Differential decay rates of CP even and odd Higgs bosons to massive quarks at NNLO in α_s

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- $h \to b\bar{b}$ is the most probable decay channel of the $h(125)$, and finally observed recently at the LHC (in the *VH*-events)!

  [→talks by D.Schaefer, A.De Wit, G.Zanderighi]

- Many BSM predict heavy scalars and pseudo-scalars coupled to heavy quarks.

  [→talks by R.Santos, J.Steggemann]

- Much work done previously on neutral scalar bosons decay into quarks.
  - ***inclusive***:
    known up to N^4LO for CP-even Higgs into massless quarks;
    [Baikov,Chetyrkin,Kuhn,06; Davies,Steinhauser,Wellmann,17; Herzog,Ruijl,Ueda,Vermaseren,Vogt,17]
    $NNLO$ corrections for CP-even/odd Higgs in power expansion of $\frac{m_Q}{m_h}$;
    [Surguladze, 94; Chetyrkin,Kwiatkowski,96; Chetyrkin,Kniehl,Steinhauser,97; Larin,Ritbergen,Vermaseren,95;Harlander,Steinhauser, 97; Chetyrkin, Harlander,Steinhauser,97,98]
    . . . . . .
  - ***differential***:
    CP-even Higgs decay into massless quarks at $NNLO$
    [Anastasiou, Herzog, Lazopoulos, 2012; Duca, Duhr, Somogyi, Tramontano, Trocsanyi, 2015]

Aim of the work

We consider the decay of a neutral Higgs boson h of arbitrary *CP* to a massive quark antiquark pair at NNLO order in perturbative QCD, i.e. $h \to Q\bar{Q}X \ \ (Q = t, b)$, at the *fully differential* level (using the *antenna subtraction method*).

- BSM heavy CP-even/odd Higgs boson decay into $t\bar{t}$ pair:
  inclusive decay width as a function of m_h, and $M_{t\bar{t}}$ distributions, etc
- the SM $h(125)$ Higgs boson decay into a massive $b\bar{b}$ pair:
  inclusive decay width, 2-jet, 3-jet, and 4-jet decay width,
  and the energy distribution of the leading jet for two-jet events.

UV-renormalization and (antenna) IR subtraction

The *hybrid* UV-renormalization in *pQCD* with $n_f + 1$ quarks:

- External-fields (and their masses): **on-shell** scheme;
- α_s: $\overline{\mathrm{MS}}$-scheme;
- The Yukawa-coupling y_Q: $\overline{\mathrm{MS}}$-scheme

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The general idea of IR-subtraction methods:
insert an *identity* $0 = \int d\sigma^{\mathcal{S}} - \int d\sigma^{\mathcal{S}}$ into $\sigma_{\textbf{NLO}} = d\sigma^{\mathcal{R}} + d\sigma^{\mathcal{V}}$.

The $d\sigma^{\mathcal{S}}$ in the *antenna* subtraction method are constructed according to the *universal IR-factorization formulae* of *color-ordered* partial QCD-amplitudes,

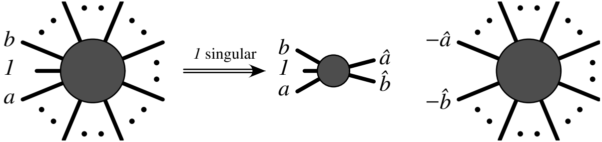


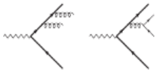
Figure: The antenna factorization of a *color-ordered* partial amplitude (PRD 71, 045016)

Organizations of ingredients for $h \to Q\bar{Q} + X$ ($Q = t, b$)

Schematically the NNLO corrections with *antenna* IR subtraction terms:

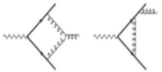
$$
\begin{aligned}
d\sigma_{\mathrm{NNLO}} &= \int_{d\Phi_4} \left(d\sigma^{RR}_{\mathrm{NNLO}} - d\sigma^{S}_{\mathrm{NNLO}} \right) \\
&+ \int_{d\Phi_3} \left(d\sigma^{RV}_{\mathrm{NNLO}} - d\sigma^{T}_{\mathrm{NNLO}} \right) \\
&+ \int_{d\Phi_2} d\sigma^{VV}_{\mathrm{NNLO}} + \int_{d\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\mathrm{NNLO}}
\end{aligned}
$$

- RR: Tree-level double real radiation correction: $h \to Q\bar{Q}gg$, $Q\bar{Q}q\bar{q}$ and $Q\bar{Q}Q\bar{Q}$



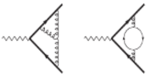
implicit IR-singularity removed by $d\sigma^{S}_{\mathrm{NNLO}}$.

- RV: One-loop correction to $h \to Q\bar{Q}g$



explicit and implicit IR-singularity removed by $d\sigma^{T}_{\mathrm{NNLO}}$

- VV: Two-loop corrections to $h \to Q\bar{Q}$



explicit IR-poles removed by $\int_{d\Phi_3} d\sigma^{T}_{\mathrm{NNLO}} + \int_{d\Phi_4} d\sigma^{S}_{\mathrm{NNLO}}$

Decay widths in terms of on-shell and $\overline{\mathrm{MS}}$ Yukawa-couplings

The *differential* decay width of a Higgs boson with a generic CP into **unpolarized** $Q\bar{Q}$:

$$d\Gamma^{Q\bar{Q}} = a_Q^2 d\Gamma_S^{Q\bar{Q}} + b_Q^2 d\Gamma_P^{Q\bar{Q}}$$

with the "*reduced*" Yukawa-couplings a_Q and b_Q as in $-y_Q h\left[a_Q \bar{Q}Q + b_Q \bar{Q} i\gamma_5 Q\right]$ (where $y_Q = \frac{m_Q}{v_{vev}}$).

Expanded to order α_s^2:

$$\begin{aligned} d\Gamma^{Q\bar{Q}} &= y_Q^2 \left[d\hat{\Gamma}_0^{Q\bar{Q}} + \frac{\alpha_s(\mu)}{\pi} d\hat{\Gamma}_1^{Q\bar{Q}} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 d\hat{\Gamma}_2^{Q\bar{Q}}\right] \\ &\equiv y_Q^2\, d\hat{\Gamma}_0^{Q\bar{Q}} \left[1 + \frac{\alpha_s(\mu)}{\pi} d\gamma_1^{Q\bar{Q}} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 d\gamma_2^{Q\bar{Q}}\right] \end{aligned} \quad (1)$$

The on-shell and $\overline{\mathrm{MS}}$ Yukawa-couplings are related by

$$y_Q^2 = \bar{y}_Q^2(\mu)\left[1 + r_1(m_Q,\mu)\frac{\alpha_s(\mu)}{\pi} + r_2(m_Q,\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\right] \quad (2)$$

Inserting (2) into (1) and **re-expanding** to order α_s^2

$$d\bar{\Gamma}^{Q\bar{Q}} = \bar{y}_Q^2(\mu) d\hat{\Gamma}_0^{Q\bar{Q}}\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(d\gamma_1^{Q\bar{Q}} + r_1\right) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2\left(d\gamma_2^{Q\bar{Q}} + r_1 d\gamma_1^{Q\bar{Q}} + r_2\right)\right].$$

We work in 6-flavor QCD:

Standard-Model inputs

$m_t^{on} = 173.34$ GeV, corresponding to $\overline{m}_t(\mu = m_t) = 163.46$ GeV;

$\alpha_s^{(5)}(m_Z) = 0.118$; $\mathbf{G}_F = 1.166379 * 10^{-5} \frac{1}{\text{GeV}^2}$

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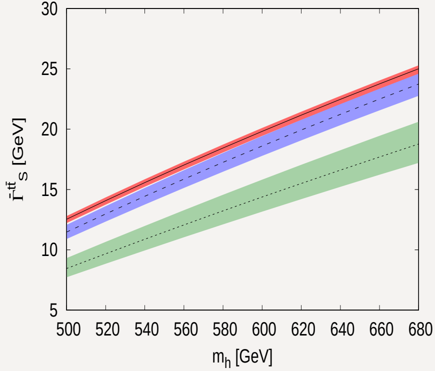
The $t\bar{t}$ inclusive decay widths at NNLO QCD, $\overline{\Gamma}_X^{t\bar{t}}$ and $\Gamma_X^{t\bar{t}}$ $(X = S, P)$

m_h [GeV]	$\overline{\Gamma}_S^{t\bar{t}}$ [GeV]	$\Gamma_S^{t\bar{t}}$ [GeV]	$\overline{\Gamma}_P^{t\bar{t}}$ [GeV]	$\Gamma_P^{t\bar{t}}$ [GeV]
500	$12.529^{+0.265}_{-0.314}$	$12.955^{+0.037}_{-0.046}$	$22.392^{+0.283}_{-0.411}$	$22.931^{+0.030}_{-0.062}$
680	$25.007^{+0.285}_{-0.408}$	$25.647^{+0.075}_{-0.101}$	$32.188^{+0.214}_{-0.397}$	$32.784^{+0.185}_{-0.225}$

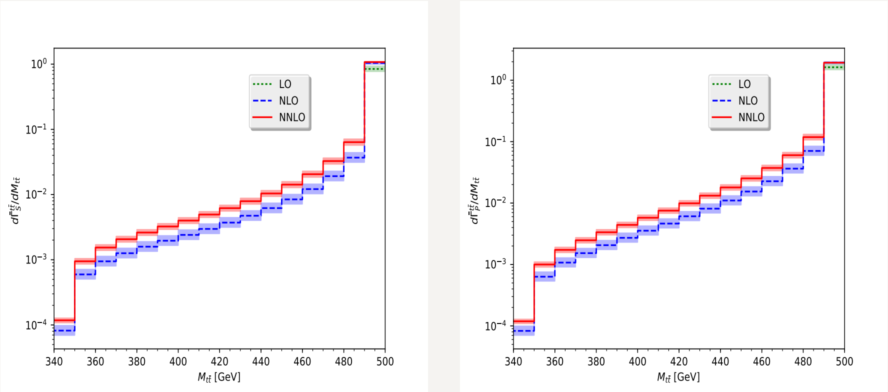
These NNLO QCD results for inclusive $t\bar{t}$-decay widths (exact in m_t) **agree** with the large m_h approximation result (to 4-th order in $(m_t/m_h)^2$) in [Harlander, Steinhauser, 1997].

The decay width into $t\bar{t}$ of scalars/pseudo-scalars at LO, NLO, and NNLO in α_s as a function of m_h.



Decays of BSM scalars/pseudo-scalars into $t\bar{t}$: $m_{t\bar{t}}$ distribution

Distribution $d\bar{\Gamma}_X^{t\bar{t}}/dM_{t\bar{t}}$ of the $t\bar{t}$ invariant mass with $m_h = 500$ GeV.



SM $h(125) \to b\bar{b} + X$: the inclusive decay width

We work in a "5-flavor" QCD:

Standard-Model inputs

$$m_h = 125.09 \text{ GeV}; \qquad \overline{m}_b(\mu = \overline{m}_b) = 4.18 \text{ GeV};$$
$$\alpha_s^{(5)}(m_Z) = 0.118; \qquad \mathbf{G}_F = 1.166379 * 10^{-5} \frac{1}{\text{GeV}^2}$$

From the 5-flavor QCD two-loop running-mass formula, it reads $m_b^{on} = 4.78$ GeV and $\overline{m}_b(\mu = m_h) = 2.80$ GeV, and hence $\overline{y}_b(\mu) = \frac{\overline{m}_b(\mu)}{v_{vev}} = 0.01137$.

We represent our result for inclusive decay width using $\overline{\text{MS}}$ Yukawa-coupling $\overline{y}_b$:

$$\overline{\Gamma}^{b\bar{b}}_{NNLO} = \overline{\Gamma}^{b\bar{b}}_{LO} \left[1 + \mathbf{g_1} \frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2} \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] ,$$

where

$$\overline{\Gamma}^{b\bar{b}}_{LO} = \overline{y}_b^2(\mu) \hat{\Gamma}^{b\bar{b}}_0, \quad \mathbf{g_1} = \gamma_1^{b\bar{b}} + r_1, \quad \mathbf{g_2} = \gamma_2^{b\bar{b}} + r_1 \gamma_1^{b\bar{b}} + r_2 \, .$$

SM $h(125) \to b\bar{b} + X$: the inclusive decay width

For the QCD-correction coefficients $\mathbf{g_1}$, $\mathbf{g_2}$ defined in

$$\overline{\Gamma}^{b\bar{b}}_{NNLO} = \overline{\Gamma}^{b\bar{b}}_{LO} \left[1 + \mathbf{g_1} \frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2} \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] ,$$

	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$\mathbf{g_1}$	3.024	5.796	8.569
$\mathbf{g_2}$	3.685	37.371	86.112
$\overline{\Gamma}^{b\bar{b}}_{LO}$ [MeV]	2.153	1.910	1.717
$\overline{\Gamma}^{b\bar{b}}_{NLO}$ [MeV]	2.413	2.307	2.196
$\overline{\Gamma}^{b\bar{b}}_{NNLO}$ [MeV]	2.425	2.399	2.353

we obtain

The known results for massless b quarks ($\mu = m_h$):

$$\mathbf{g_1}(m_b = 0) = 5.6666 \text{ and } \mathbf{g_2}(m_b = 0) = 29.1467$$

[Gorishnii, Kataev, Larin, Surguladze, 1990; K. G. Chetyrkin, 1996]

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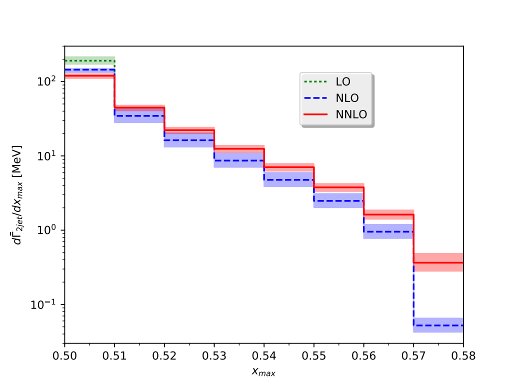
$1 + \mathbf{g_1}\alpha_s + \mathbf{g_2}\alpha_s^2 + \cdots$	total value	components
massive (α_s^2)	1.2560	$1 + 0.20789 + 0.04808$
massless (α_s^4)	1.2413	$1 + 0.203242 + 0.0374917 + 0.001927 + (-0.001366)$

$n_f = 5, \quad \mu = m_h$ [Baikov,Chetyrkin,Kuhn,06]

SM $h(125) \to b\bar{b} + X$: the x_{max} distribution

The distribution of the energy of the leading jet in two-jet events is defined w.r.t $x_{max} = \max(E_{j_1}/m_h, E_{j_2}/m_h)$ using *Durham jet-algorithm* with $y_{cut} = 0.05$.



Similar distributions were presented before for *massless* b quarks [Anastasiou, Herzog, Lazopoulos, 2012] (JADE, with $y_{cut} = 0.1$) and in [Duca, Duhr, Somogyi, Tramontano, Trocsanyi, 2015] (Durham, with $y_{cut} = 0.05$).

Summary and Outlook

- ☑ A set up is presented for calculating the fully ***differential*** decay width of a *scalar* and *pseudo-scalar* to a **massive** $Q\bar{Q}$ pair at NNLO in α_s, which can be used to compute any *infrared-safe* (differential) observable in these decays.

- ☑ The set-up is applied to the decays of heavy scalars and pseudo-scalars to $t\bar{t} + X$, and to the decay of the Standard-Model $h(125)$ Higgs boson to massive $b,\ \bar{b}$ quarks. As a check, inclusive decay rates known before are recovered.

- ☒ We expect that this set up should be useful for having a more precise (and consistent) theoretical description of the α_s^2 QCD corrections to the production of the Higgs boson in association with a massive vector boson at LHC, $\mathbf{pp} \to \mathbf{V}(W/Z) + \mathrm{H}(b\bar{b})$.

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THANK YOU

THE END

THANK YOU

Backup-Slides

Top-Triangle diagrams absent in massless b-quark computations

These amplitudes are ultraviolet- and infrared-finite (directly calculable in 4-dimension).

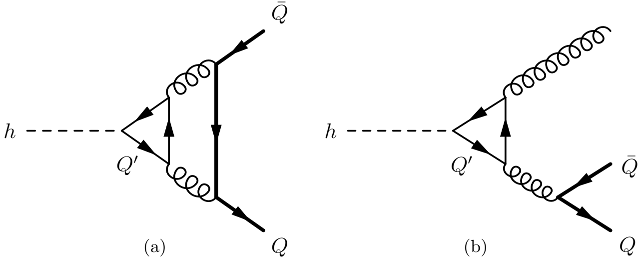


Figure: The flavor-singlet contributions to $h \to Q\bar{Q}$ and $h \to Q\bar{Q}g$, where $Q, Q' = t, b$ and h denotes a scalar or pseudoscalar Higgs boson. If the quark in a triangle (a) or (b) is taken to be massless then the contribution is zero. Diagrams with reversed fermion flow in the triangles are not shown.

These contributions can **not** be consistently added in a massless computation at NNLO [Caola, Luisoni, Melnikov, Rontsch, 17].

$h \to Q\bar{Q}$ decay width in detail

To lowest order in α_s

$$\hat{\Gamma}_0^{Q\bar{Q}} = N_c \, \frac{\beta_Q}{8\pi m_h} [a_Q^2 (m_h^2 - 4m_Q^2) + b_Q^2 m_h^2] \, ,$$

where $\beta_Q = \sqrt{1 - 4m_Q^2/m_h^2}$ and N_c denotes the number of colors.

$$\overline{\Gamma}_{NNLO}^{b\bar{b}} = \overline{\Gamma}_{LO}^{b\bar{b}} \left[1 + \mathbf{g_1} \frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2} \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] \, ,$$

where

$$\overline{\Gamma}_{LO}^{b\bar{b}} = \overline{y}_b^2(\mu) \hat{\Gamma}_0^{b\bar{b}}, \quad \mathbf{g_1} = \gamma_1^{b\bar{b}} + r_1, \quad \mathbf{g_2} = \gamma_2^{b\bar{b}} + r_1 \gamma_1^{b\bar{b}} + r_2 \, .$$

$y^{\overline{MS}}{}_b$	$\mu = m_h$
$\mathbf{g_1}$	5.796
$\mathbf{g_2}$	37.371
$\overline{\Gamma}_{LO}^{b\bar{b}}$ [MeV]	1.910
$\overline{\Gamma}_{NLO}^{b\bar{b}}$ [MeV]	2.307
$\overline{\Gamma}_{NNLO}^{b\bar{b}}$ [MeV]	2.399

$y_b^{\text{on-shell}}$	$\mu = m_h$
$\{\gamma_1^{b\bar{b}}, r_1\}$	$\{-9.93, 15.73\}$
$\{\gamma_2^{b\bar{b}}, r_2\}$	$\{-113.2, 306.7\}$
$\Gamma_{LO}^{b\bar{b}}$ [MeV]	5.578
$\Gamma_{NLO}^{b\bar{b}}$ [MeV]	3.592
$\Gamma_{NNLO}^{b\bar{b}}$ [MeV]	2.772

$h \to Q\bar{Q}$ decay width with $m_b = 0.5$ GeV

The QCD-correction coefficients $\mathbf{g_1}$, $\mathbf{g_2}$ are defined in

$$\overline{\Gamma}^{b\bar{b}}_{NNLO} = \overline{\Gamma}^{b\bar{b}}_{LO}\left[1 + \mathbf{g_1}\frac{\alpha_s^{(5)}}{\pi} + \mathbf{g_2}\left(\frac{\alpha_s^{(5)}}{\pi}\right)^2\right].$$

The known results for massless b quarks ($\mu = m_h$):

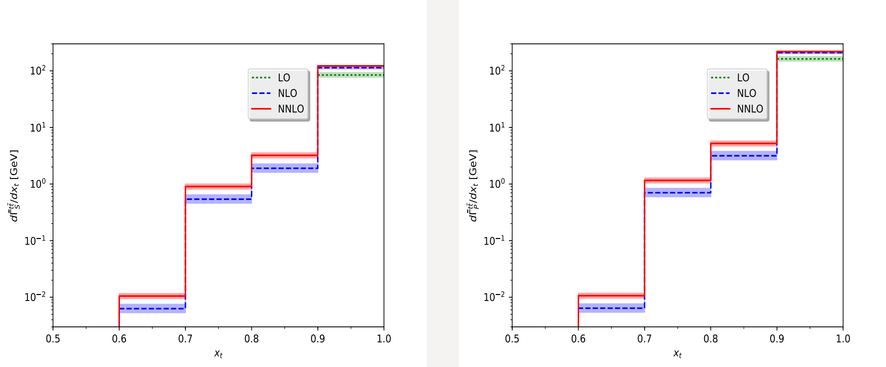
$$\mathbf{g_1}(m_b = 0) = 5.6666 \text{ and } \mathbf{g_2}(m_b = 0) = 29.1467$$

[Gorishnii, Kataev, Larin, Surguladze, 1990; K. G. Chetyrkin, 1996]

We cannot perform the limit $m_b \to 0$ analytically. Therefore we choose a value for $m_b = 0.5$ GeV that is very small compared to m_h ($\approx \frac{1}{250}$), and excluding the **top-quark triangle loop diagrams** (which contributes $g2_t = 6.898$, not sensitive to m_b), we obtain

$$\mathbf{g_1}(m_b = 0.5\text{GeV}) = 5.6685, \text{ and } \mathbf{g_2}(m_b = 0.5\text{GeV}) = 29.187.$$

The normalized top-quark energy $x_t = 2E_t/m_h$ (in the rest frame of the Higgs boson) $d\bar{\Gamma}_X^{t\bar{t}}/dx_t$ for a scalar/pseudo-scalar Higgs boson at LO, NLO, and NNLO QCD.



$h(125) \to b\bar{b} + X$: different jet rates

The n-jet rates can be represented, in analogy to the inclusive decay width, to order α_s^2 as follows:

$$\bar{\Gamma}^{b\bar{b}}_{2\,\mathrm{jet}} = \bar{\Gamma}^{b\bar{b}}_{LO} \left[1 + g_1(2\,\mathrm{jet}) \frac{\alpha_s^{(5)}}{\pi} + g_2(2\,\mathrm{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] , \tag{4}$$

$$\bar{\Gamma}^{b\bar{b}}_{3\,\mathrm{jet}} = \bar{\Gamma}^{b\bar{b}}_{LO} \left[g_1(3\,\mathrm{jet}) \frac{\alpha_s^{(5)}}{\pi} + g_2(3\,\mathrm{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \right] , \tag{5}$$

$$\bar{\Gamma}^{b\bar{b}}_{4\,\mathrm{jet}} = \bar{\Gamma}^{b\bar{b}}_{LO} \times g_2(4\,\mathrm{jet}) \left(\frac{\alpha_s^{(5)}}{\pi} \right)^2 \tag{6}$$

Table: The coefficients $g_i(\mathrm{n\,jet})$ defined above and computed with the Durham algorithm using $y_{cut} = 0.01$ and $y_{cut} = 0.05$ for three renormalization scales μ.

	$y_{cut} = 0.01$			$y_{cut} = 0.05$		
	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$g_1(2\,\mathrm{jet})$	-5.055	-2.283	0.490	0.291	3.063	5.836
$g_2(2\,\mathrm{jet})$	-56.351	-66.532	-61.658	-19.496	-0.650	33.250
$g_1(3\,\mathrm{jet})$	8.079	8.079	8.079	2.733	2.733	2.733
$g_2(3\,\mathrm{jet})$	36.873	80.741	124.609	22.256	37.096	51.937
$g_2(4\,\mathrm{jet})$	23.163	23.163	23.163	0.926	0.926	0.926