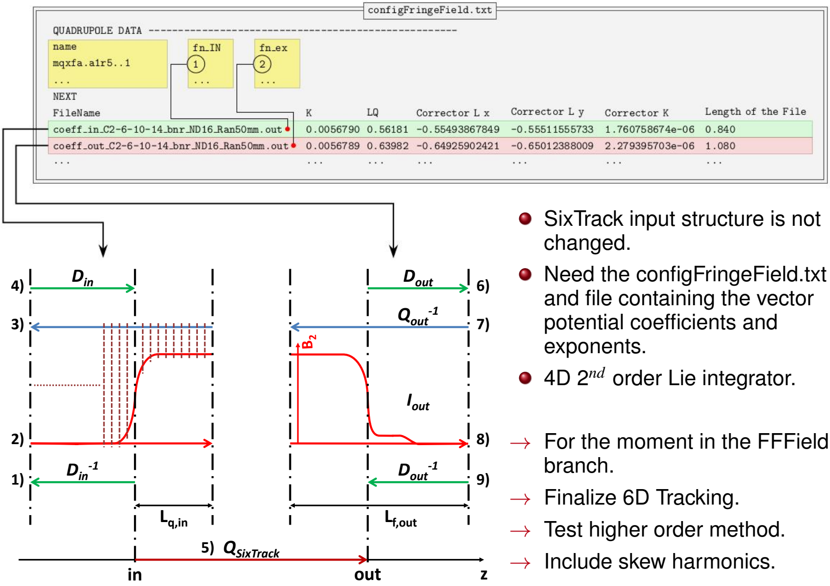
Use different initial position with different offset ($x_{in} = px_{in} = py_{in} = 0$) and only use one quadrupole for the tracking. The linear part is subtracted to the final positions and momenta, as a function of the initial coordinate.





- For a dz greater than 40 mm, information due to the longitudinal description of the field is greatly deteriorated (Ref. [2] and [5]).

Implementation in SixTrack



- SixTrack input structure is not changed.
- Need the configFringeField.txt file and file containing the vector potential coefficients and exponents.
- 4D 2^{nd} order Lie integrator.

→ For the moment in the FFField git branch.

→ Finalize 6D Tracking.

→ Test higher order method.

→ Include skew harmonics.

Contents

Dynamic aperture

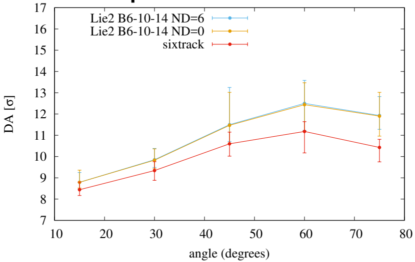
Procedure:

- *Particles:* 30 initials conditions for each interval of 2 sigma (0 to 28) and 5 phase-space angles with $\delta = 2.7e^{-4}$.
- *Optic:* HLLHCV1.0 with 60 dipole field errors seeds.
- *Number of revolution:* 10^4.
- In SixTrack, systematic $b_{6,10,14}$ only are considered and are scaled for the prototype length.

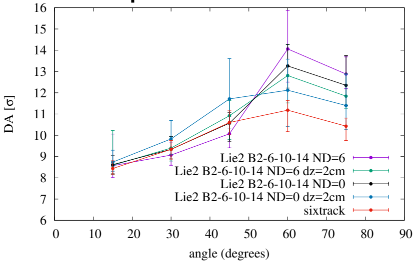
Result:

- Effect of the derivatives small compared to effect due to random field errors and to tracking precision.
- SixTrack method is robust against full tracking.

Dynamic aperture without B2 in the vector potential file



Dynamic aperture with B2 in the vector potential file



Tune vs Action

Procedure:

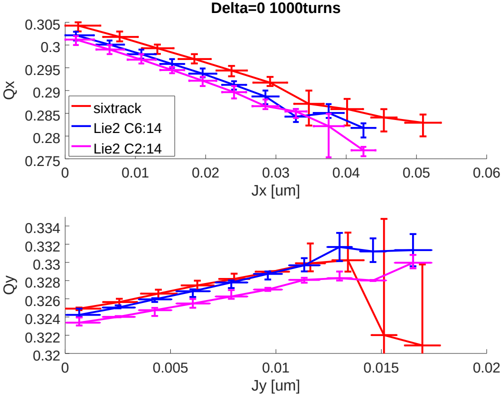
- *Particles:* 120 initials conditions with amplitude between 0.033333 mm to 4.000000 mm, the ratio between emittance in the two planes equal to 0.19281 and $\delta = 0$.
- *Optic:* HLLHCV1.0 with only one of the 60 dipole field errors seeds.
- *Number of turns:* 1000.

Result:

- Small systematic between SixTrack and the Lie2 method.
- Same result for all seeds but covered by random field errors.
- The small systematic (angle 15°) is not influencing DA result (see previous slide).

→ Test different angles.

→ Comparison with dz=2cm.



Single Quad Tracking

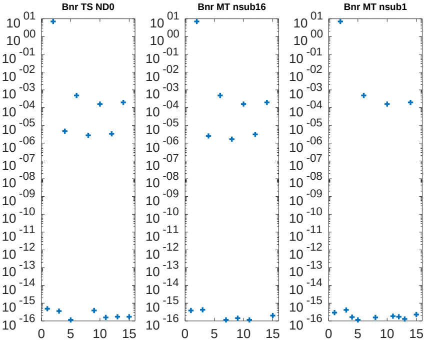
Procedure:

- *Particles:* Initial conditions on a circle for different radius and no transverse momenta.
- *Optic:* Only one quadrupole with a symmetric field. The tracking method is the Lie integrator (TS) with and without derivatives and the SixTrack multipole (MT) with and without subdivision of the thin matrix.
- *Plot:* DFT of the momenta at the end of the Quadrupole.

Result:

- An $b_{4,8,12}$ effect appear in the multipole case when the thin matrix is subdivided.
- This effect increase with the number of subdivision.
- When derivatives are included, the b_4 change sign.
- The additional b_4 increases with the radius.

→ Test with dz=2cm.



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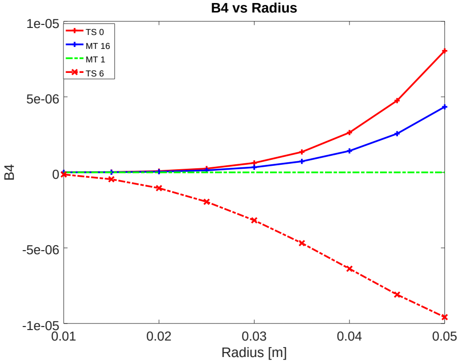


Contents

Measurements with the beam

Goal:

Search for the signature of non-linear effects in the LHC not described by the present model.

✓ July 2017: LHC IR non-linearities studies (E. Maclean, MD 2158)
- Several measurements and techniques used in LHC to evaluate non linear fields in the IR, using the beams.
- Measurements of 1^{st} and 2^{nd} order detuning with amplitude.

✓ September 2017: LHC IR non-linearites studies (E. Maclean)
- Measurement of short term DA with AC-dipole.
- Measurement of long term DA with ADT blow-up.

→ Analyse data from the previous MD (1^{st} and 2^{nd} order detunning with amplitudes, ...).

→ 2018: Non-linear MDs.
- We are particularly interested in the b_6 effects of inner triplet.

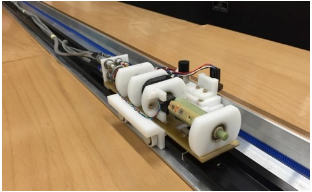
Contents

Magnetic Measurements

Goal:

Use computed or measured data for the Tracking and understand the limit of applicability of our model to measured data (filters of noise, measurements precision, etc...).

- **Collaboration with the CERN:** Brief Report of the discussions
  - Toy-train measures harmonics in step of micro-meter and provides integrated values on mm longitudinal steps (using a 6 order polynomial) (S. Roussenschuck).
  - Consider up to 6 derivatives of the Generalized Gradients in the longitudinal harmonics calculation/measurements (S. Roussenschuck).



  - Both integral and "point-like" in z measurements of harmonics (b_n and a_n) can reach a relative resolution of 10^{-6} (S. Roussenschuck).
  - For the moment, the prototypes are not representative of the HL-LHC production (E. Todesco).
    The possibility to use measurements is not given for the time of the thesis.
- **Collaboration with Fermilab**: ?

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Comparison between Integrator (2,4,6th-Gauss, 4th-RK, 2,4,6th-Lie)

- Lie methods profit more from the change of gauge than the other methods.
- Lie methods are faster with respect to other symplectic methods. The explicit, non-symplectic Runge-Kutta method is the fastest.
- All the methods display the same low accuracy for step size bigger than 4 cm for the realistic field considered.

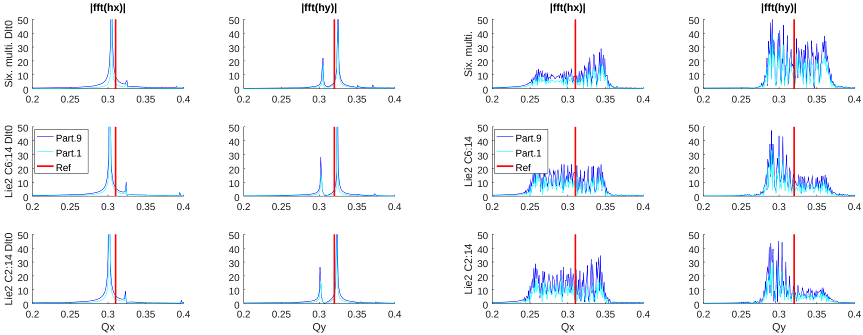
Table: Vector potential evaluation's cost (A. Simona).

	ND=2		ND=16	
	Normal	**Skew**	**Normal**	**Skew**
AF	80	68	352	330
HFC	64	52	251	225
HFC/AF	0.80	0.76	0.72	0.68

Tube Oscillation

Procedure:

- *Particles:* 2 initials conditions with amplitude respectively 0.1993 mm and 0.4599 mm, the ratio between emittance in the two planes equal to 0.19281. In the left plot, $\delta = 0$ and on the right $\delta = 2.7e - 4$.
- *Optic:* HLLHCV1.0 with only 1 dipole field errors seed.
- *Number of turns:* 10^3.



Tune vs Action

Procedure:

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Result:

- Small systematic between SixTrack and the Lie2 method.
- Same result for all seeds but covered by random field errors.
- The small systematic (angle 15°) is not influencing DA result (see previous slide).

→ Test different angles.

→ Comparison with dz=2cm.