



Top-quark mass and Yukawa coupling

Y. Sumino
(Tohoku Univ.)

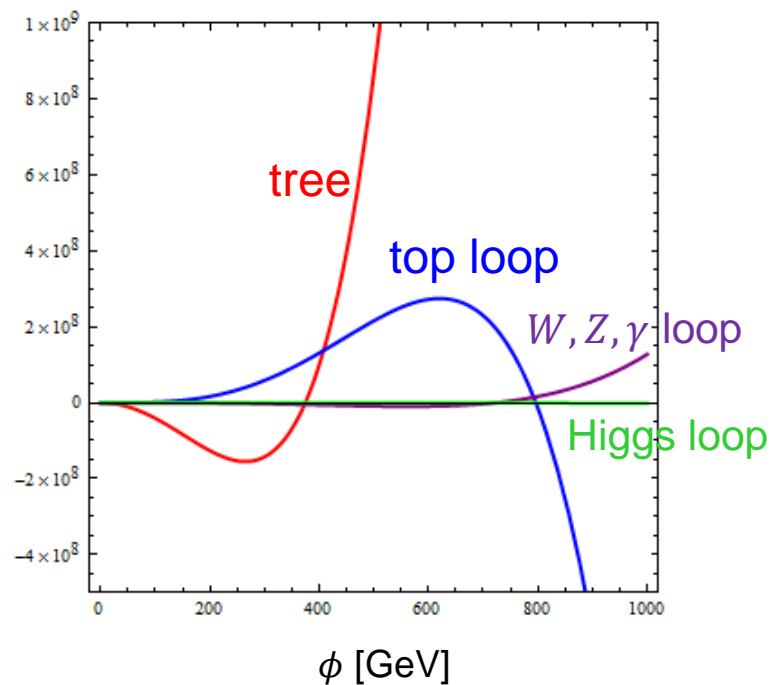


☆ Plan of Talk

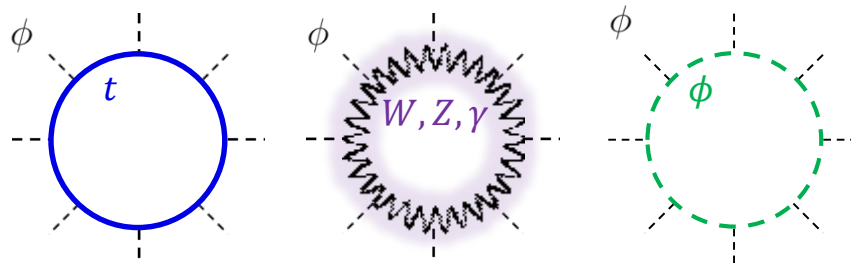
1. What's top mass(+Yukawa coupling)? [[Introduction](#)]
2. Precision top mass determination at **ILC**
3. Top mass at **LHC**: [status](#), [problems](#), [goals](#)
4. Summary and Conclusions

Role of top quark in the SM vacuum structure

Tree+1-loop Higgs potential



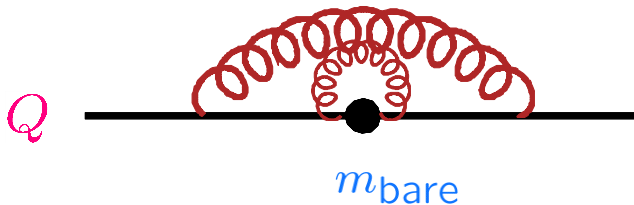
1-loop corrections



Definitions of top-quark mass in pert. QCD

Pole mass m_{pole}

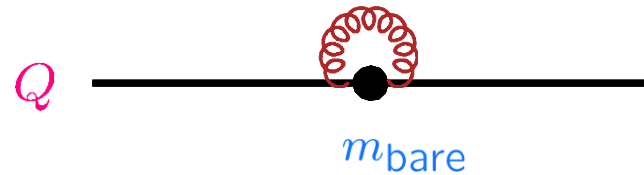
$$0 < \lambda_g < \infty$$



Not defined beyond pert. theory
 Perturbative uncertainty
 $O(\Lambda_{\text{QCD}}) \lesssim 1 \text{ GeV}$

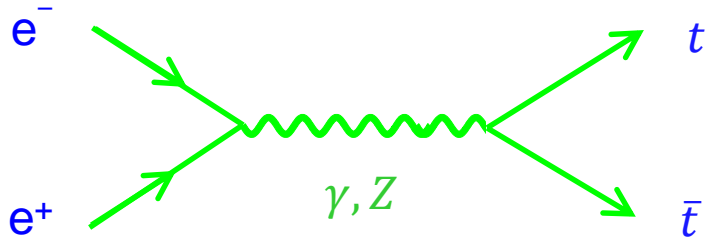
$\overline{\text{MS}}$ mass $\bar{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

$$0 < \lambda_g < 1/\bar{m}$$

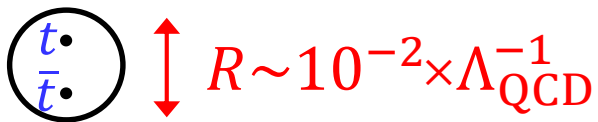


Conceptually close to
 Yukawa coupling at scale $\mu \sim \bar{m}$

Top quarks in collider experiments



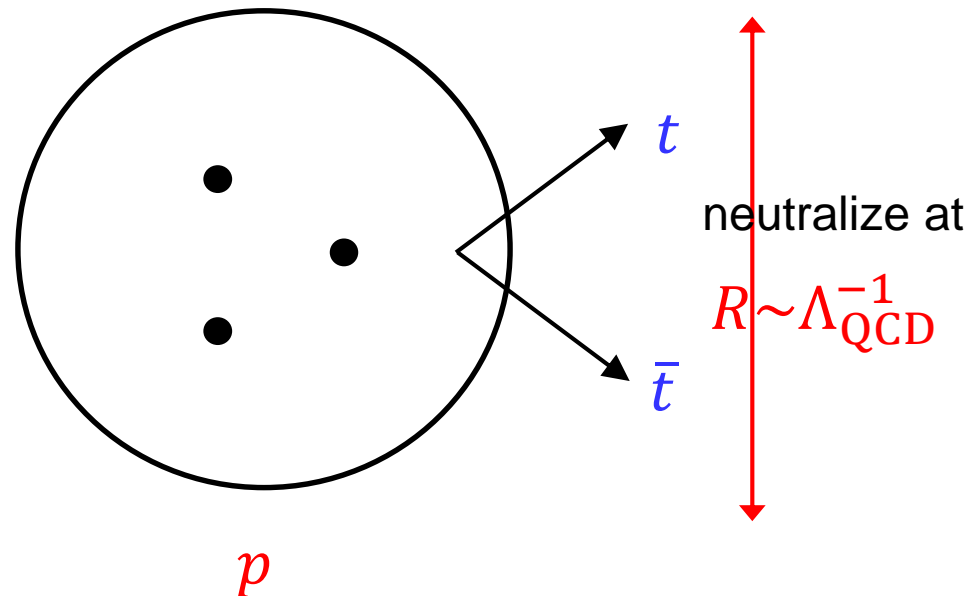
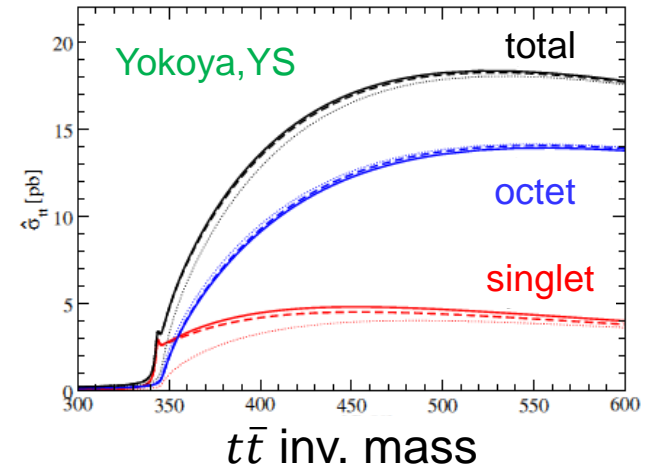
$t\bar{t}$ threshold at **ILC**



color-singlet resonance

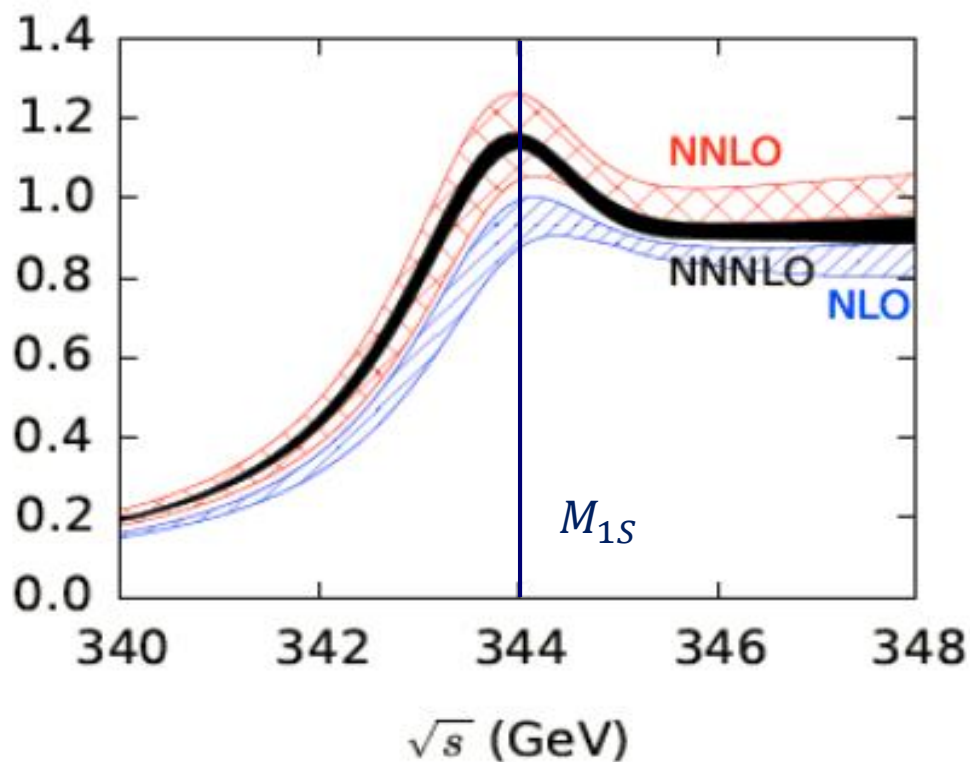
LHC

$t\bar{t}$ prod. cross section

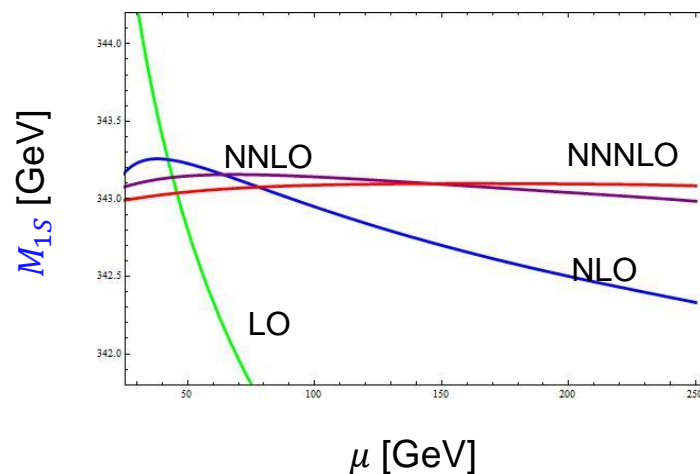
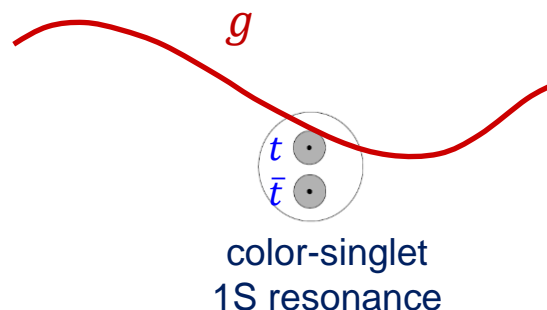


Precise m_t determination near $t\bar{t}$ threshold at e^+e^- collider

After decades of endeavor, $e^+e^- \rightarrow t\bar{t}$ cross section near threshold was computed.



Beneke, Kiyo, Marquard,
Penin, Piclum, Steinhauser



Our estimate: $\Delta m_t^{\overline{\text{MS}}} = 20\text{-}30 \text{ MeV}$

Kiyo, Mishima, YS Relative acc. order 10^{-4}

t

$$I(J^P) = 0(\frac{1}{2}^+)$$

$$\text{Charge} = \frac{2}{3} e$$

$$\text{Top} = +1$$

t-Quark Mass (Direct Measurements)

VALUE (GeV)	DOCUMENT ID	TECN	COMMENT
173.0 ± 0.4 OUR AVERAGE	Error includes scale factor of 1.3. See the ideogram below.		
172.95 ± 0.77 ^{+0.97} _{-0.93}	¹ SIRUNYAN	17L CMS	t-channel single top production
172.84 ± 0.34 ± 0.61	² AABOUD	16T ATLS	combination of ATLAS
172.44 ± 0.13 ± 0.47	³ KHACHATRY...	16AK CMS	combination of CMS
174.30 ± 0.35 ± 0.54	⁴ TEVEWWG	16 TEVA	Tevatron combination
• • • We do not use the following data for averages, fits, limits, etc. • • •			
173.72 ± 0.55 ± 1.01	⁵ AABOUD	17AH ATLS	≥ 5 jets (2b)
174.95 ± 0.40 ± 0.64	⁶ ABAZOV	17B D0	ℓ + jets and dilepton channels
170.8 ± 9.0	⁷ SIRUNYAN	17N CMS	jet mass in highly-boosted t \bar{t}

t-Quark Mass from Cross-Section Measurements

The top quark $\overline{\text{MS}}$ or pole mass can be extracted from a measurement of $\sigma(t\bar{t})$ by using theory calculations. We quote below the $\overline{\text{MS}}$ mass. See the review "The Top Quark" and references therein for more information.

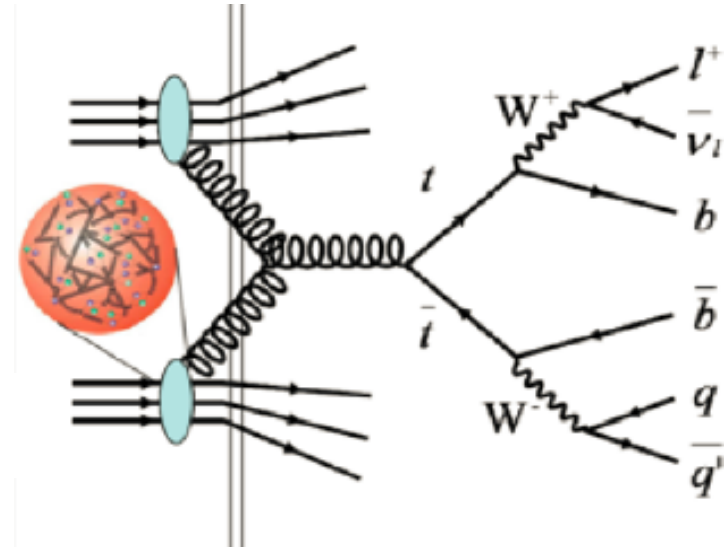
VALUE (GeV)	DOCUMENT ID	TECN	COMMENT
160.0^{+4.8}_{-4.3}	¹ ABAZOV	11s D0	$\sigma(t\bar{t})$ + theory

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t-Quark Pole Mass from Cross-Section Measurements

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173.1 ± 0.9 OUR AVERAGE			
173.2 ± 0.9 ± 0.8 ± 1.2	¹ AABOUD	17BC ATLS	e + μ + ≥ 1b jets
170.6 ± 2.7	² SIRUNYAN	17W CMS	ℓ + ≥ 1j
172.8 ± 1.1 ^{+3.3} _{-3.1}	³ ABAZOV	16F D0	ℓℓ, ℓ+jets channels
173.8 ^{+1.7} _{-1.8}	⁴ KHACHATRY...	16AW CMS	e + μ + \cancel{E}_T + ≥ 0j
173.7 ^{+2.3} _{-2.1}	⁵ AAD	15BWATLS	ℓ + \cancel{E}_T + ≥ 5j (2b-tag)
172.9 ^{+2.5} _{-2.6}	⁶ AAD	14AY ATLS	pp at $\sqrt{s} = 7, 8$ TeV

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- PDF
- hadronization

All masses measured in hadron collider exp.

t

$$I(J^P) = 0(\frac{1}{2}^+)$$

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Particle Data Group 2018

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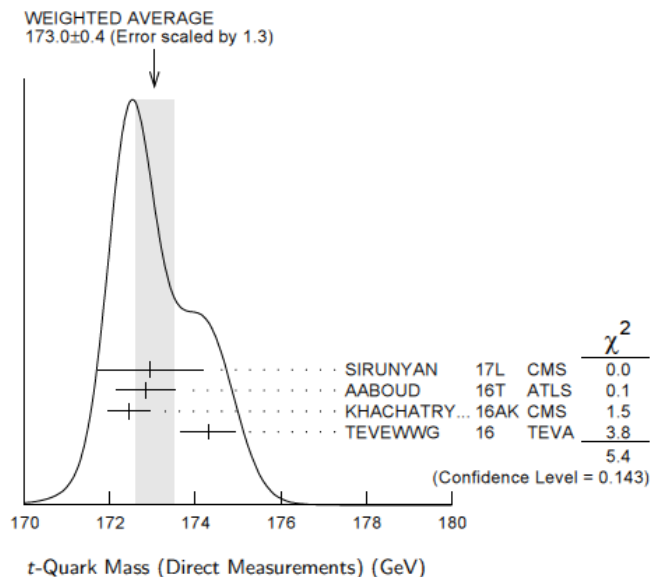
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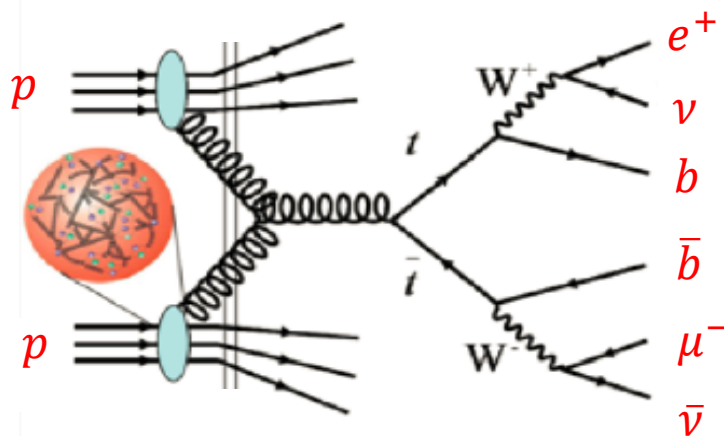
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m_t^{pole} determination at LHC using lepton distributions in dilepton channel



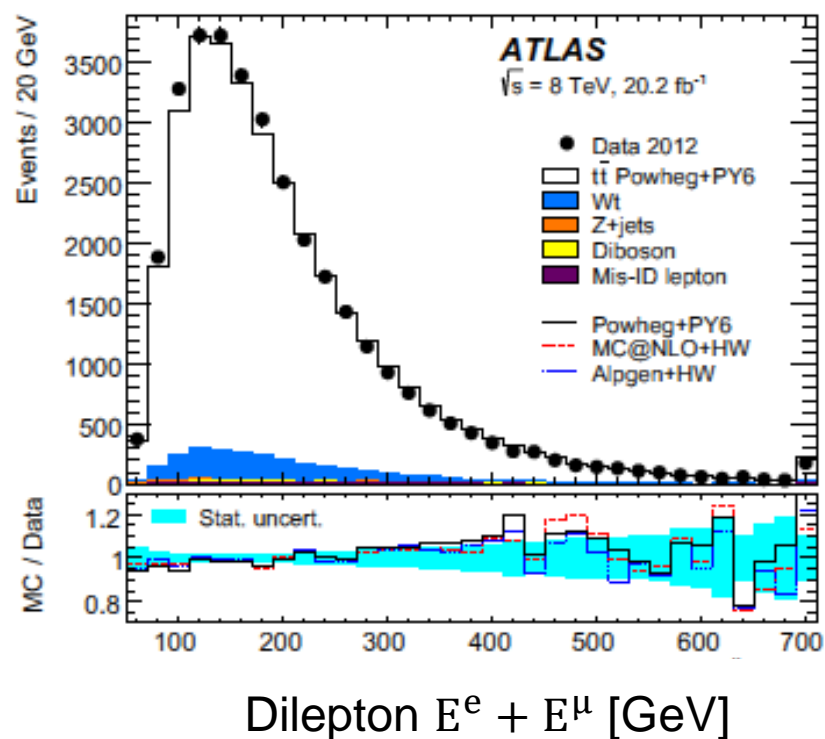
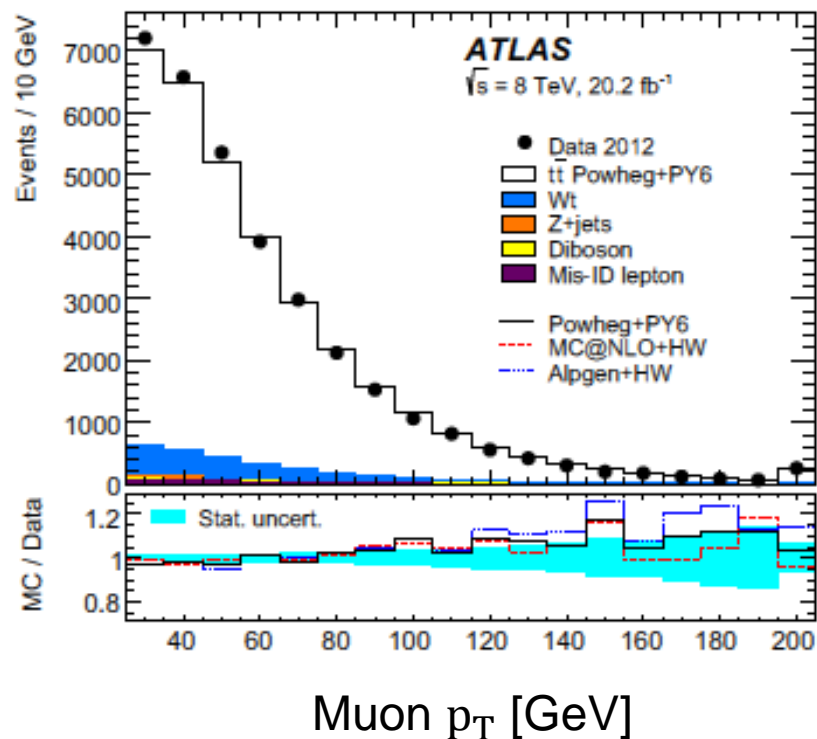
$$\sqrt{s} = 8 \text{ TeV}$$

$$m_t^{\text{pole}} = 173.2 \pm 0.9 \pm 0.8 \pm 1.2 \text{ GeV}$$
$$= 173.2 \pm 1.6 \text{ GeV}$$

The pole mass is extracted from a fit of NLO predictions to 8 lepton differential distributions in dileptonic $t\bar{t}$ events, while simultaneously constraining uncertainties due to PDFs and QCD scales.

Largely indep. of jet profiles
(e.g. Missing p_T cut is not used)

Leptonic observables vs. MC predictions



MC input $m_t = 172.5 \text{ GeV}$

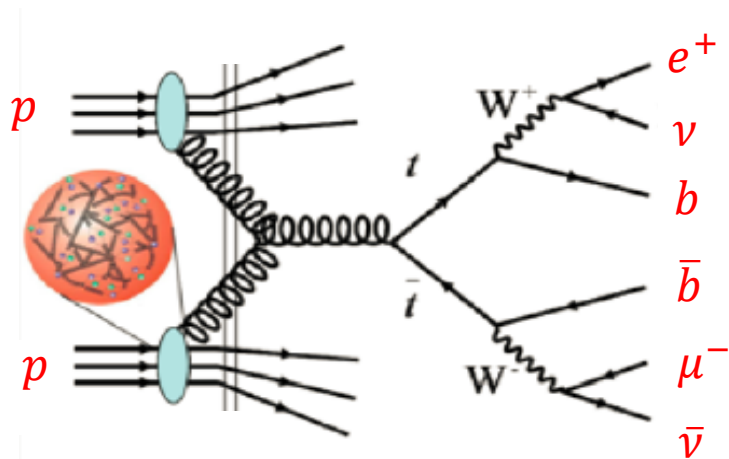
Pert. QCD prediction for $t\bar{t}$ cross sections at LHC

NNLO+NNLL predictions

Czakon, Heymes, Mitov (prod)

Gao, Papanastasiou (prod+decay)

More to come soon.



How to ideally compare with pert. QCD?


Model dependences should be largely eliminated.

hadronization, PDF

- Hadronic inclusive obs. $\sum |had.\rangle \langle had.| = \sum |q, g\rangle \langle q, g| = \mathbf{1}$
(More accurately, OPE formulation desirable.) \Rightarrow Leptonic obs.
- Uncertainty due to PDF can be eliminated (in principle).
 \Rightarrow Weight fn. method Kawabata, Shimizu, YS, Yokoya
- Roles of MC simulation needs to be reconsidered.

OPE formulation (idea)

$$\frac{1}{\sigma} \frac{d\sigma(pp \rightarrow t\bar{t} \rightarrow lvX)}{d\Phi(l)} = \frac{1}{\sigma} \int d\Phi(had) \langle pp|lvX\rangle \langle lvX|pp\rangle$$
$$= \frac{1}{\sigma} \int d\Phi(had) H^{\mu\nu} L_{\mu\nu}$$


 $\int d\Phi(had) H^{\mu\nu}(pp \rightarrow X) =$ Identify OPE in terms of EFT
by integr.-by-region method

Similar to DIS in *ep* collision

- Precise treatment of IR contributions
- Simplification by EFT and inclusive calc.

Summary and Conclusions

- Accurate view on current status of top mass determination is important.
def./assumptions w.r.t. pert. QCD
- $\overline{\text{MS}}$ mass determination at $e^+e^- \rightarrow t\bar{t}$ near threshold is ideal. $O(10^{-4})$ acc.
- At LHC Δm_t much below 1 GeV is currently challenging.
Color charge distribution around octet $t\bar{t}$ introduces unsuppressed/uncontrolled IR physics.
- For determination of well-defined top mass at LHC, use of leptonic obs. combined with NNLO prediction analysis is optimal.
PDF indep. property can be exploited in addition. *(Weight fn. method)*
OPE is desirable to control IR contributions precisely.

⇒ Model indep. analysis

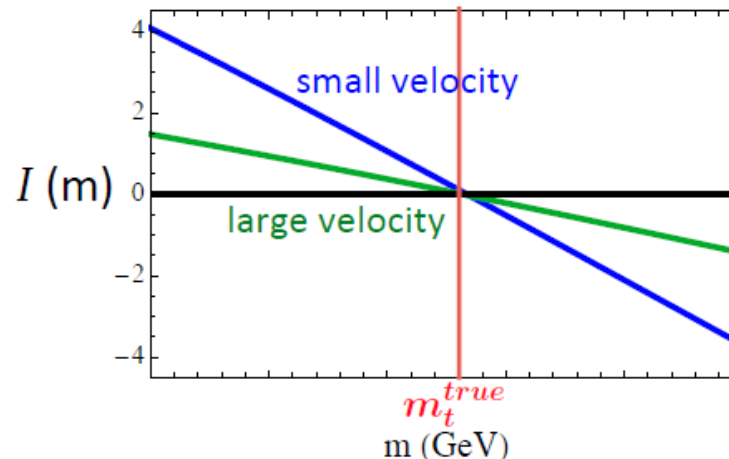
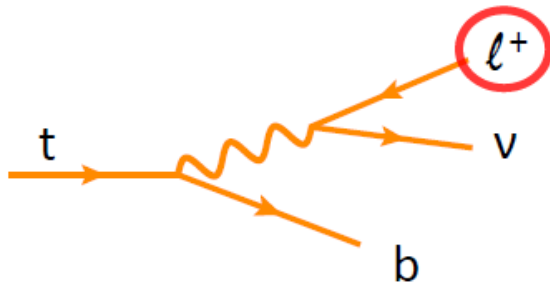


Weight functions and the weighted integrals

$$I(m) \equiv \int dE_l \underset{\substack{\uparrow \\ \text{Lepton energy distribution in the lab. frame}}}{D(E_l)} \overset{\substack{\text{Weight function}}}{\boxed{W(E_l, m)}}$$

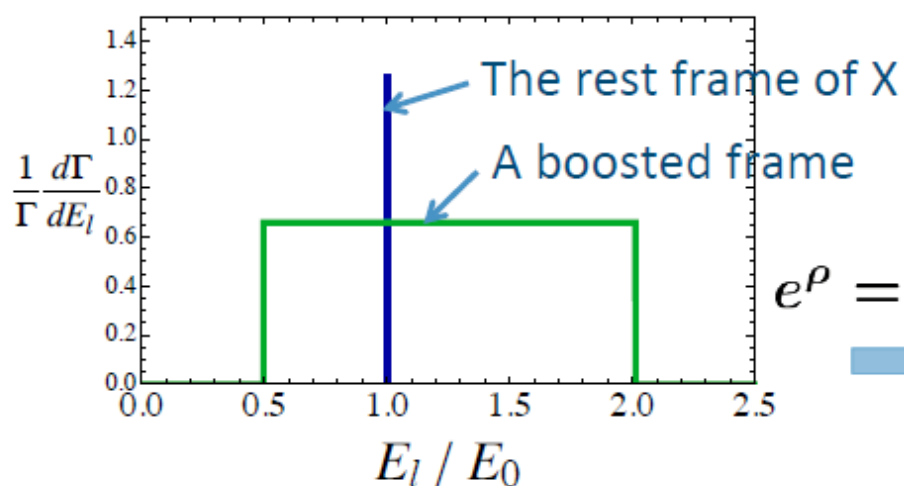
There exist an infinite number of weight functions which satisfy

$$I(m = m_t^{\text{true}}) = 0 \text{ for an arbitrary velocity distribution of top quarks}$$



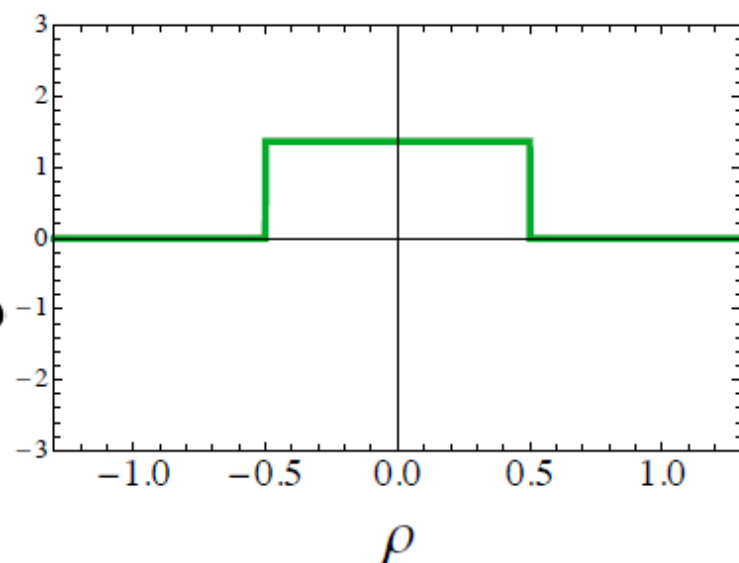
Construction of weight functions

For a two-body decay : $X \rightarrow \ell + Y$ (X is scalar or unpolarized)



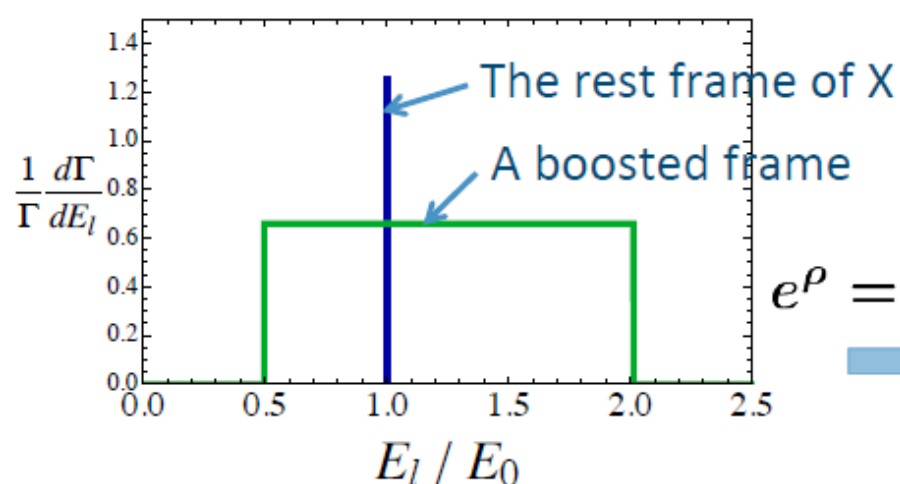
Lepton energy distribution

$$e^\rho = E_\ell / E_0$$

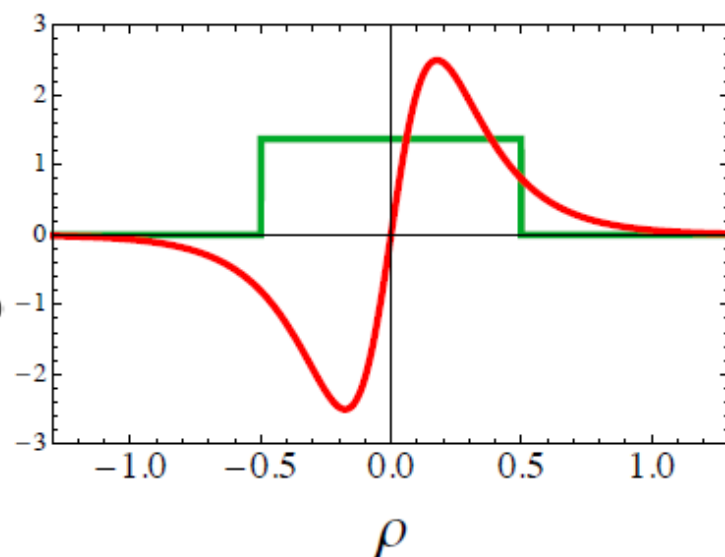


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Lepton energy distribution

$$\int dE_\ell D(E_\ell) W(E_\ell, m_X^{true}) = 0 \iff \int d\rho (\text{even func. of } \rho) (\text{odd func. of } \rho) = 0$$

$$d\rho \propto e^{-\rho} dE_\ell$$



$$W(E_\ell, m_X^{true}) = e^{-\rho} (\text{odd func. of } \rho) \big|_{e^\rho = E_\ell / E_0}$$

Possible OPE formulation (idea)

$$\frac{d\sigma(pp \rightarrow t\bar{t} \rightarrow lvX)}{d\Phi(l)} = \int d\Phi(had) \langle pp|lvX\rangle \langle lvX|pp\rangle$$

$$= \int d\Phi(had) H^{\mu\nu} L_{\mu\nu}$$



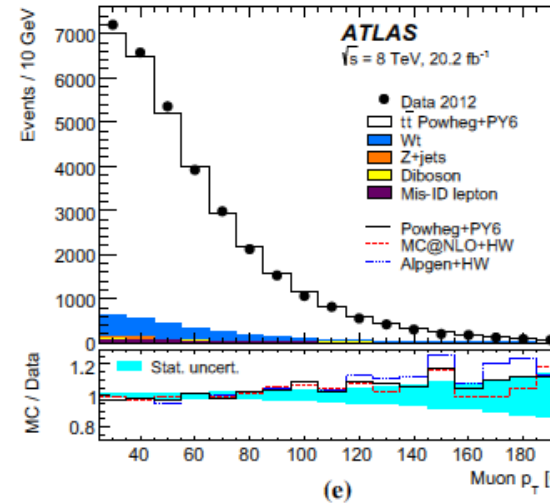
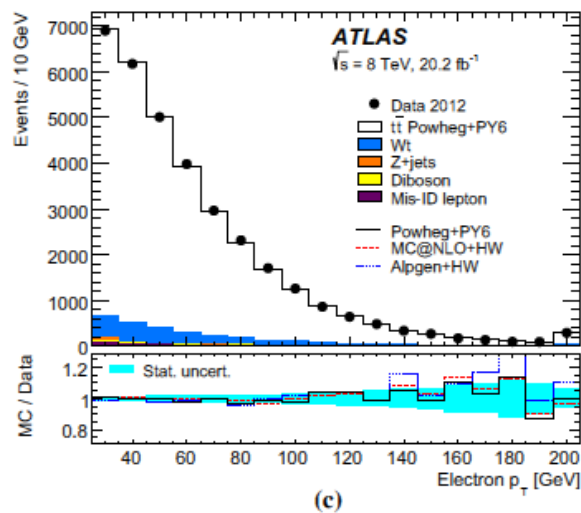
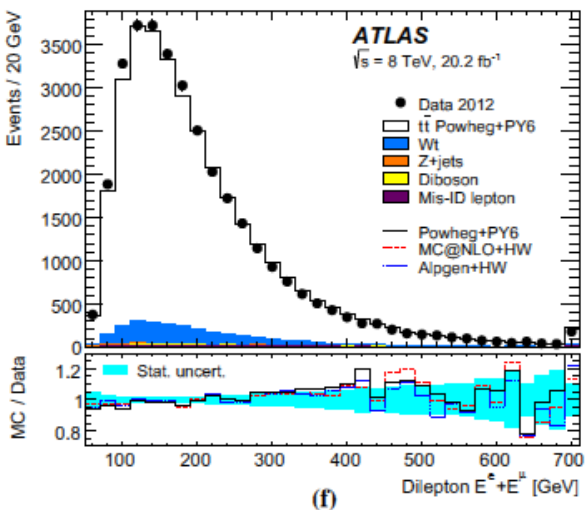
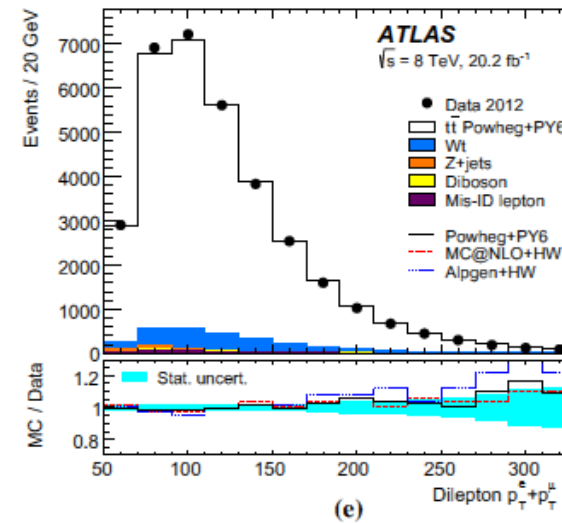
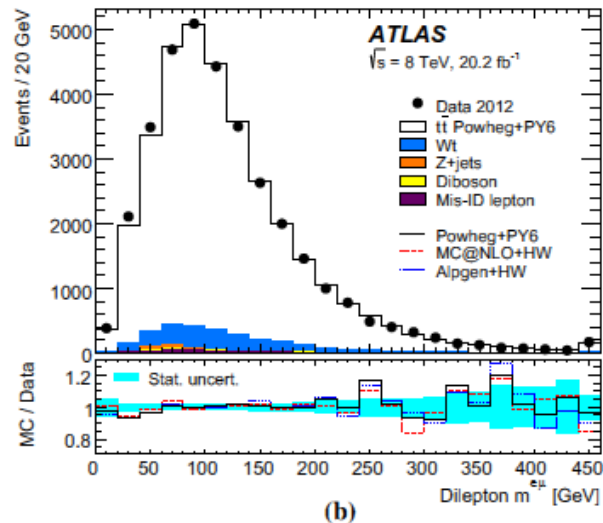
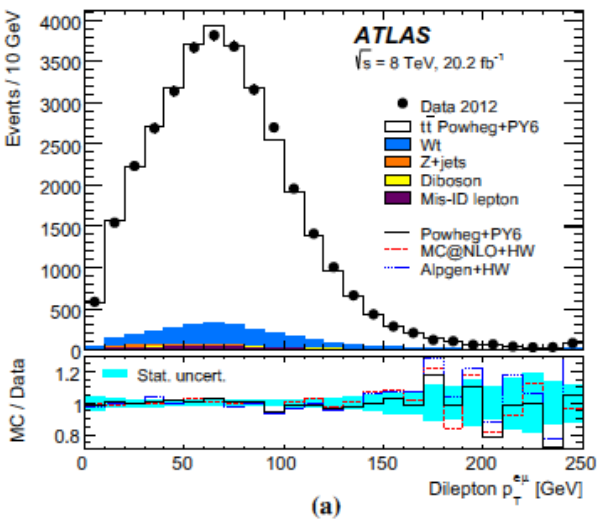
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similar to DIS in ep collision



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2. Top mass determination at ILC
3. Top mass determination at LHC: [Use of leptonic observables](#)
- (4. A future direction for precision QCD)
5. Summary and Conclusions



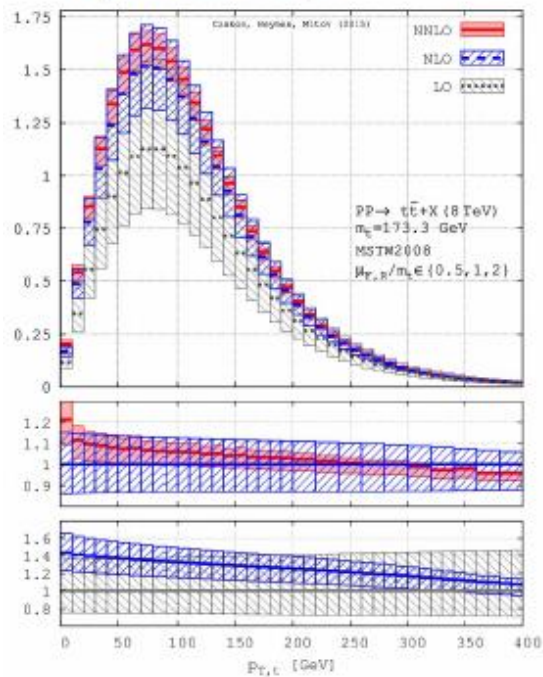
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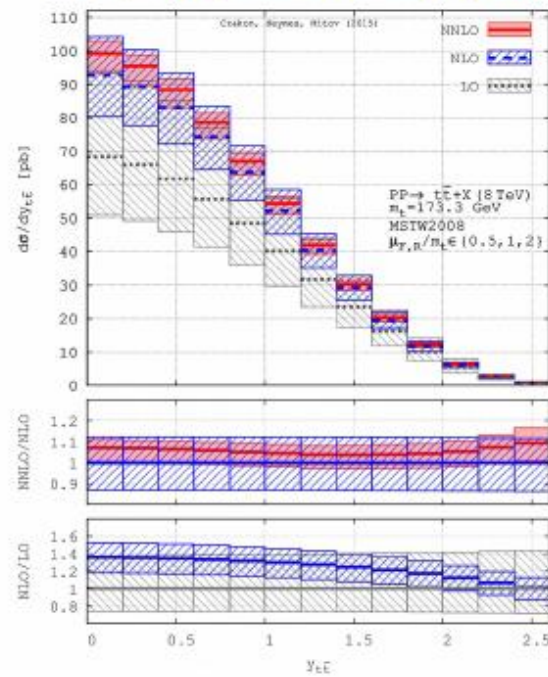
NNLO predictions

Czakon, Heymes, Mitov

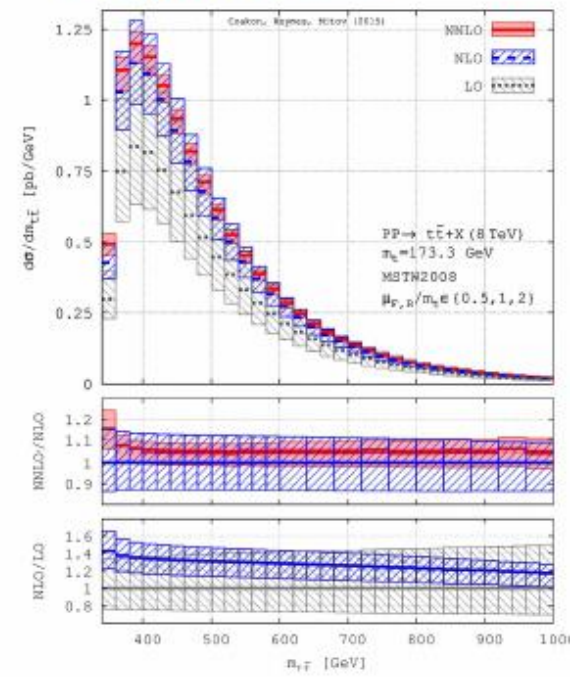
P_T of the top



Rapidity of the top-pair



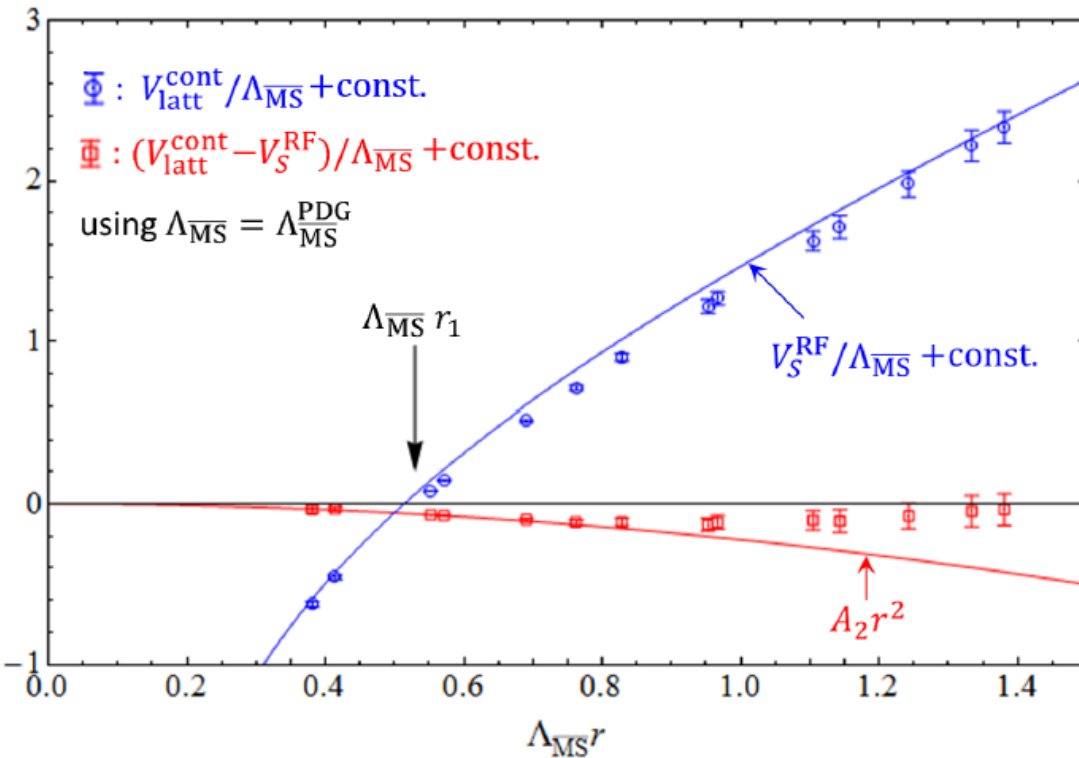
Invariant mass



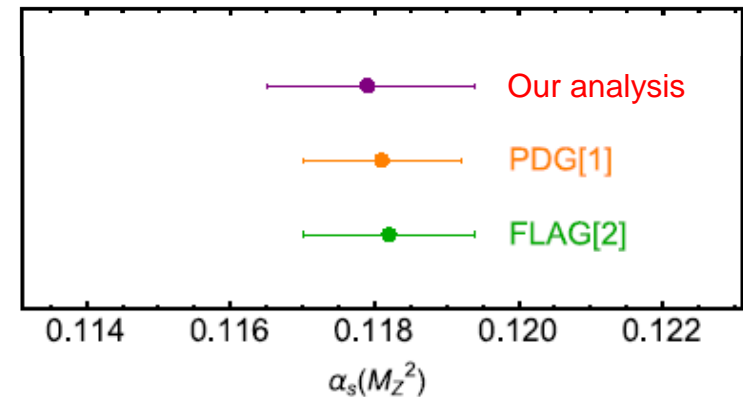
Direction for Precision QCD: OPE with renormalon subtraction

Consistency check

Takaura, Kaneko, Kiyo, YS



$\alpha_s(M_Z)$ determination



$V_{\text{QCD}}(r)$ [JLQCD: $n_f = 3$, $\{64^3 \times 128, 48^3 \times 96, 32^3 \times 64\}$] consistent with OPE at $r\Lambda_{\overline{\text{MS}}} \lesssim 0.8$ after renormalon subtraction.

$$V_{\text{QCD}}(r) = V_S^{\text{RF}}(r) + \delta E_{US}^{\text{RF}}(r)$$

NNLL

fit fn: $A_0 + A_2 r^2$

First time to confirm the OPE structure

Summary and Conclusions

- Need to understand current status of top mass determinations
def./assumptions
- $\overline{\text{MS}}$ mass determination at $e^+e^- \rightarrow t\bar{t}$ near threshold would be ideal.
- At LHC Δm_t much below 1 GeV would be difficult to achieve.
Color charge distribution around octet $t\bar{t}$ would introduce unsuppressed/uncontrolled IR physics.
- For measurement of well-defined top mass at LHC, use of leptonic observables combined with NNLO prediction + model indep. analysis is ideal.
- Steps taken towards high precision QCD (results in foreseeable future)