Higgs Boson Effective Field Theories

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Outline

- EFT generalities
- Standard Model Effective Field Theory (SMEFT)
- Dimension 6 operators
- Some experimental consequences of dim 6 operators
- Renormalization group evolution and consequences
- HEFT and measuring geometry of scalar sector
- LEFT

- An EFT is a quantum field theory with a Lagrangian

$$L = L_{d\leq 4} + \frac{1}{\Lambda} L_5 + \frac{1}{\Lambda^2} L_6 \ldots$$

  written as an expansion in powers of operator dimension
- Can compute order by order in powers of $1/\Lambda$ including loop corrections
- Manifest power counting in $1/\Lambda$, i.e. amplitudes to $1/\Lambda^2$ given by graphs with one insertion of L_6 or two insertions of L_5, etc.
- Finite number of Lagrangian terms to a given order in $1/\Lambda$
- Usual "renormalizable" theories correspond to $\Lambda \to \infty$.
- Historically: weak interactions (Fermi theory) and strong interactions (chiral perturbation theory) — this is how you discover the structure of new interactions

Can compute amplitudes as an expansion in powers of E/Λ without knowing the UV theory.

e.g. Weinberg-Tomozawa formula for πN scattering lengths (1966) many years before QCD.

EFTs have been used fruitfully since the 1960's and allowed us to figure out the SM. Used today in HQET, ChPT, NRQCD, SCET, . . .

Technique is well-known and is being applied to SMEFT.

SM is a renormalizable QFT. It gives a good description of the interactions of all SM particles, including the Higgs Boson h, up to an energy scale $E \sim 1$ TeV. SM is a great success!

A renormalizable QFT makes sense up to arbitrarily high energies $E \to \infty$. It contains no New Physics scale Λ indicating that it will need to be replaced by a new, more fundamental renormalizable theory.

Why consider non-renormalizable (EFT) interactions for SM particles, including the Higgs Boson?

- HEP experiments (high-energy and high-intensity) can
  1. (1) produce any accessible new particles (direct discovery).
  2. (2) probe new interactions of observed SM particles resulting from exchange of unobserved non-SM particles at an inaccessible higher energy scale Λ (indirect discovery).

  HEP pursues both (1) and (2).

- Current state of experimental HEP:
  - Discovery of Higgs boson h at LHC.
  - All couplings of h measured to date are consistent with SM predictions within current error bars.
  - No discovery of (or evidence for) BSM particles. Instead, more stringent bounds on BSM particles from LHC 13 TeV.
  - No discovery (5σ) of new BSM interactions in rare processes.

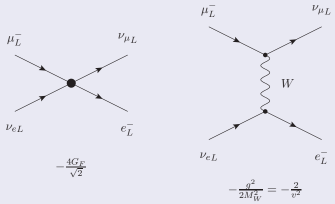
- An important goal of Experimental HEP Program

  Measurement of all couplings of h.

  Experiment needs to rule in/out SM couplings for h
  i.e. test the single Higgs doublet mechanism for EW symmetry breaking.
- Important to consider EFT generalizations of the SM with additional parameters in order to quantify the accuracy to which the SM is valid or to detect deviations from the SM consistent with constraints of QFT.
- A general model-independent analysis of EWSB can be performed using EFTs.
- A given UV theory has specific predictions for Lagrangian coefficients c_i — EFT treats them as unknown parameters consistent with power counting in E/Λ.

Indirect Discovery → Direct Discovery

Fermi Theory



μ decay gives v.

The energy dependence of parity violation in e scattering from γZ interference determines M_Z.

$$\frac{g^2}{p^2 - M_Z^2} = -\frac{g^2}{M_Z^2}\left[1 + \frac{p^2}{M_Z^2} + \ldots\right]$$

$c_6 \sim g^2/M_Z^2$, $c_8 \sim (g^2/M_Z^4)$ allows one to determine both g and M_Z.

There are 2 main EFT generalizations of SM Higgs Sector:

SMEFT = Standard Model Effective Field Theory

HEFT = Higgs Effective Field Theory

SM $\subset$ SMEFT $\subset$ HEFT

SMEFT assumes EWSB linearly realized — same as SM.

Scalar field which breaks EW gauge symmetry is one complex scalar Higgs doublet $H(x)$.

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1(x) + i\varphi_2(x) \\ h(x) + i\varphi_3(x) \end{pmatrix} \equiv \begin{pmatrix} \varphi^+(x) \\ \frac{1}{\sqrt{2}}\left(h(x) + i\varphi^0(x)\right) \end{pmatrix}$$

4 real scalar fields = 3 Goldstone bosons + Higgs boson

$\varphi^+(x), \varphi^-(x), \varphi^0(x)$ $\qquad h(x)$

Spontaneous EWSB by $\langle H(x) \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

HEFT assumes EWSB non-linearly realized — like χPT

[Warning: not the HEFT that was discussed yesterday for $gg \to h$]

There are two distinct scalar fields:
singlet $h(x)$ and triplet (adjoint) $(\varphi_1(x), \varphi_2(x), \varphi_3(x))$

$$U(\Phi(x)) = \exp\left[\frac{2i\,\Phi(x)}{v}\right],$$

$$\Phi(x) \equiv \frac{\sigma}{2}\cdot\varphi(x) = \frac{1}{2}\begin{pmatrix} \varphi_3(x) & \varphi_1(x) - i\varphi_2(x) \\ \varphi_1(x) + i\varphi_2(x) & -\varphi_3(x) \end{pmatrix}$$

Use polar coordinates, h is the radial direction, and U parameterizes angular directions.

SMEFT — relation between radial and angular directions since $H(x)$ is a single field on which $SU(2)$ acts linearly. Special case of HEFT.

SMEFT

EFT generalization of SM valid for energies $E < \Lambda$.

- Gauge symmetry: $SU(3) \times SU(2)_L \times U(1)_Y$
- SM Fields: H^i, G_μ^A, W_μ^I, B_μ, l^{ip}, q^{aip}, u^{ap}, d^{ap}, e^p
- Lagrangian:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \cdots ,$$

$$\mathcal{L}^{(5)} \propto \frac{1}{\Lambda},\ \mathcal{L}^{(6)} \propto \frac{1}{\Lambda^2}, \cdots$$

$d \leq 4$ operators: $\mathcal{L} = \mathcal{L}_{\text{SM}}$

$d > 4$ operators: all independent gauge and Lorentz invariant operators constructed from SM fields

Gives model-independent description (for energies $E < \Lambda$) of any high-energy theory with new inaccessible particles of mass $M \sim \Lambda$.

Field Redefinitions

S matrix elements are invariant under field redefinitions (e.g. Politzer 1984)

Can use this to eliminate certain terms in the Lagrangian

Used in the SM to diagonalize the kinetic and mass terms

In an EFT, more freedom to make field redefinitions since it is okay to introduce higher dimension operators, e.g.

$$\phi(x) \to \phi(x) + \frac{c}{\Lambda^2} D^2 \phi(x) + \ldots$$

SM Lagrangian

$$\mathcal{L}_{\rm SM} = -\frac{1}{4} G^A_{\mu\nu} G^{A\,\mu\nu} - \frac{1}{4} W^I_{\mu\nu} W^{I\,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$+ \sum_{\psi=q,l,u,d,e} \overline{\psi} i \not{D} \psi + \left(D_\mu H^\dagger\right)\left(D^\mu H\right) - \lambda \left(H^\dagger H - \frac{1}{2} v^2\right)^2$$

$$-\left([\bar{u}\, Y_u\, q_i\,]\, \widetilde{H}^{\dagger\, i} + [\bar{d}\, Y_d\, q_i\,]\, H^{\dagger\, i} + [\bar{e}\, Y_e\, l_i\,]\, H^{\dagger\, i} + \text{h.c.}\right)$$

$$D_\mu = \partial_\mu + i g_3 T^A G^A_\mu + i g T^I W^I_\mu + i g' Y B_\mu$$

SMEFT

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(d\leq 4)} + \mathcal{L}_{\not{L}}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}_{\not{L},\not{B}}^{(6)} + \dots$$

$$\mathcal{L}_{\not{L}}^{(5)} \propto \frac{1}{\Lambda_{\not{L}}} \qquad \mathcal{L}^{(6)} \propto \frac{1}{\Lambda^2} \qquad \mathcal{L}_{\not{L},\not{B}}^{(6)} \propto \frac{1}{\Lambda_{\not{L},\not{B}}^2}$$

Λ's do not have to be the same.

Here focus on baryon and lepton number conserving interactions $\mathcal{L}^{(6)}$

SMEFT: Dim 6

$d = 6$: $\Delta B = \Delta L = 0$ operators Leading Operators

$$\mathcal{L}^{(6)} = \sum_i \frac{C_i}{\Lambda^2} O_i^{(6)}$$

76 Hermitian operators for $n_g = 1$ 2499 for $n_g = 3$

Operators divide into 8 operator classes based on field content.

$$X = G^A_{\mu\nu}, W^I_{\mu\nu}, B_{\mu\nu} \qquad \psi = q, l, u, d, e \qquad D = D_\mu$$

$1 : X^3$ $2 : H^6$ $3 : H^4 D^2$ $4 : X^2 H^2$

$5 : \psi^2 H^3$ $6 : \psi^2 XH$ $7 : \psi^2 H^2 D$ $8 : \psi^4$

Buchmuller & Wyler (1986), Grzadkowski, Iskrzynski, Misiak and Rosiek (2010)

Warsaw basis eliminates redundant operators.

SMEFT: Warsaw Basis

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

Buchmuller & Wyler (1986)

Grzadkowski, Iskrzynski, Misiak and Rosiek (2010)

SMEFT: Warsaw Basis

$8 : \psi^4 = JJ = J_L J_L,\ J_R J_R,\ J_L J_R,\qquad (\bar{L}R)(\bar{R}L) + \text{h.c.},\qquad (\bar{L}R)(\bar{L}R) + \text{h.c.}$

$8 : (\bar{L}L)(\bar{L}L)$		$8 : (\bar{R}R)(\bar{R}R)$		$8 : (\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8 : (\bar{L}R)(\bar{R}L) + \text{h.c.}$		$8 : (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

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SMEFT: Warsaw Basis

All dim 6 operators are $1/\Lambda^2$ and are equally important. They mix under renormalization.

It is not correct to drop some of them.

Even if one is looking for, say, a triple gauge boson coupling, other operators enter.

Final state measured is not W, Z, but fermions.

SMEFT: Warsaw Basis

$$1: X^3 \quad 2: H^6 \quad 3: H^4 D^2 \quad 4: X^2 H^2$$

$$5: \psi^2 H^3 \quad 6: \psi^2 XH \quad 7: \psi^2 H^2 D$$

$$8: \psi^4$$

- ψ^0: Operators in classes 1,2,3,4 are flavor singlet
- ψ^2: Operators in classes 5,6,7 have two flavor indices p, r
- ψ^4: Operators in class 8 have four flavor indices p, r, s, t

$n_g = 1, 2, 3 \Rightarrow$ number of $d = 6$ operator coefficients is large!

For Higgs Couplings, focus on ψ^0 and ψ^2 operators.

SM parameters modified at tree-level in spontaneously broken SMEFT by $\mathcal{O}(v^2/\Lambda^2)$ corrections.

Cannot take the usual PDG values

[Here $C_i \propto 1/\Lambda^2$]

The Higgs potential is modified

$$V\left(H^\dagger H\right) = \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - C_H\left(H^\dagger H\right)^3$$

minimum at

$$\langle H^\dagger H\rangle = \frac{1}{2}v_T^2, \qquad v_T = \left(1 + \frac{3C_H v^2}{8\lambda}\right)v$$

KE terms

$$\mathcal{L} = \left(D_\mu H^\dagger\right)\left(D^\mu H\right) + C_{H\Box}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) + C_{HD}\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$$

Normalization of Higgs boson KE term after SSB necessitates rescaling of h field. In unitary gauge,

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \left[1 + c_{H,\mathrm{kin}}\right] h + v_T \end{pmatrix}, \qquad c_{H,\mathrm{kin}} = \left(C_{H\Box} - \frac{1}{4}C_{HD}\right) v^2$$

After rescaling, KE and potential terms yield h self-interactions

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - \frac{c_{H,\text{kin}}}{v_T^2}\left[h^2(\partial_\mu h)^2 + 2vh(\partial_\mu h)^2\right]$$
$$- \lambda v_T^2\left(1 - \frac{3C_H v^2}{2\lambda} + 2c_{H,\text{kin}}\right)h^2$$
$$- \lambda v_T\left(1 - \frac{5C_H v^2}{2\lambda} + 3c_{H,\text{kin}}\right)h^3$$
$$- \frac{1}{4}\lambda\left(1 - \frac{15C_H v^2}{2\lambda} + 4c_{H,\text{kin}}\right)h^4 + \frac{3}{4}C_H v\, h^5 + \frac{1}{8}C_H\, h^6$$

The Higgs boson mass is

$$m_H^2 = 2\lambda v_T^2\left(1 - \frac{3C_H v^2}{2\lambda} + 2c_{H,\text{kin}}\right)$$

$$\mathcal{L} = -\left[H^{\dagger j}\overline{d}_r\,[Y_d]_{rs}\,q_{js} + \widetilde{H}^{\dagger j}\overline{u}_r\,[Y_u]_{rs}\,q_{js} + H^{\dagger j}\overline{e}_r\,[Y_e]_{rs}\,l_{js} + \text{h.c.}\right]$$
$$+\left[C^*_{\substack{dH\\sr}}\left(H^\dagger H\right)H^{\dagger j}\overline{d}_r q_{js} + C^*_{\substack{uH\\sr}}\left(H^\dagger H\right)\tilde{H}^{\dagger j}\overline{u}_r q_{js} + C^*_{\substack{eH\\sr}}\left(H^\dagger H\right)H^{\dagger j}\overline{e}_r l_{js}\right.$$

Fermion mass matrices

$$[M_\psi]_{rs} = \frac{v_T}{\sqrt{2}}\left([Y_\psi]_{rs} - \frac{1}{2}v^2 C^*_{\substack{\psi H\\sr}}\right), \qquad \psi = u, d, e$$

The coupling matrices of the h boson to the fermions $\mathcal{L} = -h\,\overline{u}\,\mathcal{Y}\,q + \ldots$ are

$$[\mathcal{Y}_\psi]_{rs} = \frac{1}{\sqrt{2}}[Y_\psi]_{rs}\left[1 + c_{H,\text{kin}}\right] - \frac{3}{2}v^2 C^*_{\substack{\psi H\\sr}}$$
$$= \frac{1}{v_T}[M_\psi]_{rs}\left[1 + c_{H,\text{kin}}\right] - v^2 C^*_{\substack{\psi H\\sr}}, \qquad \psi = u, d, e$$

and are not simply proportional to the fermion mass matrices, as is the case in the SM.

$$\mathcal{L}_{G_F} = -\frac{4\mathcal{G}_F}{\sqrt{2}}\,(\bar{\nu}_\mu\,\gamma^\mu P_L \mu)\,(\bar{e}\,\gamma_\mu P_L \nu_e)\,.$$

$$-\frac{4\mathcal{G}_F}{\sqrt{2}} = -\frac{2}{v_T^2} + \left(C_{\substack{ll\\ \mu e e \mu}} + C_{\substack{ll\\ e\mu\mu e}}\right) - 2\left(C^{(3)}_{\substack{Hl\\ ee}} + C^{(3)}_{\substack{Hl\\ \mu\mu}}\right).$$

The C_{ll} terms are from the four-lepton interaction in $\mathcal{L}^{(6)}$, and the $C^{(3)}_{Hl}$ terms are from W exchange, where one $W\bar{l}\nu$ vertex is from the $Q^{(3)}_{Hl}$ operator, and the other is the usual SM vertex.

$$\mathcal{L}^{(6)} = C_{HG} H^\dagger H G^A_{\mu\nu} G^{A\mu\nu} + C_{HW} H^\dagger H W^I_{\mu\nu} W^{I\mu\nu} + C_{HB} H^\dagger H B_{\mu\nu} B^{\mu\nu}$$
$$+ C_{HWB} H^\dagger \tau^I H W^I_{\mu\nu} B^{\mu\nu} + C_G f^{ABC} G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho + C_W \epsilon^{IJK} W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$$

In the broken theory, the $X^2 H^2$ operators contribute to the gauge kinetic energies,

$$\mathcal{L}_{SM} + \mathcal{L}^{(6)} = -\frac{1}{2} W^+_{\mu\nu} W^{\mu\nu}_- - \frac{1}{4} W^3_{\mu\nu} W^{\mu\nu}_3 - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$
$$+ \frac{1}{2} v_T^2 C_{HG} G^A_{\mu\nu} G^{A\mu\nu} + \frac{1}{2} v_T^2 C_{HW} W^I_{\mu\nu} W^{I\mu\nu} + \frac{1}{2} v_T^2 C_{HB} B_{\mu\nu} B^{\mu\nu}$$
$$- \frac{1}{2} v_T^2 C_{HWB} W^3_{\mu\nu} B^{\mu\nu}.$$

Gauge fields are not canonically normalized, and there is kinetic mixing between W^3 and B. Gauge boson mass terms

$$\mathcal{L} = \frac{1}{4} g_2^2 v_T^2 W^+_\mu W^{-\,\mu} + \frac{1}{8} v_T^2 (g_2 W^3_\mu - g_1 B_\mu)^2$$
$$+ \frac{1}{16} v_T^4 C_{HD} (g_2 W^3_\mu - g_1 B_\mu)^2$$

Modified coupling constants are

$$\overline{g}_3 = g_3 \left(1 + C_{HG}\, v_T^2\right), \qquad \overline{g}_2 = g_2 \left(1 + C_{HW}\, v_T^2\right),$$
$$\overline{g}_1 = g_1 \left(1 + C_{HB}\, v_T^2\right).$$

Gauge boson masses

$$M_W^2 = \frac{\overline{g}_2{}^2 v_T^2}{4}$$
$$M_Z^2 = \frac{v_T^2}{4}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{8} v_T^4 C_{HWB}(\overline{g}_1{}^2 + \overline{g}_2{}^2) + \frac{1}{2} v_T^4 \overline{g}_1 \overline{g}_2 C_{HD}$$

The covariant derivative is

$$D_\mu = \partial_\mu + i \frac{\overline{g}_2}{\sqrt{2}} \left[\mathcal{W}_\mu^+ T^+ + \mathcal{W}_\mu^- T^-\right] + i \overline{g}_Z \left[T_3 - \overline{s}^2 Q\right] \mathcal{Z}_\mu + i\, \overline{e}\, Q\, \mathcal{A}_\mu$$

with effective couplings

$$\bar{e} = \frac{\bar{g}_1 \bar{g}_2}{\sqrt{\bar{g}_2{}^2 + \bar{g}_1{}^2}} \left[1 - \frac{\bar{g}_1 \bar{g}_2}{\bar{g}_2{}^2 + \bar{g}_1{}^2} v_T^2 C_{HWB} \right]$$

$$= \bar{g}_2 \sin \bar{\theta} - \frac{1}{2} \cos \bar{\theta} \, \bar{g}_2 \, v_T^2 \, C_{HWB},$$

$$\bar{g}_Z = \sqrt{\bar{g}_2{}^2 + \bar{g}_1{}^2} + \frac{\bar{g}_1 \bar{g}_2}{\sqrt{\bar{g}_2{}^2 + \bar{g}_1{}^2}} v_T^2 C_{HWB}$$

$$\sin \bar{\theta} \equiv \bar{s} = \frac{\bar{g}_1}{\sqrt{\bar{g}_1^2 + \bar{g}_2^2}} \left[1 + \frac{1}{2} \frac{\bar{g}_2}{\bar{g}_1} \left(\frac{\bar{g}_2^2 - \bar{g}_1^2}{\bar{g}_1^2 + \bar{g}_2^2} \right) v_T^2 C_{HWB} \right]$$

$$\bar{g}_Z = \frac{\bar{e}}{\sin \bar{\theta} \cos \bar{\theta}} \left[1 + \frac{1}{2} v_T^2 C_{HWB} \right]$$

$\psi^2 H^2 D$ operators modify fermion currents coupled to $W^\pm$ and Z at tree level.

$$\begin{aligned} j_{\mathcal{W}}^{\mu} &= [W_l]_{pr}\overline{\nu}_{Lp}\gamma^\mu e_{Lr} + [W_q]_{pr}\overline{u}_{Lp}\gamma^\mu d_{Lr} + [W_R]_{pr}\overline{u}_{Rp}\gamma^\mu d_{Rr}, \\ j_Z^{\mu} &= [Z_{\nu_L}]_{pr}\overline{\nu}_{Lp}\gamma^\mu \nu_{Lr} + [Z_{e_L}]_{pr}\overline{e}_{Lp}\gamma^\mu e_{Lr} + [Z_{e_R}]_{pr}\overline{e}_{Rp}\gamma^\mu e_{Rr} \\ &\quad + [Z_{u_L}]_{pr}\overline{u}_{Lp}\gamma^\mu u_{Lr} + [Z_{u_R}]_{pr}\overline{u}_{Rp}\gamma^\mu u_{Rr} + [Z_{d_L}]_{pr}\overline{d}_{Lp}\gamma^\mu d_{Lr} \\ &\quad + [Z_{d_R}]_{pr}\overline{d}_{Rp}\gamma^\mu d_{Rr}, \end{aligned}$$

where

$$[W_l]_{pr} = \left[\delta_{pr} + v_T^2 C^{(3)}_{\substack{Hl \\ pr}}\right], \qquad [W_q]_{pr} = \left[\delta_{pr} + v_T^2 C^{(3)}_{\substack{Hq \\ pr}}\right],$$

$$[W_R]_{pr} = \left[\frac{1}{2} v_T^2 C_{\substack{Hud \\ pr}}\right], \qquad \text{right-handed currents}$$

$$[Z_{\nu_L}]_{pr} = \left[\delta_{pr}\left(\frac{1}{2}\right) - \frac{1}{2} v_T^2 C^{(1)}_{\substack{Hl \\ pr}} + \frac{1}{2} v_T^2 C^{(3)}_{\substack{Hl \\ pr}}\right],$$

...

- RG evolution of $d = 6$ operator coefficients produces interesting mixing between different $d = 6$ operators and between $d = 6$ operators and SM operators. Need to do RG evolution for precision EW observables and precision Higgs boson couplings.
- RG running of SM parameters due to $d = 6$ operators. All one-loop corrections calculated.
- RG running of $d = 6$ operators. Full one-loop anomalous dimension matrix γ_{ij} for $d = 6$ operators computed. Formulae are many pages.
- RG mixing of $d = 6$ operators gives nontrivial implications for flavor structure of BSM physics.

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- RGE of SM parameters receive corrections $O(m_H^2/\Lambda^2)$ from $d = 6$ operators. For SM EFT, this running is just as important as running of $\mathcal{L}^{(6)}$.

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \lambda = \frac{m_H^2}{16\pi^2} \left[12 C_H - 32\lambda\, C_{H\Box} + 12\lambda\, C_{HD} + \cdots \right] ,$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} m_H^2 = \frac{m_H^4}{16\pi^2} \left[-4 C_{H\Box} + 2 C_{HD} \right] ,$$

$$\mu \frac{\mathrm{d}g_3}{\mathrm{d}\mu} = \frac{m_H^2}{16\pi^2} g_3 C_{HG} , \quad \mu \frac{\mathrm{d}g_2}{\mathrm{d}\mu} = \frac{m_H^2}{16\pi^2} g_2 C_{HW} , \quad \mu \frac{\mathrm{d}g_1}{\mathrm{d}\mu} = \frac{m_H^2}{16\pi^2} g_1 C_{HB} ,$$

Also for Yukawa couplings Y_e, Y_u, Y_d

- Very non-trivial flavor mixing in SMEFT.
- Strong bounds on some $d = 6$ operators, such as dipole operators. Mixing of other $d = 6$ operators into these dipole operators can yield strong constraints on flavor structure of SMEFT.
- Flavor-singlet portion of flavor non-singlet operators can mix into flavor-singlet operators.
- Interesting cancellations occur.
- Holomorphy appears for some operators, such as the dipole operators.

$\mu \to e\gamma$ and μ, e electric and magnetic dipole moments

$$\mathscr{C}_{\substack{e\gamma\\rs}} = \frac{1}{g_1} C_{\substack{eB\\rs}} - \frac{1}{g_2} C_{\substack{eW\\rs}}, \qquad \mathcal{L} = \frac{ev}{\sqrt{2}} \mathscr{C}_{\substack{e\gamma\\rs}}\, \overline{e}_r \sigma^{\mu\nu} P_R e_s F_{\mu\nu} + h.c.$$

where r and s are flavor indices $(\{e_1\,,\, e_2\,,\, e_3\} \equiv \{e\,,\, \mu\,,\, \tau\})$

$$\begin{aligned}
\dot{\mathscr{C}}_{\substack{e\gamma\\rs}} &= \left\{ Y(s) + e^2 \left(12 - \frac{9}{4}\csc^2\theta_W + \frac{1}{4}\sec^2\theta_W \right) \right\} \mathscr{C}_{\substack{e\gamma\\rs}} \\
&+ 2\mathscr{C}_{\substack{e\gamma\\rv}} [Y_e Y_e^\dagger]_{vs} + \left(\frac{1}{2} + 2\cos^2\theta_W \right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\substack{e\gamma\\ws}} + e^2 \left(12 \cot 2\theta_W\right) \mathscr{C}_{\substack{eZ\\rs}} \\
&- \left(2\sin\theta_W \cos\theta_W\right) [Y_e^\dagger Y_e]_{rw} \mathscr{C}_{\substack{eZ\\ws}} - \cot\theta_W [Y_e^\dagger]_{rs} \left(C_{HWB} + i C_{H\widetilde{W}B} \right) \\
&+ \frac{8}{3} e^2 [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma\gamma} + i\widetilde{\mathscr{C}}_{\gamma\gamma} \right) + e^2 \left(\cot\theta_W - \frac{5}{3}\tan\theta_W \right) [Y_e^\dagger]_{rs} \left(\mathscr{C}_{\gamma Z} + i\widetilde{\mathscr{C}}_{\gamma Z} \right) \\
&+ 16 [Y_u]_{wv} C^{(3)}_{\substack{lequ\\rsvw}}
\end{aligned}$$

Constraints at $\Lambda \sim 1 - 10$ TeV level from $\mu \to e\gamma$, EDM, $g-2$, etc.

HEFT

HEFT: generalization of SM valid for energies $E < \Lambda = 4\pi f$.

- Lagrangian: $\mathcal{L}_{\rm HEFT}\left[U\left(\varphi(x)\right), h, D_\mu, G^A_\mu, W^I_\mu, B_\mu, \psi_{\rm SM}\right]$
- Hybrid of $\mathcal{L}_\chi\left[U\left(\varphi(x)\right)\right]$ for $[SU(2)_L \times SU(2)_R \to SU(2)_V]$ and EFT for $\frac{h}{v}$ $\quad\Rightarrow\quad \mathcal{F}(h) \equiv 1 + c_1\frac{h}{v} + c_2\frac{h^2}{v^2} + c_3\frac{h^3}{v^3} + \cdots$

$$SU(2)_L \times U(1)_Y \to U(1)_Q \quad \text{gauge}$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{G^2}(h)\, G^A_{\mu\nu}G^{A\,\mu\nu} - \frac{1}{4}\mathcal{F}_{W^2}(h)\, W^I_{\mu\nu}W^{I\,\mu\nu} - \frac{1}{4}\mathcal{F}_{B^2}(h)\, B_{\mu\nu}B^{\mu\nu}$$

$$+\bar{\psi} i \not{D}\psi + \frac{1}{2}\partial^\mu h\,\partial_\mu h + \frac{v^2}{4}\mathcal{F}_{V^2}(h)\,{\rm Tr}\left(D_\mu U\right)U^\dagger\left(D^\mu U\right)U^\dagger - V(h)$$

$$-\frac{v}{\sqrt{2}}\left(\left[\bar{u}\,\mathcal{F}_{\mathcal{Y}_u}(h)\,U^\dagger q\right] + \left[\bar{d}\,\mathcal{F}_{\mathcal{Y}_d}(h)\,U^\dagger q\right] + \left[\,\bar{e}\,\mathcal{F}_{\mathcal{Y}_e}(h)\,U^\dagger l\right] + \text{h.c.}\right)$$

$$D_\mu U\left(\varphi(x)\right) = \partial_\mu U + igW^I_\mu T^I\,U - ig' B_\mu\,U\,T^3,\quad D^\mu h = \partial^\mu h$$

Geometry of the Scalar Sector

Global symmetry $\mathcal{G} = O(4)$

$$\mathcal{L}_{\rm SM} = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{\lambda}{4}\left(\phi \cdot \phi - v^2\right)^2$$

$$\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$$

Minimum at $\langle \phi \cdot \phi \rangle = v^2$ breaks $\mathcal{G} \to \mathcal{H} = O(3)$

In polar coordinates, 1 radial direction and 3 angular directions.

$$\phi = (v + h)\, \boldsymbol{n}(\varphi) \equiv v\, F_{\rm SM}(h)\, \boldsymbol{n}(\varphi)\,, \qquad \boldsymbol{n} \in S^3,\ \boldsymbol{n} \cdot \boldsymbol{n} = 1$$

$$F_{\rm SM}(h) = 1 + \frac{h}{v}$$

$$\mathcal{L}_{\rm KE} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j, \qquad g_{ij}^{\rm SM}(\phi) = \delta_{ij}$$

Scalar manifold $\mathcal{M}$ is flat with zero curvature.

Geometry of the Scalar Sector

- SMEFT and HEFT have non-trivial scalar metric since there are higher dimensional 2-derivative scalar operators $\Rightarrow$ scalar manifold $\mathcal{M}$ is curved.
- Riemann curvature tensor of $\mathcal{M}$ (invariant under scalar field redefinitions) determines physical observables.

$$\mathcal{L}_{\mathrm{KE}} = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i\, \partial^\mu \phi^j = v^2 F^2(h)\, \partial_\mu \boldsymbol{n}(\varphi) \cdot \partial^\mu \boldsymbol{n}(\varphi) + \frac{1}{2} (\partial_\mu h)^2$$

$$F(h) = 1 + c_1 \frac{h}{v} + \frac{1}{2} c_2 \frac{h^2}{v^2} + \cdots \qquad \text{vs.} \qquad F_{\mathrm{SM}}(h) = 1 + \frac{h}{v}$$

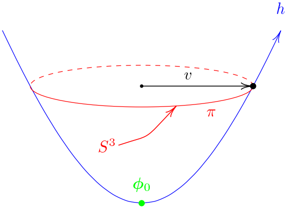
$$g_{ij}(\phi) = \begin{bmatrix} F^2(h) g_{IJ}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$$

where $g_{IJ}(\varphi)$ is metric on $\mathcal{G}/\mathcal{H} = S^3 \subset \mathcal{M}$.
3 angular coordinates of S^3 are φ/v.

SMEFT Polar coordinates

$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2} v^2 \, F(h)^2 \, (\partial_\mu \mathbf{n})^2 \qquad F(0) = 1$$

where $F(h)$ can be computed in terms of $C_{H\Box}$ and C_{HD}. Scalar manifold $\mathcal{M}$ is no longer a flat manifold. (Recover flat scalar manifold of SM for $\Lambda \to \infty$.)

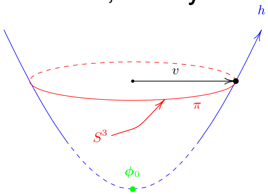


SMEFT has an unbroken symmetry point $F(h_*) = 0$

invariant under $O(4) \supset SU(2) \times U(1)$

$v + h = 0$, i.e. $\phi = 0$, $H = 0$.

For HEFT, h may have no relation to **n**. e.g. Higgs as a dilaton, GB, etc.



HEFT $\mathcal{M}$ looks like SMEFT $\mathcal{M}$, except that there may not be a point where $F(h_*) = 0$ (the green dot) which is $O(4)$ invariant

In SMEFT and HEFT, there is a scale Λ. Physical observables depend on geometric invariants (e.g. Riemann curvature) of scalar manifold.

$$\mathcal{A}\left(W_L W_L \to W_L W_L\right) = -4\lambda + \frac{s+t}{v^2}\,\mathfrak{r}_4\,, \qquad \mathfrak{r}_4 \propto R_{\varphi^a \varphi^b \varphi^c \varphi^d}$$

$$\mathcal{A}\left(W_L W_L \to hh\right) = 2\lambda - \frac{2s}{v^2}\,\mathfrak{r}_{2h}\,. \qquad \mathfrak{r}_{2h} \propto R_{\varphi^a h \varphi^b h}$$

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- Low-energy EFT of SM at low-energy $E \ll M_W$ needed to compare SM predictions with rare decays, flavor mixing.
- Construct Weak EFT of SM at low-energy by integrating out particles $t, W^{\pm}, Z, h$ with masses $\sim v$.
- SMEFT, HEFT $\rightarrow$ LEFT
- Generalizes usual weak interaction analysis to include higher dimension operators from SMEFT, HEFT
- RG equations for LEFT operators from $\mu \sim v$ to $\mu \sim E \ll v$.

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Conclusions

- EFT parametrizes all possible interactions for a given set of fields.
- Effective way to quantify whether h is SM Higgs boson with SM interactions, or parametrize deviations
- Allows for a model-independent analysis
- If deviations are found, then can provide guidance on the scale and structure of new interactions