

# Direct measurement of the Higgs self-coupling in $e^+e^- \rightarrow ZH$ .

---

Junya Nakamura

Universität Tübingen

in collaboration with A. Shivaji.

Higgs Couplings 2018, Tokyo.

26 - 30 November 2018.

EBERHARD KARLS  
UNIVERSITÄT  
TÜBINGEN

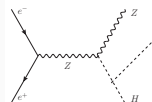


# Introduction

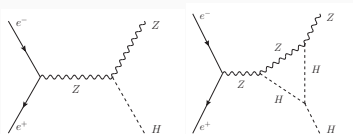
---

## Introduction: the trilinear Higgs self-coupling measurement.

- ♠ The capabilities of the LHC and future  $e^+e^-$  colliders to measure the **trilinear Higgs self-coupling**  $\lambda$  have been seriously studied in recent years.
- ♠ The "**direct**" measurement from the di-Higgs productions (such as  $gg \rightarrow HH$  and  $e^+e^- \rightarrow ZHH$ ) is challenging, because of their very small cross sections.



- ♠ "**Indirect**" constraint on  $\lambda$  may be obtained from the single-Higgs productions, because  $\lambda$  contributes to the EW one-loop correction (McCullough Phys.Rev.D90 (2014)).

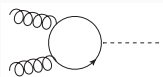


## Introduction: direct and indirect.

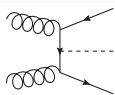
How can one distinguish "direct" from "Indirect"?

→ "Indirect" if the coupling constitutes a loop.

Examples:



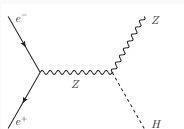
: Indirect  $ttH$ .



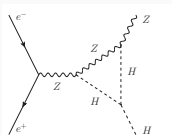
: Direct  $ttH$ .



: Indirect  $ttH$  & Direct  $HHH$ .



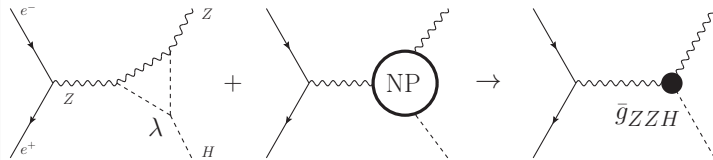
: Direct  $ZZH$ .



: Indirect  $HHH$ .

## Introduction: the weak point of the indirect method.

The weak point of the "indirect" method is that the result highly depends on assumptions about unknown NP at UV scale:



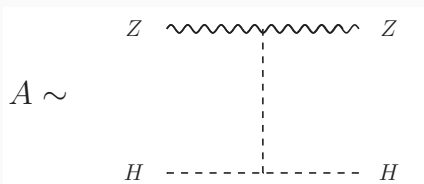
- ♠ Both the  $ZHH$  triangle-loop (left) and the one-loop correction due to heavy NP particles (middle) reduce to the same effective  $ZZH$  coupling (right).
- ♠ "NP at UV scale" and " $\lambda$ " may contribute to the cross section with the same magnitude.
- ♠ A virtual heavy fermion does not decouple from the cross section measured at low energy (Fleischer et al Nucl.Phys.B216 (1983)).

Message: a "indirect" constraint is very model-dependent.

## Introduction: a "direct" $\lambda$ measurement in $e^+e^- \rightarrow ZH$ .

In this work, a method of measuring "directly" the Higgs self-coupling  $\lambda$  in  $e^+e^- \rightarrow ZH$  is proposed:

- ♠ we consider the process  $e^+e^- \rightarrow Z(\rightarrow f\bar{f}) + H$ .
- ♠ we use **time-reversal-odd (T-odd) asymmetries**.
- ♠ the **T-odd asymmetries directly** probe  $\lambda$ , because the former measure the tree-level diagram for the  $ZH \rightarrow ZH$  scattering:



### Outline:

- ♠ General idea of time-reversal-odd (T-odd) quantities.
- ♠ T-odd asymmetries in  $e^+e^- \rightarrow Z(\rightarrow f\bar{f}) + H$ .
- ♠ Direct constraint on  $\lambda_H$  from the T-odd asymmetries.
- ♠ Conclusion.

**General idea of time-reversal-odd  
(T-odd) quantities.**

---



## General idea of T-odd quantities (I).

**T-odd observables** are generally define by (De Rujula et al Nucl.Phys.B35 (1971))

$$\mathcal{O} \equiv |\mathcal{M}_{fi}|^2 - |\mathcal{M}_{\tilde{f}\tilde{i}}|^2.$$

where  $\tilde{i}$  ( $\tilde{f}$ ) denotes the state obtained from  $i$  ( $f$ ) by reversing momenta and spins. Using unitarity of S-matrix ( $SS^\dagger = 1$ ), we may derive

$$\mathcal{O} = \underbrace{|\mathcal{M}_{if}|^2 - |\mathcal{M}_{\tilde{f}\tilde{i}}|^2}_{\text{vanishes when T-conserved}} - 2\text{Im}(\mathcal{M}_{fi}^* \mathcal{A}_{fi}) - |\mathcal{A}_{fi}|^2,$$

where

$$\mathcal{A}_{fi} \equiv (2\pi)^4 \sum_n \left[ \left( \prod_j \int \frac{d^3 p_j}{(2\pi)^3} \frac{1}{2E_j} \right) \delta^4(p_n - p_i) \mathcal{M}_{nf}^* \mathcal{M}_{ni} \right]$$

is called the **absorptive part of  $\mathcal{M}_{fi}$**  and the sum over all the possible asymptotic state  $n$  is performed. (also called scattering effects.)

Message: If T (or equally CP) is conserved, **T-odd observables** are proportional to **the absorptive part  $\mathcal{A}_{fi}$** .



**T-odd asymmetries in**

$$e^+e^- \rightarrow Z(\rightarrow f\bar{f}) + H.$$

---

## T-odd asymmetries in $e^+e^- \rightarrow Z(\rightarrow f\bar{f}) + H$ (I).

The differential cross section is

$$\frac{d^3\sigma}{d\cos\Theta d\cos\theta d\phi} = F_1(1 + \cos^2\theta) + F_2(1 - 3\cos^2\theta) + F_3 \sin 2\theta \cos\phi + F_4 \sin^2\theta \cos 2\phi \\ + F_5 \cos\theta + F_6 \sin\theta \cos\phi + F_7 \sin\theta \sin\phi + F_8 \sin 2\theta \sin\phi + F_9 \sin^2\theta \sin 2\phi,$$

where  $F_i$  ( $i = 1$  to  $9$ ) are functions of only  $s$ ,  $\cos\Theta$ . Note that

$$\frac{d\sigma}{d\cos\Theta} = \int_{-1}^1 d\cos\theta \int_0^{2\pi} d\phi \frac{d^3\sigma}{d\cos\Theta d\cos\theta d\phi} = \frac{16\pi}{3} F_1(\cos\Theta).$$

Under **T transformation without interchanging the initial and final states**,

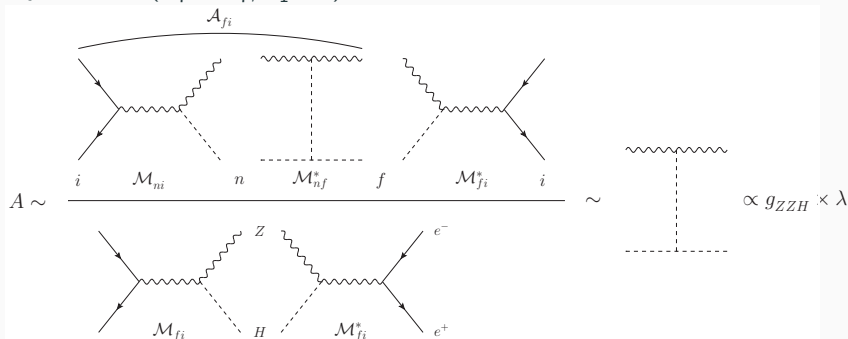
$$\frac{d^3\sigma}{d\cos\Theta d\cos\theta d\phi} \rightarrow \underbrace{F_1(1 + \cos^2\theta) + F_2(1 - 3\cos^2\theta) + F_3 \sin 2\theta \cos\phi + F_4 \sin^2\theta \cos 2\phi}_{\text{T-even}} \\ + \underbrace{F_5 \cos\theta + F_6 \sin\theta \cos\phi}_{\text{T-even}} - \underbrace{F_7 \sin\theta \sin\phi + F_8 \sin 2\theta \sin\phi + F_9 \sin^2\theta \sin 2\phi}_{\text{T-odd}},$$

Define **T-odd asymmetries** ( $A_7, A_8, A_9$ ) by

$$A_{(7,8,9)} \equiv \frac{F_{(7,8,9)}}{F_1}, \quad A_7 \propto \frac{N(\sin\phi > 0) - N(\sin\phi < 0)}{N(\sin\phi > 0) + N(\sin\phi < 0)} \text{ etc}$$

## T-odd asymmetries in $e^+e^- \rightarrow Z(\rightarrow f\bar{f}) + H$ (II).

Diagrams contributing to the numerator and denominator in the T-odd asymmetries ( $A_7 = F_7/F_1$  etc):



- ♠ The tree amplitude for  $e^+e^- \rightarrow ZH$  drops from the ratio and only the tree diagram for the  $ZH \rightarrow ZH$  scattering is left.
- ♠ The T-odd asymmetries measure " $g_{ZZH} \times \lambda$ ", and this is a "direct" probe of the Higgs self-coupling  $\lambda$ .

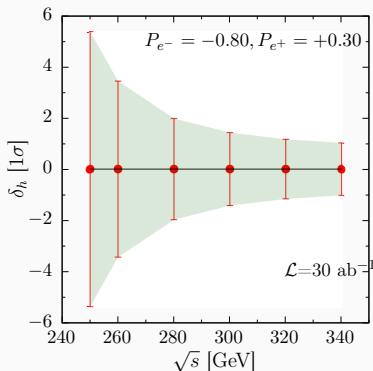
## **Direct constraint on $\lambda$ from the T-odd asymmetries**

---

## Direct constraint on $\lambda$ from the T-odd asymmetries.

Introduce a real parameter  $\delta_h$  as

$$\lambda = \frac{m_H^2}{2v^2}(1 + \delta_h).$$



The results have been obtained with FeynArts, FormCalc, LoopTools (FF) and BASES.

## Summary

---



## Summary.

- ♠ The indirect Higgs self-coupling  $\lambda$  measurement suffers from large dependence on unknown NP scenarios that do not necessarily influence  $\lambda$  itself.
- ♠ In this work, a method of measuring  $\lambda$  "directly" in  $e^+e^- \rightarrow ZH$  is discussed.
- ♠ The T-odd asymmetries directly probe  $\lambda$ , because (1) the former measure the tree-level diagram for the  $ZH \rightarrow ZH$  scattering and (2) NP at higher scale completely decouples from the former.
- ♠ The method is found very difficult. But!, this is probably the only method to constrain  $\lambda$  directly in  $e^+e^-$  collisions, when a beam energy below the  $ZHH$  threshold is only available. (Imagine the case that Japan faces an economical crisis when the ILC is running at 340 GeV...)