The Future Frontier of Precision Higgs Physics

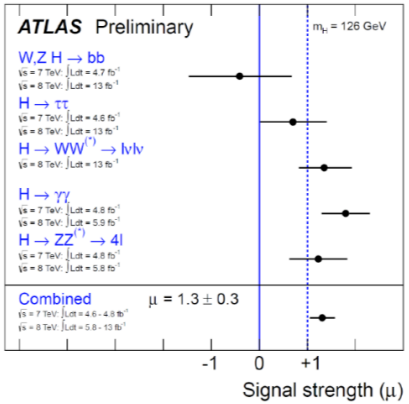
Ian Low
Argonne/Northwestern
November 28, 2018

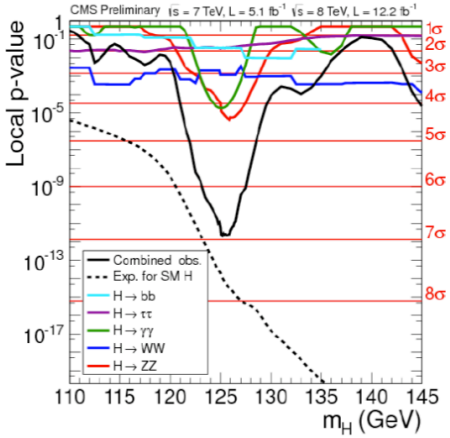
The Higgs Coupling workshop was held in Tokyo back in 2012:

Higgs Identification with Current Data

Ian Low
Argonne/ Northwestern/ KITP Santa Barbara

Nov 19, 2012
Higgs Coupling 2012, Tokyo





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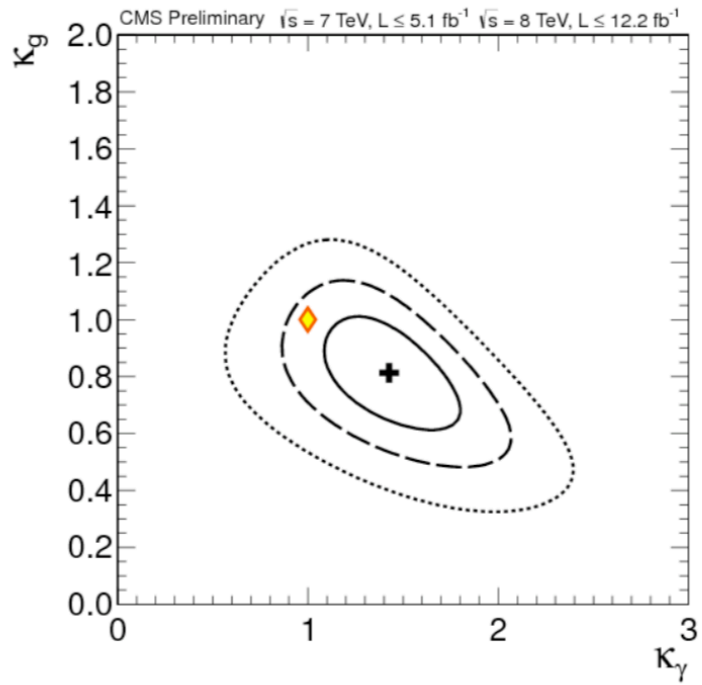
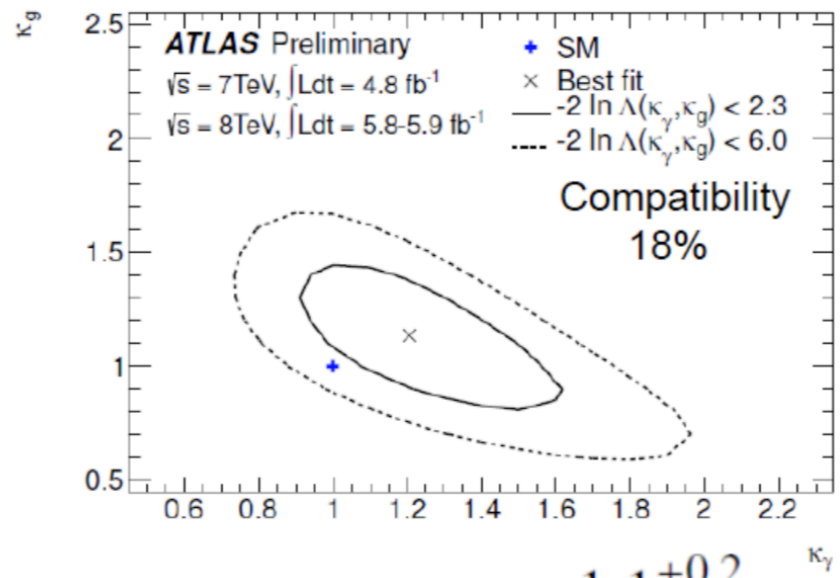


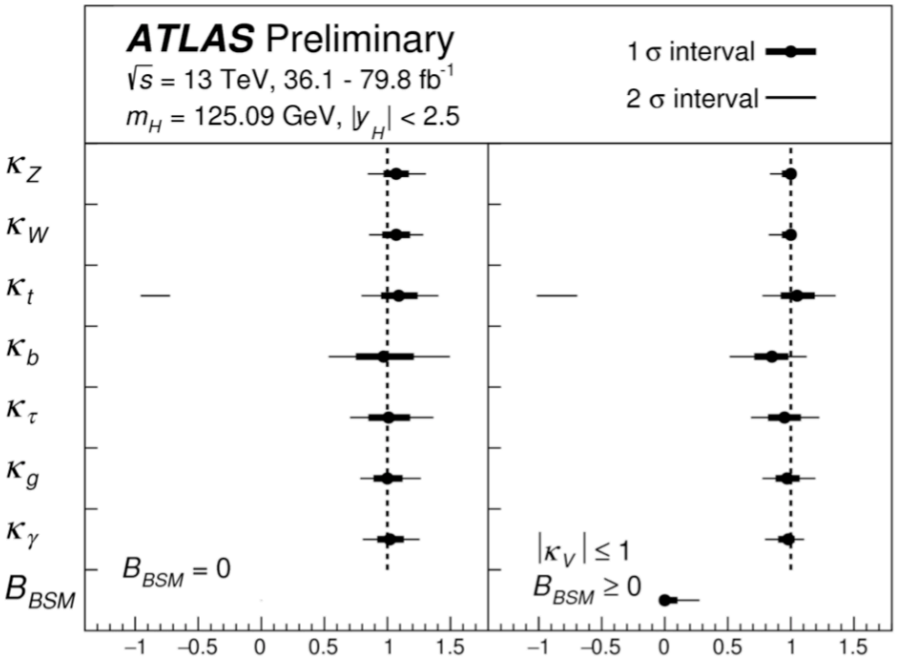


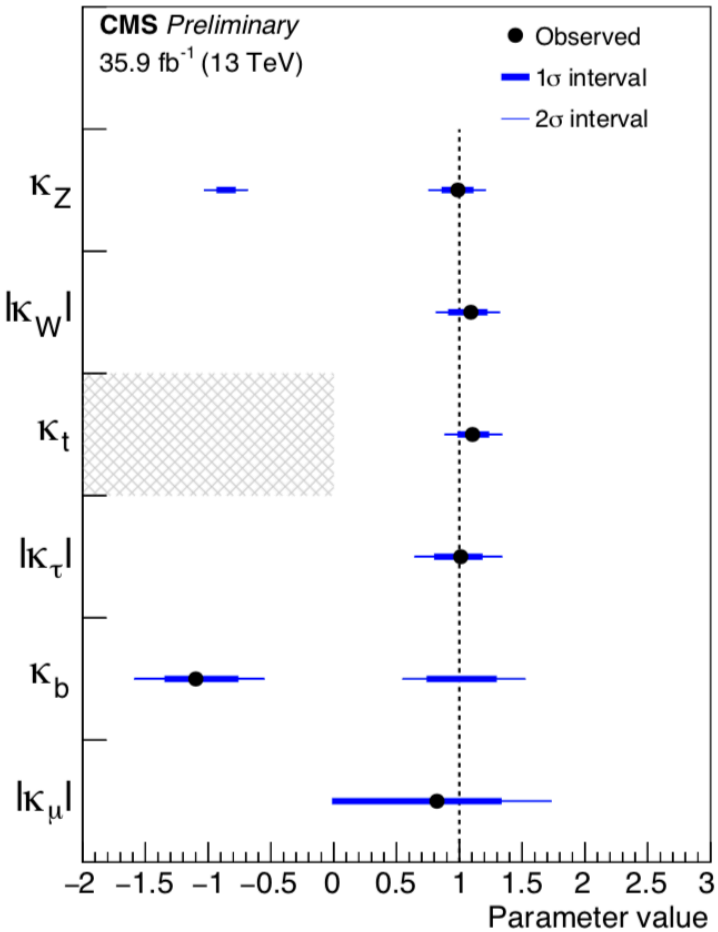
68% CL

$$\kappa_g = 1.1^{+0.2}_{-0.3}$$

$$\kappa_\gamma = 1.2^{+0.3}_{-0.2}$$

It has been a long way since 2012:





We have recently established ttH and bbH couplings!

Our understanding has gotten pretty sophisticated -- 10-parameter fit has been performed:

Table 15: Results of the ten-parameter fit of $\mu_F^f = \mu_{gg\mathrm{F}+ttH}^f$ and $\mu_V^f = \mu_{\mathrm{VBF}+VH}^f$ for each of the five decay channels, and of the six-parameter fit of the global ratio $\mu_V/\mu_F = \mu_{\mathrm{VBF}+VH}/\mu_{gg\mathrm{F}+ttH}$ together with μ_F^f for each of the five decay channels. The results are shown for the combination of ATLAS and CMS, together with their measured and expected uncertainties. The measured results are also shown separately for each experiment.

Parameter	ATLAS+CMS Measured	ATLAS+CMS Expected uncertainty	ATLAS Measured	CMS Measured
Ten-parameter fit of μ_F^f and μ_V^f				
$\mu_V^{\gamma\gamma}$	$1.05^{+0.44}_{-0.41}$	$^{+0.41}_{-0.38}$	$0.69^{+0.63}_{-0.58}$	$1.37^{+0.62}_{-0.56}$
μ_V^{ZZ}	$0.47^{+1.37}_{-0.92}$	$^{+1.16}_{-0.84}$	$0.24^{+1.60}_{-0.93}$	$1.45^{+2.32}_{-2.29}$
μ_V^{WW}	$1.38^{+0.41}_{-0.37}$	$^{+0.38}_{-0.35}$	$1.56^{+0.52}_{-0.46}$	$1.08^{+0.65}_{-0.58}$
$\mu_V^{\tau\tau}$	$1.12^{+0.37}_{-0.35}$	$^{+0.38}_{-0.35}$	$1.29^{+0.58}_{-0.53}$	$0.88^{+0.49}_{-0.45}$
μ_V^{bb}	$0.65^{+0.31}_{-0.29}$	$^{+0.32}_{-0.30}$	$0.50^{+0.39}_{-0.37}$	$0.85^{+0.47}_{-0.44}$
$\mu_F^{\gamma\gamma}$	$1.16^{+0.27}_{-0.24}$	$^{+0.25}_{-0.23}$	$1.30^{+0.37}_{-0.33}$	$1.00^{+0.33}_{-0.30}$
μ_F^{ZZ}	$1.42^{+0.37}_{-0.33}$	$^{+0.29}_{-0.25}$	$1.74^{+0.51}_{-0.44}$	$0.96^{+0.53}_{-0.41}$
μ_F^{WW}	$0.98^{+0.22}_{-0.20}$	$^{+0.21}_{-0.19}$	$1.10^{+0.29}_{-0.26}$	$0.84^{+0.27}_{-0.24}$
$\mu_F^{\tau\tau}$	$1.06^{+0.60}_{-0.56}$	$^{+0.56}_{-0.53}$	$1.72^{+1.24}_{-1.12}$	$0.89^{+0.67}_{-0.63}$
μ_F^{bb}	$1.15^{+0.99}_{-0.94}$	$^{+0.90}_{-0.86}$	$1.52^{+1.16}_{-1.09}$	$0.11^{+1.85}_{-1.90}$

Run 1 Legacy paper

We have measured many of the following couplings with uncertainties of 10 – 30 % or larger:

Couplings to massive gauge bosons → $\left(\frac{2m_W^2}{v}\, h\, W_\mu^+ W^{-\,\mu} + \frac{m_Z^2}{v}\, h\, Z_\mu Z^\mu\right)$

Couplings to massless gauge bosons →

$$+c_g \frac{\alpha_s}{12\pi v} h\, G^a_{\mu\nu} G^{a\,\mu\nu} + c_\gamma \frac{\alpha}{8\pi v} h\, F_{\mu\nu} F^{\mu\nu} + c_{Z\gamma} \frac{\alpha}{8\pi v s_w}\, h\, F_{\mu\nu} Z^{\mu\nu}$$

$$c_g^{(SM)}(125\ \mathrm{GeV}) = 1\ , \qquad c_\gamma^{(SM)}(125\ \mathrm{GeV}) = -6.48\ , \qquad c_{Z\gamma}^{(SM)}(125\ \mathrm{GeV}) = 5.48\ .$$

Couplings to fermions → $\sum_f \frac{m_f}{v} h \bar{f} f$ for *bb*, *tt*, and *ττ* only!

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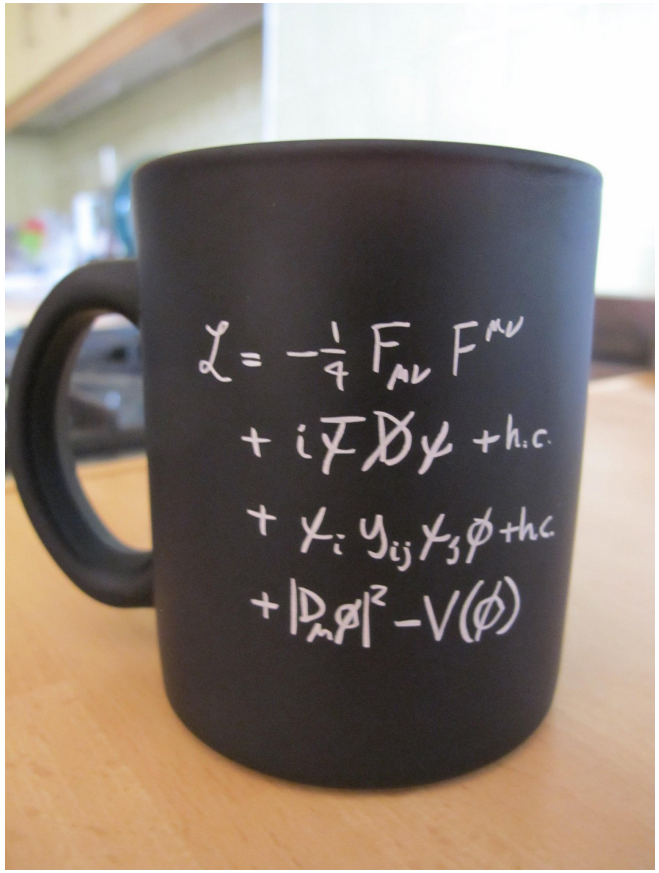
Self-couplings is being probed in the double Higgs production channel:

$$\frac{1}{2} m_h^2 h^2 + \frac{m_h^2}{v} h^3 + \frac{2m_h^2}{v^2} h^4$$

- Limits at 95% CL on self-coupling scale factor κ_λ:
  - ATLAS: $-5.0<\kappa_\lambda<12.1$
  - CMS: $-11.8<\kappa_\lambda<18.8$

Rosati's talk on Monday

What are we missing still?

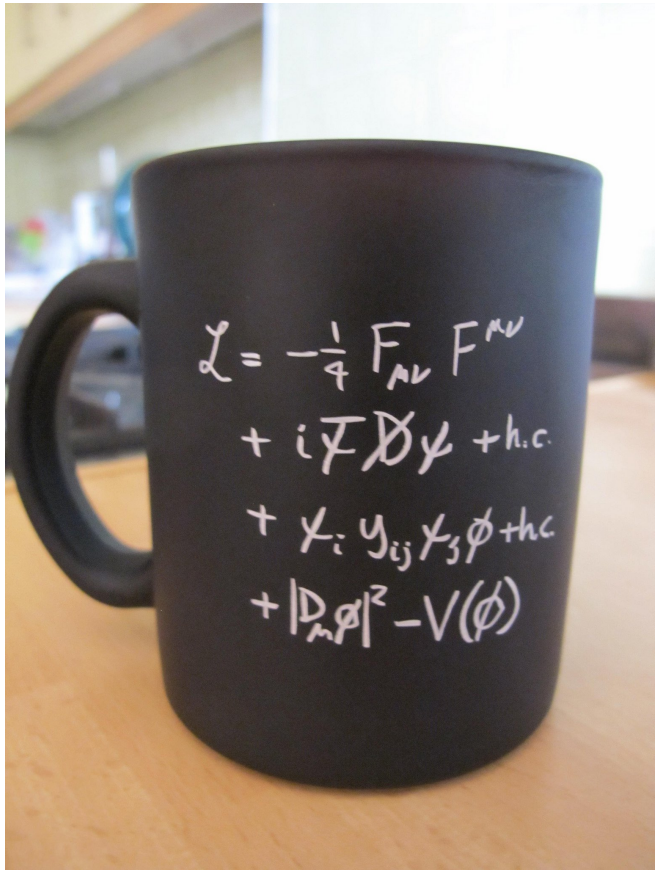


What are we missing still?

One obvious answer is Higgs couplings to first two generation fermions.

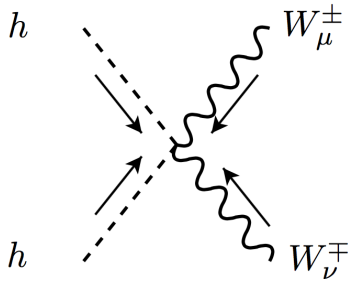
→ Quite a number of theoretical studies already.

Anything else??

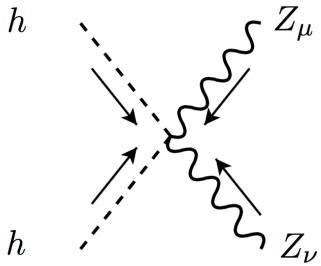


In my view, one class of couplings that has not received enough attention is the HHVV coupling:

$$D_\mu H^\dagger D^\mu H \;\supset\; g^2 h^2 V_\mu V^\mu$$



$$\frac{i}{2}\, g^2\, g_{\mu\nu}$$



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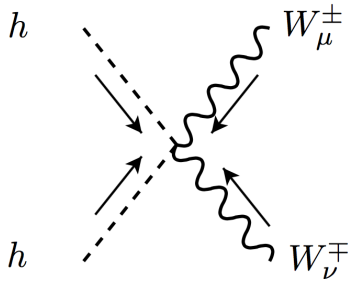

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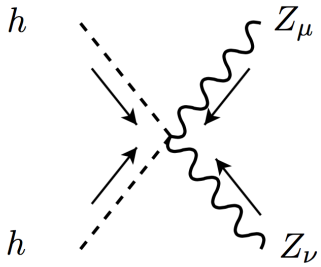
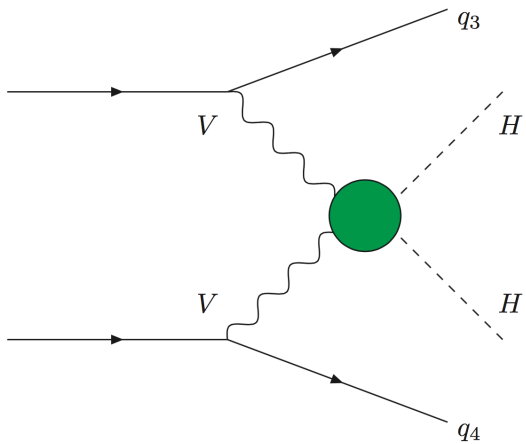
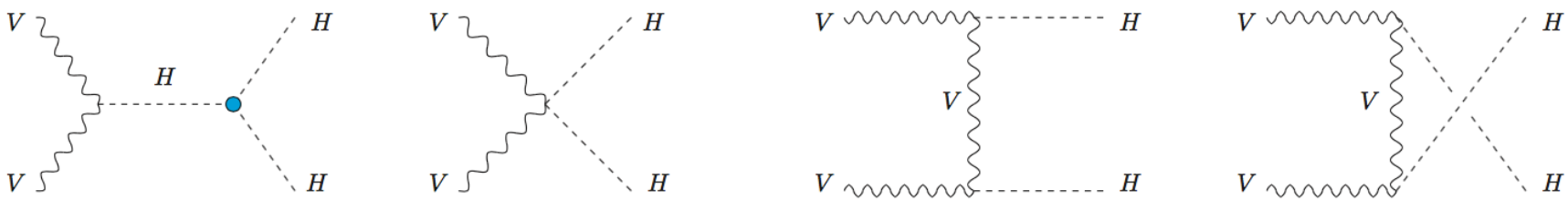


$$\frac{i}{2}\frac{g^2}{\cos^2\theta_W} g_{\mu\nu}$$

This coupling can be probed by double Higgs production in the VBF channel!



HH in VBF can be viewed as part of the program to study VV → VV scattering. There are a few contributing diagrams:



The NNLO production rate:

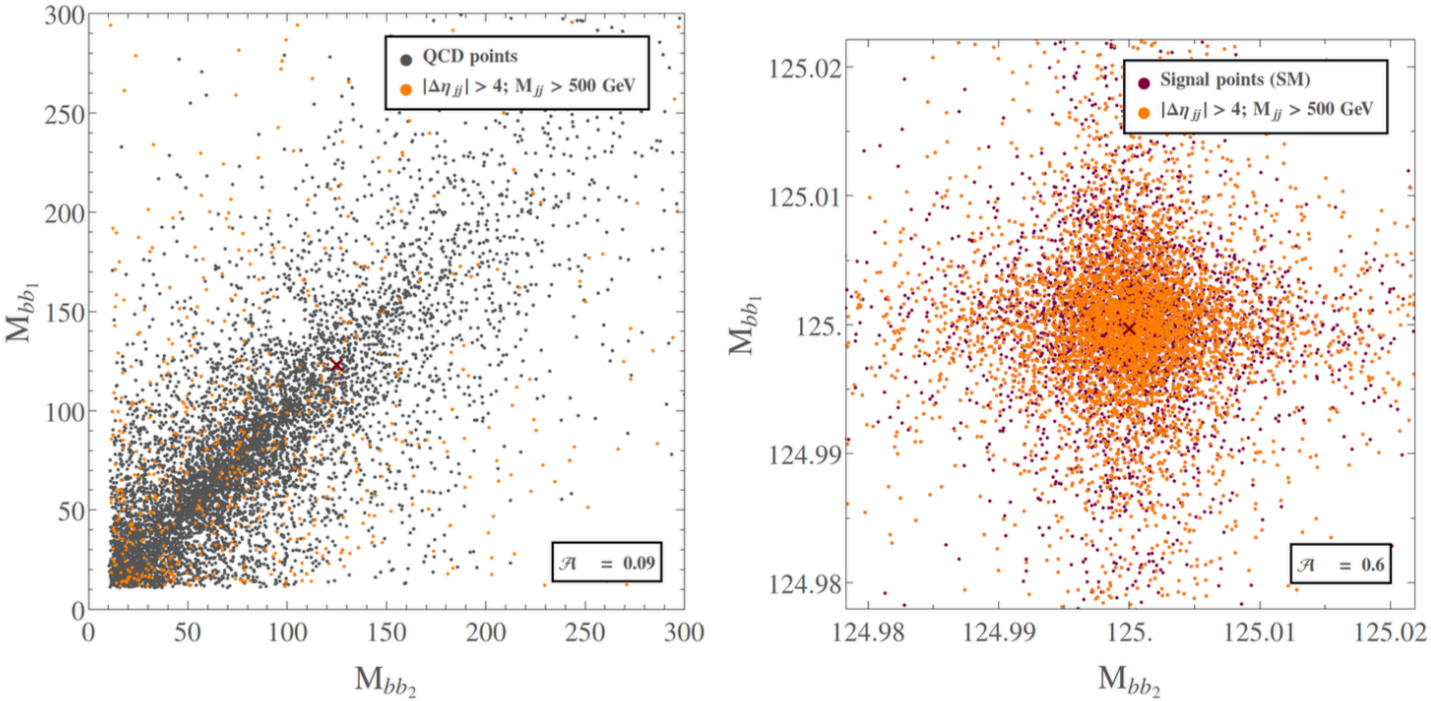
$\sqrt{S}$	LO [fb]	NLO [fb]	NNLO [fb]
14 TeV	$1.858^{+0.374}_{-0.270}$	$1.976^{+0}_{-0.078}$	$1.986^{+0.045}_{-0}$
33 TeV	$11.234^{+0.878}_{-0.830}$	$12.002^{+0.190}_{-0.562}$	$12.041^{+0.359}_{-0.060}$
100 TeV	$75.36^{+4.91}_{-6.34}$	$79.82^{+3.92}_{-5.26}$	$80.05^{+3.92}_{-0.80}$

Table 1: The central values of the total cross section ($\kappa = 1$) and the errors due to scale uncertainty with $\kappa \in [1/4, 4]$ at the $\sqrt{S} = 14$, 33, and 100 TeV hadron colliders by using the MSTW2008 (68% C.L.) PDFs.

Ling et. al.: 1401.7754

There has been a number of collider studies on HH in VBF both at the LHC and at future colliders:

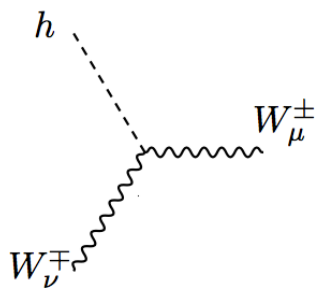
Dolan et. al.: 1310.1084; Dolan et. al.: 1506.08008; Bishara et. al.: 1611.03860; Arganda et. al.: 1807.09736; Killian et. al.: 1808.05534



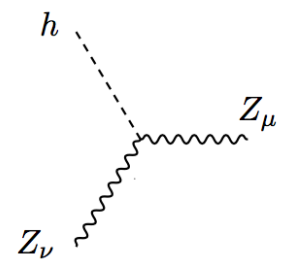
1807.09736

These studies are useful, and could be refined further, for a number of reasons.

Recall that establishing and studying HVV couplings is among the top priority of the Higgs program at the LHC:



$$i g\, m_W\, g_{\mu\nu}$$



$$i\,\frac{g}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

The goal of the program includes both the coupling strength and the tensor structure:

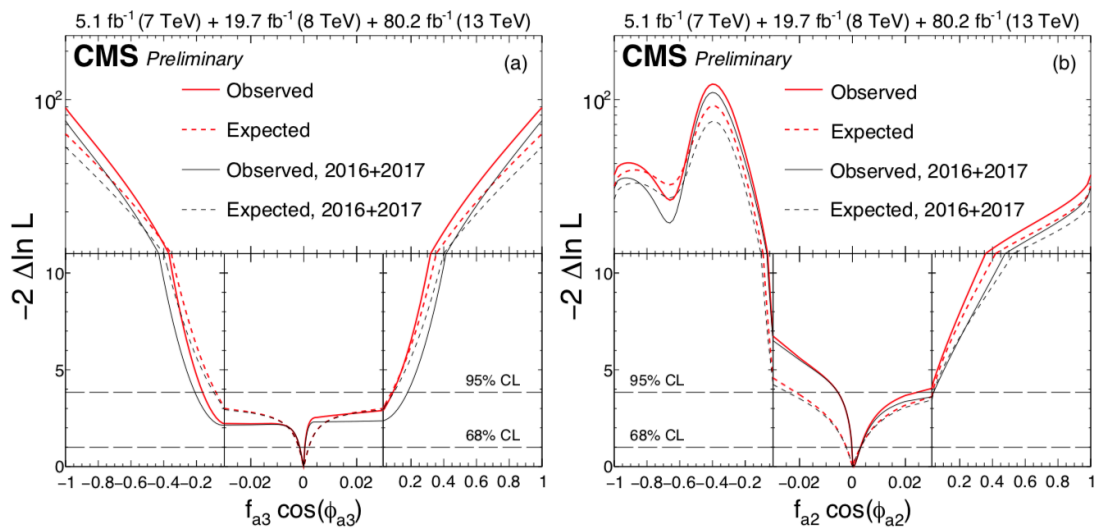
$$\begin{aligned}
L(\mathrm{HVV}) \sim\ & a_1 \frac{m_Z^2}{2} \mathrm{HZ}^\mu \mathrm{Z}_\mu - \frac{\kappa_1}{(\Lambda_1)^2} m_Z^2 \mathrm{HZ}_\mu \Box \mathrm{Z}^\mu - \frac{1}{2} a_2 \mathrm{HZ}^{\mu\nu} \mathrm{Z}_{\mu\nu} - \frac{1}{2} a_3 \mathrm{HZ}^{\mu\nu} \tilde{\mathrm{Z}}_{\mu\nu} \\
& + a_1^{\mathrm{WW}} m_{\mathrm{W}}^2 \mathrm{HW}^{+\mu} \mathrm{W}_\mu^- - \frac{1}{(\Lambda_1^{\mathrm{WW}})^2} m_{\mathrm{W}}^2 \mathrm{H} \left(\kappa_1^{\mathrm{WW}} \mathrm{W}_\mu^- \Box \mathrm{W}^{+\mu} + \kappa_2^{\mathrm{WW}} \mathrm{W}_\mu^+ \Box \mathrm{W}^{-\mu} \right) \\
& - a_2^{\mathrm{WW}} \mathrm{HW}^{+\mu\nu} \mathrm{W}_{\mu\nu}^- - a_3^{\mathrm{WW}} \mathrm{HW}^{+\mu\nu} \tilde{\mathrm{W}}_{\mu\nu}^- \\
& + \frac{\kappa_2^{\mathrm{Z}\gamma}}{\left(\Lambda_1^{\mathrm{Z}\gamma}\right)^2} m_{\mathrm{Z}}^2 \mathrm{HZ}_\mu \partial_\nu \mathrm{F}^{\mu\nu} - a_2^{\mathrm{Z}\gamma} \mathrm{HF}^{\mu\nu} \mathrm{Z}_{\mu\nu} - a_3^{\mathrm{Z}\gamma} \mathrm{HF}^{\mu\nu} \tilde{\mathrm{Z}}_{\mu\nu} - \frac{1}{2} a_2^{\gamma\gamma} \mathrm{HF}^{\mu\nu} \mathrm{F}_{\mu\nu} - \frac{1}{2} a_3^{\gamma\gamma} \mathrm{HF}^{\mu\nu} \tilde{F}_{\mu\nu},
\end{aligned}$$

CMS-HIG-14-018

There's still significant room for anomalous HVV couplings even today:

Table 6: Summary of allowed 68% CL (central values with uncertainties) and 95% CL (in square brackets) intervals on anomalous coupling parameters $f_{ai}\cos(\phi_{ai})$ obtained from the on-shell data analysis of the Run 1 and Run 2 combined dataset.

Parameter	Observed	Expected
$f_{a3}\cos(\phi_{a3})$	$-0.0001^{+0.0005}_{-0.0015}$ $[-0.16, 0.09]$	$0.0000^{+0.0019}_{-0.0019}$ $[-0.082, 0.082]$
$f_{a2}\cos(\phi_{a2})$	$0.0004^{+0.0026}_{-0.0007}$ $[-0.006, 0.025]$	$0.0000^{+0.0030}_{-0.0023}$ $[-0.021, 0.035]$
$f_{\Lambda 1}\cos(\phi_{\Lambda 1})$	$0.0000^{+0.0035}_{-0.0008}$ $[-0.21, 0.09]$	$0.0000^{+0.0012}_{-0.0006}$ $[-0.059, 0.032]$
$f_{\Lambda 1}^{Z\gamma}\cos\left(\phi_{\Lambda 1}^{Z\gamma}\right)$	$0.000^{+0.355}_{-0.009}$ $[-0.17, 0.61]$	$0.000^{+0.009}_{-0.010}$ $[-0.10, 0.34]$



CMS-HIG-18-002

A similar program to study HHVV coupling should be pursued --

Reason 1:

Among the least understood/measured coupling of the 125 GeV Higgs!

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Reason 2:

HVV and HHVV couplings together sheds light on the UV nature of the 125 GeV Higgs.

Let me elaborate –

Suppose the SM is just an effective description:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^n} \mathcal{O}_i^{(n-4)}$$

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At the weak scale, the HVV and HHVV couplings deviate from their SM expectations, both in <u>coupling strength</u> and the <u>tensor structure</u>,

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + b_{nh}\left(\frac{h}{v}\right)^n \left(m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right)$$

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There are also operators carrying "four-derivative":

$$\frac{h}{v} V_{1\,\mu} \mathcal{D}^{\mu\nu} V_{2\,\nu}\ , \qquad \frac{h}{v} V_{1\,\mu\nu} V_2^{\mu\nu}\ , \qquad \mathcal{D}^{\mu\nu} = \partial^\mu \partial^\nu - \eta^{\mu\nu}\partial^2$$

$$\frac{h^2}{v^2} V_{1\,\mu} \mathcal{D}^{\mu\nu} V_{2\,\nu}\ , \qquad \frac{h^2}{v^2} V_{1\,\mu\nu} V_2^{\mu\nu}\ , \qquad \frac{\partial_\mu h \partial_\nu h}{v^2} V_1^\mu V_2^\nu$$

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However, there is a special class of models where these anomalous HVV and HHVV couplings are controlled by only a small number of parameters

→ because there is a symmetry relating the coefficients.

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$$b_h = 1 - 2\xi \qquad b_{2h} = 2\sqrt{1-\xi}\ ,$$

$$b_{3h} = -\frac{4}{3}\xi\sqrt{1-\xi}\ , \qquad b_{4h} = \frac{1}{3}\xi(2\xi - 1)\ ,$$

$$b_{5h} = \frac{4}{15}\xi^2\sqrt{1-\xi}\ , \qquad b_{6h} = \frac{2}{45}\xi^2(1-2\xi)\ ,$$

$$\ldots \qquad \ldots$$

For this class of theories, the two-derivative Lagrangian can be written in a compact way:

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4}\sin^2(\theta + h/f)\left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu\right)$$

$$\sin^2\theta = \xi = \frac{v^2}{f^2}$$

In the unitary gauge, the “symmetry” that enforces this particular form is highly disguised and non-trivial.

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In the unitary gauge, the "symmetry" that enforces this particular form is highly disguised and non-trivial.

One way to "detect" the presence of such disguised symmetry is to measure HVV and HHVV couplings to see if they are controlled by the same parameter.

More concretely, consider the following "anomalous" HVV and HHVV couplings:

$$\mathcal{L}_{\rm NL} = \sum_i \frac{m_W^2}{m_\rho^2}\left(C_i^h \mathcal{I}_i^h + C_i^{2h}\mathcal{I}_i^{2h} + C_i^{3V}\mathcal{I}_i^{3V}\right)$$

$\mathcal{I}_i^h$
(1) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$
(2) $\frac{h}{v} Z_{\mu\nu} Z^{\mu\nu}$
(3) $\frac{h}{v} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$
(4) $\frac{h}{v} Z_{\mu\nu} A^{\mu\nu}$

$\mathcal{I}_i^{2h}$
(1) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} Z_\nu$
(2) $\frac{h^2}{v^2} Z_{\mu\nu} Z^{\mu\nu}$
(3) $\frac{h^2}{v^2} Z_\mu \mathcal{D}^{\mu\nu} A_\nu$
(4) $\frac{h^2}{v^2} Z_{\mu\nu} A^{\mu\nu}$

Some examples of "Universal Relations" are

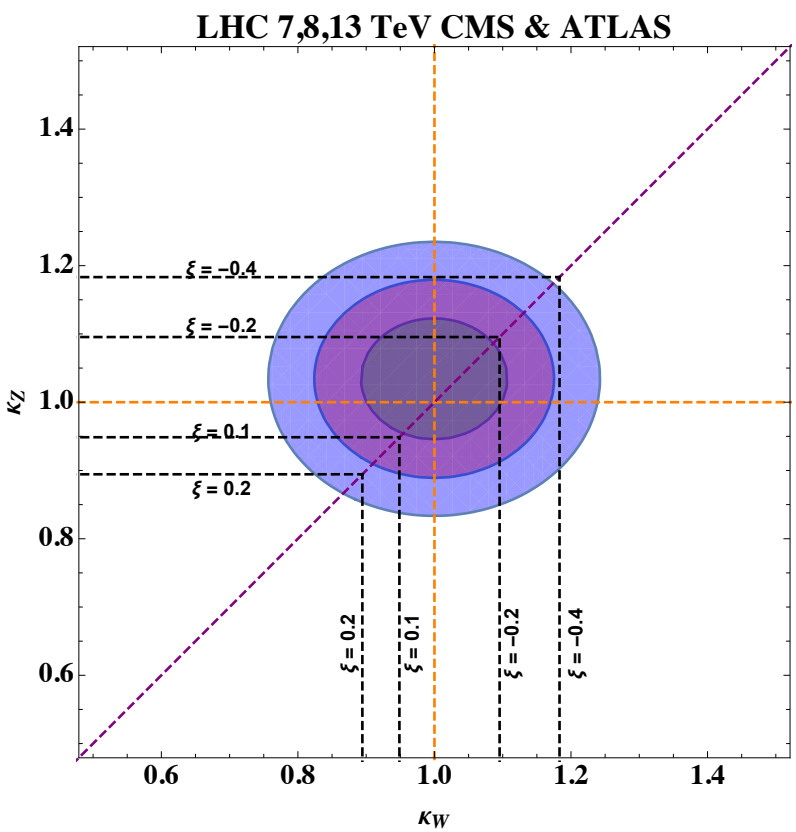
$$\frac{C_3^{2h}}{C_3^h} = \frac{C_4^{2h}}{C_4^h} = \frac{1}{2}\cos\theta \;= \frac{1}{2}\sqrt{1-\xi}$$

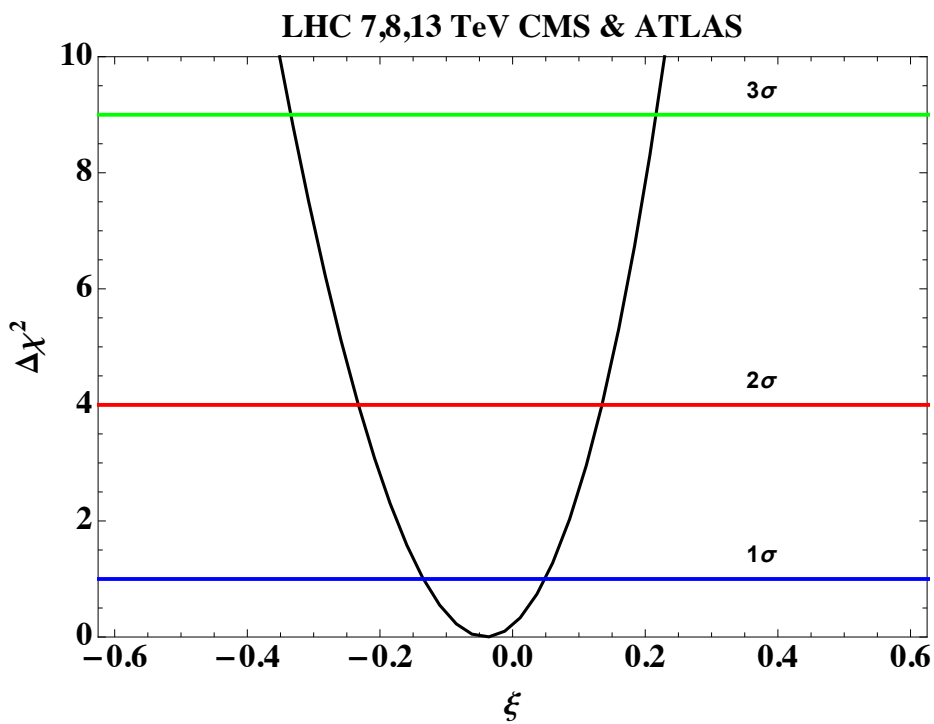
$$\frac{s_{2w}\, C_1^{2h} - c_{2w}\, C_3^{2h}}{s_{2w}\, C_1^h - c_{2w}\, C_3^h} = \frac{s_{2w}\, C_2^{2h} - c_{2w}\, C_4^{2h}}{s_{2w}\, C_2^h - c_{2w}\, C_4^h} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right)$$

Z. Yin, D. Liu and IL: 1805.00489; 1809.09126

There are several more "Universal Relations" among HVV and HHVV couplings that are controlled by a single parameter ξ.

In particular, ξ is already constrained by current measurements on HZZ and HWW coupling strengths:





ξ can be either positive or negative!

Z. Yin, D. Liu and IL: 1809.09126

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Observation:

Secretly this is a symmetry relating multi-Higgs self-interactions –

Recall in the unitary gauge the longitudinal components of the W/Z gauge bosons are related to the 125 GeV Higgs by SU(2)xU(1).

In fact, everyone knows an example of such a symmetry.

(Sometimes, you don’t know what you know…)

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The example is pions in low-energy QCD. There are many ways to write down the effective Lagrangian of pions. One possibility is

$$U(x) = \frac{1}{f}\left[\sigma(x) + \mathrm{i}\vec{\tau}\cdot\vec{\pi}(x)\right], \quad \sigma(x) = \sqrt{f^2 - \vec{\pi}^2(x)},$$

$$\mathcal{L}^{(2)} = \tfrac{1}{4} f^2 \,\mathrm{Tr}[\mathrm{D}_\mu U (\mathrm{D}^\mu U)^\dagger]$$

When expanding the two-derivative Lagrangian order-by-order in "f", all "multi-pion" vertices are controlled by one single parameter "f".

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When expanding the two-derivative Lagrangian order-by-order in "f", all "multi-pion" vertices are controlled by one single parameter "f".

This is similar to the case we discussed in Higgs:

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{g^2 f^2}{4}\sin^2(\theta + h/f)\left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu\right)$$

Pions are (pseudo)-Nambu-Goldstone bosons arising from the chiral symmetry breaking:

$$SU(2)_L x SU(2)_R \rightarrow SU(2)_V$$

The “symmetry” enforcing relations among multi-pion vertices is the result of degenerate vacua and the unbroken $SU(2)_V$ isospin symmetry.

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Similarly, if we detect relations among HVV and HHVV couplings, it's the clearest signal that the 125 GeV Higgs is a (pseudo-)Nambu-Goldstone boson!

Nowadays this class of theories is generically referred to as "composite Higgs models."

Here is a survey of existing models from 5 years ago. They differ in the symmetry breaking pattern (G/H) they choose.

$\mathcal{G}$	$\mathcal{H}$	C	N_G	$\mathbf{r}_{\mathcal{H}} = \mathbf{r}_{\mathrm{SU}(2)\times\mathrm{SU}(2)}\,(\mathbf{r}_{\mathrm{SU}(2)\times\mathrm{U}(1)})$	Ref.
SO(5)	SO(4)	✓	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$	[11]
SU(3) × U(1)	SU(2) × U(1)		5	$\mathbf{2}_{\pm 1/2} + \mathbf{1}_0$	[10, 35]
SU(4)	Sp(4)	✓	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[29, 47, 64]
SU(4)	$[\mathrm{SU}(2)]^2 \times \mathrm{U}(1)$	✓*	8	$(\mathbf{2},\mathbf{2})_{\pm 2} = 2\cdot(\mathbf{2},\mathbf{2})$	[65]
SO(7)	SO(6)	✓	6	$\mathbf{6} = 2\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	G_2	✓*	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	[66]
SO(7)	SO(5) × U(1)	✓*	10	$\mathbf{10}_0 = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$	–
SO(7)	$[\mathrm{SU}(2)]^3$	✓*	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\cdot(\mathbf{2},\mathbf{2})$	–
Sp(6)	Sp(4) × SU(2)	✓	8	$(\mathbf{4},\mathbf{2}) = 2\cdot(\mathbf{2},\mathbf{2})$	[65]
SU(5)	SU(4) × U(1)	✓*	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+5} = 2\cdot(\mathbf{2},\mathbf{2})$	[67]
SU(5)	SO(5)	✓*	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$	[9, 47, 49]
SO(8)	SO(7)	✓	7	$\mathbf{7} = 3\cdot(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	–
SO(9)	SO(8)	✓	8	$\mathbf{8} = 2\cdot(\mathbf{2},\mathbf{2})$	[67]
SO(9)	SO(5) × SO(4)	✓*	20	$(\mathbf{5},\mathbf{4}) = (\mathbf{2},\mathbf{2}) + (\mathbf{1}+\mathbf{3},\mathbf{1}+\mathbf{3})$	[34]
$[\mathrm{SU}(3)]^2$	SU(3)		8	$\mathbf{8} = \mathbf{1}_0 + \mathbf{2}_{\pm 1/2} + \mathbf{3}_0$	[8]
$[\mathrm{SO}(5)]^2$	SO(5)	✓*	10	$\mathbf{10} = (\mathbf{1},\mathbf{3}) + (\mathbf{3},\mathbf{1}) + (\mathbf{2},\mathbf{2})$	[32]
SU(4) × U(1)	SU(3) × U(1)		7	$\mathbf{3}_{-1/3} + \bar{\mathbf{3}}_{+1/3} + \mathbf{1}_0 = 3\cdot\mathbf{1}_0 + \mathbf{2}_{\pm 1/2}$	[35, 41]
SU(6)	Sp(6)	✓*	14	$\mathbf{14} = 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{3}) + 3\cdot(\mathbf{1},\mathbf{1})$	[30, 47]
$[\mathrm{SO}(6)]^2$	SO(6)	✓*	15	$\mathbf{15} = (\mathbf{1},\mathbf{1}) + 2\cdot(\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$	[36]

Table 1: Symmetry breaking patterns $\mathcal{G} \to \mathcal{H}$ for Lie groups. The third column denotes whether the breaking pattern incorporates custodial symmetry. The fourth column gives the dimension N_G of the coset, while the fifth contains the representations of the GB's under $\mathcal{H}$ and $\mathrm{SO}(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ (or simply $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ if there is no custodial symmetry). In case of more than two SU(2)'s in $\mathcal{H}$ and several different possible decompositions we quote the one with largest number of bi-doublets.

Bellazzni, Csaki and Serra:1401.2457

The “universal relations” we proposed are independent of the symmetry breaking pattern G/H, under some minimal and widely adopted assumptions. (Goldstone self-interactions only depend on H, but not on G.)

There are many of them:

$$\mathrm{UR1}: \frac{C_6^h - C_4^h/t_w}{\delta\tilde{\kappa}_\gamma} = \frac{2c_w^2\, C_2^h - c_{2w}\, C_4^h/t_w}{\delta\tilde{\kappa}_\gamma} = \cos\theta \approx 1 - \frac{1}{2}\xi,$$

$$\mathrm{UR2}: \frac{c_{2w}}{t_w} C_3^h - 2c_w^2\, C_1^h = 4c_w^2\, \delta\tilde{g}_1^Z\, \cos\theta + \frac{1}{t_w} C_3^h\, \cos^2\theta\ ,$$

$$\mathrm{UR3}: C_5^h = -2c_w^2\, \delta\tilde{g}_1^Z\, \cos\theta + \frac{1}{2t_w} C_3^h\, \sin^2\theta\ ,$$

$$\mathrm{UR4}: \frac{C_3^{2h}}{C_3^h} = \frac{C_4^{2h}}{C_4^h} = \frac{1}{2}\cos\theta\ ,$$

$$\mathrm{UR5}: \frac{C_5^{2h} - C_3^{2h}/2t_w}{C_5^h - C_3^h/2t_w} = \frac{C_6^{2h} - C_4^{2h}/t_w}{C_6^h - C_4^h/t_w} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right)\ ,$$

$$\mathrm{UR6}: \frac{s_{2w}\, C_1^{2h} - c_{2w}\, C_3^{2h}}{s_{2w}\, C_1^h - c_{2w}\, C_3^h} = \frac{s_{2w}\, C_2^{2h} - c_{2w}\, C_4^{2h}}{s_{2w}\, C_2^h - c_{2w}\, C_4^h} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right)$$

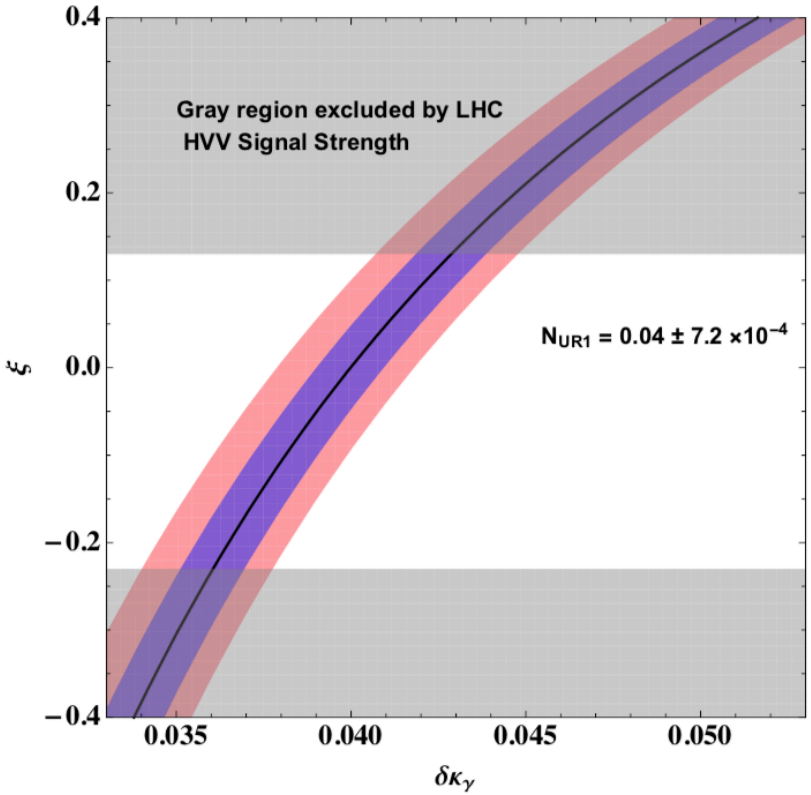
Z. Yin, D. Liu and IL: 1805.00489; 1809.09126

As a preliminary study, let's consider UR1 which only involves HVV and TGC:

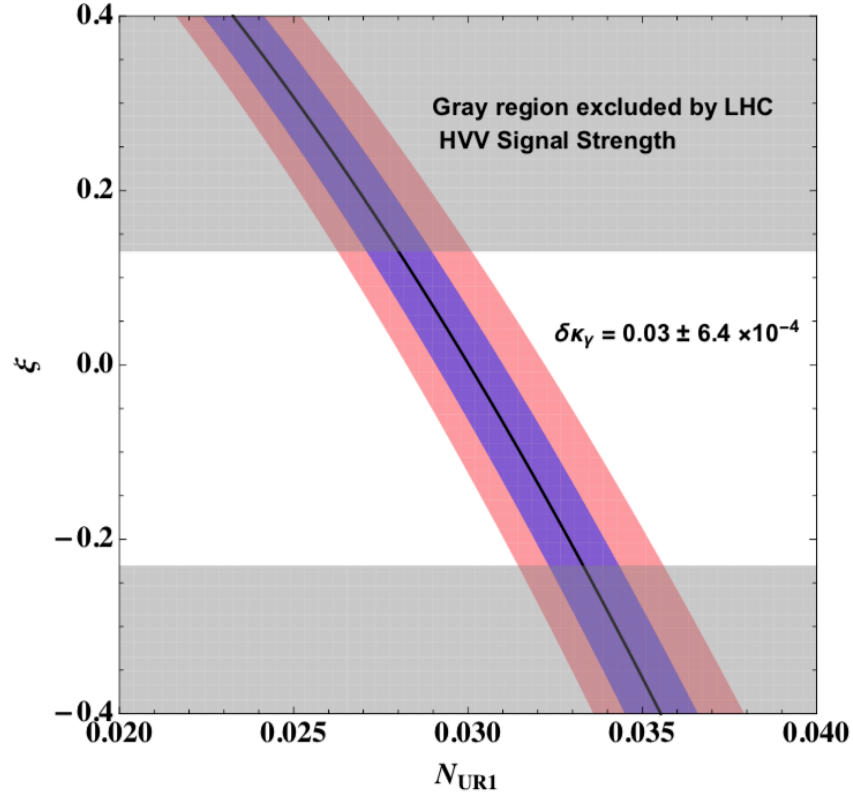
$$\mathrm{UR1}: \frac{N_{\mathrm{UR1}}}{\delta\kappa_\gamma} = \sqrt{1-\xi}$$

$\mathcal{I}_i^h$	$\frac{m_W^2}{m_\rho^2} C_i^h$(ILC)
(1) $hZ_\mu \mathcal{D}^{\mu\nu} Z_\nu / v$	5.83×10^{-4}
(2) $hZ_{\mu\nu} Z^{\mu\nu} / v$	3.93×10^{-4}
(3) $hZ_\mu \mathcal{D}^{\mu\nu} A_\nu / v$	
(4) $hZ_{\mu\nu} A^{\mu\nu} / v$	3.88×10^{-4}
$\mathcal{I}_i^{3V}$	$\frac{m_W^2}{m_\rho^2} C_i^{3V}$(ILC)
$(\delta g_1^Z) i\, g\, c_w W^{+\mu\nu} W_\mu^- Z_\nu + h.c.$	6.1×10^{-4}
$(\delta\kappa_\gamma) i\, e\, W_\mu^+ W_\nu^- A^{\mu\nu}$	6.4×10^{-4}

Table 7: Prospective 1σ uncertainty at the future lepton colliders, taken from Ref. [50].



(a) Measured ξ as a function of $\delta\kappa_\gamma$, using $N_{\rm UR1}$ as an input.



(b) Measured ξ as a function of $N_{\rm UR1}$, using $\delta\kappa_\gamma$ as an input.

Figure 5: Using UR1 to measure ξ, where the blue and red bands correspond to the 1σ and 2σ region on the measurement.

To test these relations requires high precision, high enough to be sensitive to effects of dim-8 operators –

Predictions from SMEFT with unconstrained coefficients:

$$\frac{s_{2w}C_1^{2h} - c_{2w}C_3^{2h}}{s_{2w}C_1^{h} - c_{2w}C_3^{h}} \approx \frac{1}{2}\left(1 + \frac{\alpha_W^8 + \alpha_B^8 + \alpha_{HW}^8 + \alpha_{HB}^8}{\alpha_W + \alpha_B + \alpha_{HW} + \alpha_{HB}}\xi\right),$$
$$\frac{s_{2w}C_2^{2h} - c_{2w}C_4^{2h}}{s_{2w}C_2^{h} - c_{2w}C_4^{h}} \approx \frac{1}{2}\left(1 + \frac{\alpha_{HW}^8 + \alpha_{HB}^8}{\alpha_{HW} + \alpha_{HB}}\xi\right).$$

Prediction from Universal Relations:

$$\text{UR7}: \frac{s_{2w}\,C_1^{2h} - c_{2w}\,C_3^{2h}}{s_{2w}\,C_1^{h} - c_{2w}\,C_3^{h}} = \frac{s_{2w}\,C_2^{2h} - c_{2w}\,C_4^{2h}}{s_{2w}\,C_2^{h} - c_{2w}\,C_4^{h}} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right)$$

To test these relations requires high precision, high enough to be sensitive to effects of dim-8 operators –

Predictions from SMEFT with unconstrained coefficients:

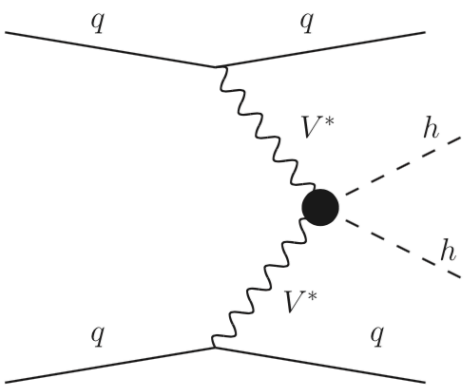
$$\frac{s_{2w}C_1^{2h} - c_{2w}C_3^{2h}}{s_{2w}C_1^{h} - c_{2w}C_3^{h}} \approx \frac{1}{2}\left(1 + \frac{\alpha_W^8 + \alpha_B^8 + \alpha_{HW}^8 + \alpha_{HB}^8}{\alpha_W + \alpha_B + \alpha_{HW} + \alpha_{HB}}\xi\right),$$
$$\frac{s_{2w}C_2^{2h} - c_{2w}C_4^{2h}}{s_{2w}C_2^{h} - c_{2w}C_4^{h}} \approx \frac{1}{2}\left(1 + \frac{\alpha_{HW}^8 + \alpha_{HB}^8}{\alpha_{HW} + \alpha_{HB}}\xi\right).$$

Prediction from Universal Relations:

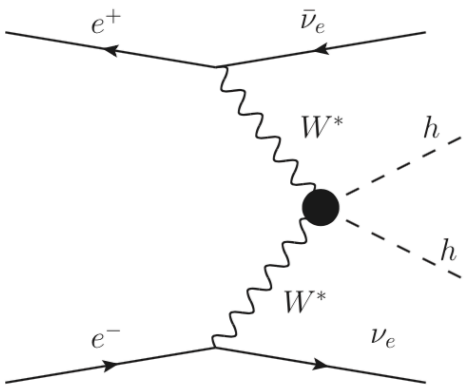
$$\mathrm{UR7}: \frac{s_{2w}\,C_1^{2h} - c_{2w}\,C_3^{2h}}{s_{2w}\,C_1^{h} - c_{2w}\,C_3^{h}} = \frac{s_{2w}\,C_2^{2h} - c_{2w}\,C_4^{2h}}{s_{2w}\,C_2^{h} - c_{2w}\,C_4^{h}} = \frac{\cos 2\theta}{2\cos\theta} \approx \frac{1}{2}\left(1 - \frac{3}{2}\xi\right)$$

This would be a nice playground to introduce new analysis techniques such as AI and ML?

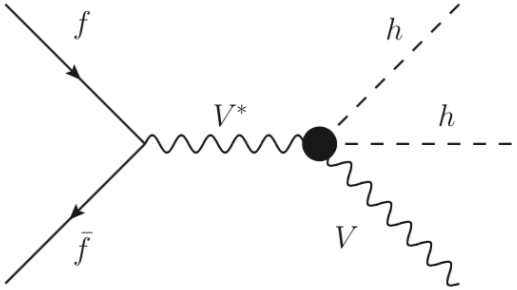
More generally, the HHVV coupling can be probed in the following channels:



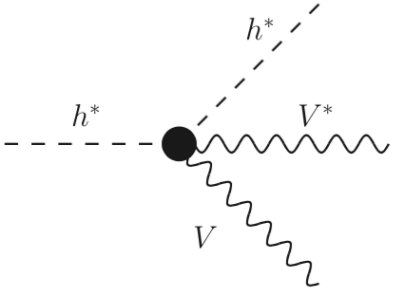
(a) Double Higgs production through vector boson fusion at a hadron collider.



(b) Double Higgs production through vector boson fusion at a lepton collider.

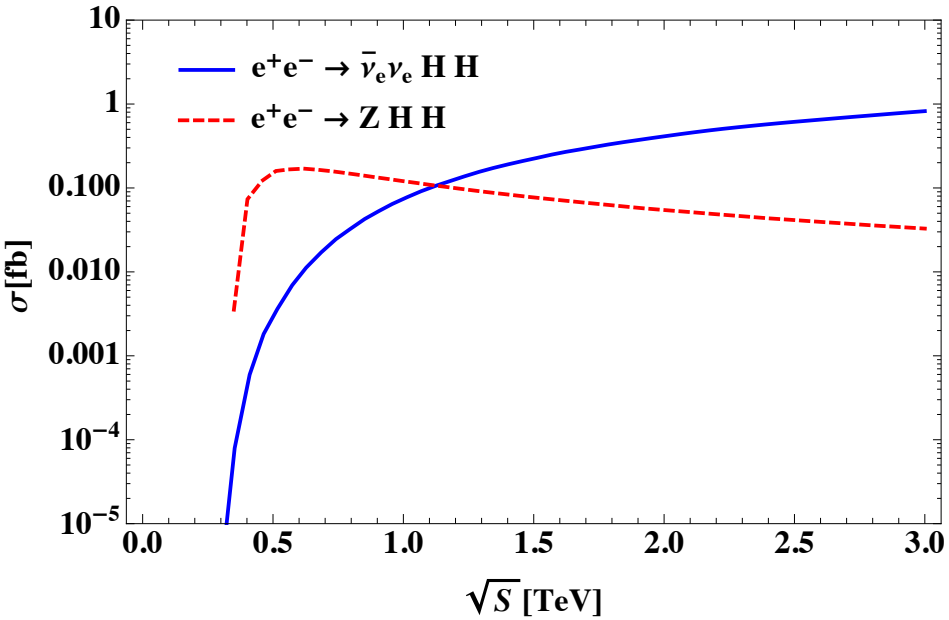


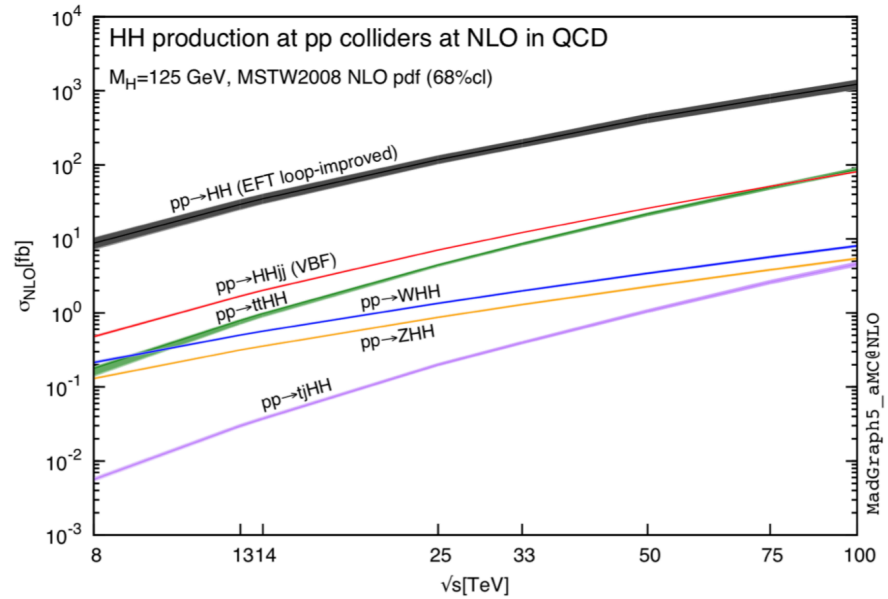
(c) Double Higgs production in association with a vector boson.



(d) Off-shell Single Higgs decay.

The rate at future colliders:





1401.7340

This is the future frontier of precision Higgs physics!

Concluding Remarks:

- HHVV coupling is the least studied coupling in Higgs physics. HH production in VBF channel will play a central role.

- Need to verify the tensor structure of the coupling, in the same fashion as in the studies of HVV coupling.

- Simultaneous measurements of HVV, HHVV and TGCs provide a unique window into the pNGB nature of the 125 GeV Higgs.

- The required precision to test the universal relations is high. Need to introduce advanced analysis techniques.