Symmetry and geometry in generalized Higgs sector

~ Finiteness of oblique corrections v.s. perturbative unitarity ~

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(Nagoya U.)

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Introduction

We believe that the SM is not complete   Hierarchy problem? Dark Matter?, etc…

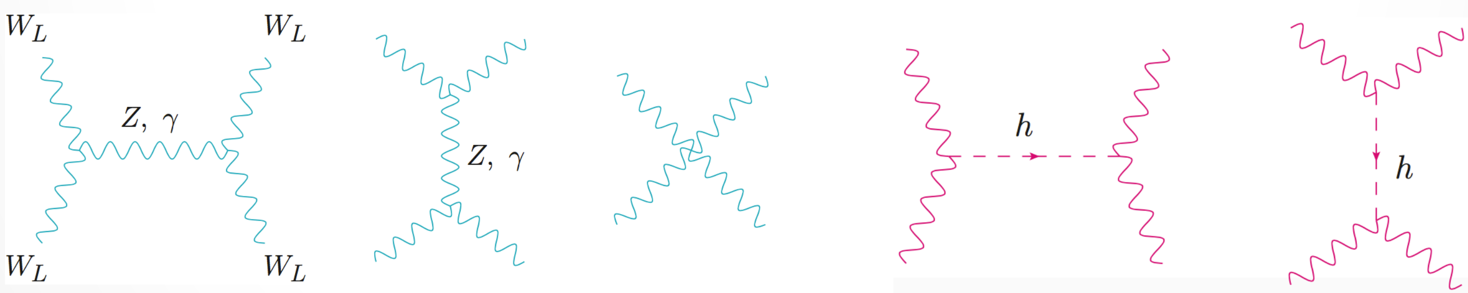
→ Beyond the SM is needed !

Correct understanding of the EWSB sector is the key for NP search

<Standard Model>

I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1-(\kappa_V^h)^2\right) \quad (\ \kappa_V^h = 1 \ \text{ in SM })$$



II. Higgs cancels the divergence in oblique corrections

Peskin Takeuchi
Phys. Rev. Lett. 65 (1990) 964

$$S \simeq \frac{1}{12\pi}\left(1-(\kappa_V^h)^2\right)\ln\frac{\Lambda^2}{\mu^2}$$

Introduction

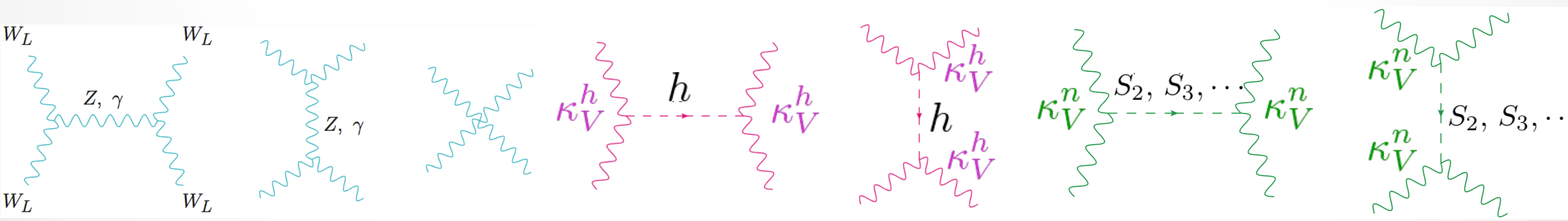
$$\mathcal{L} = \frac{v^2}{4} F(h, S_n) \mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \mathcal{L}_{kinetic} - V(h, S_n)$$

$$F(h, S_n) = 1 + 2\kappa_V^h \frac{h}{v} + 2\sum_n \kappa_V^n \frac{S_n}{v} + \cdots \qquad U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$$

< Singlet extension of the SM (w/ custodial sym.) >

I. Higgs unitalize W_L scattering amplitude at tree level (tree level unitarity)

$$\mathcal{M}_{W_L W_L \to W_L W_L} \simeq \frac{s+t}{v^2}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N} (\kappa_V^n)^2\right)$$



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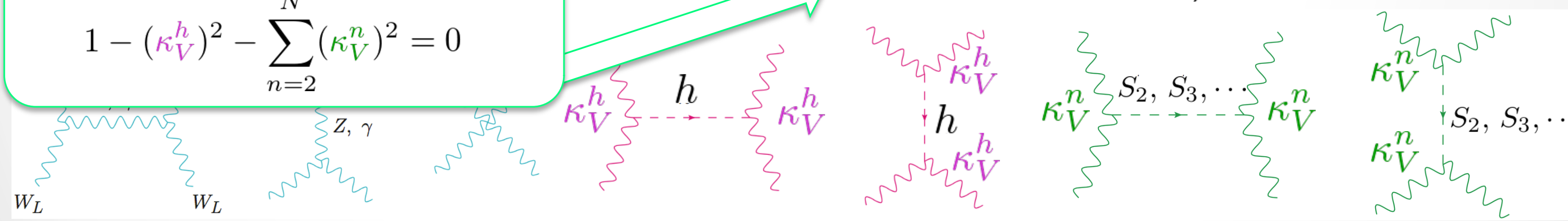
unitarity sum rules

$$1-(\kappa_V^h)^2-\sum_{n=2}^{N}(\kappa_V^n)^2 = 0$$



finiteness conditions

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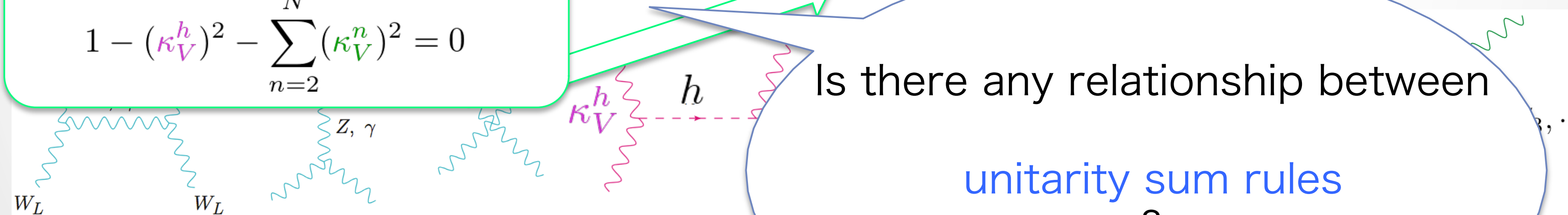

Is there any relationship between

unitarity sum rules

&

finiteness conditions

?

finiteness conditions

$$1 - (\kappa_V^h)^2 - \sum_{n=2}^N (\kappa_V^n)^2 = 0$$

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$$\frac{}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^N (\kappa_V^n)^2\right) \ln\frac{\Lambda^2}{\mu^2}$$

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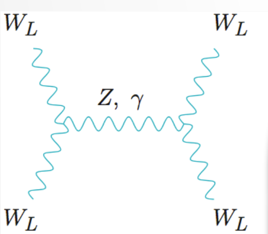

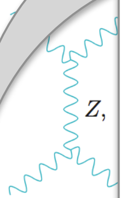

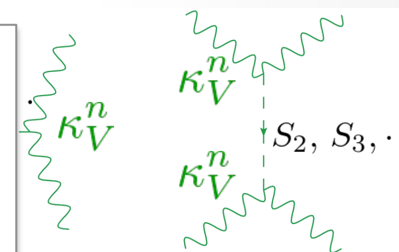

If we impose the unitarity sum rules
in singlet extension,

S, T, U parameter's one loop finiteness is
automatically guaranteed

R Nagai, M Tanabashi, K Tsumura Phys. Rev. D 91, 034030 (2015)

II. Higgs ca cels

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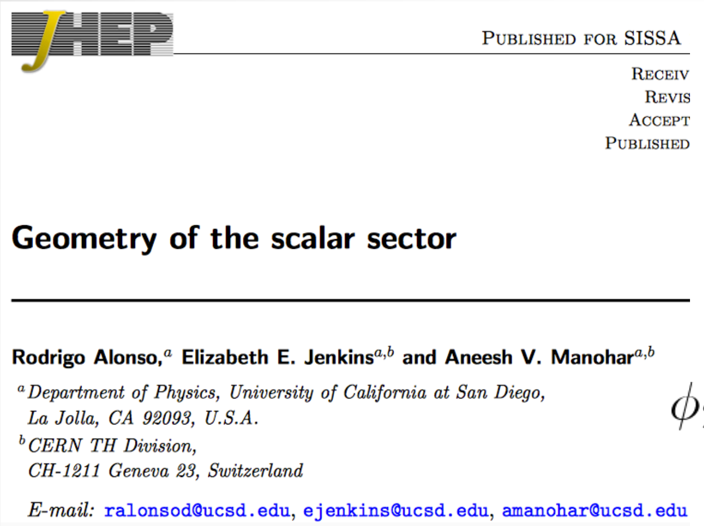
Rev. Lett. 65 (1990) 964

$$S \simeq \frac{1}{12\pi}\left(1 - (\kappa_V^h)^2 - \sum_{n=2}^{N} (\kappa_V^n)^2\right) \ln\frac{\Lambda^2}{\mu^2}$$

We have succeeded to verify the following relation in arbitrary model !

tree level unitarity $\Rightarrow$ S,U parameter 1-loop finiteness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Geometry of the scalar sector

Rodrigo Alonso,[a] Elizabeth E. Jenkins[a,b] and Aneesh V. Manohar[a,b]

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[b] *CERN TH Division, CH-1211 Geneva 23, Switzerland*

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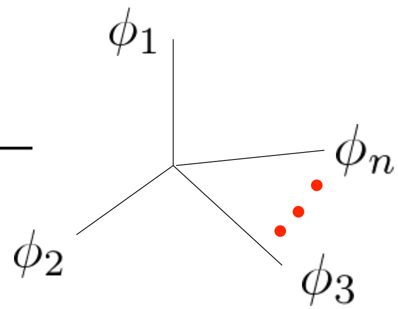
Let's focus on the internal space of scalar fields…



$$\mathcal{L} = \frac{1}{2}\underbrace{g_{ij}(\phi)}_{\text{metric tensor}}(\partial_\mu\phi)^i(\partial^\mu\phi)^j - V(\phi)$$

R. Alonso, E. E. Jenkins and A V Manohar JHEP08(2016)101

ex.) Singlet extension w/ custodial sym.

$$U = \exp\left(i\frac{\pi^a}{v}\frac{\tau^a}{2}\right)$$

$$\mathcal{L} = \frac{v^2}{4}F(h,S_n)\mathrm{Tr}[(\partial_\mu U)^\dagger\partial^\mu U] + \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{1}{2}\sum_{n=2}^{N}\partial_\mu S_n\partial^\mu S_n$$

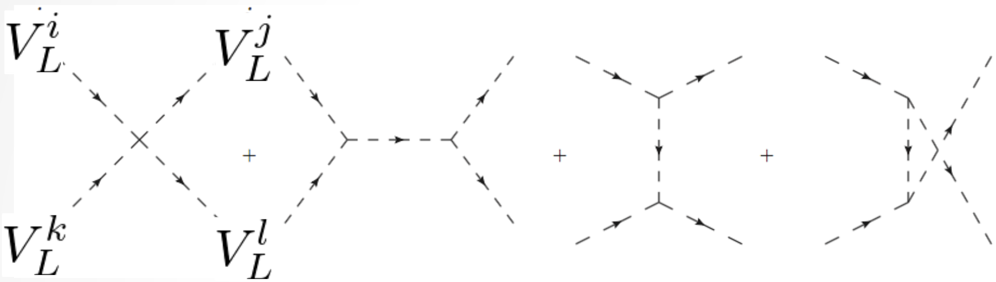
$$= \frac{1}{2}\partial_\mu(\pi^a,\, h,\, S_2,\, S_3,\cdots)\underbrace{\begin{pmatrix} F(h,S_n)(\delta_{ab}+\mathcal{O}(\pi^2)) & 0 \\ 0 & 1_{N\times N}\end{pmatrix}}_{\text{metric tensor}}\partial^\mu\begin{pmatrix}\pi^b \\ h \\ S_2 \\ S_3 \\ \vdots\end{pmatrix}$$

$$= \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j$$

We have succeeded to verify the following relation in arbitrary model !

$$\boxed{\text{tree level unitarity} \Rightarrow \text{S,U parameter 1-loop finiteness}}$$

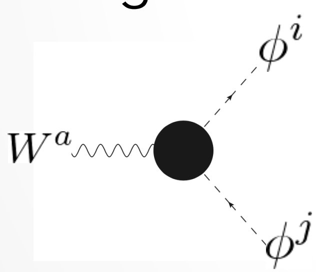
① Unitarity-violating amplitudes can be written as Riemann tensor



$$= -i(t\,\mathcal{R}_{ijkl} + u\,\mathcal{R}_{iljk})$$

Riemann tensor = 0 $\Longleftrightarrow$ Tree level unitary is respected

② Charged and neutral current can be written as Killing vector



$$= (w_a^i)_{;j}$$

w_a^i : Killing vector for SU(2)_L sym. (a=1~3)

y^i : Killing vector for U(1)_Y sym.

S and U parameter can be written in terms of $\underbrace{\text{charged and neutral currents}}_{\text{Killing vector}}$

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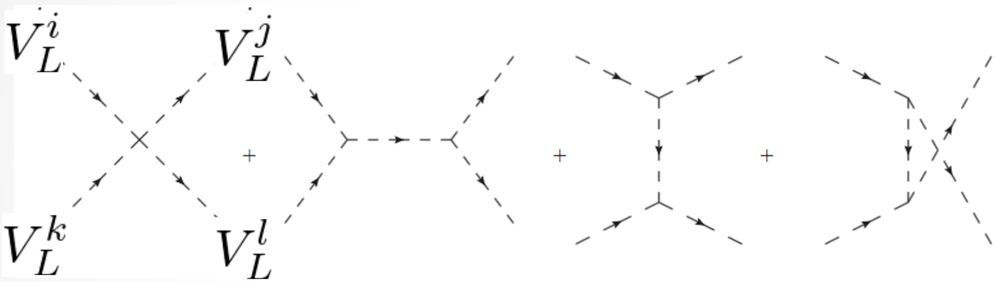

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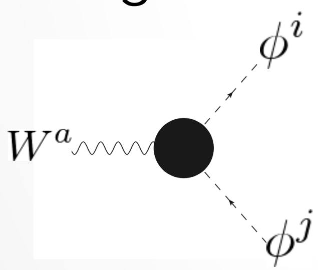

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$$S_{\rm div} = -\frac{1}{6\pi}\sum_{i,j} w^j_{3;i}\, y^i_{;j} \times \ln(\Lambda/\mu)$$

$$U_{\rm div} = \frac{1}{6\pi}\sum_{i,j}\left((w_1^j)_{;i}(w_1^i)_{;j} - (w_3^j)_{;i}(w_3^i)_{;j} \right)\ln(\Lambda/\mu)$$

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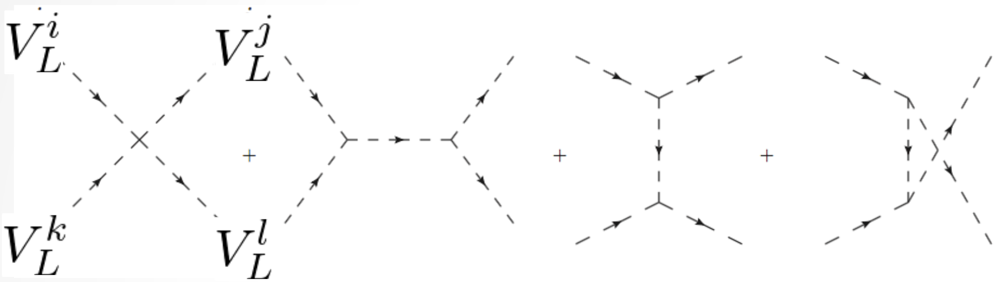

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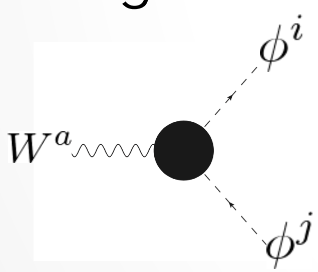

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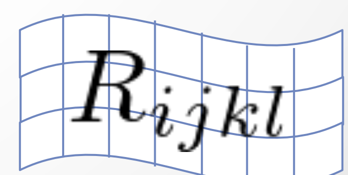
We can relate Killing vector with Riemann tensor by Killing equation

$$g_{il}(w_a^l)_{;j;k} = w_a^l R_{lkji}$$



https://ja.wikipedia.org/wiki/キリングベクトル場

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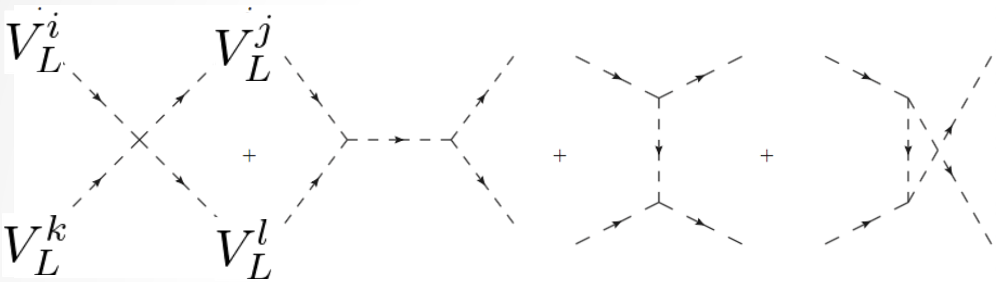

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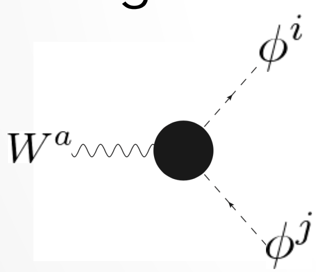

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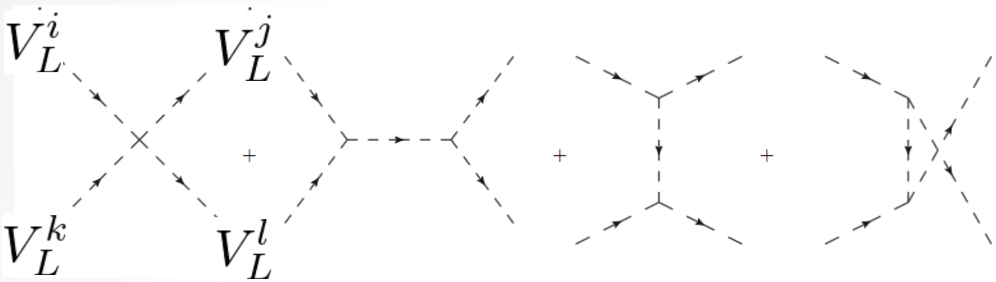

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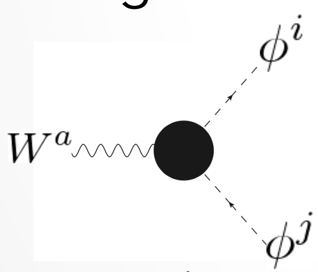

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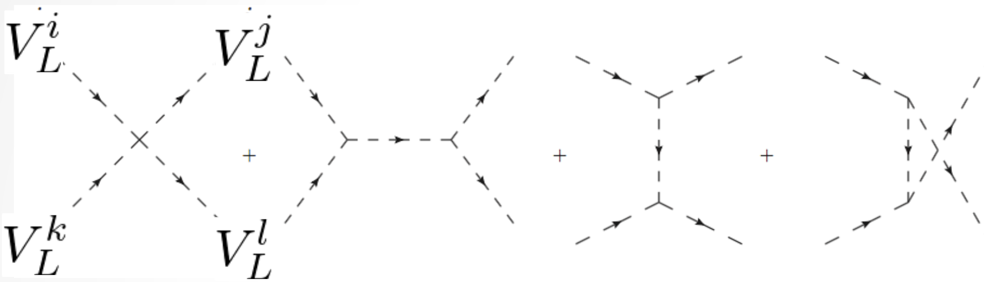

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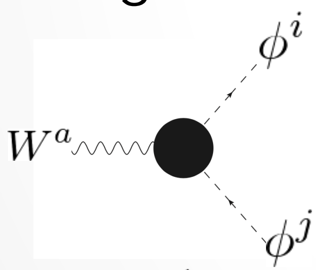

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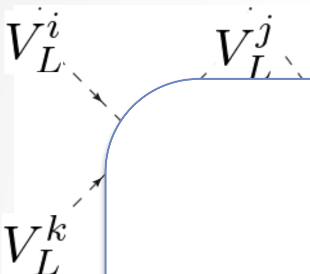


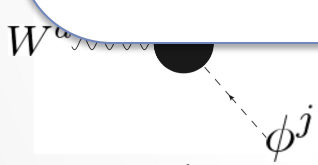
If tree level unitary is satisfied
(scalar manifold is flat)

$\Downarrow$

S, U parameter is 1-loop finite
(product of killing vectors = 0)

② C … or

(a=1~3)



$= (w_a)_{;j}$

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Main statement

The most general EFT in terms of SM particles is

Higgs Effective Field Theory (HEFT)

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{G^2}(h)\, G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4}\mathcal{F}_{W^2}(h)\, W^I_{\mu\nu} W^{I\mu\nu} - \frac{1}{4}\mathcal{F}_{B^2}(h)\, B_{\mu\nu} B^{\mu\nu}$$

$$+\bar{\psi} i \not{D} \psi + \frac{1}{2}\partial^\mu h \partial_\mu h + \underline{\frac{v^2}{4}\mathcal{F}_{V^2}(h)\, \mathrm{Tr}\,(D_\mu U) U^\dagger (D^\mu U) U^\dagger - V(h)}$$

$$-\frac{v}{\sqrt{2}}\left(\left[\bar{u}\,\mathcal{F}_{\mathcal{Y}_u}(h)\, U^\dagger q\right] + \left[\bar{d}\,\mathcal{F}_{\mathcal{Y}_d}(h)\, U^\dagger q\right] + \left[\bar{e}\,\mathcal{F}_{\mathcal{Y}_e}(h)\, U^\dagger l\right] + \mathrm{h.c.}\right)$$

$$D_\mu U(\varphi(x)) = \partial_\mu U + i g W^I_\mu T^I\, U - i g' B_\mu\, U\, T^3, \quad D^\mu h = \partial^\mu h$$

From Prof. E. Jenkins' slide

If we extend the scalar sector of HEFT in any way, the following statement is universal.

If tree level unitary is satisfied
(scalar manifold is flat)

$\Downarrow$

S, U parameter is 1-loop finite
(product of killing vectors = 0)

Summary

- Tree level unitarity and S,T,U parameters' 1-loop finiteness is deeply related

tree level unitary $\Rightarrow$ S,T,U parameter 1-loop finiteness

In "neutral singlet extension of SM"

- Dose the relationship still hold in the models with arbitral Higgs sector?

tree level unitary $\Rightarrow$ S,U parameter 1-loop finiteness

In arbitral Higgs sector

- By listing the Riemann tensors contributing the S parameter, we can provide the list of amplitudes preferred to be small to keep S,U parameter << 1.

Back Up

T parameter

Quadratic div.

$$\left(v_Z^2\Pi_{11}(0) - v^2\Pi_{33}(0)\right)\Big|_{\Lambda^2} = -\frac{1}{(4\pi)^2}\frac{v^2v_Z^2}{4}(\bar{R}_{1I1J} - \bar{R}_{3I3J})\delta^{IJ}\Lambda^2$$

Coefficient of quadratic div. is proportional to Riemann tensor

If tree level unitary is satisfied (scalar manifold is flat) $\Rightarrow$ T parameter has no quadratic divergence

Logarithmic div.

$$\begin{aligned}&\left(v_Z^2\Pi_{11}(0) - v^2\Pi_{33}(0)\right)\Big|_{\ln\Lambda^2}\\ &= \frac{v^2v_Z^2}{(4\pi)^2}\Bigg[\frac{g^2}{16}v^2(\bar{R}_{1212} - \bar{R}_{3232}) + \frac{1}{16}(g^2v^2 + g_Z^2v_Z^2)\bar{R}_{1313} + \frac{1}{4}(\bar{R}_{1I1J} - \bar{R}_{3I3J})\bar{V}_{;IJ}\\ &\quad -g^2\sum_{A=1}^{3}\left\{(\bar{w}_A^1)_{;I}(\bar{w}_A^1)_{;J} - (\bar{w}_A^3)_{;I}(\bar{w}_A^3)_{;J}\right\}\delta^{IJ} - g_Y^2\left\{(\bar{y}^1)_{;I}(\bar{y}^1)_{;J} - (\bar{y}^3)_{;I}(\bar{y}^3)_{;J}\right\}\delta^{IJ}\Bigg]\ln\frac{\Lambda^2}{\mu^2}\end{aligned}$$

Coefficient of logarithmic div. cannot be written only with Riemann tensor

Ex.) Georgi Machacek model is perturbative unitary, but has logarithmic divergent T parameter

Main statement

The most general EFT in terms of SM particles is

Higgs Effective Field Theory (HEFT)

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{G^2}(h)\, G^A_{\mu\nu}G^{A\,\mu\nu} - \frac{1}{4}\mathcal{F}_{W^2}(h)\, W^I_{\mu\nu}W^{I\,\mu\nu} - \frac{1}{4}\mathcal{F}_{B^2}(h)\, B_{\mu\nu}B^{\mu\nu}$$

$$+\bar{\psi} i \not{D} \psi + \frac{1}{2}\partial^\mu h\, \partial_\mu h + \underline{\frac{v^2}{4}\mathcal{F}_{V^2}(h)\, \mathrm{Tr}\,(D_\mu U)U^\dagger (D^\mu U)U^\dagger - V(h)}$$

$$-\frac{v}{\sqrt{2}}\left(\left[\bar{u}\,\mathcal{F}_{\mathcal{Y}_u}(h)\, U^\dagger q\right] + \left[\bar{d}\,\mathcal{F}_{\mathcal{Y}_d}(h)\, U^\dagger q\right] + \left[\,\bar{e}\,\mathcal{F}_{\mathcal{Y}_e}(h)\, U^\dagger l\right] + \text{h.c.}\right)$$

$$D_\mu U(\varphi(x)) = \partial_\mu U + igW^I_\mu T^I\, U - ig' B_\mu\, U\, T^3, \quad D^\mu h = \partial^\mu h$$

From Prof. E. Jenkins' slide

If we extend the scalar sector of HEFT in any way, the following statement is universal.

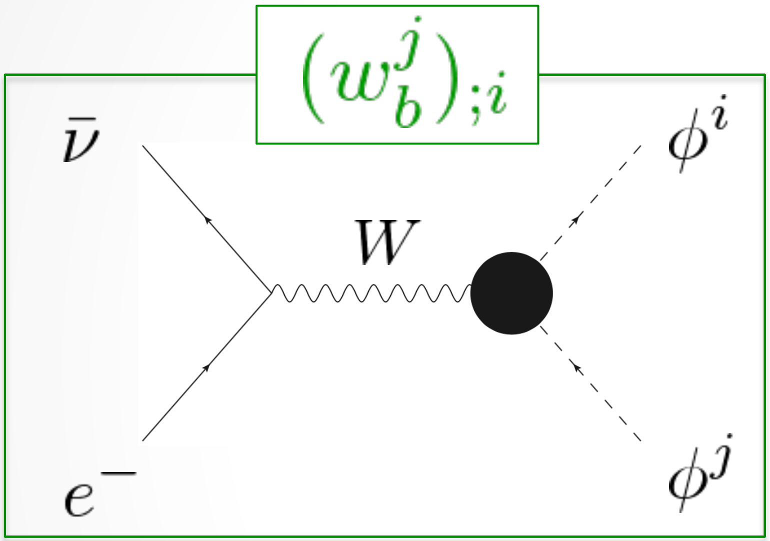
If tree level unitary is satisfied
(scalar manifold is flat)

$\Downarrow$

S, U parameter is 1-loop finite
(product of killing vectors = 0)

Interpretation

$$w^{j}_{3;i}\, y^{i}_{;j} = \frac{1}{2}\epsilon_{3ab} w^{k}_{c} w^{l}_{3} (w^{j}_{b})_{;i} (\mathcal{R}^{i}_{jkl}) + \cdots$$



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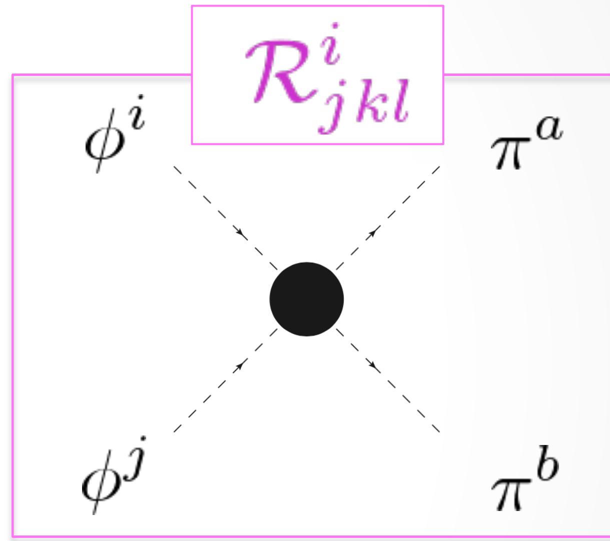


ϕ^i : scalar field (ex. $H, A, H^{\pm}, \pi, \cdots$)

For keeping the consistency with EWPT, It is not necessary for all the components of Riemann tensor to be zero.

<interesting scenario>

Tree level unitarity is broken in certain amplitudes with keeping the consistency with EWPT.

What is the most general form of the scalar sector ?

If we assume SM matter contents ⋯

① linear form (SMEFT)

$$\Phi = \begin{pmatrix} \pi^+ \\ \frac{v+h+\pi^0}{\sqrt{2}} \end{pmatrix}$$

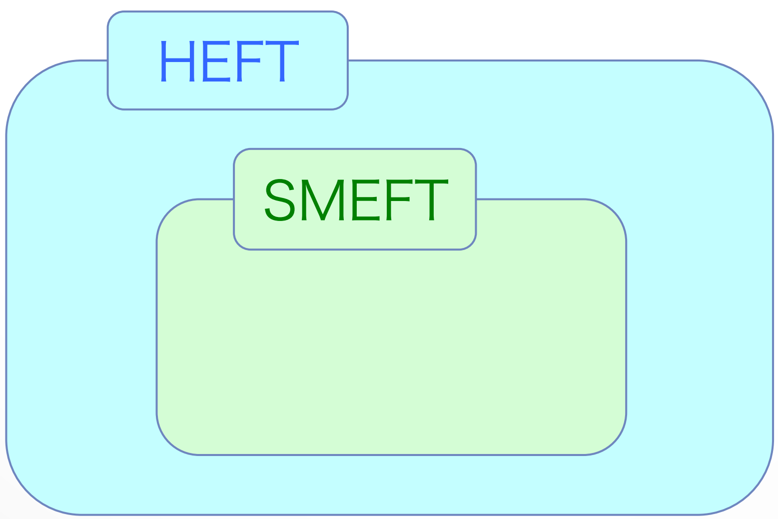
$$\mathcal{L} = (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 |\Phi|^2 - \lambda |\Phi|^4 + \sum_{d \geq 5} c_i \mathcal{O}_i(\Phi)$$

More general

② non-linear form (HEFT)

$$\mathcal{L} = \frac{v^2}{4} F(h) \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) + \cdots$$

$F(h)$: arbitrary function of h $\quad F_{\mathrm{SM}}(h) = \left(1 + \frac{h}{v}\right)^2 \quad U = \exp\left(i \frac{\pi^a}{v} \frac{\tau^a}{2}\right)$



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If we consider the most general Higgs sector (including new particle)

$$\boxed{\mathcal{L} = \frac{1}{2} g_{ij}(\phi) (D_\mu \phi)^i (D^\mu \phi)^j - V(\phi)}$$

ϕ^i : scalar field

Ex.) SM ($\phi^i = h, \pi^a$)

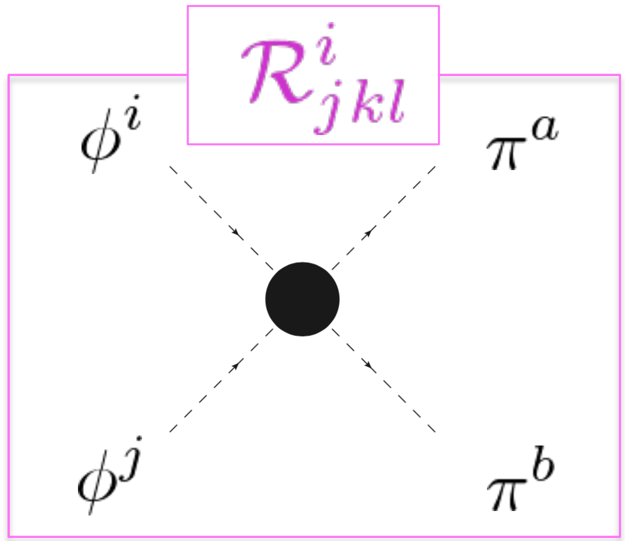
$$g_{ij}(\phi) = \begin{pmatrix} \hat{g}_{ab} & 0 \\ 0 & 1 \end{pmatrix} \qquad \hat{g}_{ab} = \left(1 + \frac{h}{v}\right)^2 \left[\delta^{ab} - \frac{1}{3v^2}\{(\vec{\pi}\cdot\vec{\pi})\delta^{ab} - \pi^a \pi^b\} + \cdots\right]$$

Ex.) neutral singlet extension w/ custodial sym. ($\phi^i = h(= S_0), S_1, S_2 \cdots, \pi^a$)

$$g_{ij}(\phi) = \begin{pmatrix} \hat{g}_{ab} & 0 \\ 0 & \delta_{nm} \end{pmatrix} \qquad \hat{g_{ab}} = \left(1 + 2\sum_{n=1}^{N} \kappa_n \frac{S_n}{v}\right) \left[\delta^{ab} - \frac{1}{3v^2}\{(\vec{\pi}\cdot\vec{\pi})\delta^{ab} - \pi^a \pi^b\} + \cdots\right]$$

Interpretation

$$w^{j}_{3;i}\, y^{i}_{;j} = \frac{1}{2}\epsilon_{3ab} w^{k}_{c} w^{l}_{3} (w^{j}_{b})_{;i} (\mathcal{R}^{i}_{jkl}) + \cdots$$



These amplitudes are preferred to be zero to keep S,U parameter << 1

Even if the unitarity-violating amplitude are nonzero,
it is (at least) consistent with the bound on S,U parameter.

$$w^j_{3;i} y^i_{;j} = \frac{1}{2}\epsilon_{3ab} w^k_c w^l_3 (w^j_b)_{;i} (\mathcal{R}^i_{jkl}) + \frac{1}{2}\epsilon_{3bc} w^k_b w^l_c (y^j)_{;i} (\mathcal{R}^i_{jkl})$$

<derivation>

In Riemann Normal Coordinate

Solution of Killing eq. $(w^j_a)_{;k;l} = \mathcal{R}_{ilkj} w^i_a$

$$w^i_a = \overline{w}^i_a + \overline{w}^i_{a,j}\phi^j + \frac{1}{3}\overline{\mathcal{R}}^i_{jkl}\overline{w}^l_a \phi^j \phi^k + \cdots \qquad \cdots①$$

Commutation relation of Killing vector

$$w^i_a (y^j)_{,i} - y^i (w^j_a)_{,i} = 0 \qquad \cdots②$$

$$w^i_a (w^j_b)_{,i} - w^i_b (w^j_a)_{,i} = \epsilon_{abc} w^j_c \qquad \cdots③$$

Substituting ① into ②, ③ and comparing the coefficient of ϕ
we can get some formulas about $[T_a, T_Y]_j{}^i$ where $(T_a)_j{}^i = \overline{w}^i_{a,j}$, $(T_Y)_j{}^i = \overline{y}^i_{,j}$

Substituting these formula into

$$\mathrm{tr}(T_3 T_Y) = \frac{1}{2}\epsilon_{3bc}\mathrm{tr}([T_c, T_Y]T_b) + \frac{1}{2}\epsilon_{3bc}\overline{w}^k_b \overline{w}^l_c \overline{\mathcal{R}}^i_{jkl}(T_Y)_i{}^j$$

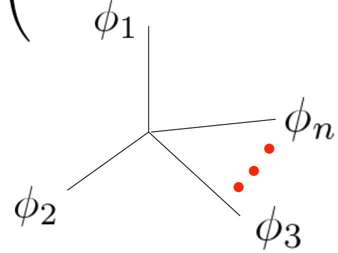
and writing in the covariant form, we get above formula.

We succeeded to verify the previous relation in the arbitrary scalar sector !

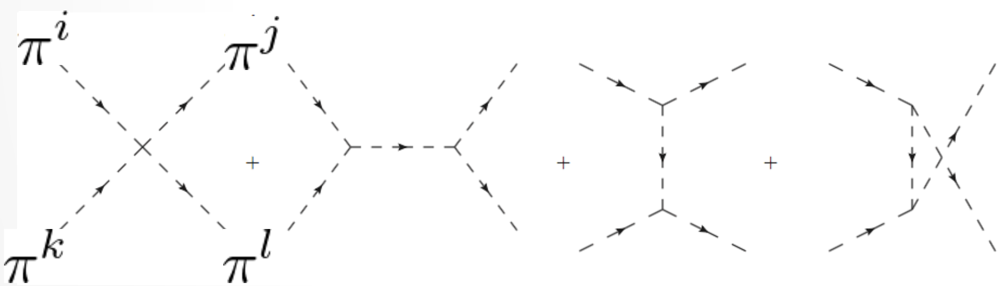
$$\mathcal{L} = \frac{v^2}{4}\mathrm{tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_V^h \frac{h}{v} + 2\sum_{n=2}^{N} \kappa_V^n \frac{S_n}{v}\right) + \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{2}\sum_{n=2}^{N}(\partial_\mu S_n)(\partial^\mu S_n)$$

$$= \frac{1}{2}\partial_\mu(\pi^a,\, h,\, S_1,\, S_2, \cdots)\begin{pmatrix} \left(1 + 2\kappa_V^h \frac{h}{v} + 2\sum_{n=2}^{N} \kappa_V^n \frac{S_n}{v}\right)(\delta_{ab} + \mathcal{O}(\pi^2)) & 0 \\ 0 & 1_{N\times N} \end{pmatrix}\partial^\mu \begin{pmatrix} \pi^a \\ h \\ S_1 \\ S_2 \\ \vdots \end{pmatrix}$$

$$= \frac{1}{2} g_{ij}(\phi) D_\mu \phi^i D^\mu \phi^j$$



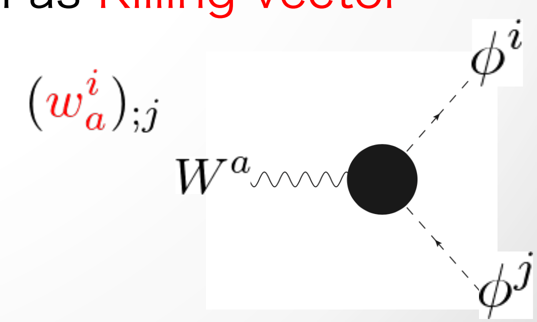
① Unitarity-violating amplitudes can be written as Riemann tensor



$$= -i(t\,\mathcal{R}_{ijkl} + u\,\mathcal{R}_{iljk})$$

② Charged and neutral current can be written as Killing vector

$\boxed{\text{Global sym. = isometry of the scalar manifold}}$



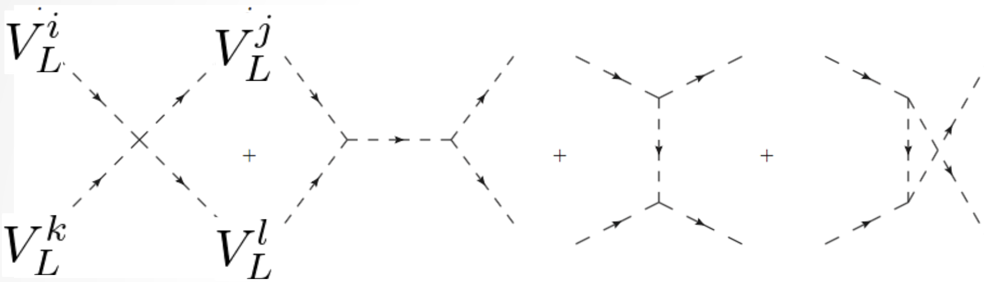
w_a^i : Killing vector for SU(2)_L symmetry (a=1~3)

y^i : Killing vector for U(1)_Y symmetry

We have succeeded to verify the following relation in arbitrary model !

tree level unitarity $\Rightarrow$ S,U parameter 1-loop finiteness

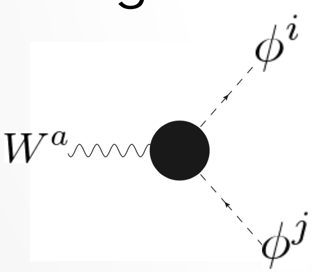
① Unitarity-violating amplitudes can be written as Riemann tensor



$$= -i(t\,\mathcal{R}_{ijkl} + u\,\mathcal{R}_{iljk})$$

Riemann tensor = 0 $\Longleftrightarrow$ Tree level unitary is respected

② Charged and neutral current can be written as Killing vector

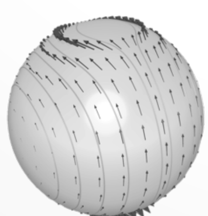
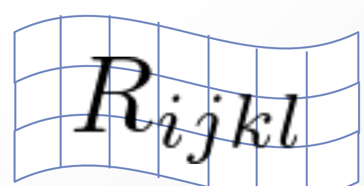


$$= (w_a^i)_{;j}$$

w_a^i : Killing vector for SU(2)_L sym. (a=1~3)

y^i : Killing vector for U(1)_Y sym.

We can relate Killing vector with Riemann tensor by Killing equation

w_a^i  y^i $\Longleftrightarrow$ 

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