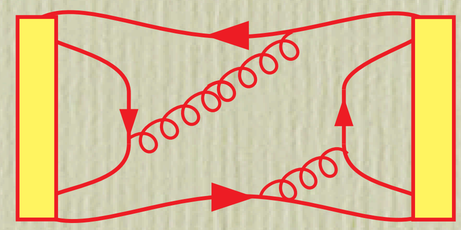
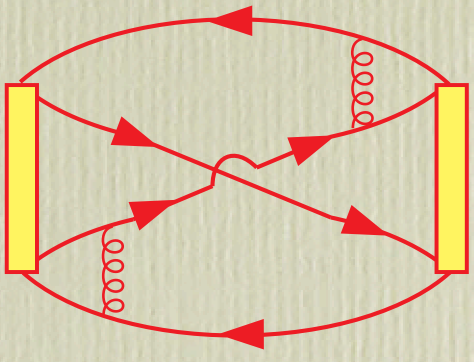
Ordinary and Extraordinary Hadrons (Mesons)

R L Jaffe





RLJ, AIP Conf. Proc. **964**, 1 (2007) [hep-ph/0701038];

J. R. Pelaez, PRL **92**, 102001 (2004) [hep-ph/0309272], etc.;
C. Hanhart, J. R. Pelaez, G. Rios, PL **739**, 375 (2014) [1407.7452];
J. R. Pelaez, Phys. Rept. **658**, 1 (2015) [1510.00653]

Other references: S. Weinberg, PRL **110**, 261601 (2013);
T. D. Cohen & R. F. Lebed, Phys. Rev. **D89**, 054018 (2014), etc.

Ordinary: $(Q\overline{Q})$ "quark model" mesons

Extraordinary: "tetraquarks", "meson-molecules", threshold enhancements, ...

I. A useful qualitative distinction motivated by large N_c

II. Supported by unitarized chiral dynamics applied to light scalar and vector mesons

III. Ordinary mesons – Feshbach Resonances
Decouple as $N_c \to \infty$

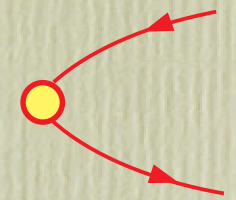
IV. Extraordinary mesons – Open channel enhancements
Subside into the continuum as $N_c \to \infty$

V. Follow the poles

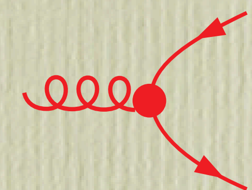
VI. The S – wave is special

Expectations at large N_c

Rules



$$\frac{1}{\sqrt{N_c}}$$



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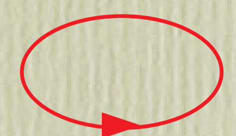
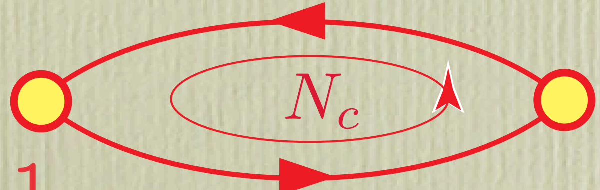


$$N_c$$

Meson source properly normalized

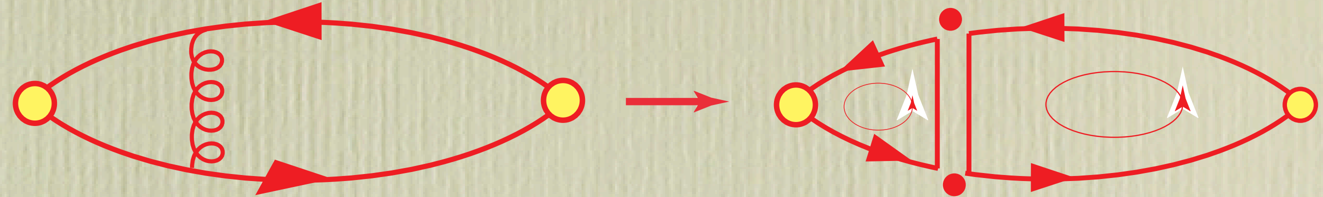


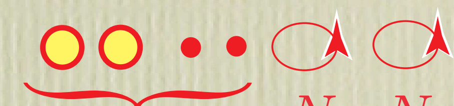
$$\frac{1}{\sqrt{N_c}} \qquad \frac{1}{\sqrt{N_c}}$$

$$\mathcal{O}\left(\frac{1}{\sqrt{N_c}}\right)^2 N_c \sim 1$$

Standard meson results as $N_c \to \infty$

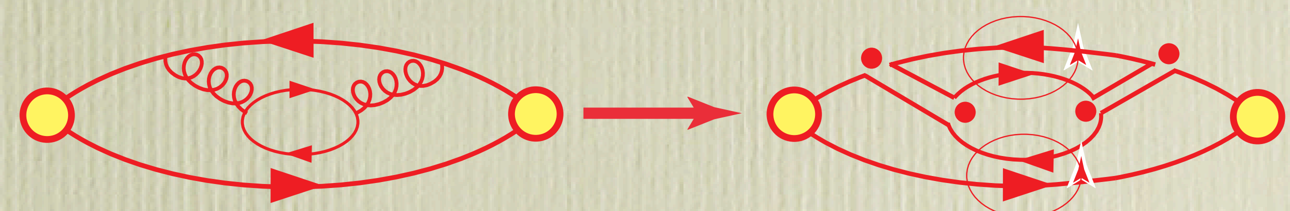
Planar gluon interactions $\mathcal{O}(1)$

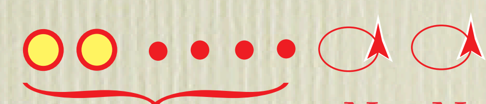




$$\left[\frac{1}{\sqrt{N_c}}\right]^4 \to 1$$

Quark loops $\mathcal{O}(1/N_c)$



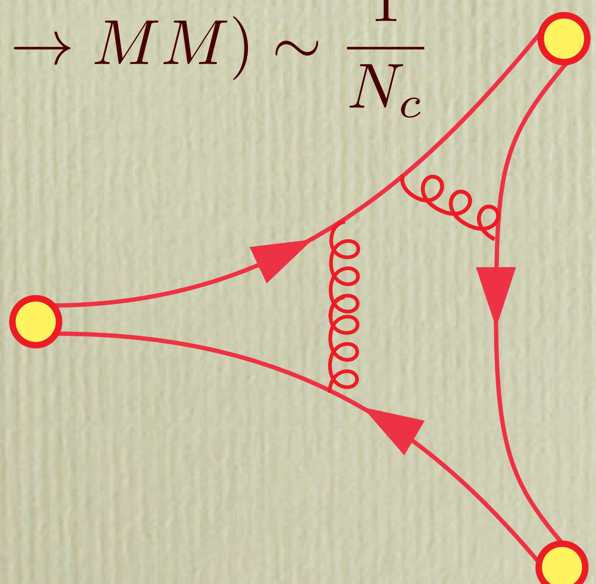


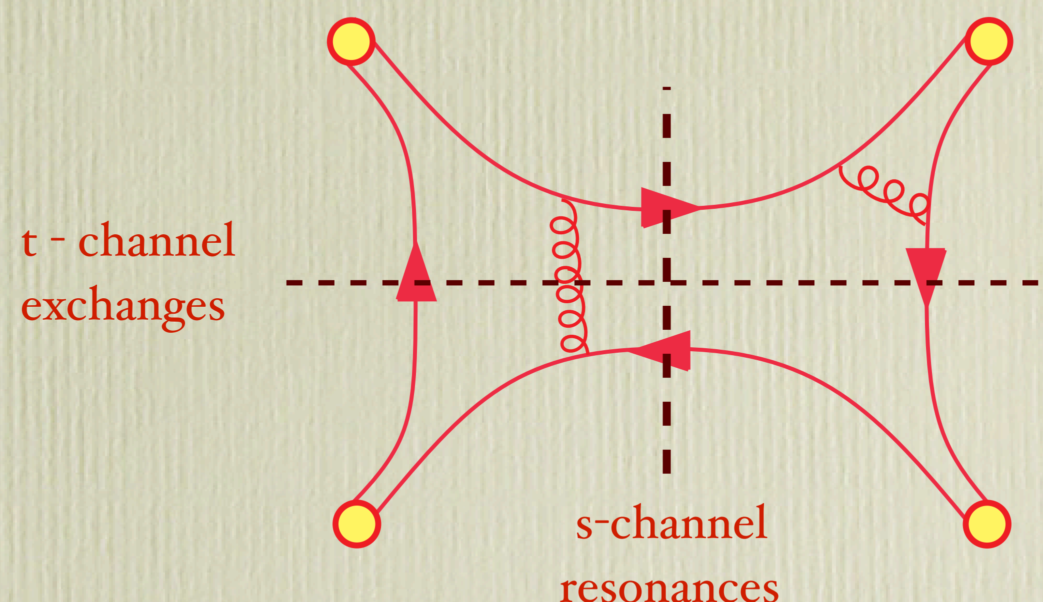
$$\left[\frac{1}{\sqrt{N_c}}\right]^6 \to \frac{1}{N_c}$$

Classic results:

- Meson widths vanish $\Gamma \sim \mathcal{O}(1/N_c)$ as $N_c \to \infty$
- Quark content becomes pure $Q\bar{Q}$
- Meson-meson scattering vanishes as $\mathcal{O}(1/N_c)$; ordinary mesons appear as narrow s-channel resonances dual to t-channel exchanges.

$$\Gamma(M \to MM) \sim \frac{1}{N_c}$$





Closer look at meson-meson interactions as $N_c \to \infty$

Generic normalized two meson source

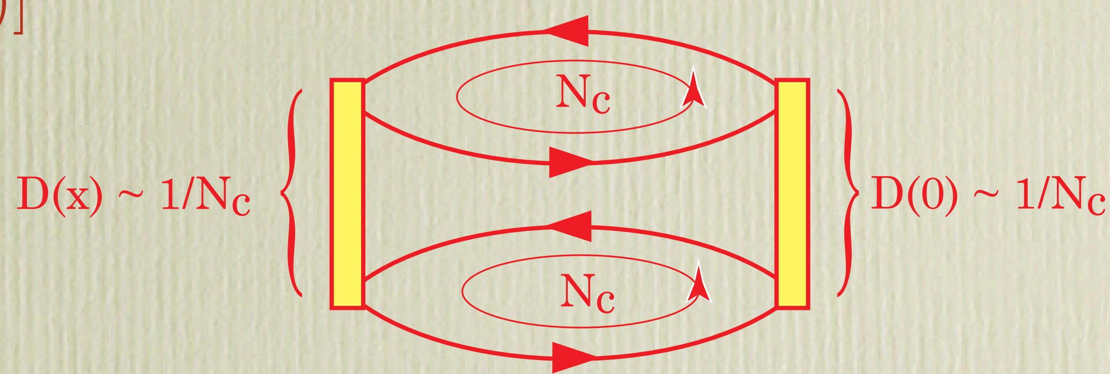
$$D(x) \equiv \frac{1}{N_c}\bar{q}\bar{q}qq(x)$$

- Large N_c counting and qualitative dynamics do not depend on the internal coupling of quark fields in the source.
- Any such source can always be Fierz transformed to a sum of products of color singlet meson sources...

$$D(x) = \cos\theta\, M_{12}(x)M_{34}(x) + \sin\theta\, M_{14}(x)M_{23}(x)$$

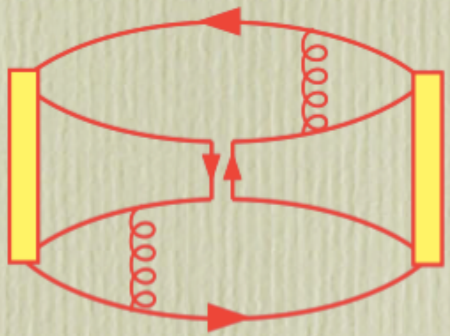
$$M_{ij}(x) = [\bar{q}_i(x)q_j(x)]^{\mathbf{1}}$$

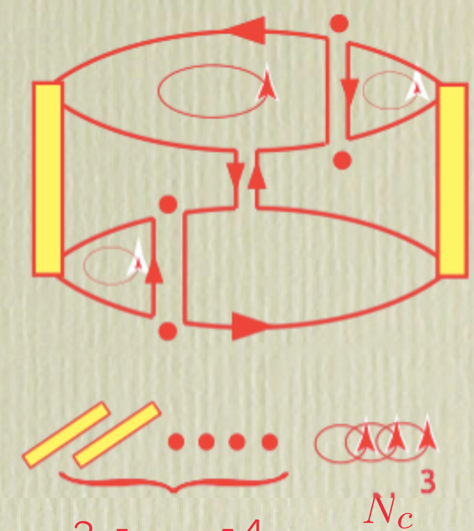
$$\langle 0|D(x)D(0)|0\rangle \sim 1 \quad \text{as } N_c \to \infty$$



Meson-meson interactions vanish as $N_c \to \infty$, but what processes are least suppressed?

- The best known residual meson-meson interaction at large N_c is t-channel meson exchange, which is down by $1/N_c$





$$\left[\frac{1}{N_c}\right]^2 \left[\frac{1}{\sqrt{N_c}}\right]^4 \longrightarrow \boxed{\frac{1}{N_c}}$$

- However this interaction is dual to the resonances (with widths of order $1/N_c$). So it would be ______________ narrow resonances and t-channel meson exchange.

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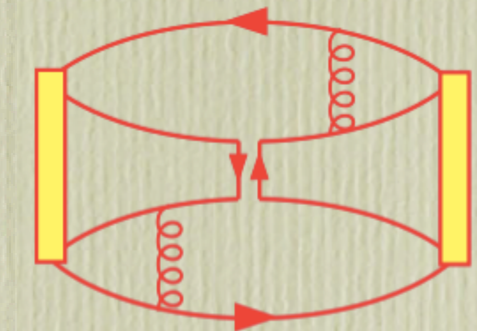

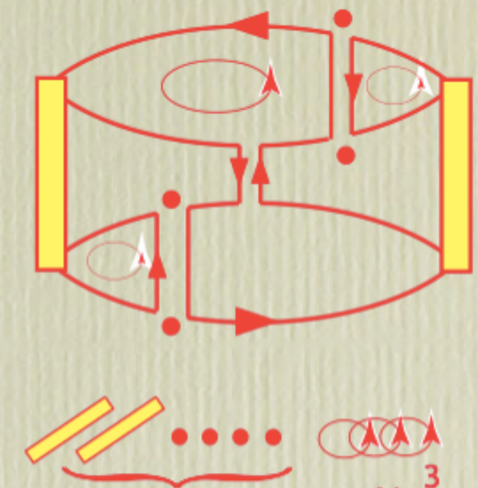

$$N_c^3 \left[\frac{1}{N_c}\right]^2 \left[\frac{1}{\sqrt{N_c}}\right]^4 \to \boxed{\frac{1}{N_c}}$$

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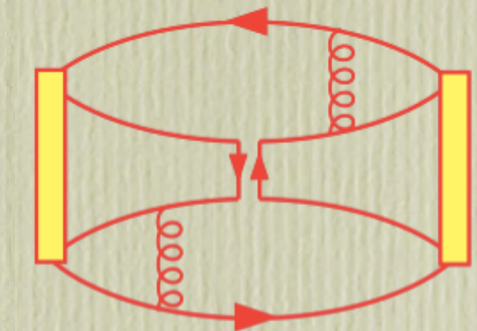

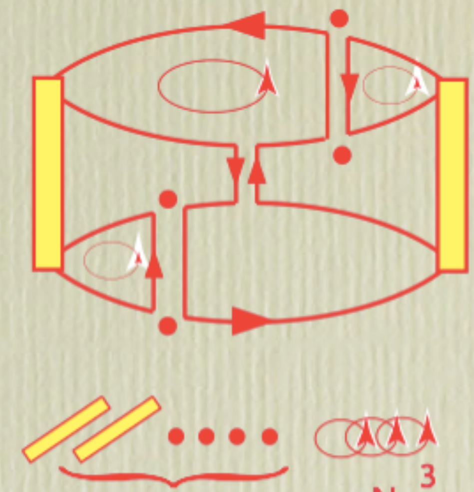

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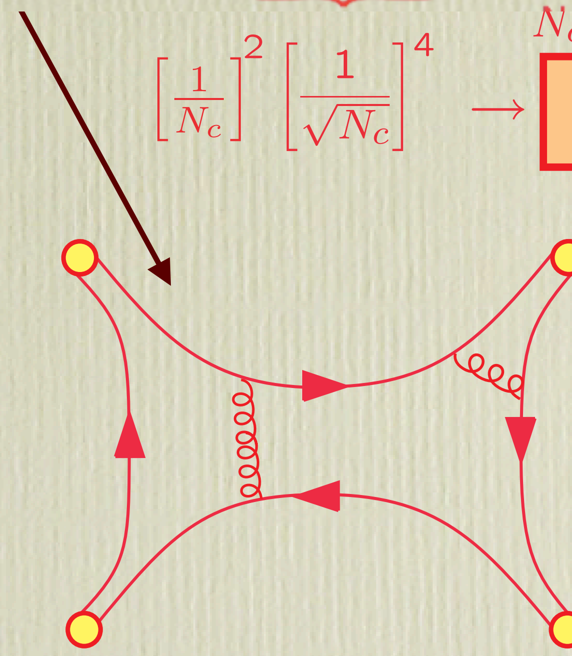
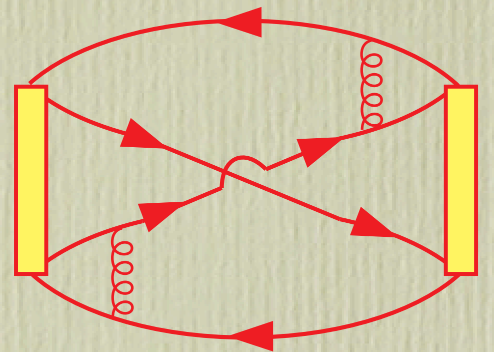
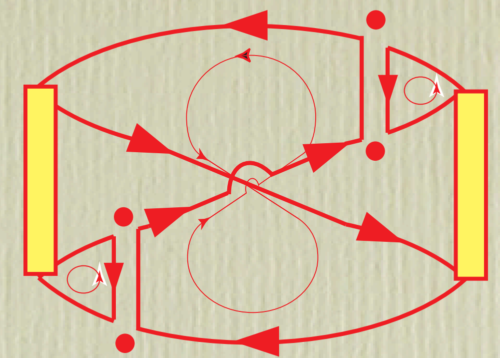
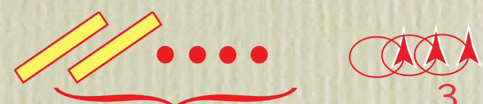


Another residual interaction at large N_c – and the only one away from the narrow resonances – is quark exchange

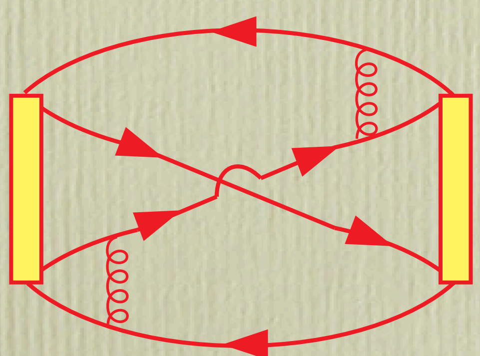






N_c^3

$$\left[\frac{1}{N_c}\right]^2 \left[\frac{1}{\sqrt{N_c}}\right]^4 \longrightarrow \boxed{\frac{1}{N_c}}$$



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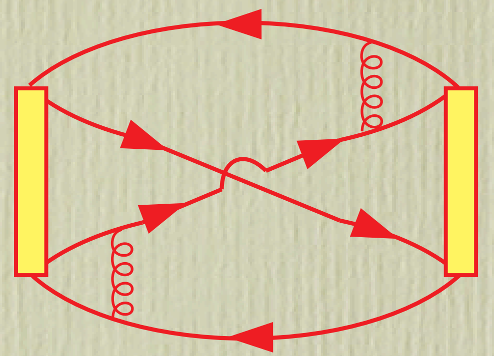
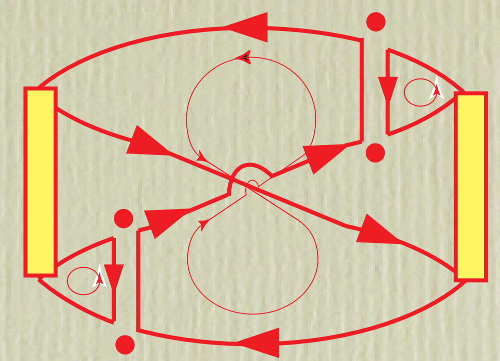
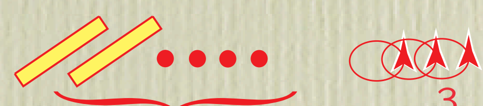
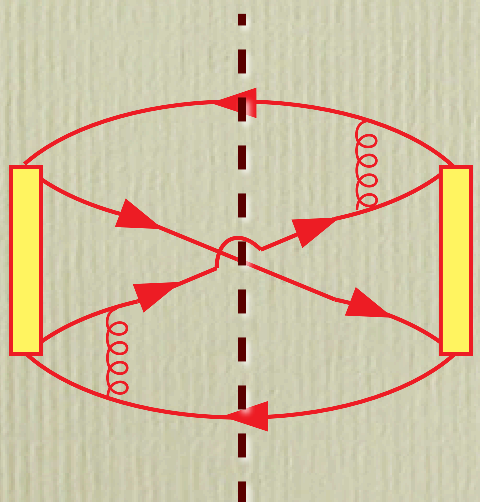






$$\left[\frac{1}{N_c}\right]^2\left[\frac{1}{\sqrt{N_c}}\right]^4 \longrightarrow \boxed{\frac{1}{N_c}}$$

No coupling to $Q\bar{Q}$ mesons



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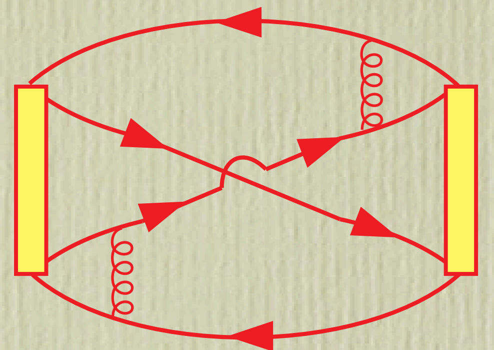

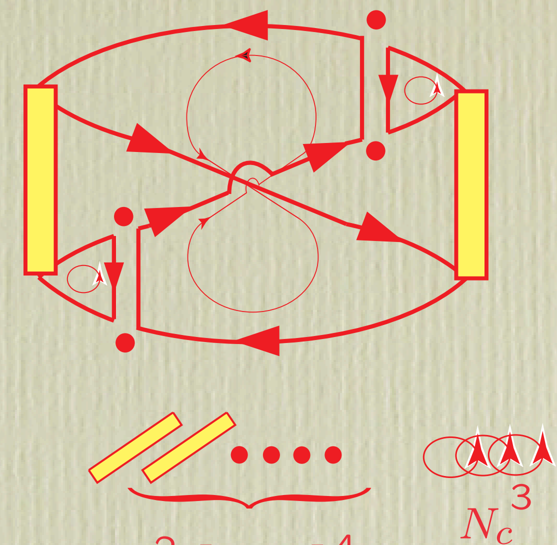

$$\left[\frac{1}{N_c}\right]^2\left[\frac{1}{\sqrt{N_c}}\right]^4 \rightarrow \boxed{\frac{1}{N_c}}$$

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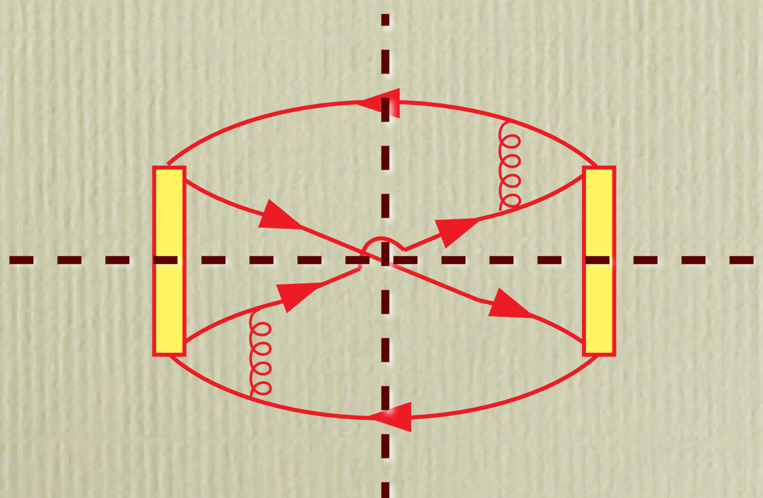
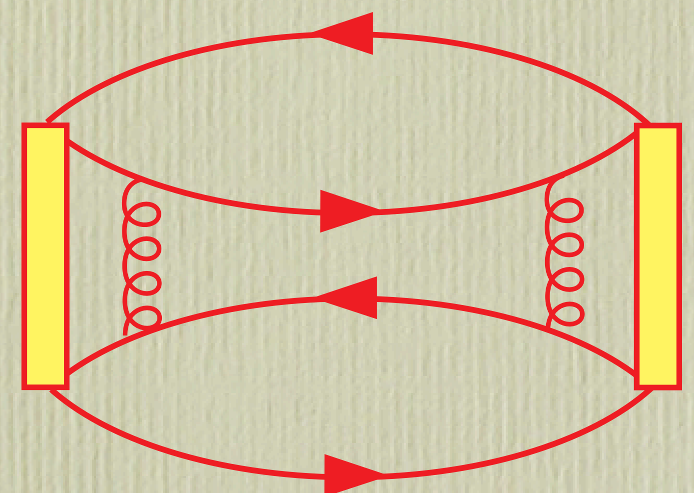
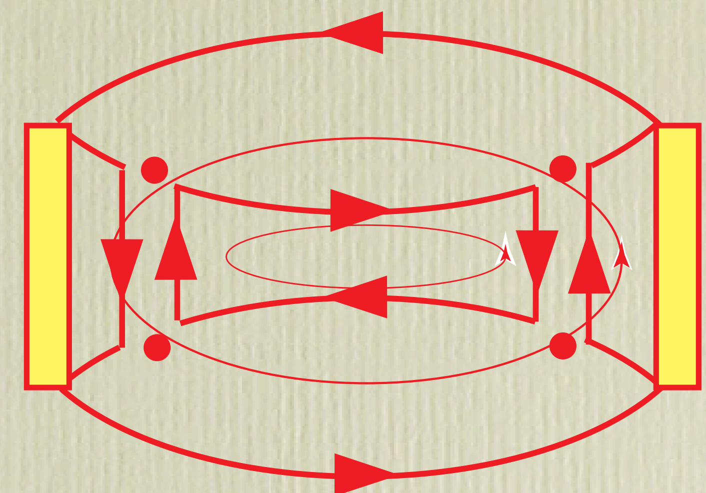


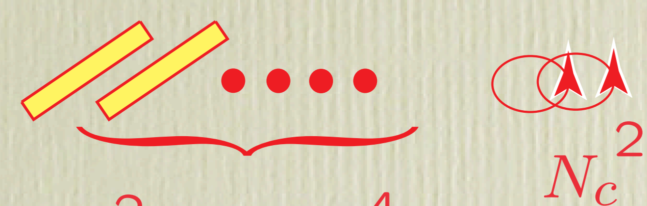
Quark exchange “forces”

Scattering state projects entirely onto meson-meson continuum.

Further contributions to meson-meson scattering are down by high powers of N_c (exchange of vacuum quantum numbers shown).



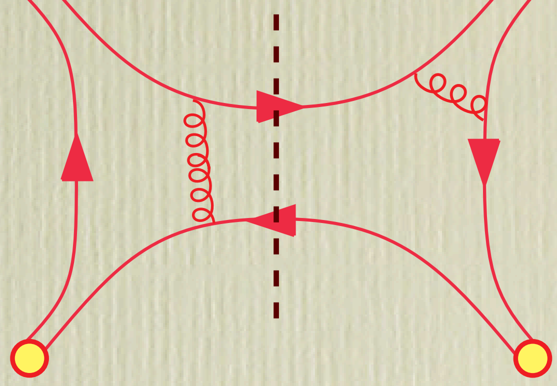




$$\left[\frac{1}{N_c}\right]^2 \left[\frac{1}{\sqrt{N_c}}\right]^4 \longrightarrow \boxed{\frac{1}{N_c^2}}$$

Summary of expectations for meson-meson scattering at large N_c

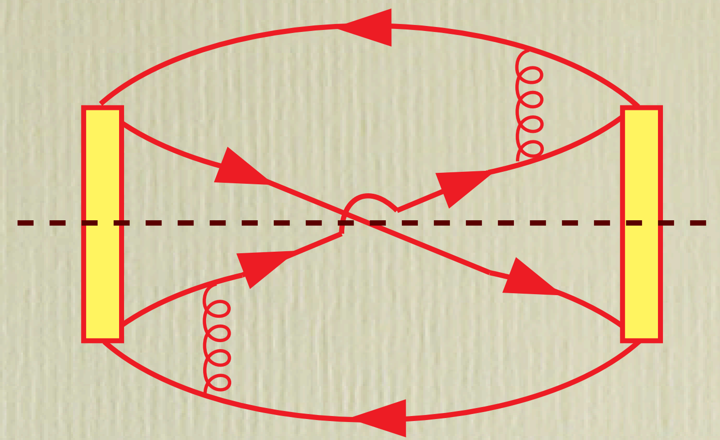
- Overall scattering amplitude is $\mathcal{O}(1/N_c)$



Ordinary mesons

- s-channel resonances $\longleftrightarrow \bar{Q}Q$ mesons with widths that vanish like $1/N_c$.
- Bound states in the meson-meson continuum.
- Do not lie within the space of meson-meson scattering states.

Potentially extraordinary mesons



Quark exchange "forces"

- In any fixed basis, eg. $M_{12}M_{34}$ quark exchange mixes color octet components into the wavefunction, so the force is fundamentally chromodynamic.

- The range of the force is determined by the distance at which hadrons overlap, of order 1 fermi.

- Attractive? repulsive? Capable of generating bound/virtual states and resonances.

- No coupling to confined channels, so the interactions are "potential-like". Non-relativistic analog would be simply the Schrödinger equation with an open channel potential.

- Extraordinary hadrons --- if they exist at all --- disappear as $N_c \to \infty$; they merely subside into the hadron-hadron continuum.

Corroboration of N_c dependence: unitarized chiral dynamics

J. Pelaez & collaborators

J. R. Pelaez, PRL **92**, 102001 (2004) [hep-ph/0309272], etc.];
J. R. Pelaez, Phys. Rept. **658**, 1 (2015) [1510.00653]

Low energy $\pi\pi$ scattering can be computed in a power series in p^2/Λ_ξ^2 using chiral perturbation theory. In limit of exact $SU(2)_L \times SU(2)_R$ only parameter at order p^2 is f_π. At order p^4, eight parameters enter: $L_1 ... L_8$ (Gasser, Leutwyler).

No finite expansion in powers of p^2 can uniquely locate a pole, however "unitarization" methods allow approximate analysis.

Imaginary part from unitarity

Inverse amplitude method†

$$t_{IJ}^{-1}(p) = g_{IJ}(p^2) - \frac{ip}{\sqrt{p^2+m_\pi^2}}$$

Extrapolate to energies where interactions become strong.

Real part from chiral perturbation theory

Compute N_c dependence from underlying QCD; fit $N_c = 3$ to the data and then vary N_c

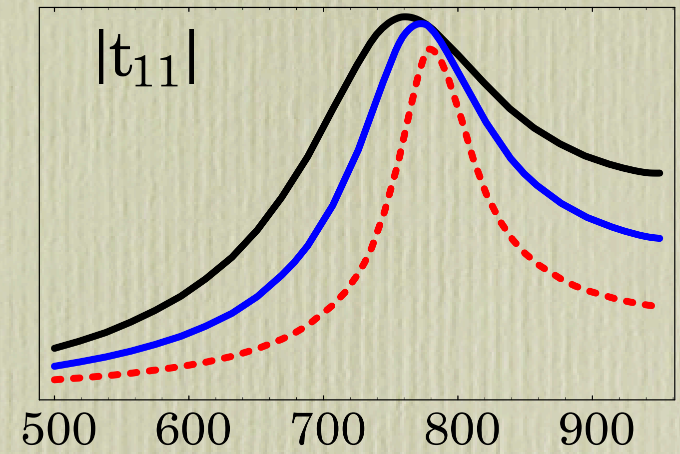
$$\begin{aligned} t_{IJ}(p) &= \frac{\sqrt{p^2+m^2}}{p} f_{IJ}(p) \\ \text{where} \quad f_{IJ}(p) &= \sin\delta_{IJ}(p) e^{i\delta_{IJ}(p)} \end{aligned}$$

†I understand that there is model dependence here!

Ordinary mesons – masses independent of N_c and widths → 0 as $N_c \to \infty$

$O(p^4)$ Parameter	N_c Scaling
L_1	$O(N_c)$
L_2	$O(N_c)$
L_3	$O(N_c)$
L_4	$O(1)$
L_5	$O(N_c)$
L_6	$O(1)$
L_7	$O(1)$
L_8	$O(N_c)$
$2L_1 - L_2$	$O(1)$

ρ(770)



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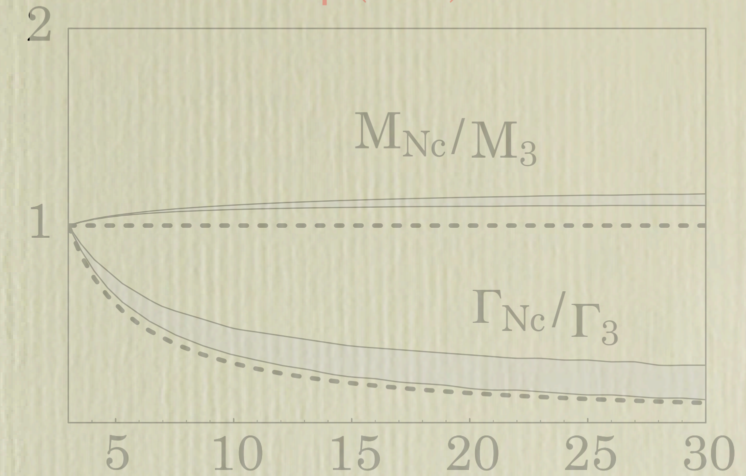
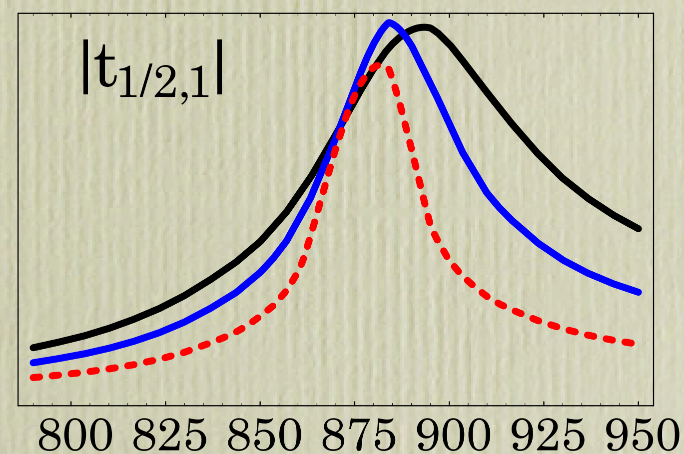


K*(892)



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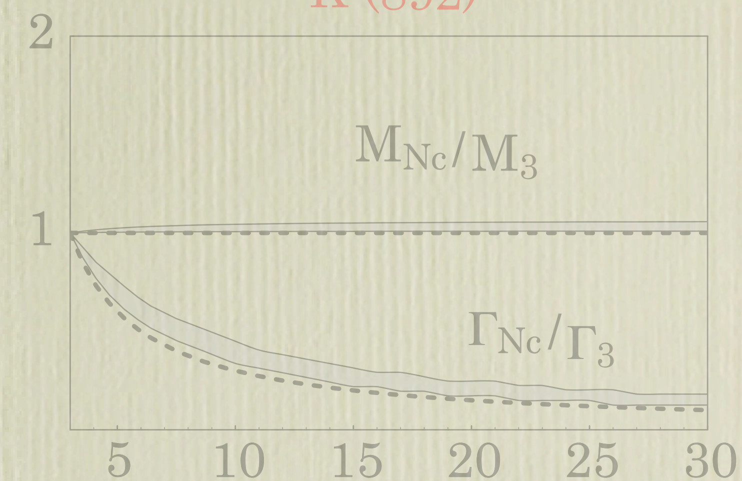

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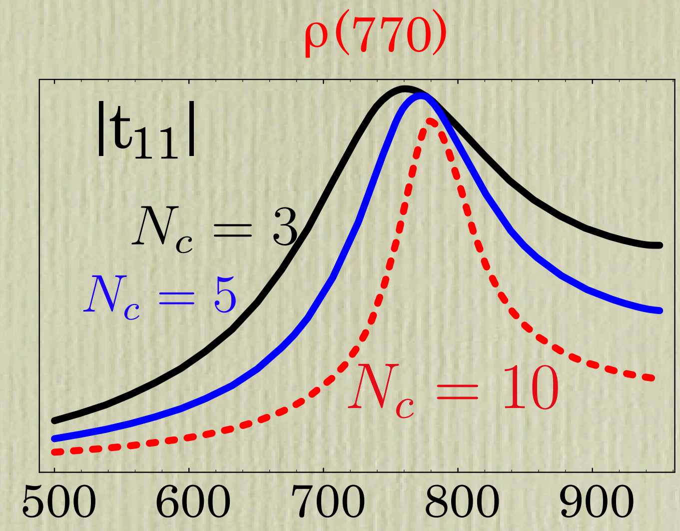

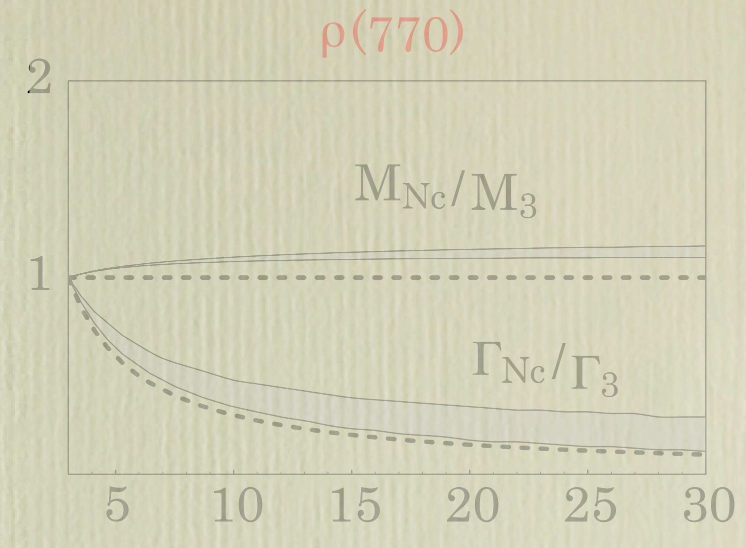

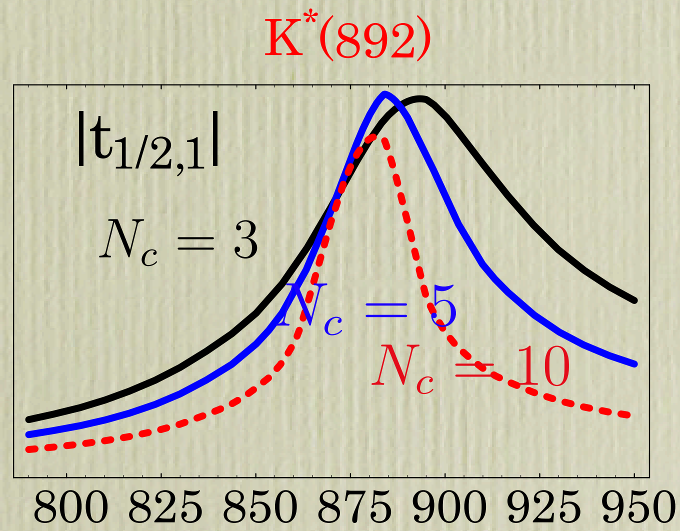

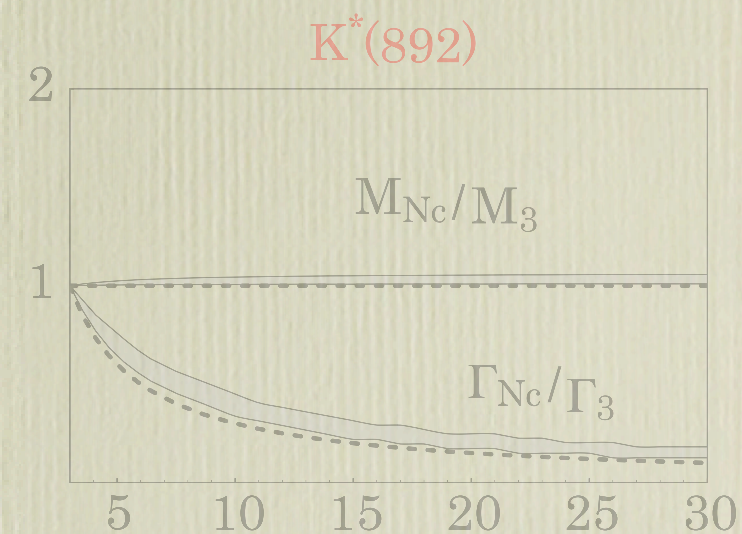

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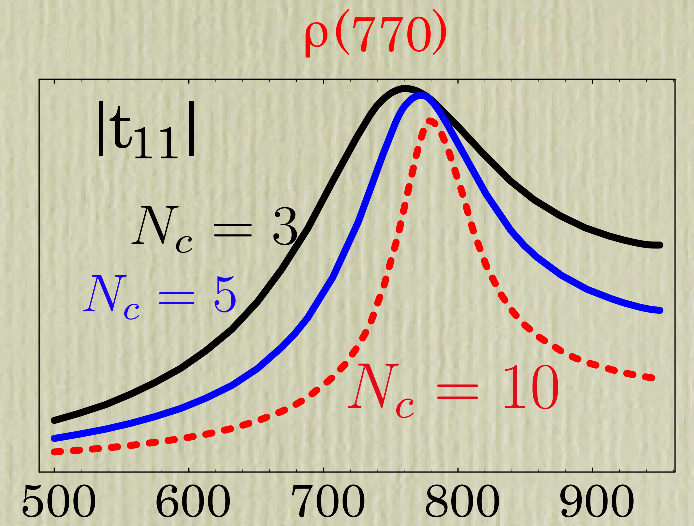
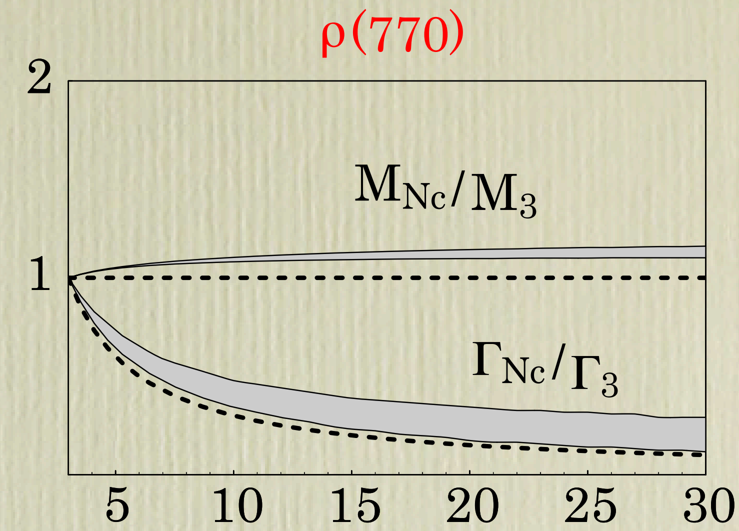
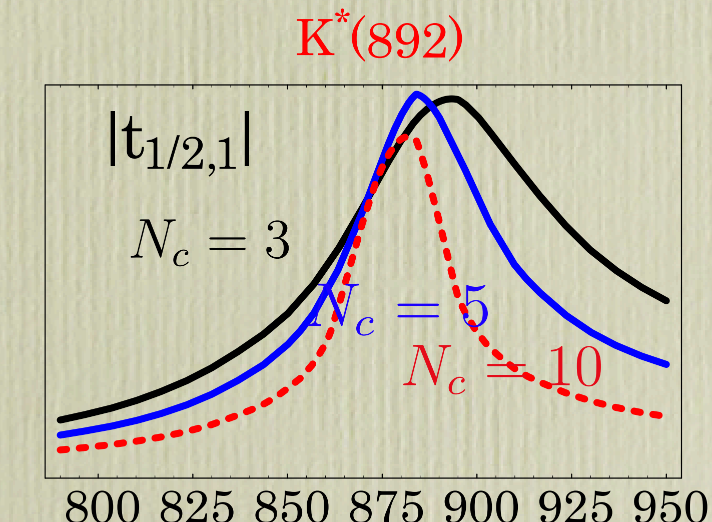
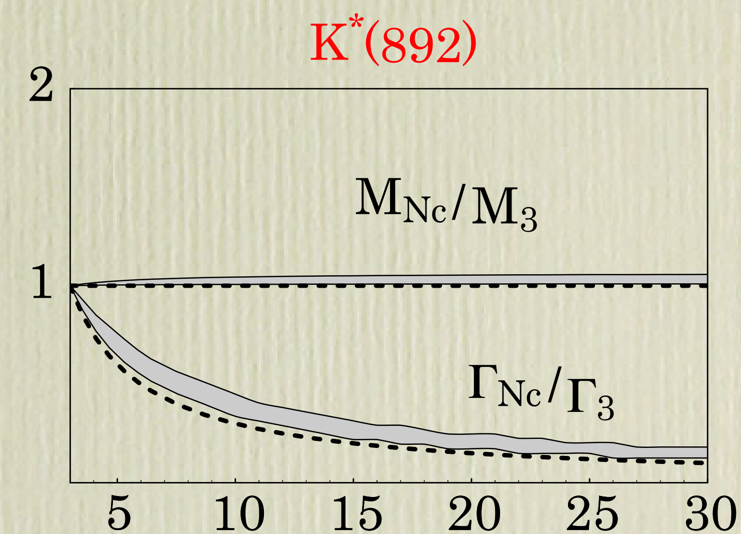


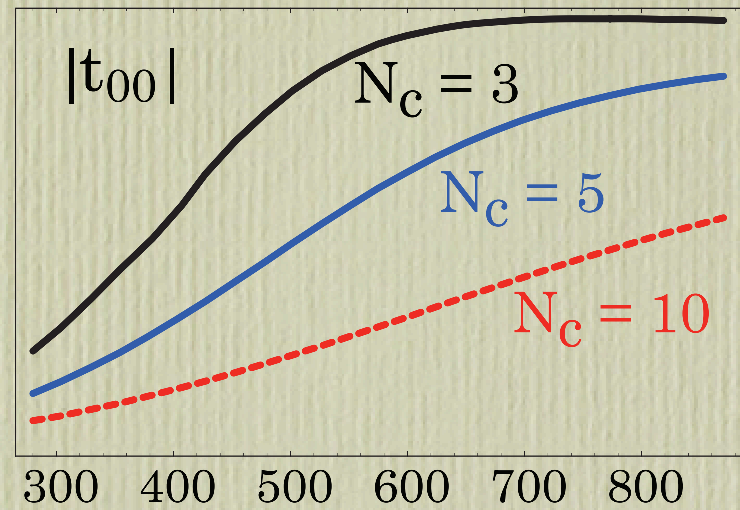




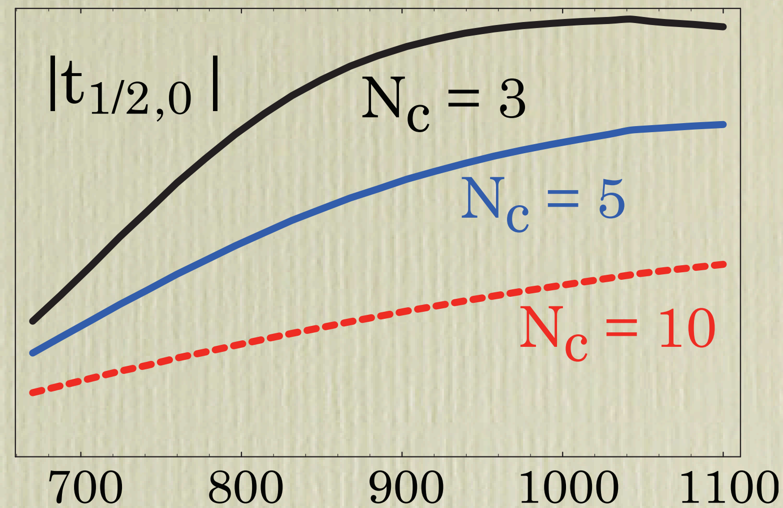


Extraordinary mesons subside into the continuum as $N_c \to \infty$

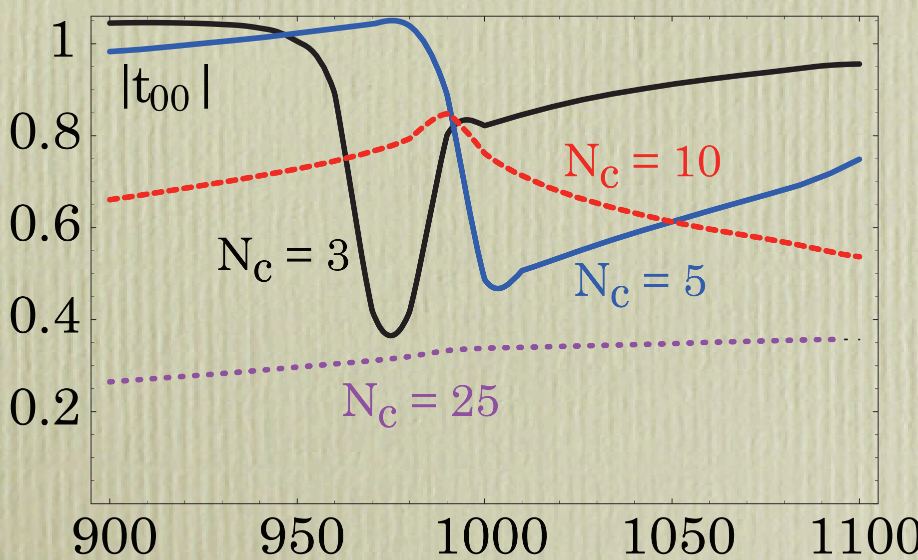
σ Region



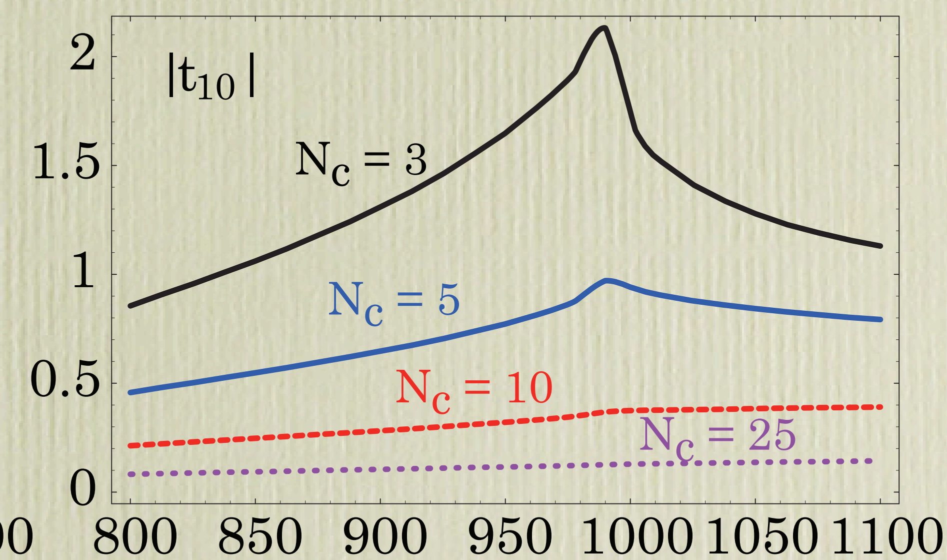
κ Region



$f_0(980)$region



$a_0(980)$region

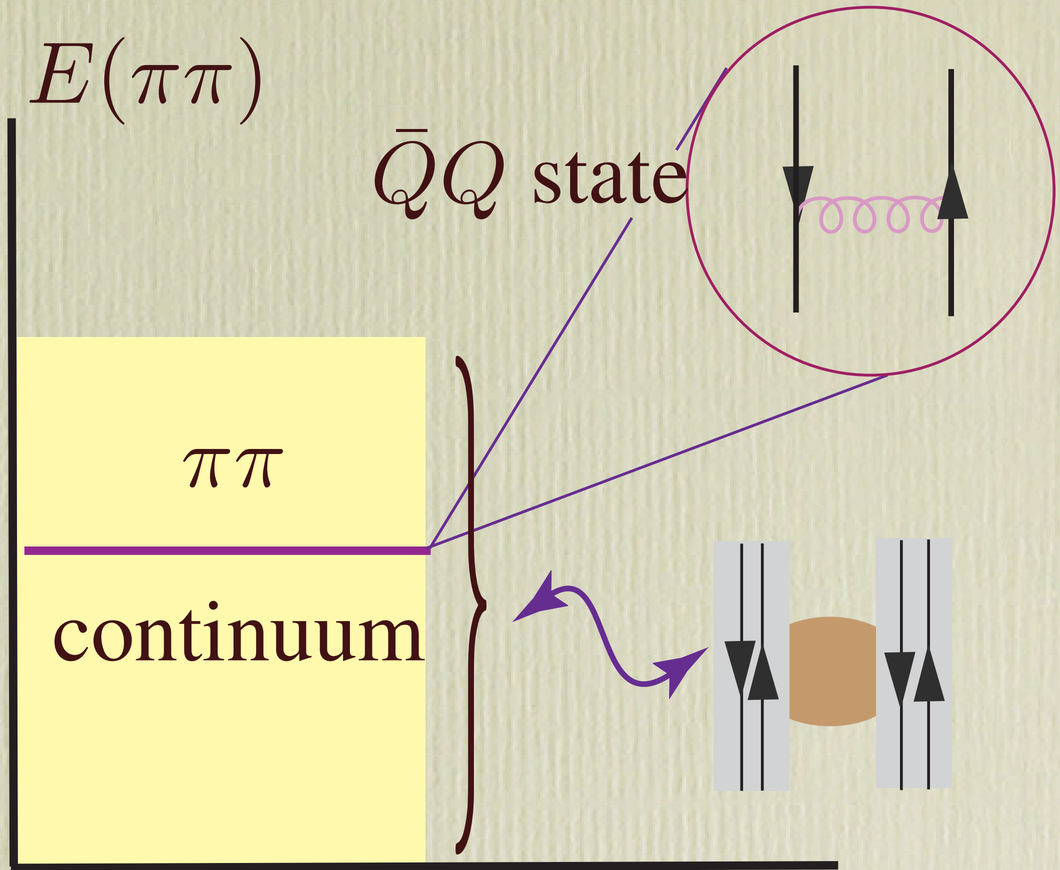


Ordinary hadrons as Feshbach resonances

O. K. Rice, J. Chem. Phys. **1**, 375 (1933)
U. Fano, Nuovo Cimento **12**, 154 (1935)
Phys. Rev. **124**, 1866 (1961)
H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958).

General idea:

- Two channels, one open, the other closed.
- Closed channel has discrete spectrum that overlaps the continuum spectrum of the open channel.
- Turn on coupling between channels: The closed channel bound state appears as resonance in the open channel.



Open channel – meson-meson (eg. ππ)
Closed (in fact confined) channel – $\bar{Q}Q$ (eg. ρ)

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \begin{matrix} \longleftrightarrow \pi\pi \\ \longleftrightarrow \rho \end{matrix}$$

Ordinary hadrons as Feshbach resonances

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \begin{matrix} \longleftrightarrow \pi\pi \\ \longleftrightarrow \rho \end{matrix}$$

$$\left.\begin{aligned} h_0|\psi_1\rangle + \frac{1}{\sqrt{N_c}} V|\psi_2\rangle &= E|\psi_1\rangle \\ h_0|\psi_2\rangle + \frac{1}{\sqrt{N_c}} V|\psi_1\rangle + \boxed{V_{\text{confining}}}|\psi_2\rangle &= E|\psi_2\rangle \end{aligned}\right\}$$

No interaction in open channel except for transition to confined channel. Confined channel couples back to open channel allowing "bound state in the continuum" to decay back to open channel.

Only a discrete spectrum

$$\mathcal{H} = \begin{pmatrix} h_0 & V/\sqrt{N_c} \\ V/\sqrt{N_c} & h \end{pmatrix} \qquad \mathcal{G}(p) = \frac{\mathbb{I}}{k^2 - h} \approx \frac{|\phi\rangle\langle\phi|}{k^2 - k_0^2} \quad \text{for } k^2 \approx k_0^2$$

$$h|\phi\rangle = E_0|\phi\rangle = k_0^2|\phi\rangle \qquad \boxed{h_\ell|u_\ell\rangle + \frac{V}{\sqrt{N_c}}\left(\frac{|\phi\rangle\langle\phi|}{k^2 - k_0^2}\right)\frac{V}{\sqrt{N_c}}|u_\ell\rangle = k^2|u_\ell\rangle}$$

Confined channel "bound state" appears as a pole in the effective separable potential in the open channel. Easily solved using Greens function methods.

Scattering near a Feshbach resonance

$$h_\ell|u_\ell\rangle + \frac{V}{\sqrt{N_c}}\left(\frac{|\phi\rangle\langle\phi|}{k^2-k_0^2}\right)\frac{V}{\sqrt{N_c}}|u_\ell\rangle = k^2|u_\ell\rangle$$

Special case of a separable potential $\mathcal{H} = h_0 - \lambda|\chi\rangle\langle\chi|$ with $|\chi\rangle = V|\phi\rangle$

$\langle\chi|\chi\rangle = \langle\phi V|V\phi\rangle = 1$ and $\lambda = -\frac{1}{N_c}\frac{1}{k^2-k_0^2}$

$$h_\ell|u_\ell(k)\rangle - \lambda|\chi\rangle\langle\chi|u_\ell(k)\rangle = k^2|u_\ell(k)\rangle$$

General solution for the Argand amplitude $f_\ell(k) = \sin\delta_\ell(k)e^{i\delta_l(k)}$

$$f_\ell(k) = \frac{1}{k}\frac{\lambda|\langle\chi|u_\ell^0(k)\rangle|^2}{1-\frac{\lambda}{\pi}\int_{-\infty}^{\infty}dq\frac{|\langle\chi|u_\ell^0(q)\rangle|^2}{q^2-k^2-i\varepsilon}}$$

with $\langle r|u_\ell^0(k)\rangle = rj_\ell(kr)$

- For Feshbach resonance $\langle\chi|u_\ell^0\rangle \Rightarrow \langle\phi|V|u_\ell^0\rangle$ is overlap of open channel scattering state with confined state mediated by transition potential.

- And $\lambda \to -\frac{1}{N_c}\frac{1}{k^2-k_0^2}$ feeds confined channel state into open channel resonance.

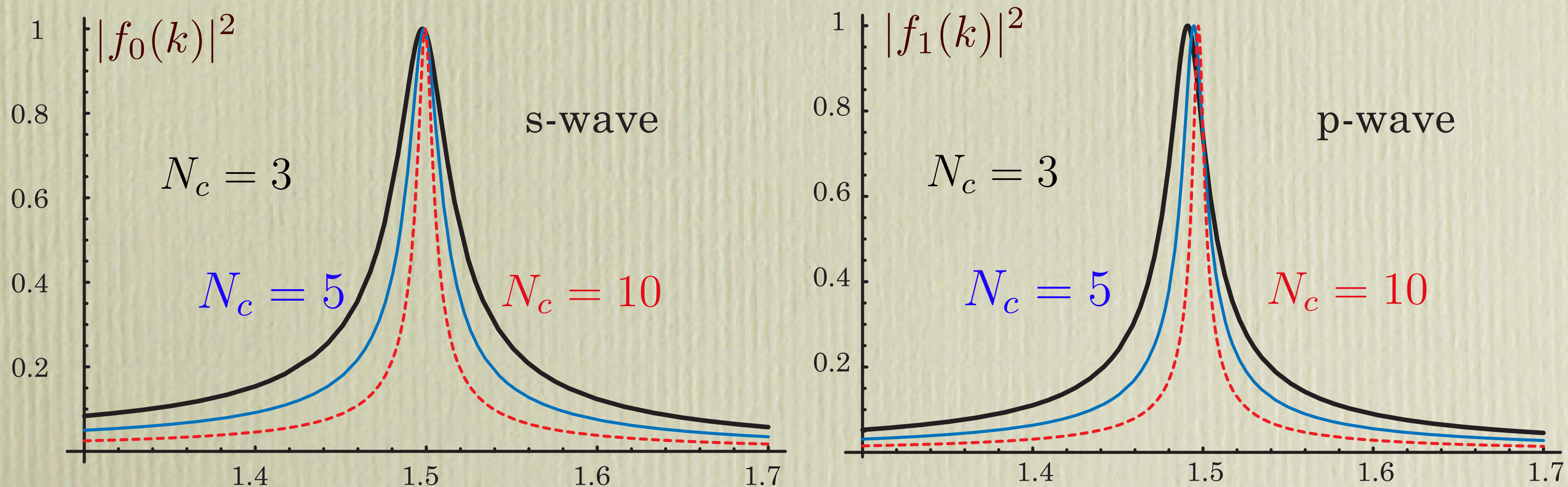
Scattering near a Feshbach resonance

$$f_\ell(k) = \frac{1}{k}\,\frac{\frac{1}{N_c}\xi_\ell(k)^2}{k_0^2 - k^2 - \frac{1}{N_c\pi}\int_{-\infty}^{\infty} dq\,\frac{\xi_\ell(q)^2}{q^2-k^2-i\varepsilon}}$$

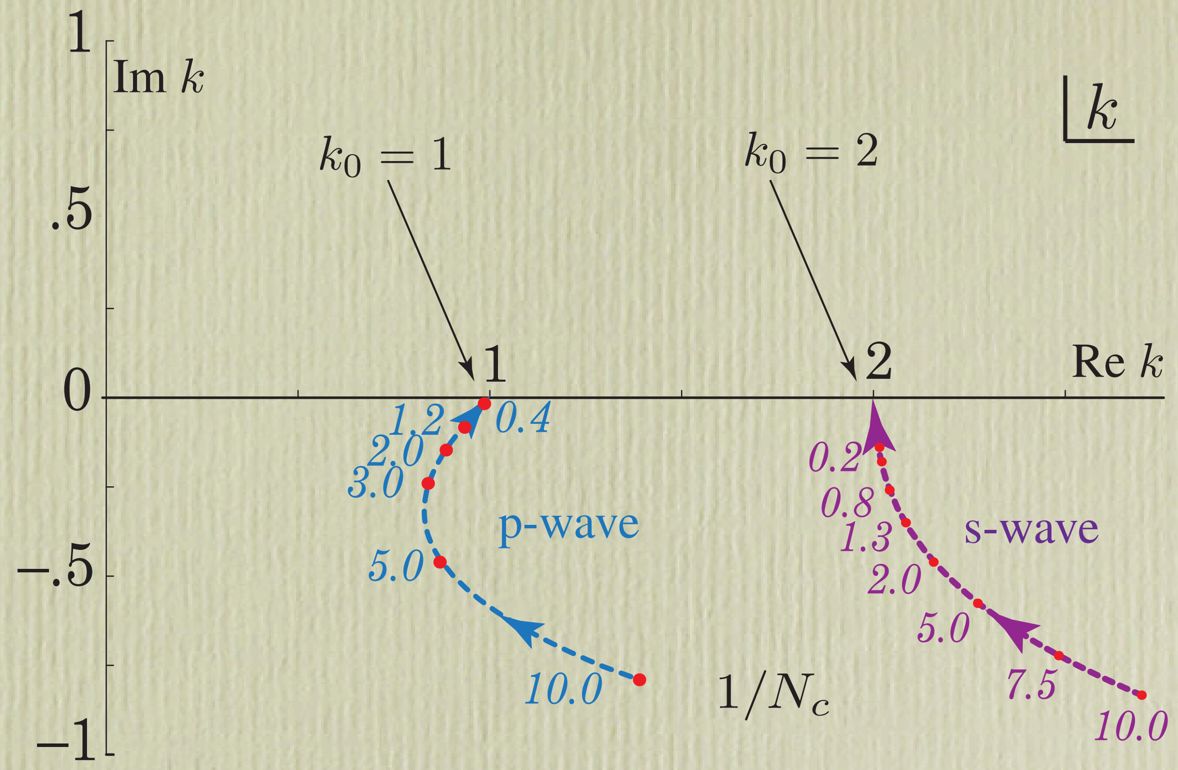
- Unitary
- Relativistic with $s \equiv E^2 = 4(m^2+k^2)$
- In non-relativistic limit

$$\xi(k) = \langle\phi|V|u_\ell^0(k)\rangle = \int_0^\infty dr\phi(r)V(r)rj_\ell(kr)$$

- Scattering amplitude



Ordinary meson pole trajectories in the complex plane



- Poles below Re k axis are resonances
- Ordinary hadrons decouple as $N_c \to \infty$. They become stable states in the meson-meson continuum
- Poles and associated resonances have no particular association with thresholds

Modelling extraordinary hadrons

- Effects generated by open channel (meson-meson) potentials
- Respect relativity and unitarity with partial wave N/D method.
- Equivalent to solving Schrödinger equation with separable potential in open channel

$$h_\ell |u_\ell(k)\rangle - \lambda |\chi\rangle\langle\chi|u_\ell(k)\rangle = k^2 |u_\ell(k)\rangle$$

General solution for the Argand amplitude $f_\ell(k) = \sin\delta_\ell(k) e^{i\delta_l(k)}$

$$f_\ell(k) = \frac{1}{k}\frac{\frac{\lambda}{N_c}|\langle\chi|u_\ell^0(k)\rangle|^2}{1 - \frac{\lambda}{N_c\pi}\int_{-\infty}^{\infty} dq \frac{|\langle\chi|u_\ell^0(q)\rangle|^2}{q^2-k^2-i\varepsilon}}$$

$$= \frac{1}{k}\frac{\frac{\lambda}{N_c}\xi_\ell(k)^2}{1 - \frac{\lambda}{N_c\pi}\int_{-\infty}^{\infty} dq \frac{\xi_\ell(q)^2}{q^2-k^2-i\varepsilon}}$$

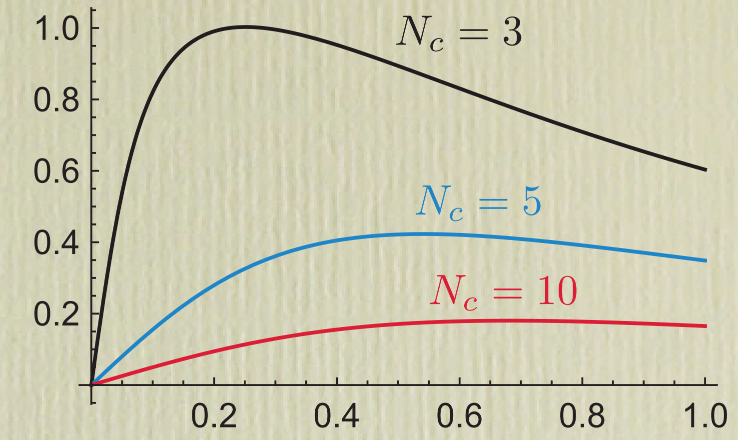
- $\lambda > 0 \Rightarrow$ attraction; $\lambda < 0 \Rightarrow$ repulsion.
- Note N_c dependence as motivated earlier.
- Nature of possible enhancements depend on character of quark-exchange interaction. No guarantee of resonance, certainly not in s-wave.
- Examine N_c dependence.

Low energy scattering – extraordinary hadrons

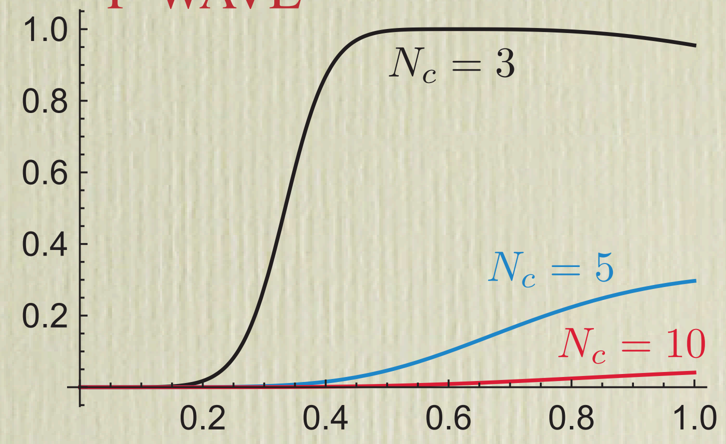
Choose example of attractive single channel interaction giving rise to enhancement/ resonance when λ is large enough

- No resonance in the s-wave –there is no barrier and no confined channel state to drive resonance formation, only virtual or bound state as λ increases
- p-wave shows attractive enhancement, leading to resonance and bound state as λ increases. Angular momentum barrier is responsible for resonance.
- Enhancements in both s- and p-waves vanish as $N_c \to \infty$

S-WAVE

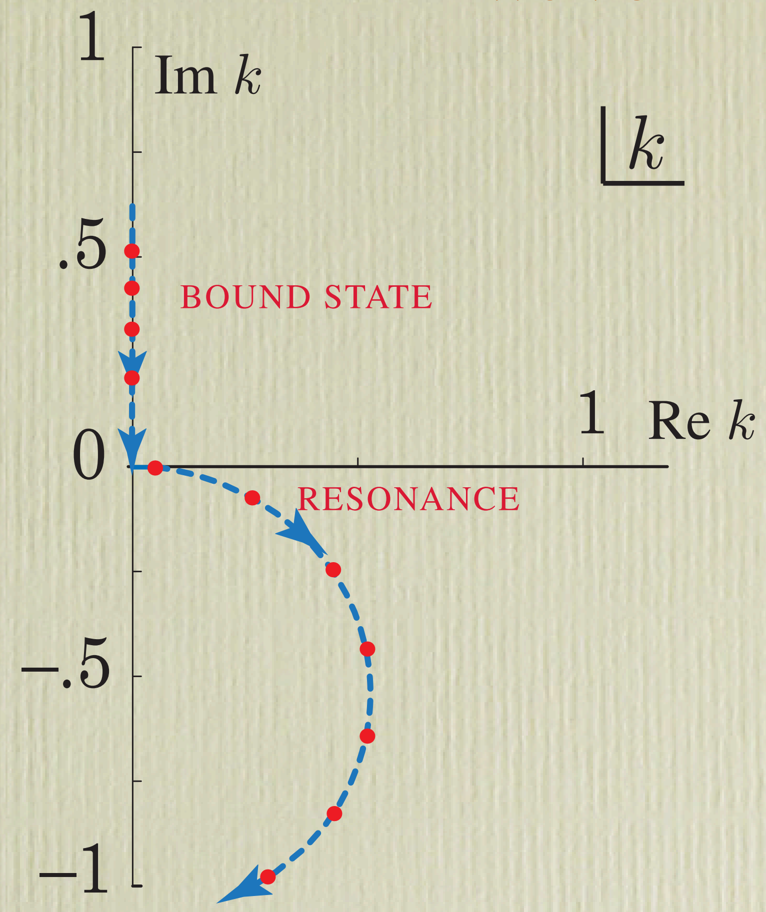


P-WAVE

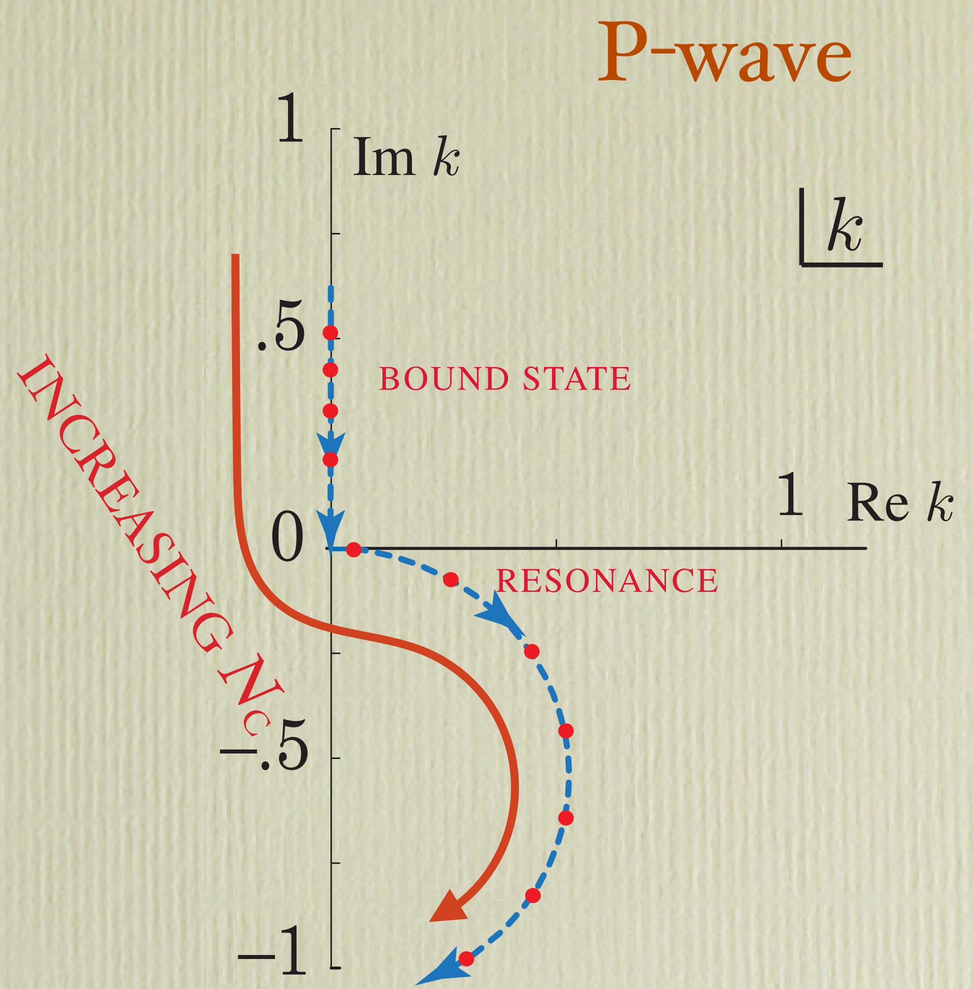


Extraordinary meson pole trajectories in the complex plane

P-wave

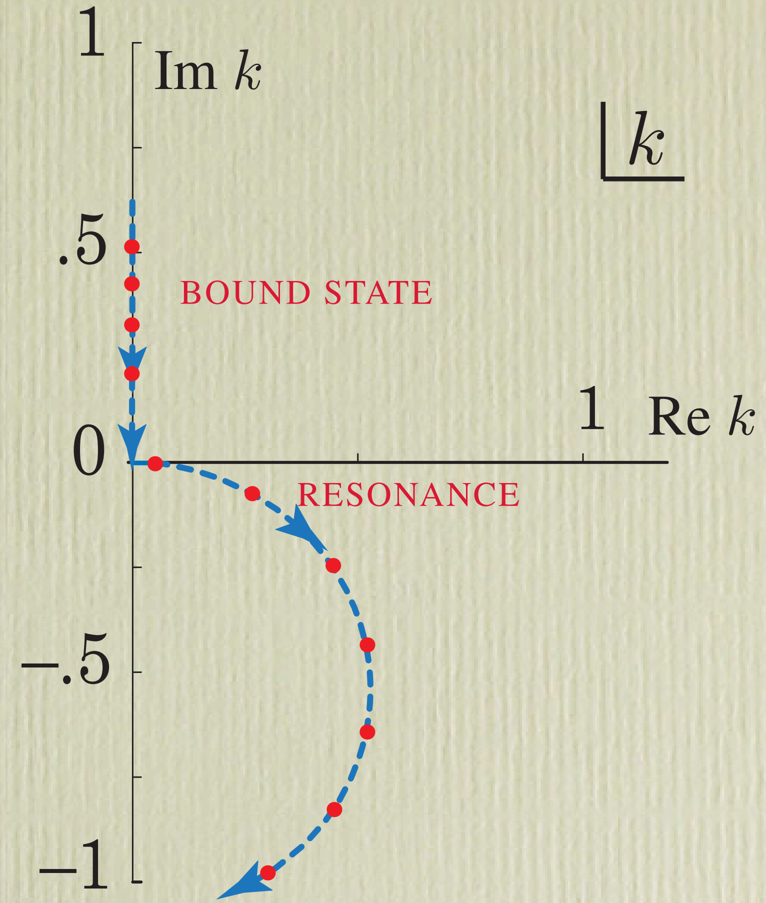


Extraordinary meson pole trajectories in the complex plane



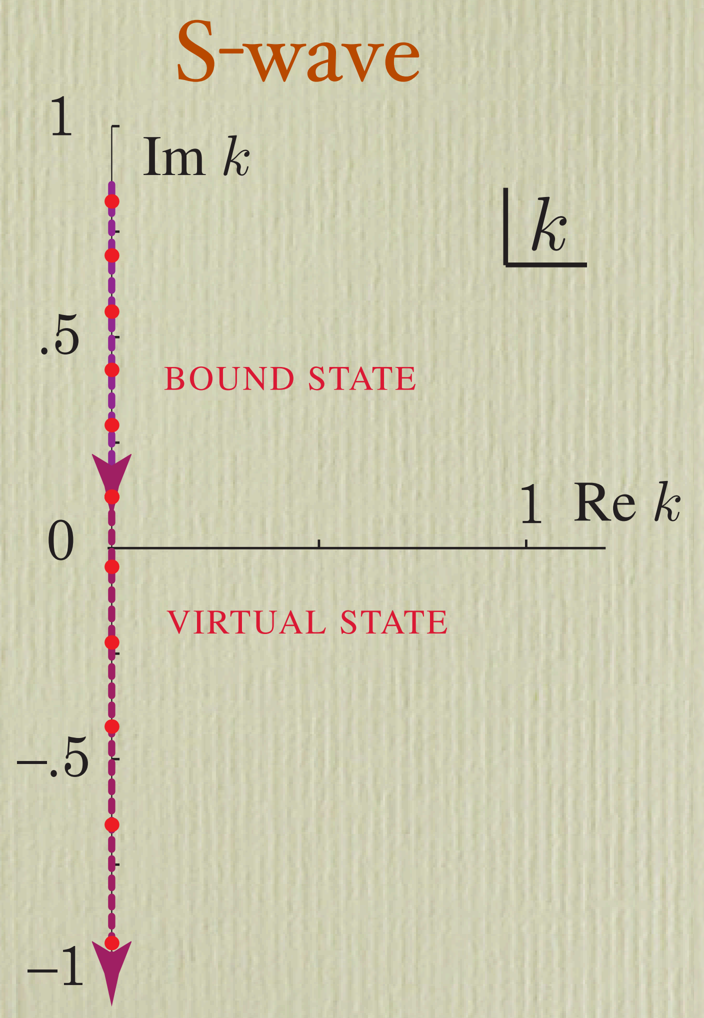
Extraordinary meson pole trajectories in the complex plane

P-wave



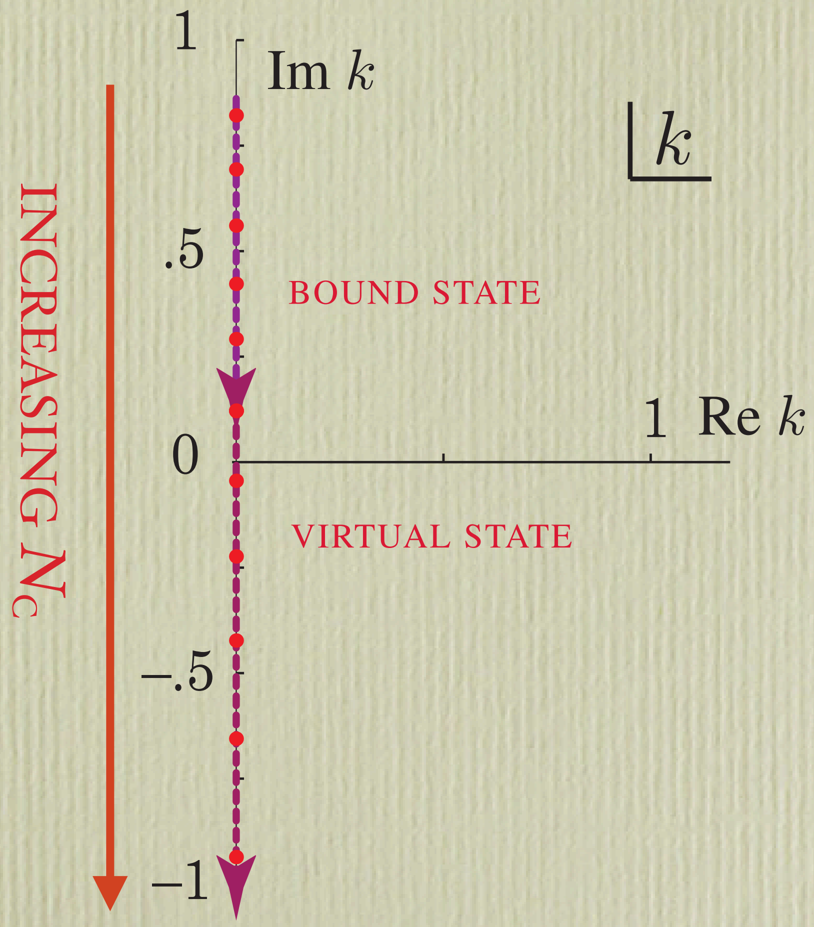
Extraordinary meson pole trajectories in the complex plane

Extraordinary meson pole trajectories in the complex plane



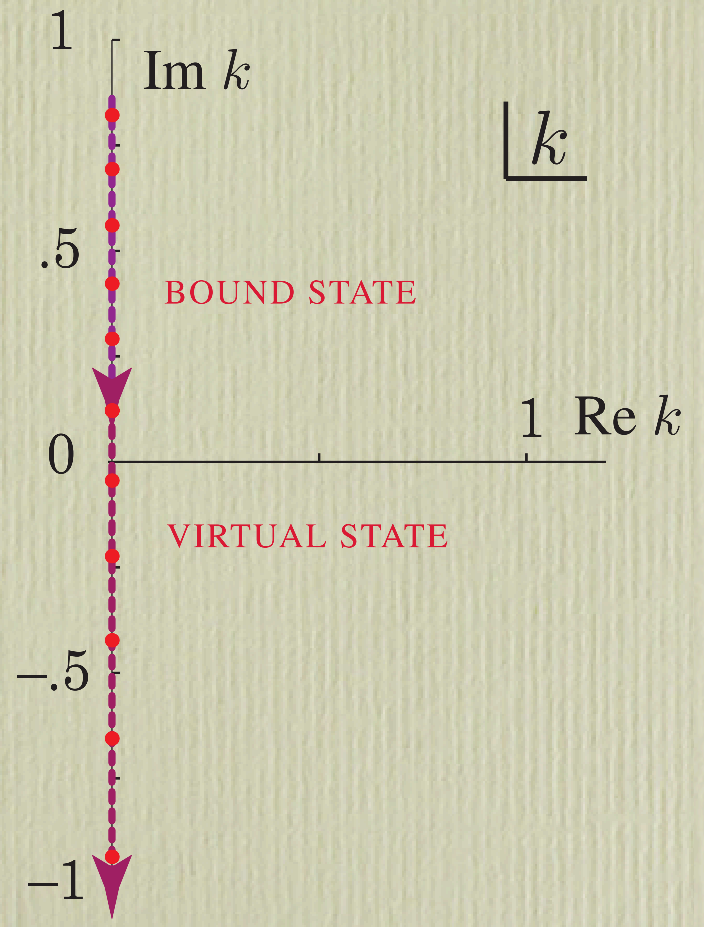
Extraordinary meson pole trajectories in the complex plane

S-wave

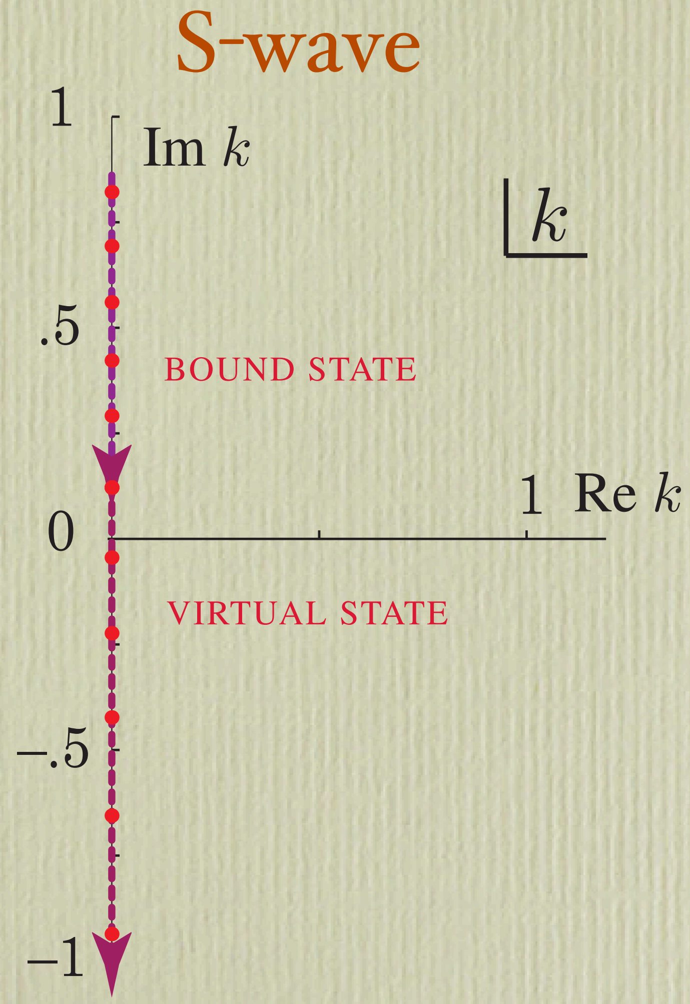


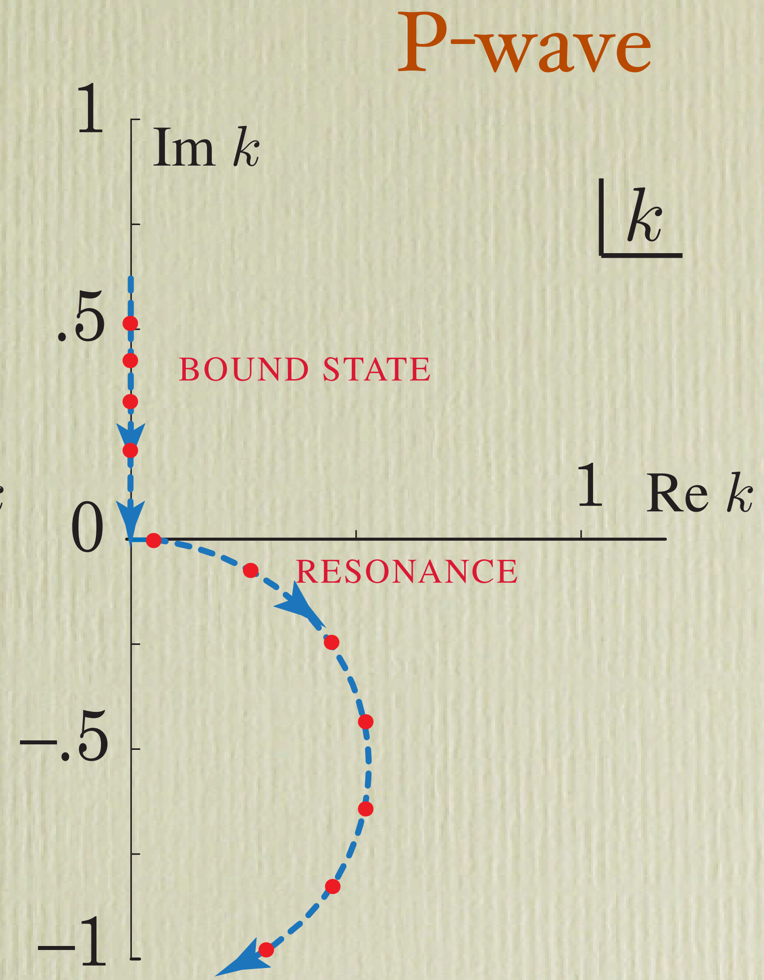
Extraordinary meson pole trajectories in the complex plane

S-wave

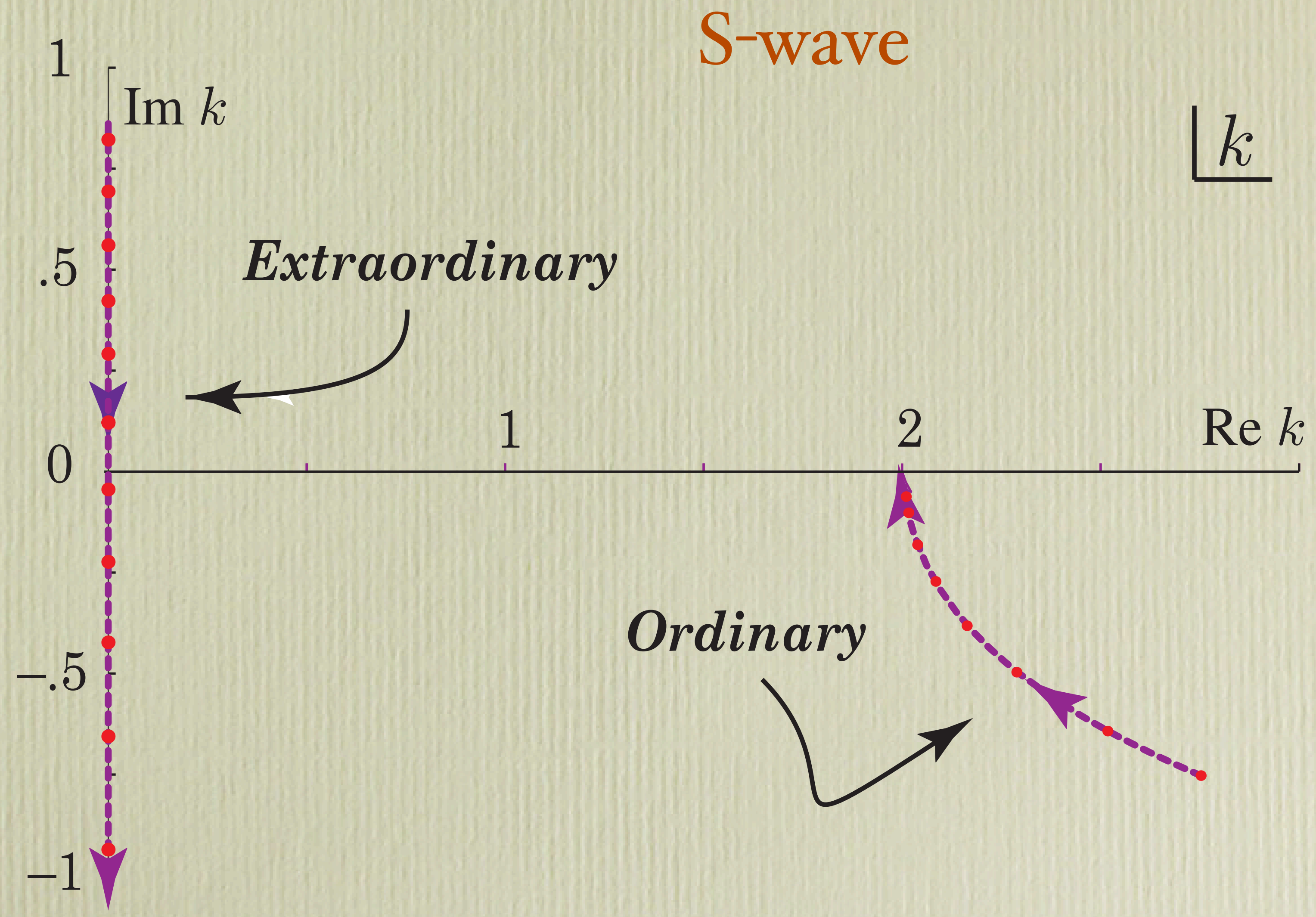


Extraordinary meson pole trajectories in the complex plane

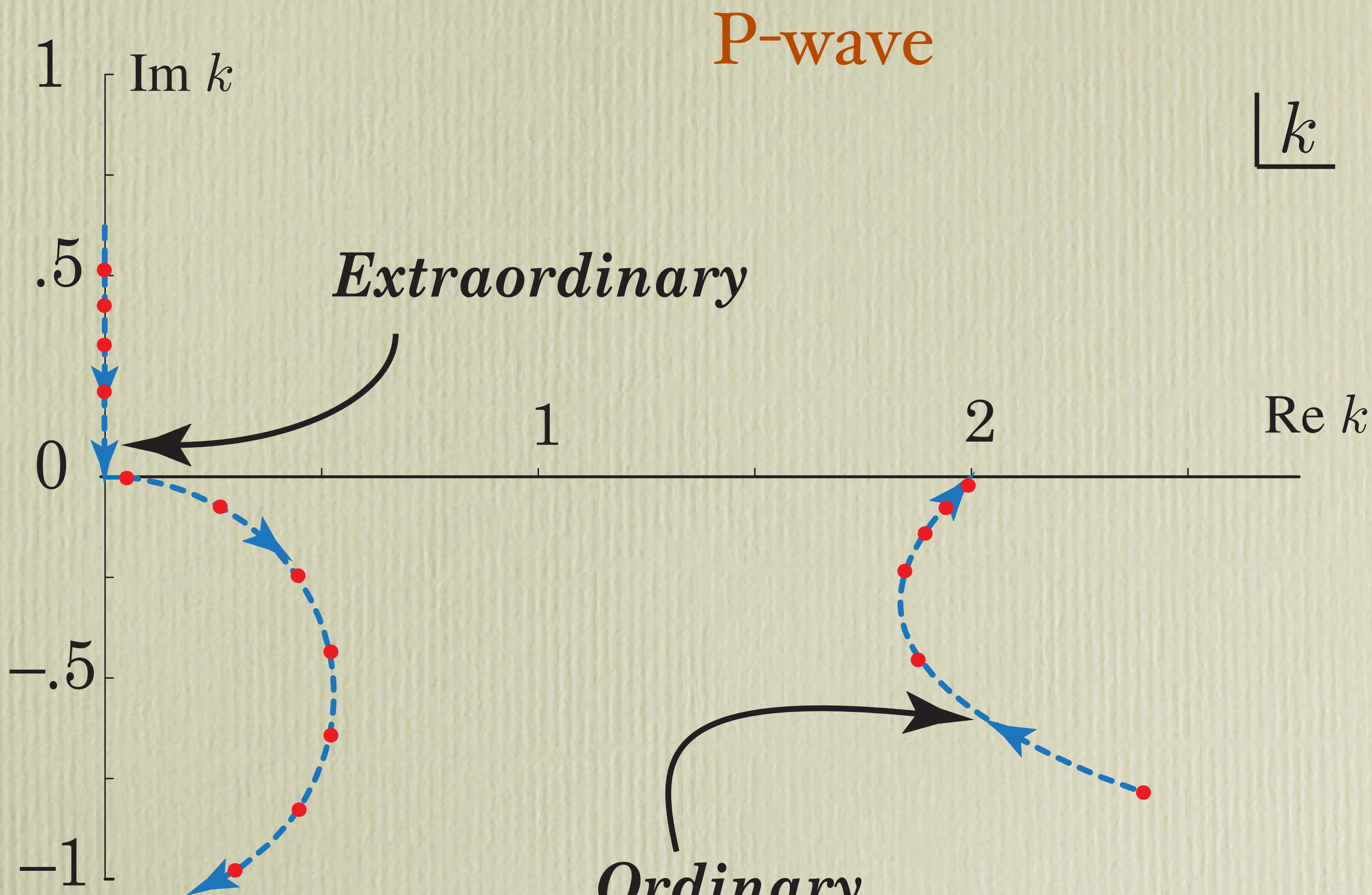




Ordinary and extraordinary mesons could hardly be more different!



Ordinary and extraordinary mesons could hardly be more different!

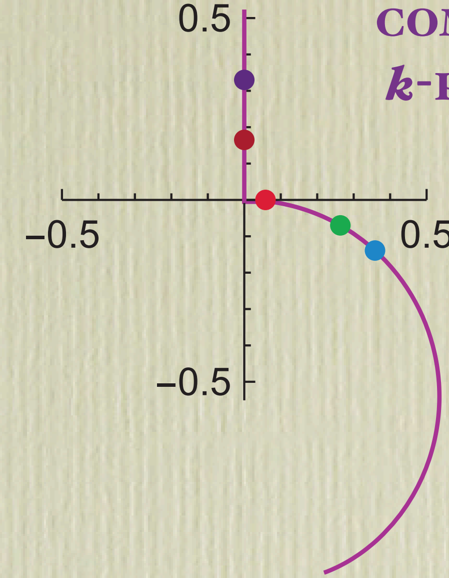


S-wave extraordinary hadrons are different from other partial waves

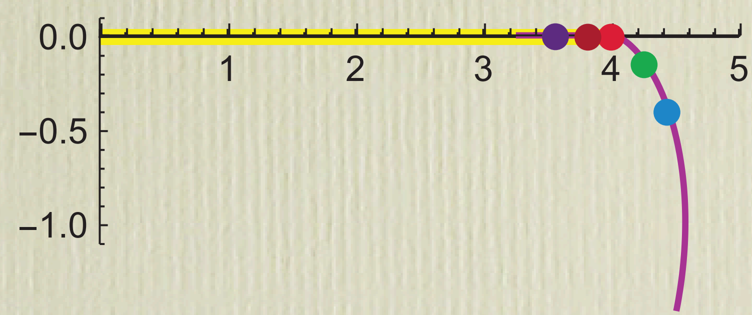
P-wave

STRONG (SMALL N_C) ● ● ● ● ● ● **WEAK (LARGE N_C)**

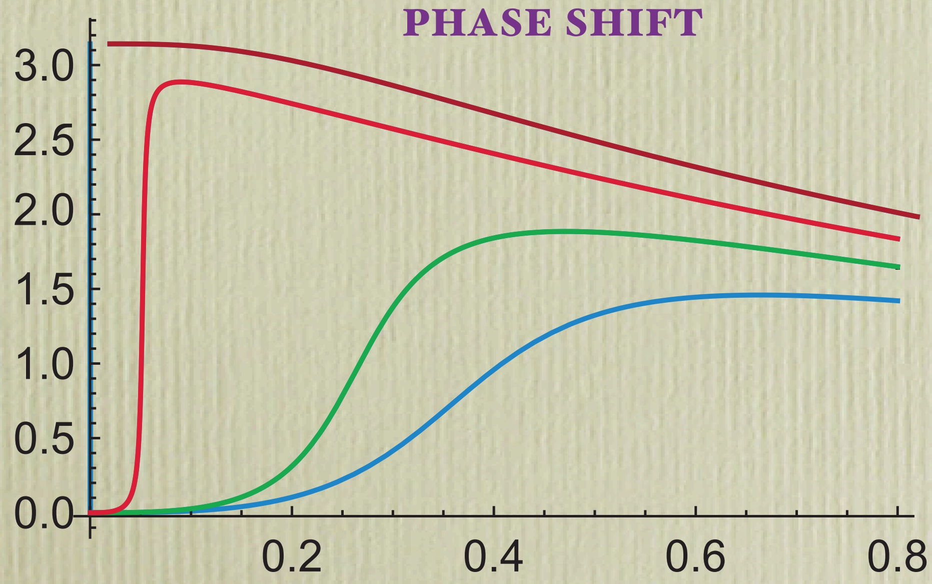
COMPLEX *k*-PLANE



COMPLEX *s*-PLANE



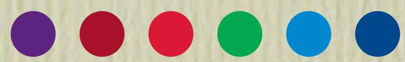
PHASE SHIFT



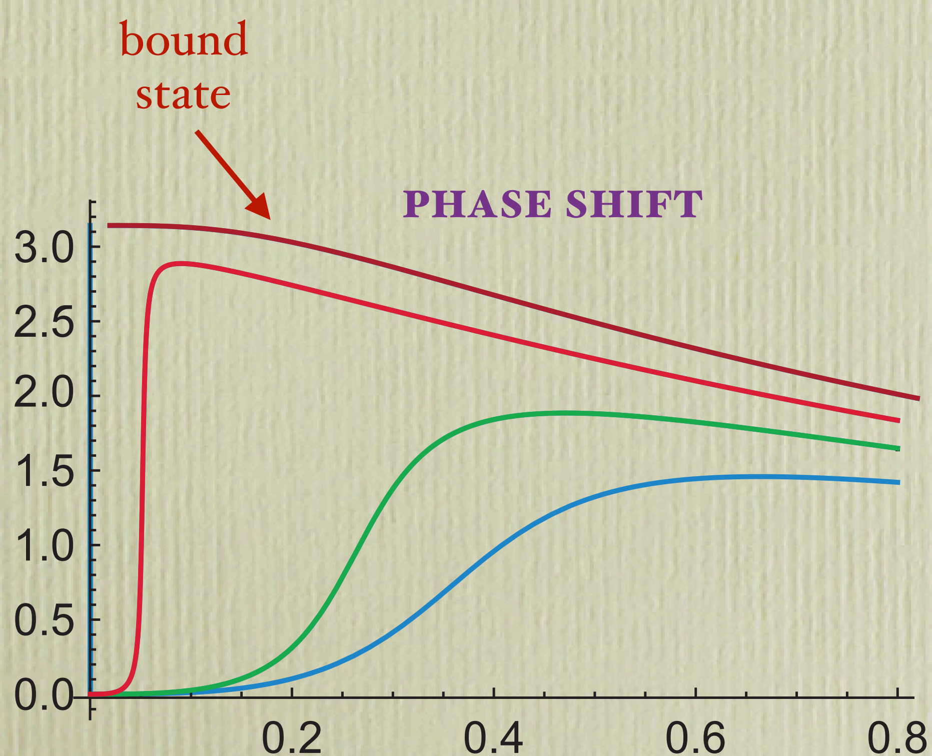
S-wave extraordinary hadrons are different from other partial waves

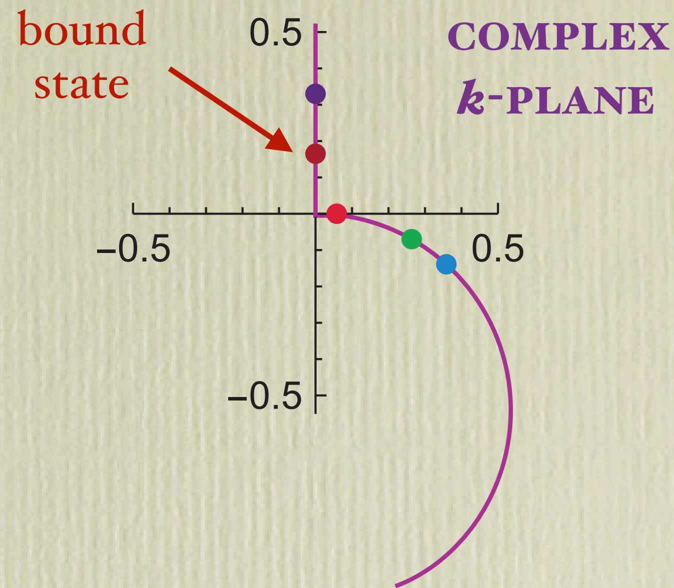
P-wave

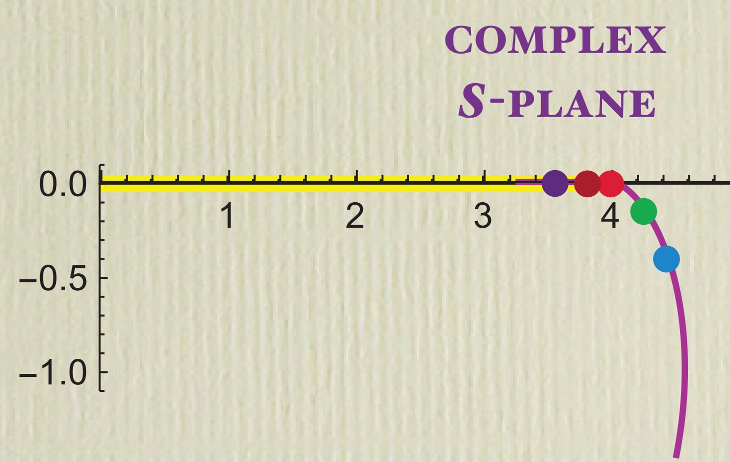
STRONG (SMALL N_C)



WEAK (LARGE N_C)



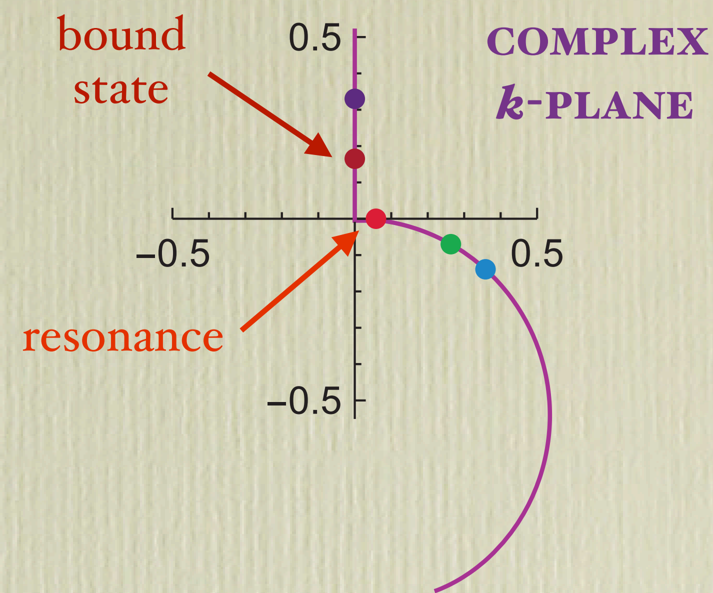


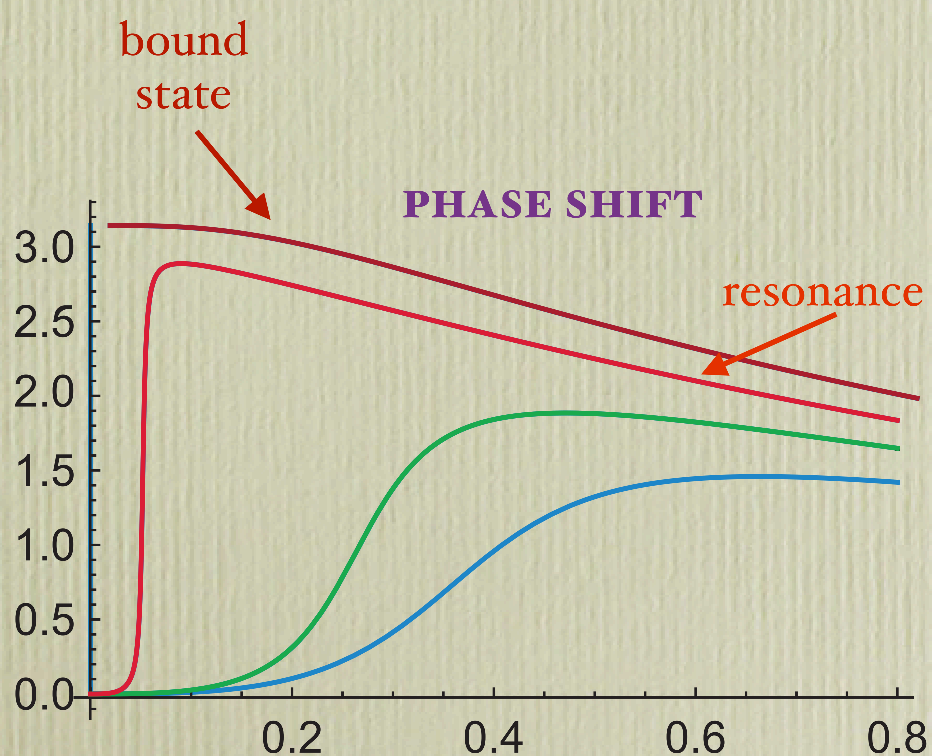


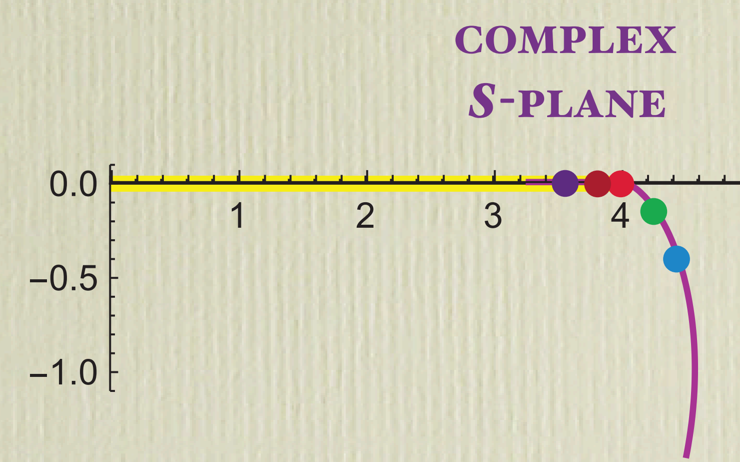
S-wave extraordinary hadrons are different from other partial waves

P-wave

STRONG (SMALL N_C) ● ● ● ● ● ● WEAK (LARGE N_C)

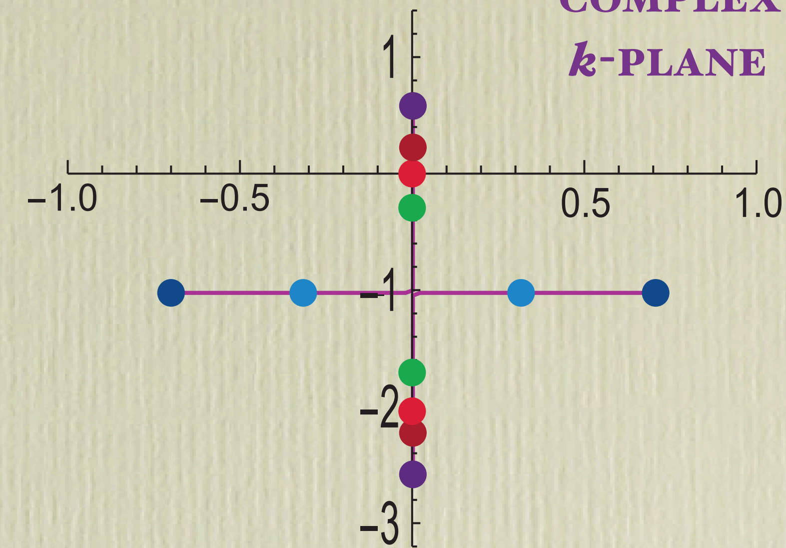






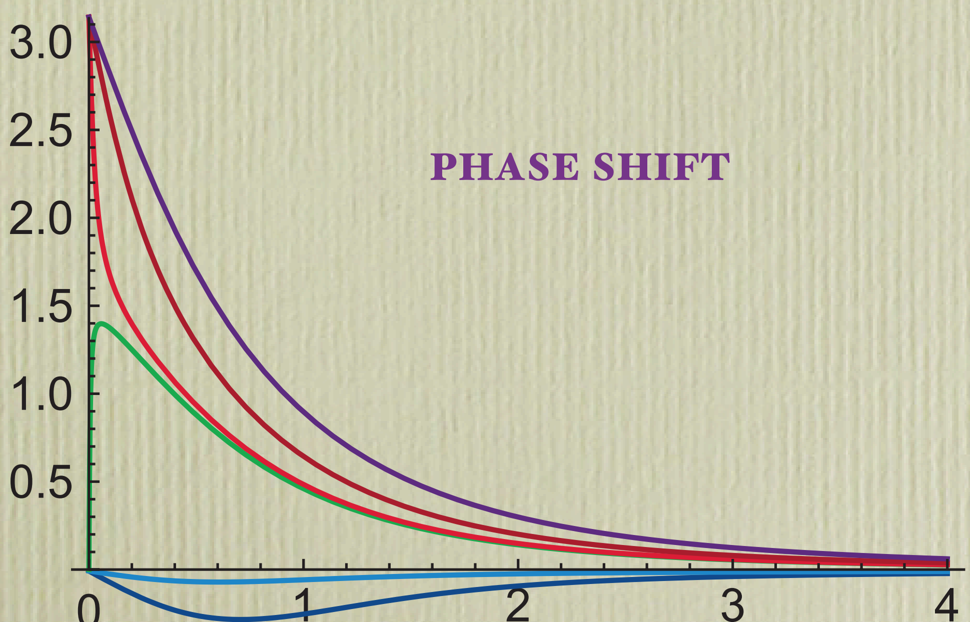
S-wave extraordinary hadrons are different from other partial waves

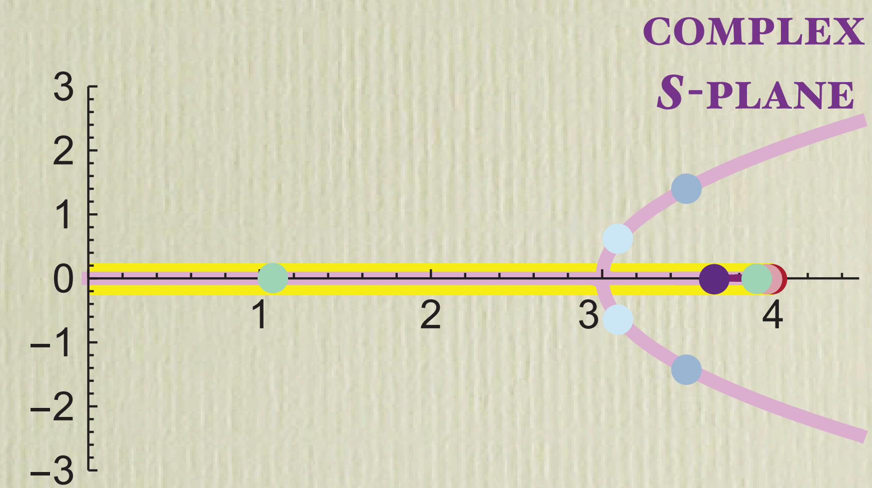
S-wave



STRONG (SMALL N_C)

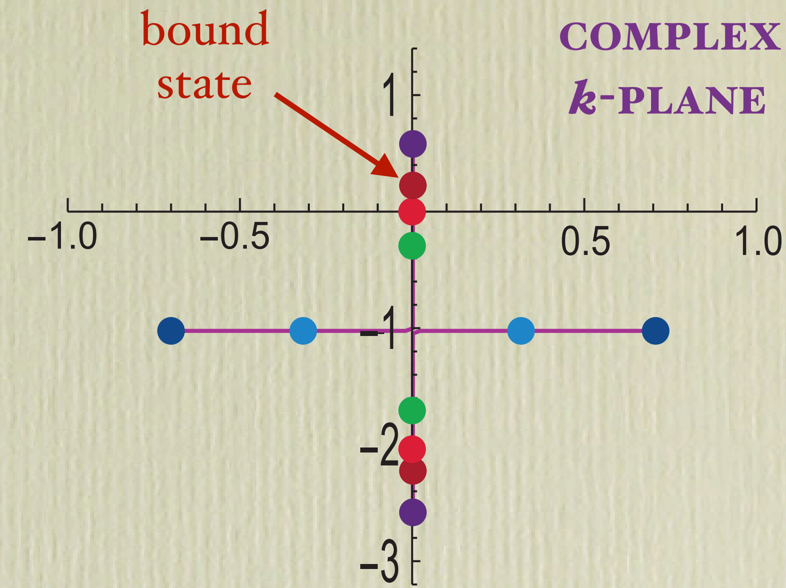
WEAK (LARGE N_C)



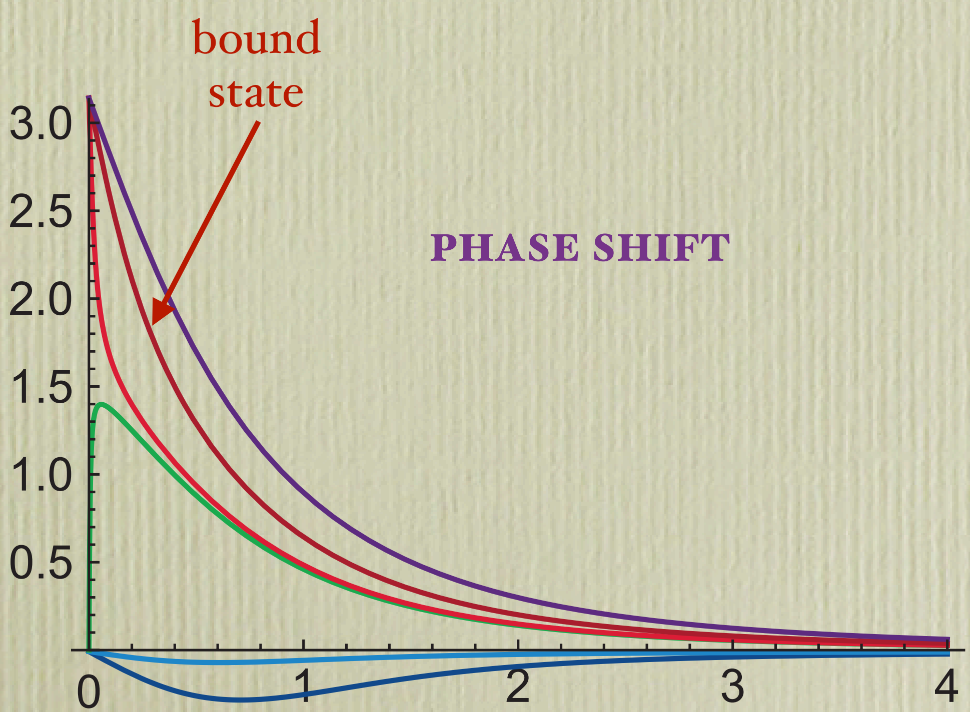


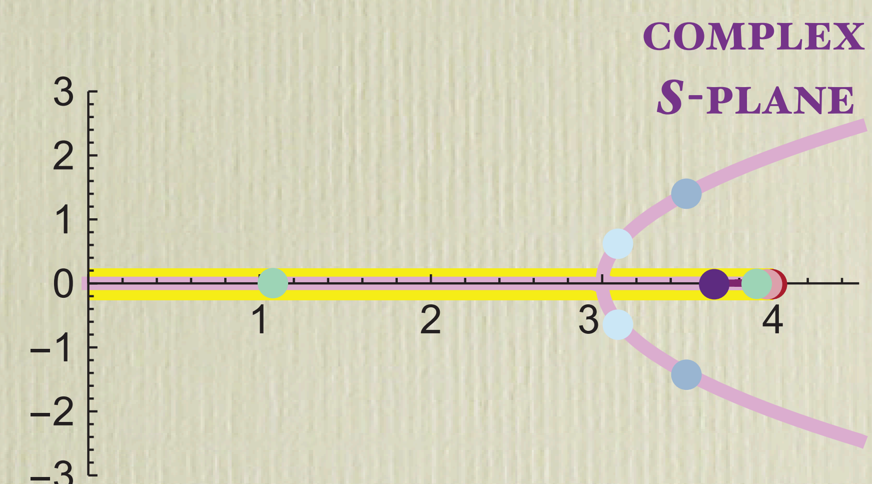
S-wave extraordinary hadrons are different from other partial waves

S-wave



STRONG (SMALL N_C) WEAK (LARGE N_C)

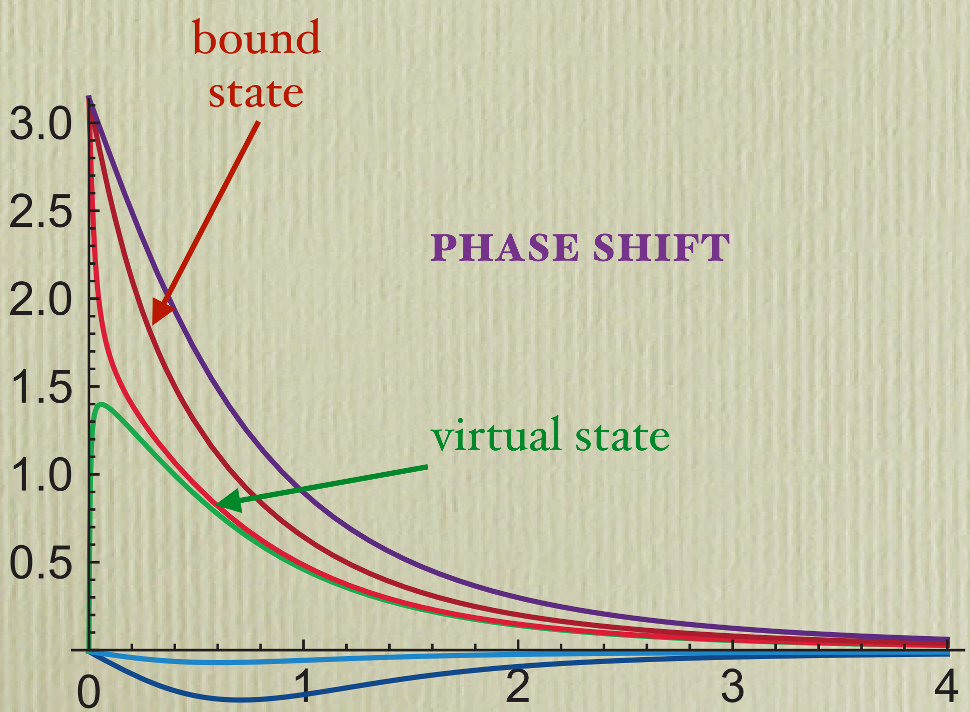


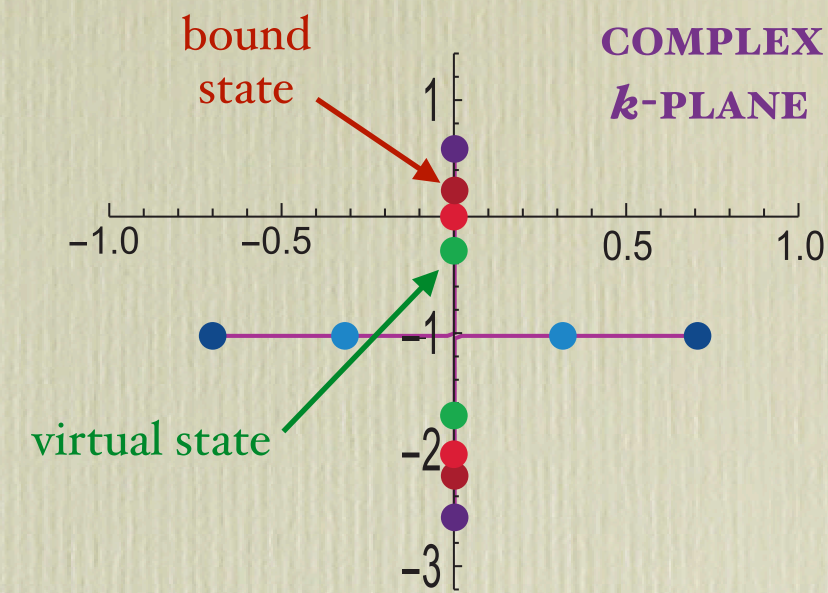


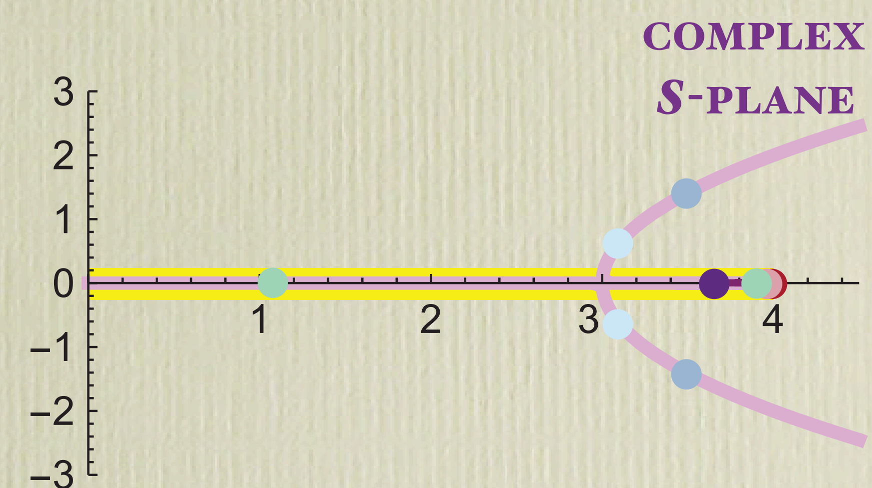
S-wave extraordinary hadrons are different from other partial waves

S-wave

STRONG (SMALL N_C) ● ● ● ● ● ● **WEAK (LARGE N_C)**







Summary

- Large N_c distinguishes ordinary ($\bar{Q}Q$) mesons from possible extraordinary ($\geq \bar{Q}\bar{Q}QQ$) mesons.
- Unitarized chiral dynamics $\Rightarrow$ vector mesons are ordinary and light scalar mesons are extraordinary
- Ordinary mesons $=$ Feshbach resonances
- Extraordindary mesons $=$ open-channel enhancements, resonances, bound or virtual states.
- Watch out for unique behavior of S-wave "states" near threshold.