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Beyond the Standard Model

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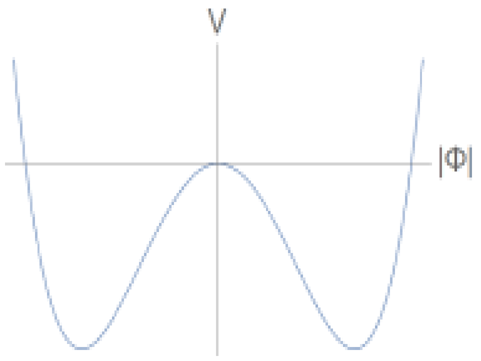
Two-Higgs Doublet Model

- In the SM, we have one complex scalar doublet

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

- The scalar potential is given by

$$V = -\mu^2 \Phi^\dagger \Phi + \frac{1}{2}\lambda \left(\Phi^\dagger \Phi\right)^2$$



where μ^2 and λ are real.

- We assume that the scalar potential has a non-zero vev:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

- Expanding about the vev

$$\Phi = \begin{pmatrix} G^+ \\ (v + H + iG^0)/\sqrt{2} \end{pmatrix}$$

where G^- is a complex field and H, G^0 are real fields
- The potential becomes

$$V = -\frac{\mu^4}{2\lambda} + \mu^2 H^2 + \mathcal{L}_{\rm Int}$$

Two-Higgs-Doublet Model

- Now we add a second Higgs doublet
- The most general, renormalizable two-doublet scalar potential is

$$
\begin{aligned}
V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left(m_{12}^2 \Phi_1^\dagger \Phi_2 + \mathrm{H.C.} \right) \\
& + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\
& + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \left[\frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2 \right)^2 \right. \\
& \left. + \lambda_6 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_1^\dagger \Phi_2 \right) + \lambda_7 \left(\Phi_2^\dagger \Phi_2 \right) \left(\Phi_1^\dagger \Phi_2 \right) + \mathrm{H.C.} \right]
\end{aligned}
$$

- where m_{11}^2, m_{22}^2 and $\lambda_{1,2,3,4}$ are real and m_{12}^2 and $\lambda_{5,6,7}$ are in general complex
  - For illustration and simplicity, we will work with a CP conserving vacua and a scalar potential which is CP conserving, in which the quartic terms obeys a Z_2 symmetry:

$$Z_2: \quad \Phi_1 \to -\Phi_1 \quad \text{and} \quad \Phi_2 \to \Phi_2$$

$$
\begin{aligned}
V = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\
& + \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) \\
& + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \frac{\lambda_5}{2} \left(\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right)
\end{aligned}
$$

Expanding about these vev's the fields are

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ (v_a + \rho_a + i\eta_a)/\sqrt{2} \end{pmatrix}, \qquad a \in \{1, 2\}$$

Two-Higgs-Doublet Model

- There are 8 fields. Three fields get "eaten" up by the $W_\mu^\pm$ and the Z_μ^0
- The remaining content is: two neutral scalars, a pseudo scalar and a charged scalar.
- To determine mass matricies, one needs to enfore extrema of the potential: $\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = 0$, which occurs only when

$$0 = m_{11}^2 v_1 - \mathrm{Re}\left(m_{12}^2\right) v_2 + v_1^3 \frac{\lambda_1}{2} + v_1 v_2^2 \frac{\lambda_{345}}{2}$$
$$0 = m_{22}^2 v_2 - \mathrm{Re}\left(m_{12}^2\right) v_1 + v_2^3 \frac{\lambda_2}{2} + v_2 v_1^2 \frac{\lambda_{345}}{2}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$

Charged Higgs

- For the charged scalars, the mass matrix is

$$\left(M_{\phi^\pm}^2\right)_{ij} = \frac{\partial^2 V}{\partial \phi_i^- \partial \phi_j^+} = \left[m_{12}^2 - (\lambda_4 + \lambda_5) v_1 v_2\right] \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix}$$

- This matrix has one zero eigenvalue corresponding to the charged Goldstone boson (which gets eaten by the W). The mass of the charged Higgs is

$$H^+ = \frac{v_2 \phi_1^+ - v_1 \phi_2^+}{\sqrt{v_1^2 + v_2^2}}$$
$$M_{H^+}^2 = \frac{v^2}{v_1 v_2}\left[m_{12}^2 - v_1 v_2 (\lambda_4 + \lambda_5)\right]$$

Two-Higgs-Doublet Model

Pseudo Scalar Higgs

- The mass matrix for the pseudo scalar Higgs is

$$(M_A^2)_{ij} = \frac{\partial^2 V}{\partial \eta_i \partial \eta_j} = \left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5\right) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}$$

- Diagonalizing this, we find one massless pseudo scalar (which gets eaten by the Z boson) and a massive pseudo scalar with mass M_A^2 :

$$A = \frac{v_2 \eta_1 - v_1 \eta_2}{\sqrt{v_1^2 + v_2^2}}$$

$$M_A^2 = (v_1^2 + v_2^2)\left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5\right)$$

Neutral Scalar Higgs

- Lastly we have the mass matrix for the neutral scalar Higgs

$$(M_h^2)_{ij} = \frac{\partial^2 V}{\partial \rho_i \partial \rho_j} = \begin{pmatrix} A & C \\ C & B \end{pmatrix}$$

where

$$A = m_{11}^2 + \frac{3\lambda_1}{2} v_1^2 + \frac{\lambda_{345}}{2} v_2^2$$

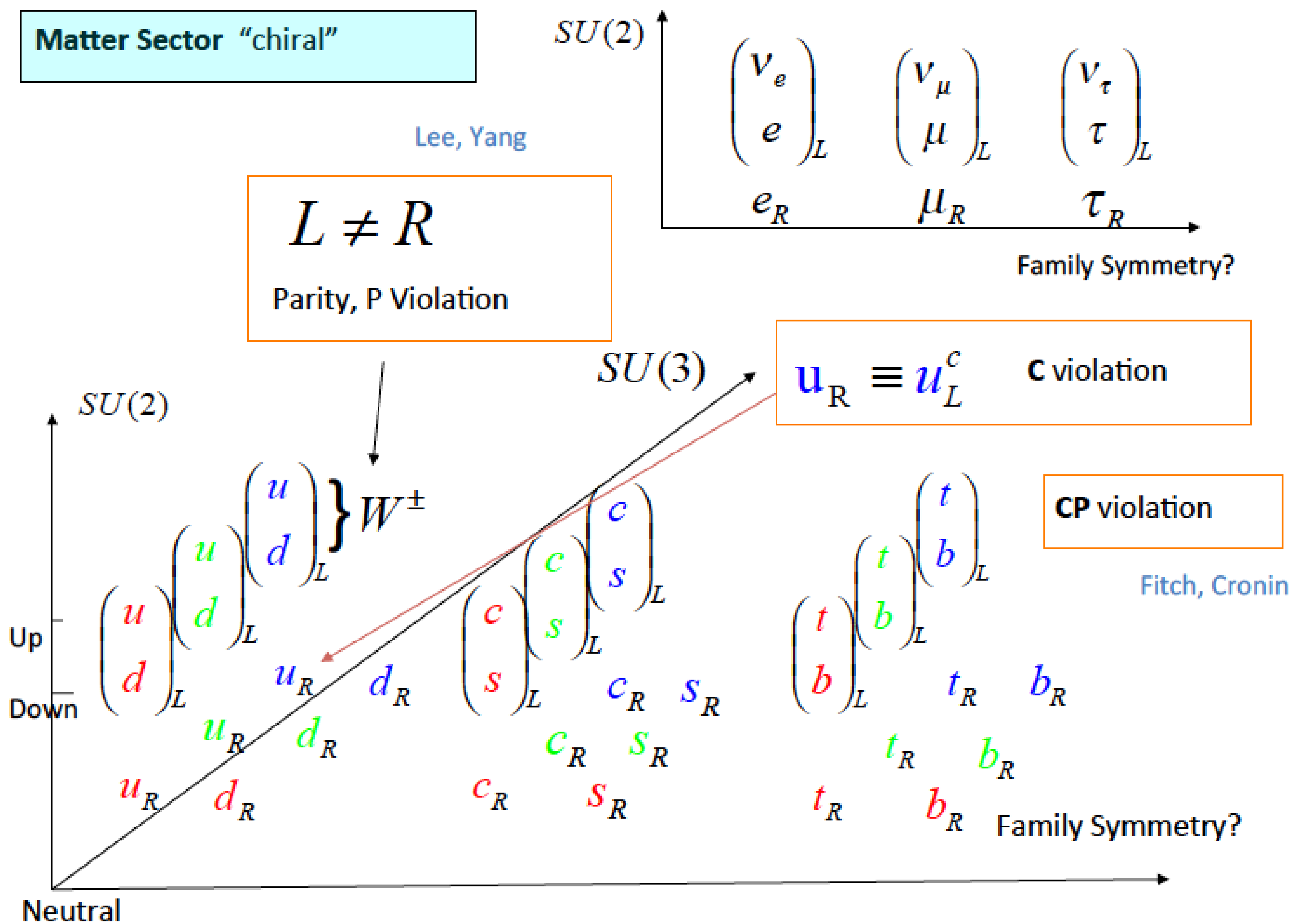
$$B = m_{22}^2 + \frac{3\lambda_2}{2} v_2^2 + \frac{\lambda_{345}}{2} v_1^2$$

$$C = -m_{12}^2 + \lambda_{345} v_1 v_2$$

- This can be diagonalized by and angle α, resulting in the physical fields h and H:

$$H = (-\rho_1 \cos\alpha - \rho_2 \sin\alpha) \qquad h = (\rho_1 \sin\alpha - \rho_2 \cos\alpha)$$

Left-Right Asymmetry in the SM



Left Right Model

- Left Right Model of elementary particles is described by $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge invariant theory.

Where

$SU(2)_R$:

is describe RH particles same as $SU(2)_L$ for LH particles

- isospin defined in LRM as $\boldsymbol{T_L^3}$ for $S(2)_L$ and $\boldsymbol{T_R^3}$ for $SU(2)_R$ And hypercharge in LRM defined as

$$\mathbf{Y = T_R^3 + (B - L)/2}$$

- So electric charge in LRM

$$\mathbf{Q = T_L^3 + T_R^3 + (B - L)/2}$$

- **(B – L) :**

  is (baryon - Lepton) number for particles

  e.g $L(e^-) = 1$, $L(e^+) = -1$, $L(q) = 0$, $B(q) = \frac{1}{3}$, $B(\bar{q}) = \frac{-1}{3}$, $B(e) = 0$

Left Right Model

Then in LRM lepton sector:

$L_L = \begin{pmatrix} e_L \\ \nu_{e\,L} \end{pmatrix} \begin{pmatrix} \mu_L \\ \nu_{\mu\,L} \end{pmatrix} \begin{pmatrix} \tau_L \\ \nu_{\tau\,L} \end{pmatrix}$ $\qquad$ $L_R = \begin{pmatrix} e_R \\ \nu_{e\,R} \end{pmatrix} \begin{pmatrix} \mu_R \\ \nu_{\mu\,R} \end{pmatrix} \begin{pmatrix} \tau_R \\ \nu_{\tau\,R} \end{pmatrix}$

and quark sector:

$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ $\qquad$ $Q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix} \begin{pmatrix} c_R \\ s_R \end{pmatrix} \begin{pmatrix} t_R \\ b_R \end{pmatrix}$

We must note that At low energies, it is required that the LRSM gauge group be broken spontaneously to the SM gauge group

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \xrightarrow{scale > TeV} SU(2)_L \otimes U(1)_Y \xrightarrow{scale > 100\ GeV} U(1)_{EM}$$

- So we expect extra Higgs to make this Breaking
- in LRM We expect three extra gauge bosons, two charged $W_R^{\pm}$ and one neutral Z_R

Left Right Model

The Higgs fields

- As fermions in the LRM are grouped in doublets under the SU(2) groups, a gauge invariant Yukawa term $\overline{\Psi}_{\mathrm{L}}\phi\Psi_{R}$ implies that the scalar field ϕ is a 2×2 matrix Hence U $(1)_{B-L}$ would be an exact symmetry after SSB by ϕ only and So we have to add an extra Higgs field because we need Breaking $SU(2)_R$ to give mass for neutrino Then must add RH Higgs triplet Δ_R for Breaking $SU(2)_R$ in Yukawa term $\overline{\Psi}_{\mathrm{R}}\Delta_{\mathrm{R}}\Psi_{R}$ and by analogy must add LH Higgs triplet (Δ_L) is only responsible for maintaining the discrete parity symmetry in Yukawa term $\overline{\Psi}_{\mathrm{L}}\Delta_{\mathrm{L}}\Psi_{L}$
- So now we have 3 Higgs one bidouplet and two triplet

$$\emptyset = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \qquad \Delta_{\mathrm{R}} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & \frac{\Delta^-}{\sqrt{2}} \end{pmatrix} \qquad \Delta_{\mathrm{L}} = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & \frac{\Delta^-}{\sqrt{2}} \end{pmatrix}$$

Left Right Model

Spontaneous Symmetry Breaking in the LRSM

- As a consequence of the LRM Lagrangian gauge invariance, all the fermions and gauge bosons are massless before SSB. The symmetry breaking of the LRSM is spontaneously achieved in two stages and the symmetry breaking mechanism is similar to that of the SM.
- The VEV's of Higgs scalars are given to their neutral components

$$\langle\phi\rangle = \begin{pmatrix} v_u & 0 \\ 0 & v_d \end{pmatrix} \qquad \langle\Delta_R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \qquad \langle\Delta_L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}$$

Where $v_R \gg (v_u, v_d) \gg v_L$ Since the EW analysis lead to the constraint $v_L \leq 10\ GeV$ and the seesaw mechanism for small LH neutrino masses requires $v_L \leq$ a few GeV If we work at the limit $v_L = 0$ then $\sqrt{v_u 2 + v_d 2} = v = 246$ GeV like in SM

Higgs sector of The LR

- the Higgs sector of the LRM is very rich consists of one bidoublet (ϕ) and

two triplets ($\Delta_{L,R}$) complex scalar Higgs fields after breaking is go to doublet scalar field in SM

- Neutral component of Higgs fields are expanded around the vacuum as $\phi_1^0 = \frac{v_u + h_1^0 + i\varphi_1^0}{\sqrt{2}}$ $\phi_2^0 = \frac{v_d + h_2^0 + i\varphi_2^0}{\sqrt{2}}$ $\Delta_H^0 = \frac{v_H + h_H^0 + i\varphi_H^0}{\sqrt{2}}$

Where there are four neutral scalar (h_1^0, h_2^0, h_L^0, h_R^0)

four neutral pseudo scalar (φ_1^0, φ_2^0,φ_L^0,φ_R^0)

Singly Charged Scalars (ϕ_1^+,ϕ_2^{-*}, Δ_L^+ , Δ_R^+)

Doubly Charged Scalars (Δ_L^{++}, Δ_R^{++})

- But have problem (Changing neutral current)

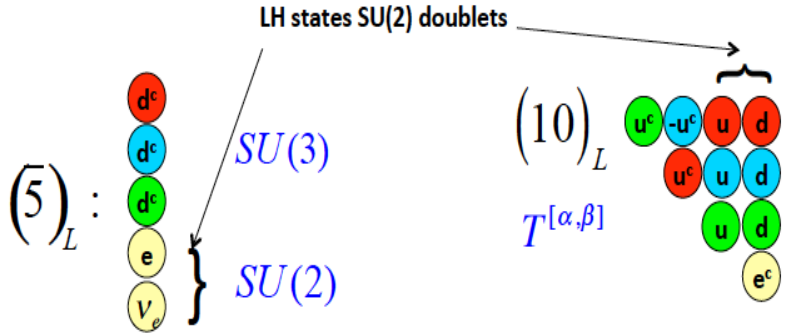
SU(5) GUT Model

$SU(5) \supset SU(3) \times SU(2) \times U(1).$

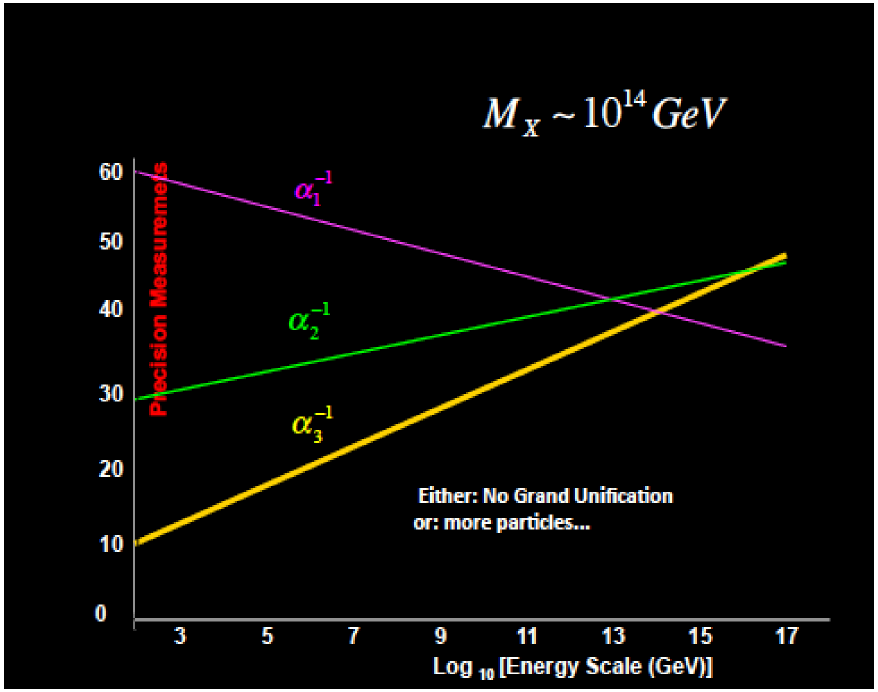
If $SU(5)$ is a local gauge symmetry there are 24 gauge bosons, $V_\mu^{a=1..24}$ which couple to matter via the covariant derivative. Defining $V_\mu = \frac{1}{\sqrt{2}} \sum_{a=1}^{24} V_\mu^a L^a$ the action of the covariant derivative on a 5 dimensional representation of fermions is given by

$$(D_\mu \psi^5)_i = [\delta_i^j \partial_\mu - \frac{\imath g}{\sqrt{2}} V_{\mu,i}^j] \psi_5^j$$

$$V_\mu = \begin{bmatrix} G_1^1 - \frac{2B}{\sqrt{30}} & G_2^1 & G_3^1 & \overline{X}_1 & \overline{Y}_1 \\ G_1^2 & G_2^2 - \frac{2B}{\sqrt{30}} & G_3^2 & \overline{X}_2 & \overline{Y}_2 \\ G_1^3 & G_2^3 & G_3^3 - \frac{2B}{\sqrt{30}} & \overline{X}_3 & \overline{Y}_3 \\ X_1 & X_2 & X_3 & \frac{W_\mu^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_\mu^3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{bmatrix}_\mu ,$$



Here $G_{\mu,j}^i, W_{\mu,j}^i, B_\mu$ are the $SU(3) \times SU(2) \times U(1)$ gauge boson fields; $SU(3)$ ($SU(2)$) has been embedded in the first 3 (last 2) rows and columns of V_μ. The 12 new gauge bosons in $SU(5)$ but not in the SM are $X_i, \bar{X}_i, Y_i, \bar{Y}_i$.



The evolution of the gauge couplings below the $SU(5)$ unification scale

Since the down quarks and leptons belong to the same $\bar{5} + 10$ representations the first term leads to a relation between their masses given by

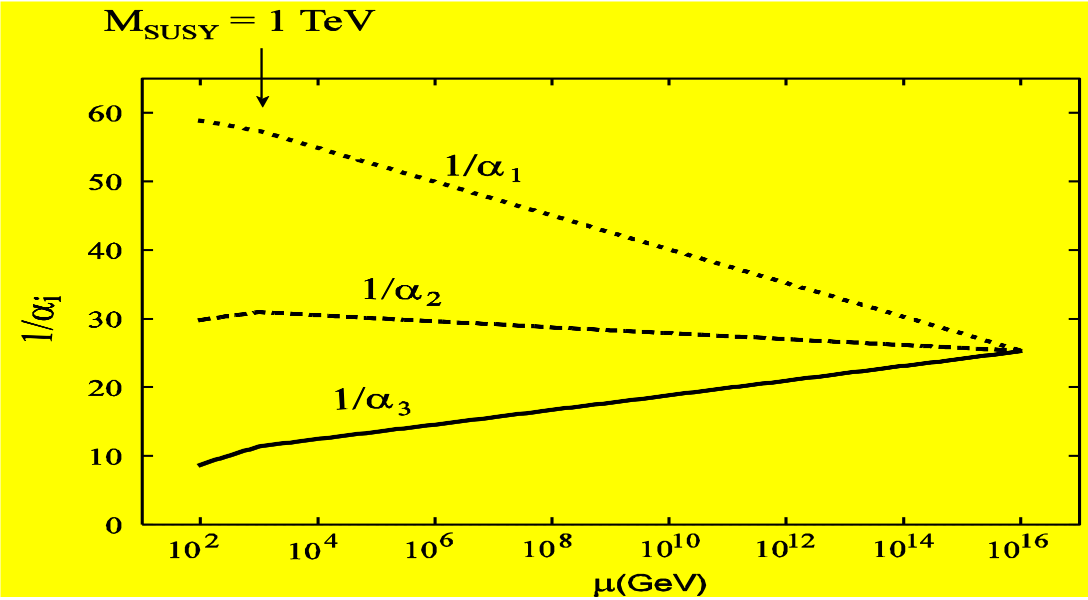
$$m_d = m_e,\ m_s = m_\mu,\ m_b = m_\tau.$$

Supersymmetry relates particles of different spin with equal quantum numbers and identical masses.

$$\begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix} \sim \begin{pmatrix} q\ (quark) \\ \tilde{q}\ (squark) \end{pmatrix} \qquad \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} g\ (gluon) \\ \tilde{g}\ (gluino) \end{pmatrix}$$

Chiral supermultiplet   Gauge supermultiplet

With supersymmetry, the SM gauge couplings are unified at GUT scale $M_G \approx 2 \times 10^{16}$ GeV.



What is Supersymmetry

- **Supersymmetry (SUSY): a symmetry between bosons and fermions.**

$$Q_\alpha|\text{Boson}\rangle = |\text{Fermion}\rangle \qquad Q_\alpha|\text{Fermion}\rangle = |\text{Boson}\rangle$$

- **Introduced in 1973 as a part of an extension of the special relativity.**
- **Super Poincare algebra.**

$$\begin{array}{ll} P_\mu & (translation), \\ M_{\mu,\nu} & (rotation\ and\ Lorentz\ tranformation), \\ Q_\alpha & (SUSY transformation) \end{array}$$

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu)_{\alpha\beta} P_\mu$$

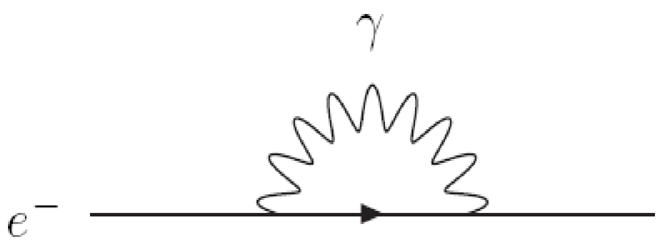
- **SUSY = a translation in Superspace**.

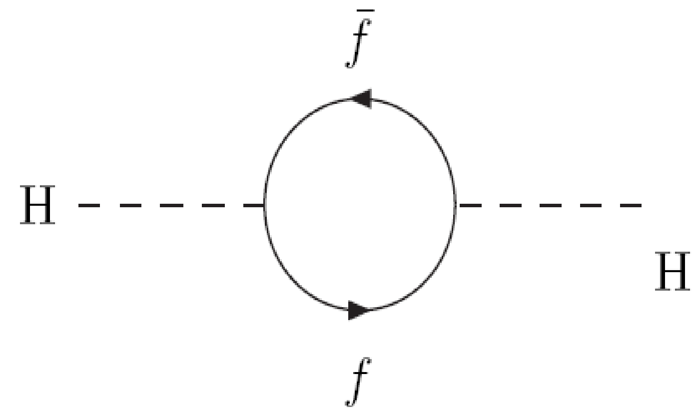
Space-time (x^μ) $\rightarrow$ Superspace (x^μ, θ)

SUSY transformation:

$$\begin{aligned} x^\mu &\rightarrow x'^\mu = x^\mu + \frac{i}{2}\bar{\epsilon}\gamma^\mu\theta \\ \theta &\rightarrow \theta' = \theta + \epsilon \end{aligned}$$

Hierarchy problem





$$\delta m_e = 2\frac{\alpha_{em}}{\pi} m_e \log\frac{\Lambda}{m_e} + \ \ldots$$

$m_e \to 0$, Chiral Symmetry

$$\delta m_H^2(f) = -2N_f \frac{|\lambda_f|^2}{16\pi^2}[\Lambda^2 - 2m_f^2 \ln\frac{\Lambda}{m_f} + \ldots]$$

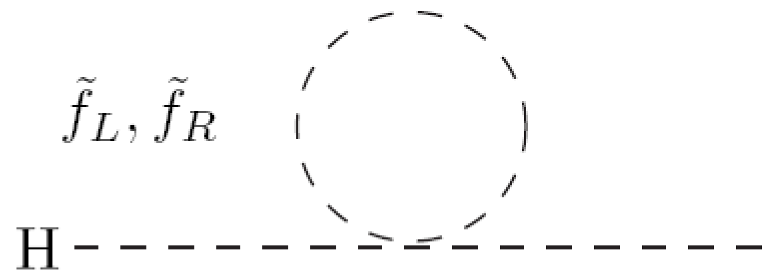
m_H^2 no symmetry, Quadratic div.!!

$$\Lambda = 10^{19}\ \text{GeV}$$

$$\delta m_e \ = \ 0.24\ m_e$$

$$\delta m_H^2 \ \simeq \ 10^{30}\ \text{GeV}$$

Hierarchy problem and SUSY



$$\delta m_H^2(\tilde{f}) = -2N_{\tilde{f}} \frac{\lambda_{\tilde{f}}}{16\pi^2}\left[\Lambda^2 - 2m_{\tilde{f}}^2 \ln \frac{\Lambda}{m_{\tilde{f}}} + \ldots\right]$$

if $N_f = N_{\tilde{f}}$, $|\lambda_f|^2 = -\lambda_{\tilde{f}}$ and $m_f = m_{\tilde{f}} \Rightarrow \delta m_H^2(f) + \delta m_H^2(\tilde{f}) = 0$

Supersymmetry + Chiral symmetry

solve the hierarchy problem.