

## Heavy-light exotics (hadronic molecules)

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*Exotic Hadrons and Flavor Physics*

Simons Center for Geometry and Physics, 28 May – 1 June, 2018

- Charm-strange mesons:  $D_{s0}^*(2317)$ ,  $D_{s1}(2460)$
- Charm-nonstrange mesons:  $D_0^*(2400)$ ,  $D_1(2430)$
- Predictions:
  - ↳ bottom-strange and nonstrange mesons
  - ↳ doubly charmed baryons with  $J^P = 1/2^-$
  - ↳ an explanation of  $D_{s1}^*(2860)$

# Beginning of the interesting story: $D_{s0}^*$ (2317) and $D_{s1}$ (2460)

## Charm-strange mesons

- $D_{s0}^*$  (2317):  $0^+$  BaBar (2003)

$$M = (2317.7 \pm 0.6) \text{ MeV},$$

$$\Gamma < 3.8 \text{ MeV}$$

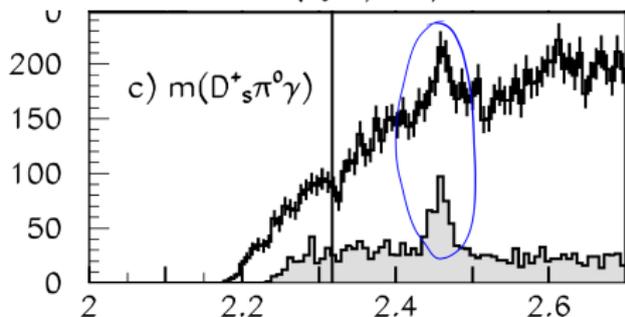
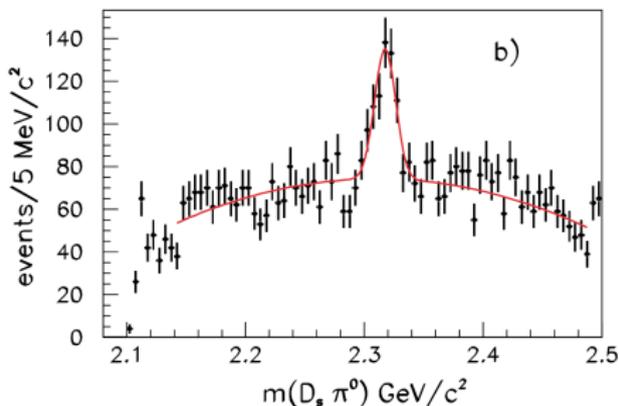
The only hadronic decay:  $D_s \pi$

- $D_{s1}$  (2460):  $1^+$  BaBar, CLEO (2003)

$$M = (2459.5 \pm 0.6) \text{ MeV},$$

$$\Gamma < 3.5 \text{ MeV}$$

- no isospin partner observed, tiny widths  $\Rightarrow I = 0$



BABAR, PRL90(2003)242001

# $D_0^*(2400)$ and $D_1(2430)$

- $D_0^*(2400)^0: J^P = 0^+, \Gamma = (247 \pm 67) \text{ MeV}$

Belle (2004)

## PDG2017:

2318 $\pm$ 29	OUR AVERAGE	Error includes scale factor of 1.7.			
2297 $\pm$ 8 $\pm$ 20	3.4k	AUBERT	2009AB	BABR	$B^- \rightarrow D^+ \pi^- \pi^-$
2308 $\pm$ 17 $\pm$ 32		ABE	2004D	BELL	$B^- \rightarrow D^+ \pi^- \pi^-$
2407 $\pm$ 21 $\pm$ 35	9.8k	LINK	2004A	FOCS	$\gamma A$

New measurements by LHCb:  $(2360 \pm 34) \text{ MeV}$

LHCb, PRD92(2015)012012

- $D_1(2430)^0: J^P = 1^+, \Gamma = 384_{-110}^{+130} \text{ MeV}$

VALUE (MeV)	DOCUMENT ID	TECN	COMMENT
2427 $\pm$ 26 $\pm$ 25	ABE	2004D	BELL $B^- \rightarrow D^{*(0)+} \pi^- \pi^-$
••• We do not use the following data for averages, fits, limits, etc. •••			
2477 $\pm$ 28	1 AUBERT	2006L	BABR $\bar{B}^0 \rightarrow D^{*+} \omega \pi^-$

- Notice: all these experiments used a Breit–Wigner to extract the resonance

☞  $D^{(*)} \pi - D^{(*)} \eta - D_s^{(*)} \bar{K}$  coupled-channel effects are absent

☞ chiral symmetry constraint on soft pions is absent

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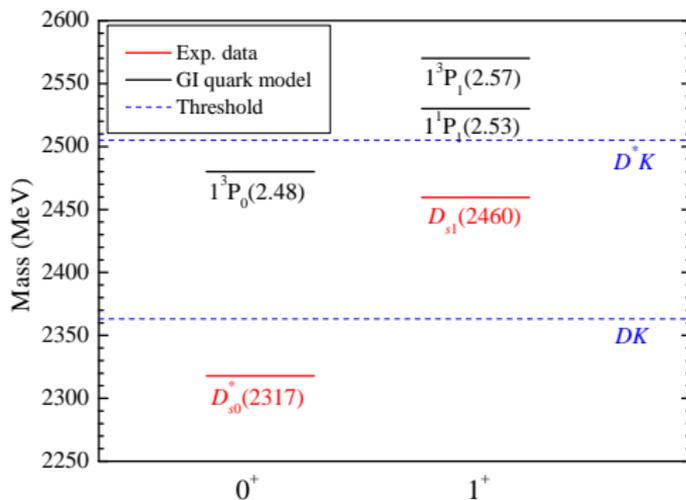
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# Why are they interesting?



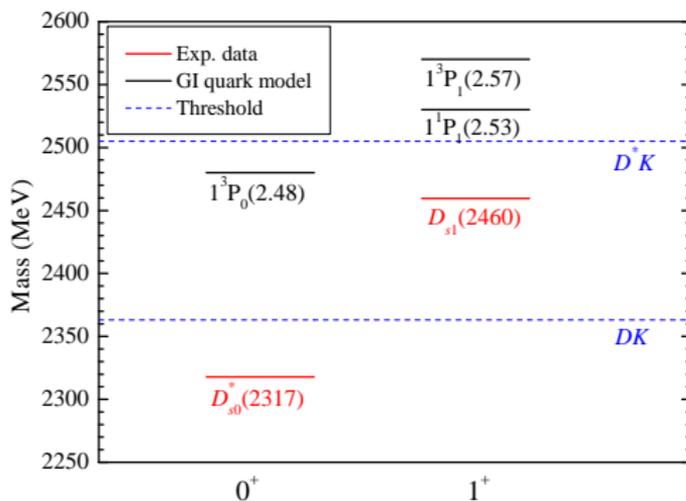
GI quark model: Godfrey, Isgur (1985)

👉 Why are the masses of  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  much lower than quark model predictions for  $c\bar{s}$  mesons ?

👉 Why  $\underbrace{M_{D_{s1}(2460)\pm} - M_{D_{s0}^*(2317)\pm}}_{=(141.8 \pm 0.8) \text{ MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^\pm}}_{=(140.67 \pm 0.08) \text{ MeV}}$  within 2 MeV?

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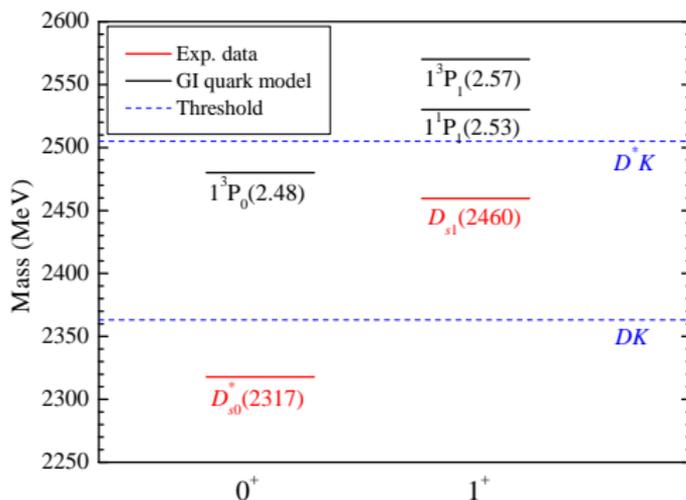
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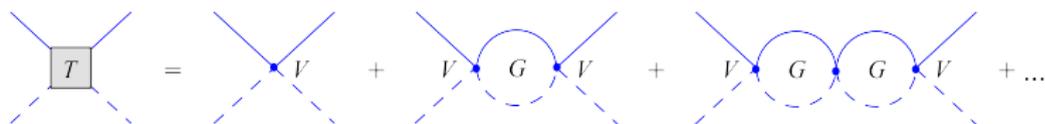
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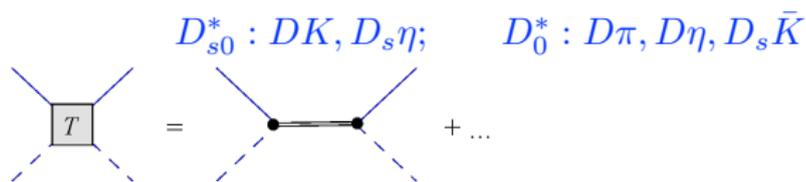
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# Interactions between charm and light mesons

- $S$ -wave interactions between charm mesons ( $D, D_s$ ) and light pseudoscalar mesons ( $\pi, K, \eta$ )



- not far from the thresholds  $\Rightarrow$  chiral EFT for matter field
- $D_{s0}^*/D_0^*$  should appear as poles in scattering amplitudes:



$\Rightarrow$  needs a nonperturbative treatment: ChPT + unitarization

$$T^{-1}(s) = V^{-1}(s) - G(s)$$

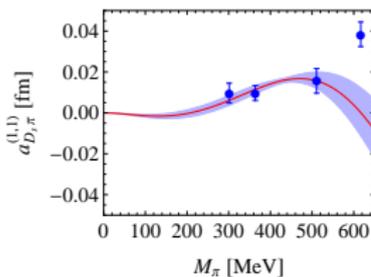
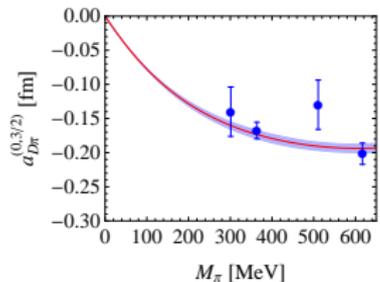
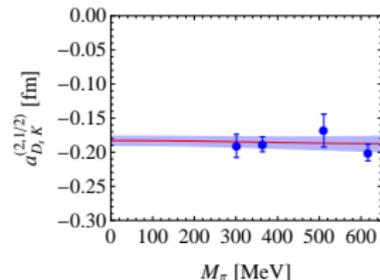
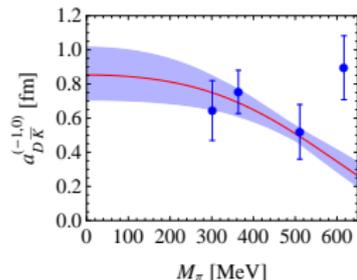
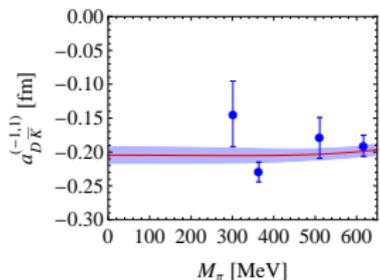
$V(s)$ : to be derived from SU(3) chiral Lagrangian, 6 LECs up to NLO

$G(s)$ : 2-point scalar loop functions, regularized with a subtraction constant  $a(\mu)$

- Fit to lattice data on scattering lengths in 5 simple channels:

$D\bar{K}(I=1, I=0)$ ,  $D_s K$ ,  $D\pi(I=3/2)$ ,  $D_s\pi$ : no disconnected contribution

5 parameters:  $h_2, h_3, h_4, h_5$  and  $a(\mu)$

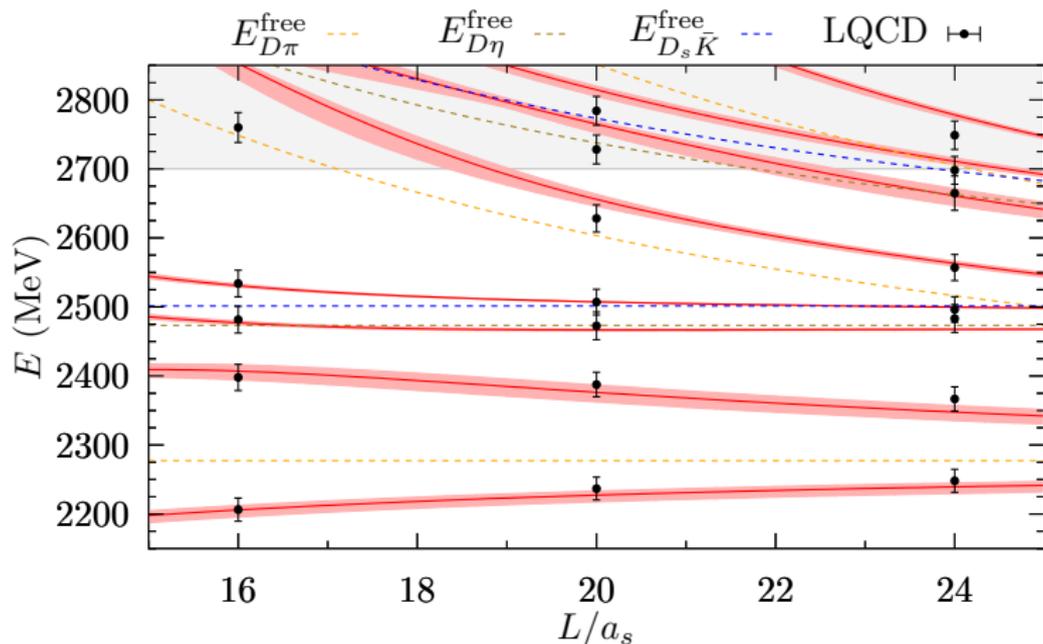


# Postdictions versus recent lattice results: charm-nonstrange

- In a finite volume:  $\vec{q} = \frac{2\pi}{L}\vec{n}$ ,  $\vec{n} \in \mathbb{Z}^3$ ; loop integral  $G(s)$ :  $\int d^3\vec{q} \rightarrow \frac{1}{L^3} \sum_{\vec{q}}$
- Postdicted  $I = 1/2 D\pi, D\eta, D_s\bar{K}$  finite volume energy levels versus lattice QCD results by [G. Moir *et al.* [Hadron Spectrum Collaboration], JHEP1610(2016)011]

NOT a fit!

M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465

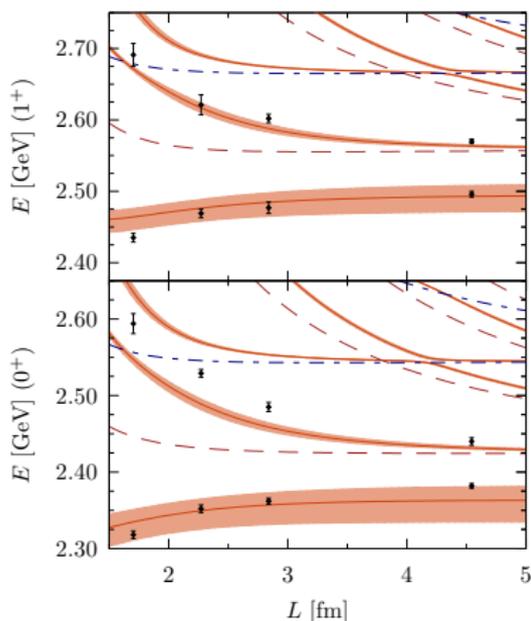


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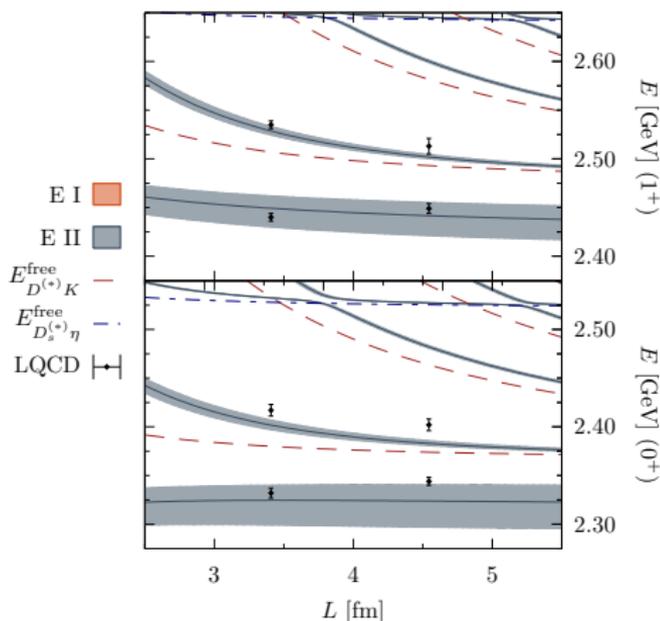
- **Postdicted** finite volume energy levels for  $(S, I) = (1, 0)$   $D^{(*)}K$ ,  $J^P = 1^+ & 0^+$  versus lattice QCD results by [G. Bali, S. Collins, A. Cox, A. Schäfer, PRD96(2017)074501]

M. Albaladejo, P. Fernandez-Soler, J. Nieves, P. G. Ortega, arXiv:1805.07104

E I:  $M_\pi = 290$  MeV



E II:  $M_\pi = 150$  MeV



- Heavy-strange

meson	$J^P$	prediction (MeV)	PDG2017 (MeV)	lattice (MeV)
$D_{s0}^*$	$0^+$	$2315_{-28}^{+18}$	$2317.7 \pm 0.6$	$2348_{-4}^{+7}$ [1]

- Heavy-nonstrange, two  $I = 1/2$  states ( $M, \Gamma/2$ ):

	Lower (MeV)	Higher (MeV)	PDG2017 (MeV)
$D_0^*$	$(2105_{-8}^{+6}, 102_{-11}^{+10})$	$(2451_{-26}^{+36}, 134_{-8}^{+7})$	$(2318 \pm 29, 134 \pm 20)$
$D_1$	$(2247_{-6}^{+5}, 107_{-10}^{+11})$	$(2555_{-30}^{+47}, 203_{-9}^{+8})$	$(2427 \pm 40, 192_{-55}^{+65})$
$B_0^*$	$(5535_{-11}^{+9}, 113_{-17}^{+15})$	$(5852_{-19}^{+16}, 36 \pm 5)$	—
$B_1$	$(5584_{-11}^{+9}, 119_{-17}^{+14})$	$(5912_{-18}^{+15}, 42_{-4}^{+5})$	—

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$B_{s0}^*$	$0^+$	$5720_{-23}^{+16}$	—	$5711 \pm 23[2]$
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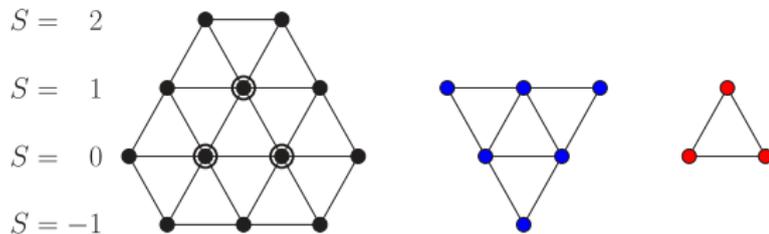
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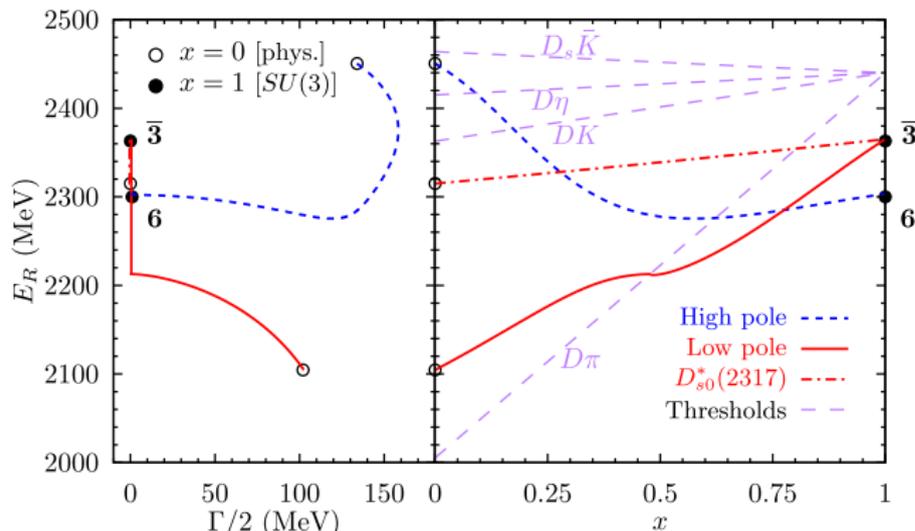
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# SU(3) analysis

- In the SU(3) limit, irreps:  $\bar{3} \otimes 8 = \bar{15} \oplus 6 \oplus \bar{3}$



- Evolution of the two poles from the physical to the SU(3) symmetric case



- Compositeness ( $1 - Z$ ) related to the  $S$ -wave scattering length: Weinberg (1965)

$$a \simeq -2 \frac{1 - Z}{2 - Z} \frac{1}{\sqrt{2\mu E_B}}$$

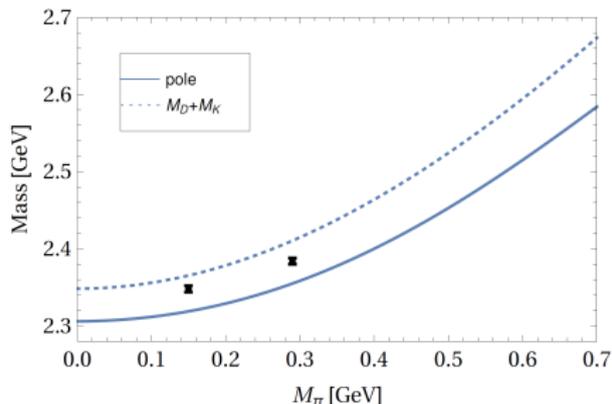
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 $D_{s0}^*(2317)$  contains  $\sim 70\%$   $DK$  Martínez Torres, Oset, Prelovsek, Ramos, JHEP1505,053
- Latest lattice results in G. Bali et al., PRD96(2017)074501

# DK component from lattice QCD

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$$1 - Z = 1.04(0.08)(+0.30)$$

$M_\pi$ [MeV]	150	290
$M_{D_{s0}^*(2317)}$ [MeV]	$2348 \pm 4$	$2384 \pm 3$
$M_{D_s}$ [MeV]	$1977 \pm 1$	$1980 \pm 1$

strong  $M_\pi$  dependence!

curves: prediction in Du et al., EPJC77(2017)728

## Solutions to all three puzzles

- Q: Why are the masses of  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  much lower than quark model predictions for  $c\bar{s}$  mesons ?

A: hadronic molecules, main components:  $D_{s0}^*(2317)[DK], D_{s1}(2460)[D^*K]$

Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); Chen, Li (2004); FKG et al. (2006); ...

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A: HQSS  $\Rightarrow DK$  and  $D^*K$  interactions almost the same  $\Rightarrow$

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$$M_D + M_K - M_{D_{s0}^*(2317)} \simeq M_{D^*} + M_K - M_{D_{s1}(2460)} \pm 4 \text{ MeV}$$

Uncertainty: binding energy (45 MeV)  $\times \frac{\Lambda_{\text{QCD}}}{m_c} \frac{M_K}{\Lambda_\chi}$

- Q: Why  $M_{D_0^*(2400)} \gtrsim M_{D_{s0}^*(2317)}$  and  $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$  ?

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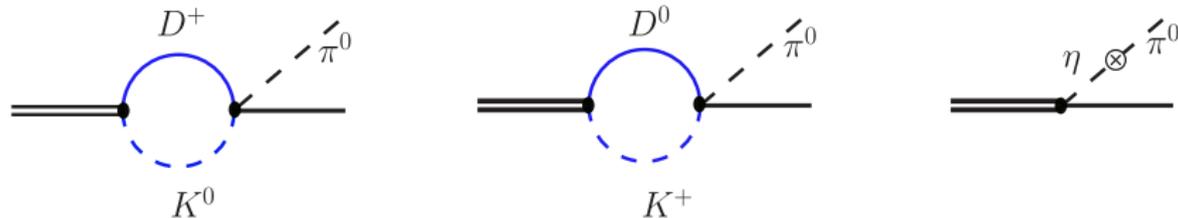
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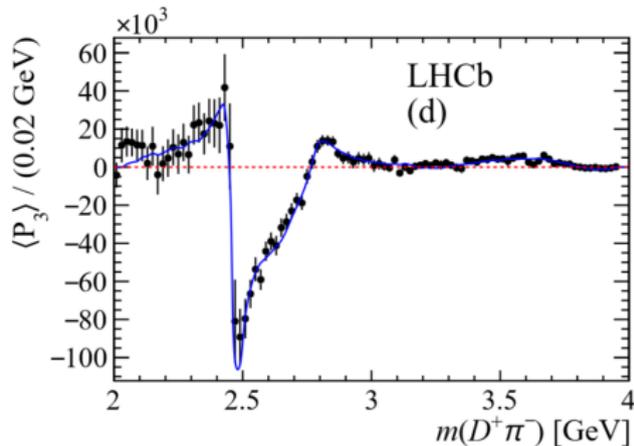
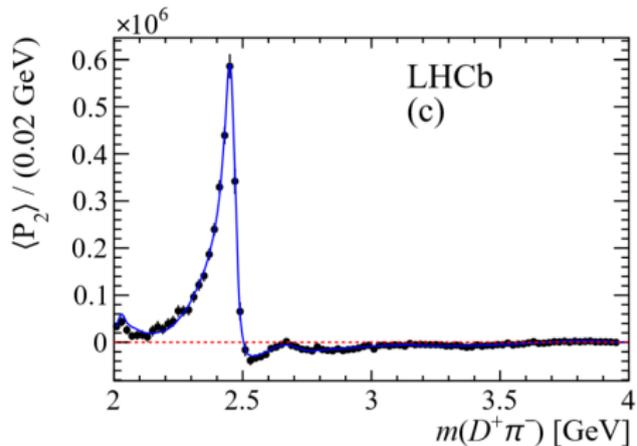
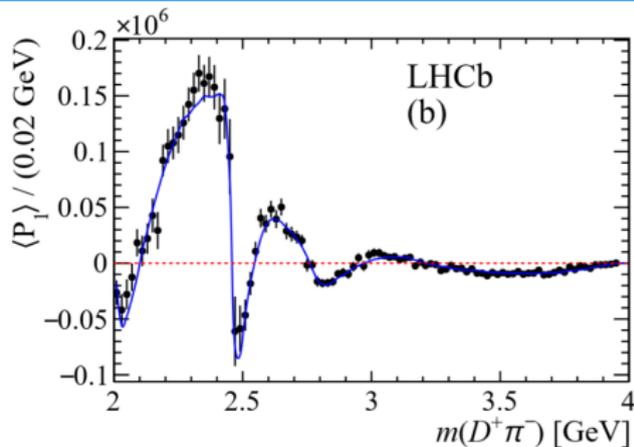
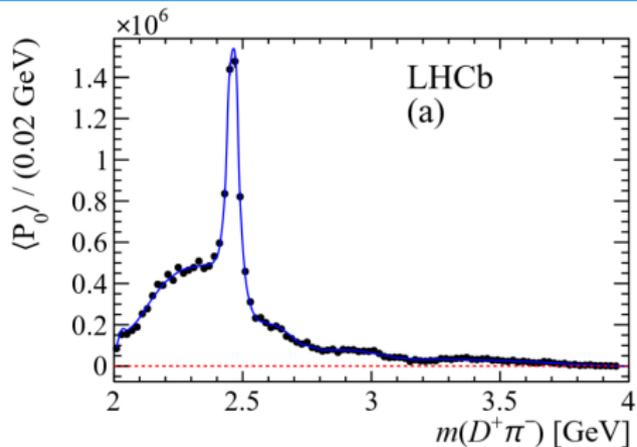
A: There are two  $D_0^*$  and two  $D_1$ , and the  $\bar{\mathbf{3}}$  ones have smaller masses.

Experimental consequences?

# Decay width of $D_{s0}^*(2317)$



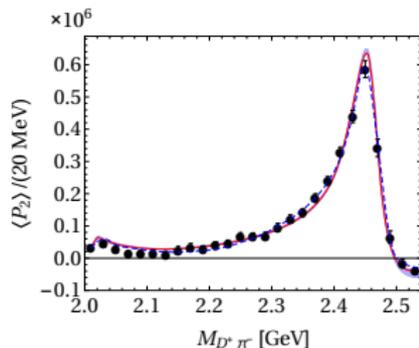
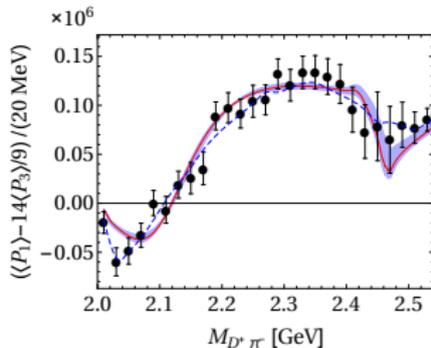
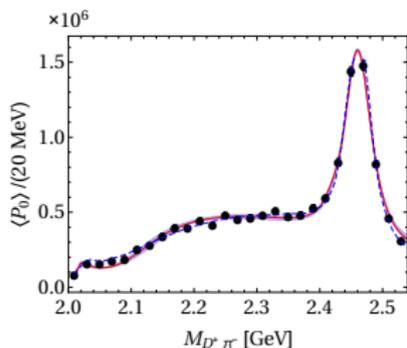
- Large isospin decay width  $\Gamma(D_{s0}^*(2317)^+ \rightarrow D_s^+ \pi^0) \sim 100 \text{ keV}$   
Faessler *et al.* (2007); Lutz, Soyeur (2007); FKG *et al.* (2008); Cleven *et al.* (2014)
- $\Gamma(D_{s0}^*(2317)) = (133 \pm 22) \text{ keV}$  L. Liu *et al.*, PRD86(2013)014508
- Recent result with terms up to  $O(p^4)$  in chiral expansion  
X.-Y. Guo, Y. Heo, M. F. M. Lutz, arXiv:1801.10122
  - 👉 LECs from fitting to the lattice results of masses and phase shifts
  - 👉  $\Rightarrow \Gamma(D_{s0}^*(2317)) = (110 \pm 6) \text{ keV}$
- To be measured at  $\bar{\text{P}}\text{ANDA}$



$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2,$$

$$\langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2| \cos(\delta_2 - \delta_0),$$

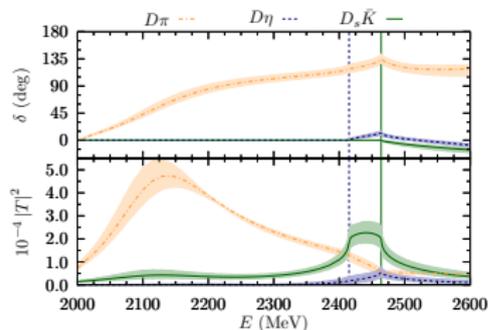
$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1| \cos(\delta_1 - \delta_0)$$



- The *S*-wave  $D\pi$  well described using our amplitudes with pre-fixed LECs (the same as before)
- **Fast variation** in [2.4, 2.5] GeV in  $\langle P_{13} \rangle$ : cusps at  $D\eta$  and  $D_s\bar{K}$  thresholds

# Searching for the higher nonstrange state

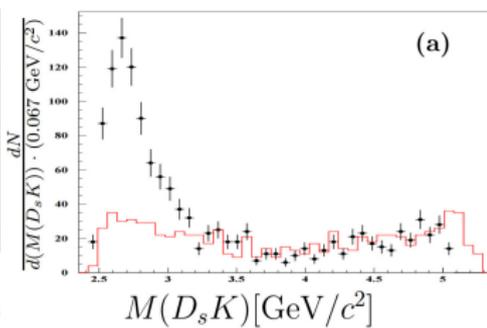
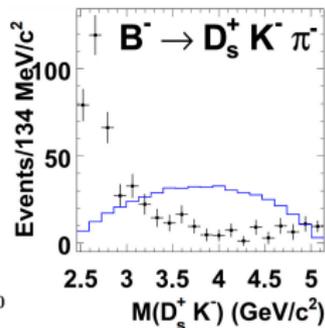
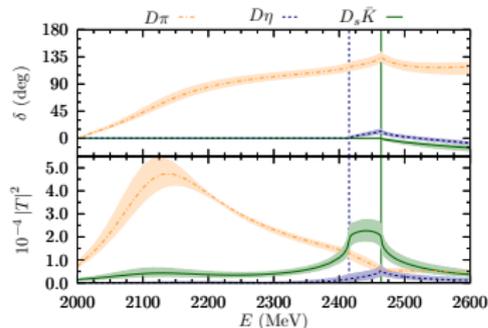
- Near-threshold enhancement in  $D_s \bar{K}$ ?



- Lattice QCD calculation with a SU(3) symmetric large quark mass:

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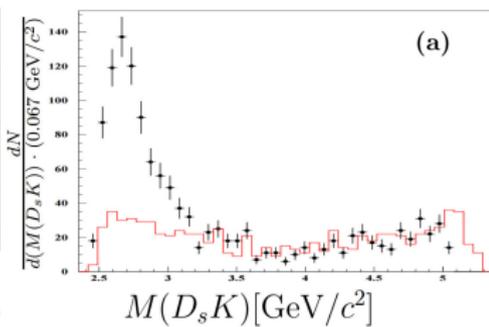
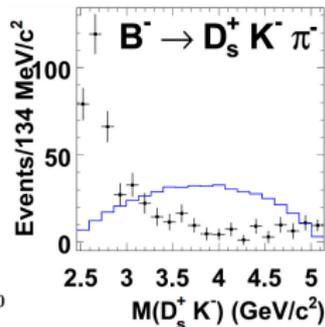
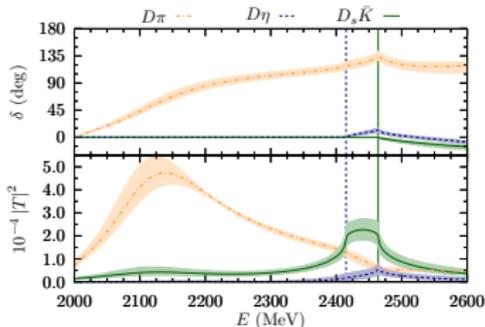
BaBar, PRL100(2008)171803;

Belle, PRD80(2009)052005

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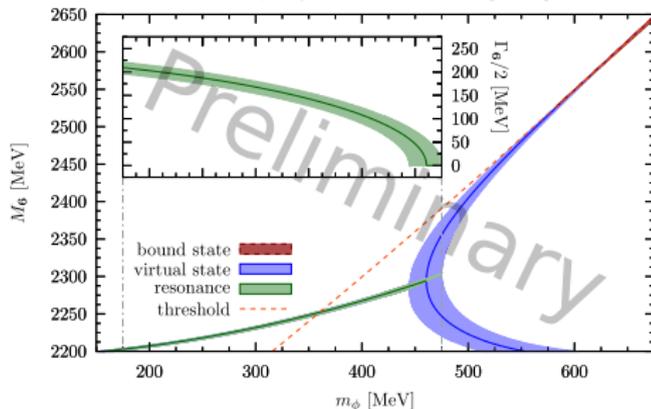
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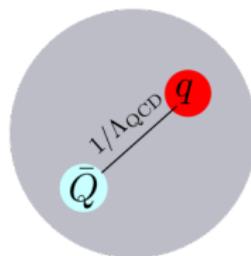
# Doubly charmed baryons

- Heavy anti-quark–diquark symmetry (HADS):

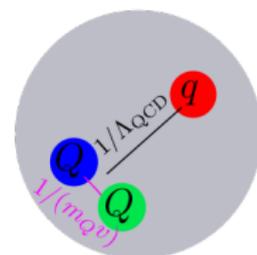
$$m_Q v \gg \Lambda_{\text{QCD}},$$

the diquark serves as a point-like color- $\bar{3}$  source, like a heavy anti-quark.

doubly-heavy baryons  $\Leftrightarrow$  anti-heavy mesons



Savage, Wise (1990)



- HADS + CHPT with virtual photons: Brodsky, FKG, Hanhart, Meißner, PLB698(2011)251

$$M_{D^+} - M_{D^0} \Rightarrow M_{\Xi_{cc}^{++}} - M_{\Xi_{cc}^+} = (1.5 \pm 2.7) \text{ MeV}$$

- LHCb observation of  $\Xi_{cc}^{++}$ :  $M = (3621.40 \pm 0.78) \text{ MeV}$  LHCb, PRL119(2017)112001

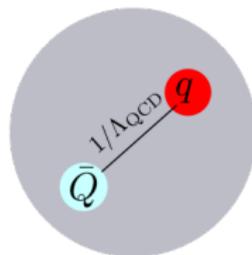
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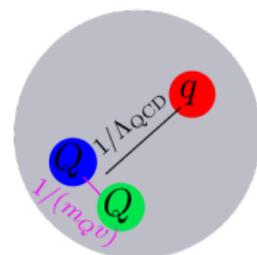
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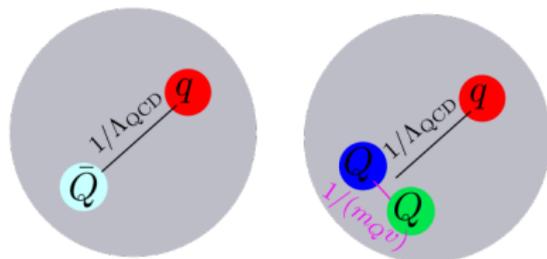
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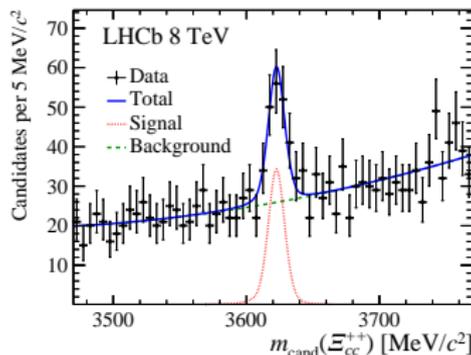
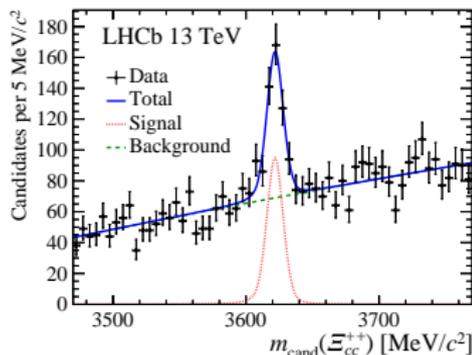


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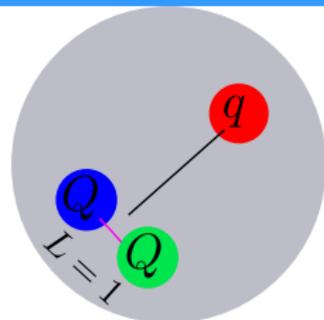
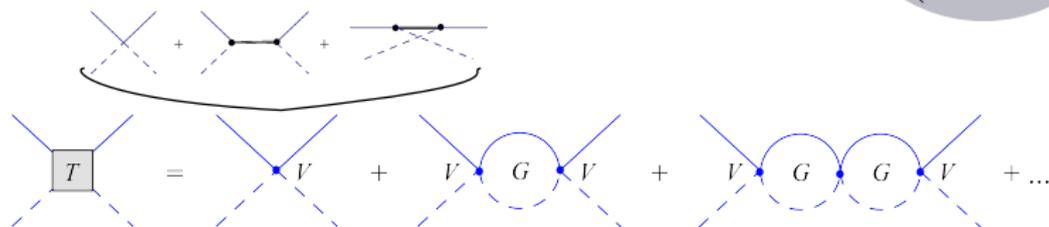
- $P$ -wave  $QQ$

excitation energy

Mehen, PRD96(2017)094028

$$\sim \frac{1}{2}(M_{h_c} - M_{J/\psi}) = \mathcal{O}(200 \text{ MeV})$$

- $\Rightarrow \Xi_{cc}^P, \Omega_{cc}^P$  as dynamical degrees of freedom



- $S$ -wave  $QQ$ : spin  $s_{QQ} = 1$ ,  $P$ -wave  $QQ$ : spin  $s_{QQ} = 0$

A diagram showing the decay of a doubly charmed baryon. A red dot represents the vertex where a doubly charmed baryon ( $\Xi_{cc}^P$ ) decays into a singly charmed baryon ( $\Xi_{cc}$ ) and a pion ( $\pi$ ). The decay is labeled with  $\lambda$  and is given by the equation:

$$\lambda = \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right)$$

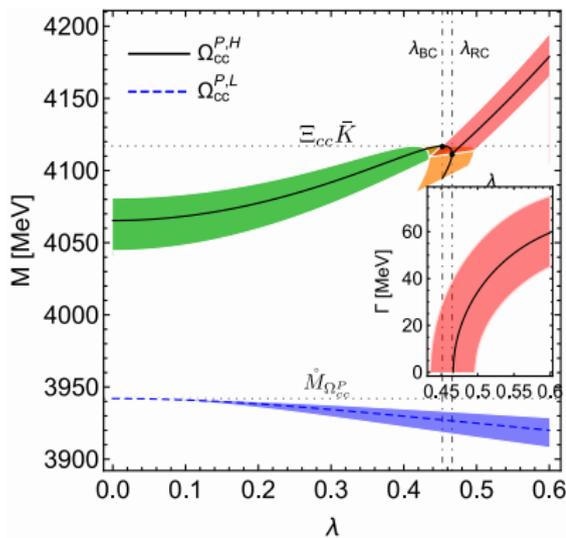
# Doubly charmed strange baryons with $J^P = 1/2^-$ : $\Omega_{cc}^P$

M.-J. Yan, X.-H. Liu, S. González-Solís, FKG et al., arXiv:1805.1xxxx

- Very likely two states below the  $\Xi_{cc}\bar{K}$  threshold

Inputs: bare  $\dot{M}_{\Xi_{cc}^P} = 3838$  MeV from quark model [D. Ebert et al., PRD96\(2002\)024008](#)

$$M_{\Omega_{cc}} - M_{\Xi_{cc}} = M_{D_s} - M_D, \quad \dot{M}_{\Omega_{cc}^P} - M_{\Omega_{cc}} = 217 \text{ MeV}$$



- Tiny width due to isospin breaking:

$$\Omega_{cc}^P \rightarrow \Omega_{cc}\pi^0$$

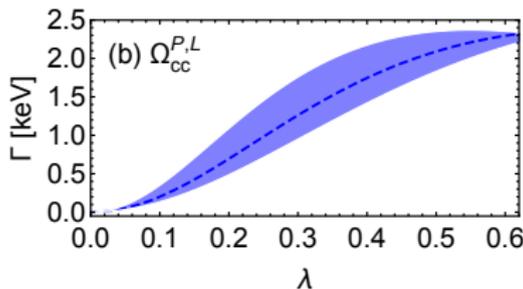
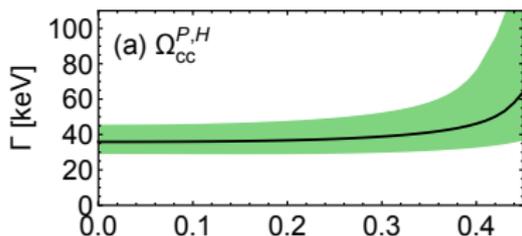
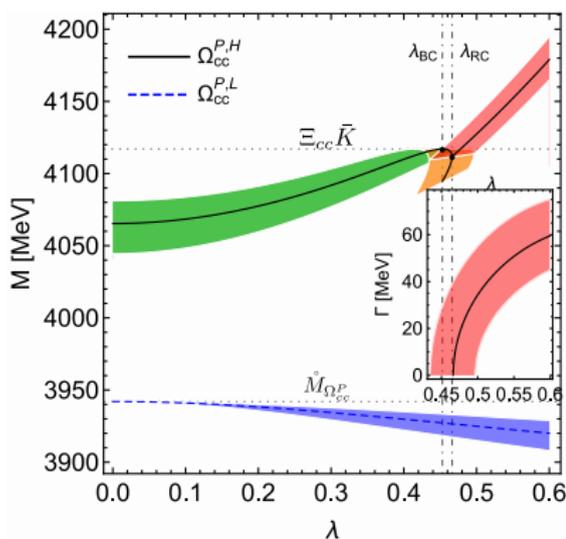
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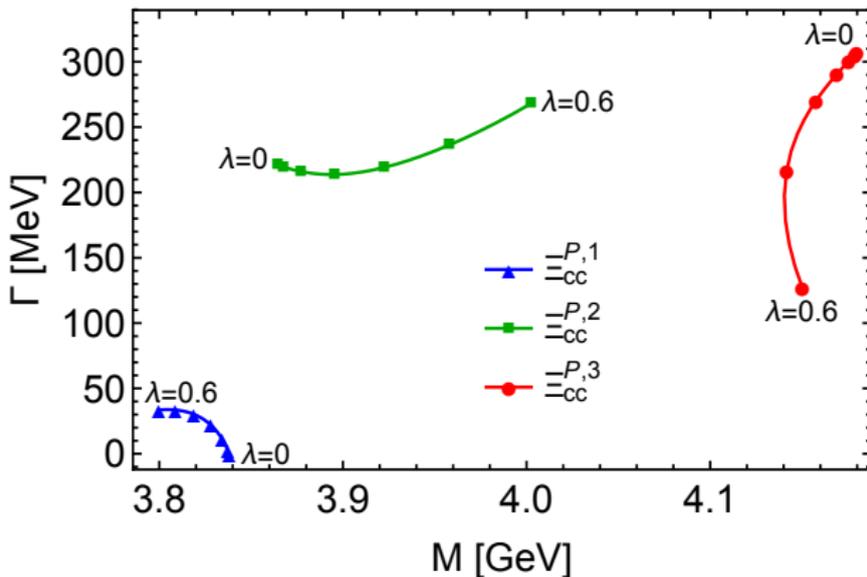
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# Doubly charmed nonstrange baryons with $J^P = 1/2^-$ : $\Xi_{cc}^P$

M.-J. Yan, X.-H. Liu, S. González-Solís, FKG et al., arXiv:1805.1xxxx

- Three  $\frac{1}{2}^-$   $\Xi_{cc}^P$  states below 4.2 GeV:

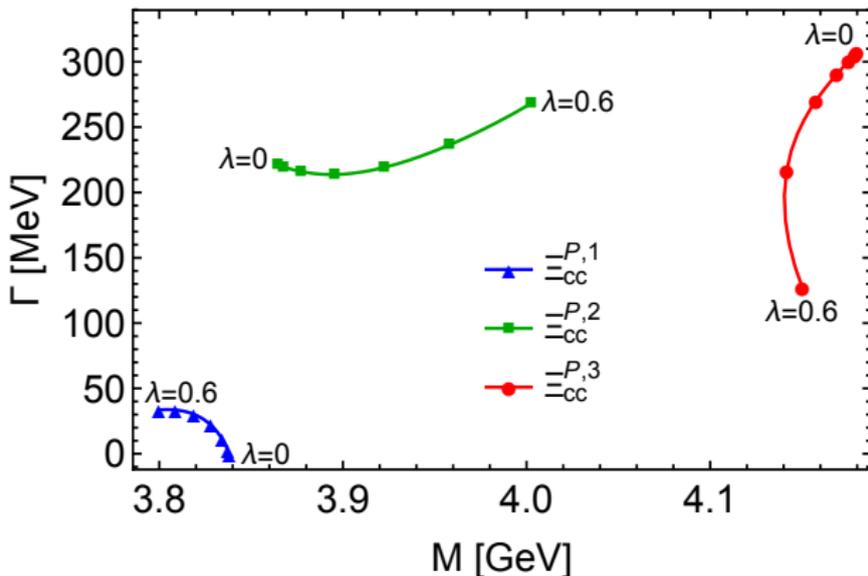


- $\mathcal{B}(\Xi_{cc}^{P,1}, \Xi_{cc}^{P,2} \rightarrow \Xi_{cc}\pi) \simeq 100\%$ , searching for  $\Xi_{cc}^{P,1}$  in  $\Xi_{cc}^{++}\pi^- \Rightarrow \lambda$

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applicable to any hadrons with **a small width  $\Gamma \ll$  inverse of force range**
- nice candidates:  $D_1(2420)$  &  $D_2(2460)$ ,  $\Gamma \sim 30$  MeV  
more speculative (using the same subtraction constant) predictions of  
 $D_1(2420)K$  and  $D_2(2460)K$  bound states

Constituents	$D_1(2420)K$	$D_2(2460)K$	$\bar{B}_1(5720)K$	$\bar{B}_2(5747)K$
$J^P$	$1^-$	$2^-$	$1^-$	$2^-$
Predictions	$2870 \pm 9$	$2910 \pm 9$	$6151 \pm 33$	$6169 \pm 33$
Decays	$D^{(*)}K, D_s^{(*)}\eta$	$D^*K, D_s^*\eta$	$\bar{B}^{(*)}K, \bar{B}_s^{(*)}\eta$	$\bar{B}^*K, \bar{B}_s^*\eta$

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# What is $D_s^*(2860)$ ?

- $D_{s1}^*(2860)$ : puzzling decay pattern:  $\Gamma(D^*K)/\Gamma(DK) = 1.10 \pm 0.24$

Predictions from HQSS:

P.Colangelo et al., PRD77(2008)014012

$D_{sJ}(2860)$	$D_{sJ}(2860) \rightarrow DK$	$\frac{\Gamma(D_{sJ} \rightarrow D^*K)}{\Gamma(D_{sJ} \rightarrow DK)}$
$s_\ell^P = \frac{1}{2}^-, J^P = 1^-, n = 1$	$p$ -wave	1.23
$s_\ell^P = \frac{1}{2}^+, J^P = 0^+, n = 1$	$s$ -wave	0
$s_\ell^P = \frac{3}{2}^+, J^P = 2^+, n = 1$	$d$ -wave	0.63
$s_\ell^P = \frac{3}{2}^-, J^P = 1^-, n = 0$	$p$ -wave	0.06
$s_\ell^P = \frac{5}{2}^-, J^P = 3^-, n = 0$	$f$ -wave	0.39

but, better candidate for  $(2S, 1^-)$ :  $D_{s1}^*(2700)$   $\Gamma(D^*K)/\Gamma(DK) = 0.91 \pm 0.18$

$$M(2P, 2^+) \sim 3.16 \text{ GeV}$$

M. Di Piero, E. Eichten, PRD64(2001)114004

- A natural explanation of the decay pattern:

$$: \frac{\Gamma(D_{s1}^*(2860) \rightarrow D^*K)}{DK} \simeq 2 \frac{M_{D^*}}{M_D} \left| \frac{\vec{k}_{D^*}}{\vec{k}_D} \right|^3 = 1.23$$

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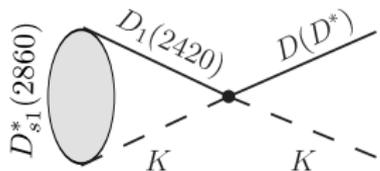
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- Parameter-free predictions agree with lattice energy levels;  $D\pi$   $S$ -wave amplitude consistent with precise LHCb data
- **Puzzles of positive-parity charmed mesons naturally understood** in the hadronic molecular picture
- New spectrum of  $J^P = 1/2^-$  doubly charmed baryons; searching for  $\Xi_{cc}^{\prime}$  with a mass about 3.82 GeV in  $\Xi_{cc}^{\prime} \rightarrow \Sigma_{cc}^{\prime} \pi^{\pm}$
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Experiments

Lattice

Thank you for your attention !

EFT, models

# Lattice studies of the charmed scalar mesons: strange

- Early studies using only  $c\bar{s}$ -type interpolators typically give mass larger than that for  $D_{s0}^*$  (2317) Bali (2003); UKQCD (2003); ...

- $c\bar{s} + DK$  interpolators:  $\sim$ right mass

Mohler et al., PRL111(2013)222001

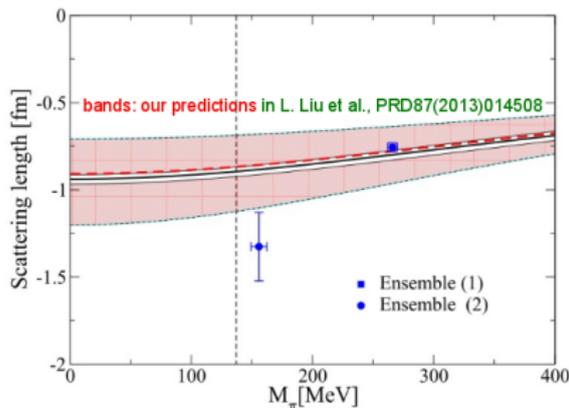
$$M_{D_{s0}^*} - \frac{1}{4} (M_{D_s} + 3M_{D_s^*}):$$

Mohler et al.

PDG2017

$(266 \pm 16)$  MeV

$(241.5 \pm 0.8)$  MeV



- New calculation with  $M_\pi = 150$  MeV

Bali et al. [RQCD Col.], PRD96(2017)074501

	Energy [MeV]	Expt [MeV]
$m_{0-}$	1976.9(2)	1966.0(4)
$m_{1-}$	2094.9(7)	2111.3(6)
$m_{0+}$	2348(4)(+6)	2317.7(0.6)(2.0)
$m_{1+}$	2451(4)(+1)	2459.5(0.6)(2.0)

# Lattice studies of the charmed scalar mesons: nonstrange (1)

- $(S, I) = (0, \frac{1}{2}): c\bar{q} + D\pi$

interpolators:

Mohler et al., PRD87(2013)034501

$$M_\pi \approx 266 \text{ MeV},$$

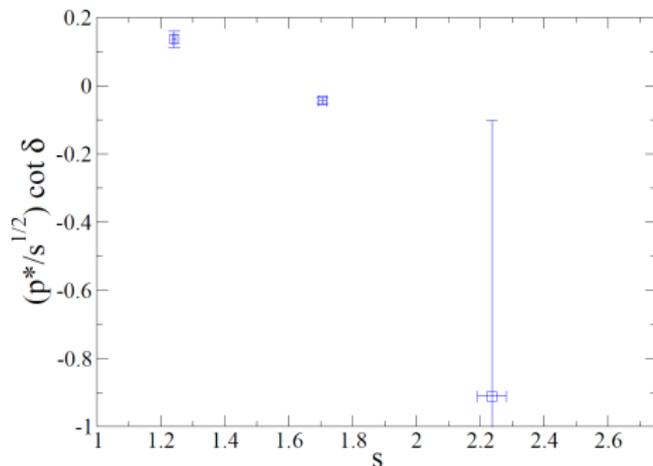
$$M_D \approx 1558 \text{ MeV},$$

$$M_{D^*} \approx 1690 \text{ MeV}$$

Lüscher's formula  $\Rightarrow D\pi$  phase

shifts

$\Rightarrow$  BW parameters of  $D_0^*(2400)$  consistent with PDG values



	Mohler et al.	PDG2017
$M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$	$(351 \pm 21) \text{ MeV}$	$(347 \pm 29) \text{ MeV}$
$M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$	$(380 \pm 21) \text{ MeV}$	$(456 \pm 40) \text{ MeV}$

## Lattice studies of the charmed scalar mesons: nonstrange (2)

- $(S, I) = (0, \frac{1}{2})$ : first coupled-channel lattice calculation including interpolating fields for  $c\bar{q} + D\pi + D\eta + D_s\bar{K}$ : Moir et al. [Hadron Spectrum Col.], JHEP1610(2016)011
- $M_\pi = 391$  MeV,  $M_D = 1885$  MeV:  $D\pi$  threshold  $(2276.4 \pm 0.9)$  MeV
- for coupled channels:  
parametrizing the  $T$ -matrix with the  $K$ -matrix formalism

$$T_{ij}^{-1}(s) = K_{ij}^{-1}(s) + I_{ij}(s)$$

$I_{ij}(s)$ : 2-point loop function evaluated with a subtracted dispersion integral

$K_{ij}(s)$ : different forms of the  $K$ -matrix were used, summarized as

$$K_{ij}(s) = \left(g_i^{(0)} + g_i^{(1)}s\right) \left(g_j^{(0)} + g_j^{(1)}s\right) \frac{1}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)}s$$

- $\Rightarrow$  a pole below threshold  $(2275.9 \pm 0.9)$  MeV. relation to  $D_0^*(2400)$ ?

# HQS for $D_{s0}^*$ (2317) and $D_{s1}$ (2460)

- Heavy quark flavor symmetry:

$$\text{for a singly-heavy hadron, } M_{HQ} = m_Q + A + \mathcal{O}(m_Q^{-1})$$

- rough estimates of bottom analogues **whatever the  $D_{sJ}$  states are**

$$M_{B_{s0}^*} = M_{D_{s0}^*(2317)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.65 \pm 0.15) \text{ GeV}$$

$$M_{B_{s1}} = M_{D_{s1}(2460)} + \Delta_{b-c} + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 \left(\frac{1}{m_c} - \frac{1}{m_b}\right)\right) \simeq (5.79 \pm 0.15) \text{ GeV}$$

here  $\Delta_{b-c} \equiv m_b - m_c \simeq \overline{M}_{B_s} - \overline{M}_{D_s} \simeq 3.33 \text{ GeV}$ , where

$\overline{M}_{B_s} = 5.403 \text{ GeV}$ ,  $\overline{M}_{D_s} = 2.076 \text{ GeV}$ : spin-averaged g.s.  $Q\bar{s}$  meson masses

 both to be discovered <sup>1</sup>

- more precise predictions can be made in a given model, e.g. **hadronic molecules**

---

<sup>1</sup>The established meson  $B_{s1}(5830)$  is probably the bottom partner of  $D_{s1}(2536)$ .

- Heavy quark flavor symmetry (HQFS) for any hadron containing **one** heavy quark: velocity remains unchanged in the limit  $m_Q \rightarrow \infty$ :  $\Delta v = \frac{\Delta p}{m_Q} = \frac{\Lambda_{\text{QCD}}}{m_Q}$   
 $\Rightarrow$  heavy quark is like a **static** color triplet source,  $m_Q$  is irrelevant
- Predicting the bottom-partner masses in 1 minute:

$$M_{B_{s0}^*} \simeq M_B + M_K - 45 \text{ MeV} \simeq 5.730 \text{ GeV}$$

$$M_{B_{s1}} \simeq M_{B^*} + M_K - 45 \text{ MeV} \simeq 5.776 \text{ GeV}$$

nice agreement with lattice results: Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

$$M_{B_{s0}^*}^{\text{lat.}} = (5.711 \pm 0.013 \pm 0.019) \text{ GeV}$$

$$M_{B_{s1}}^{\text{lat.}} = (5.750 \pm 0.017 \pm 0.019) \text{ GeV}$$

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- The leading order Lagrangian:

$$\mathcal{L}_{\phi P}^{(1)} = D_\mu P D^\mu P^\dagger - m^2 P P^\dagger$$

with  $P = (D^0, D^+, D_s^+)$  denoting the  $D$ -mesons, and the covariant derivative being

$$D_\mu P = \partial_\mu P + P \Gamma_\mu^\dagger, \quad D_\mu P^\dagger = (\partial_\mu + \Gamma_\mu) P^\dagger,$$
$$\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger),$$

where  $u_\mu = i [u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger]$ ,  $u = e^{i\lambda_a \phi_a / (2F_0)}$

Burdman, Donoghue (1992); Wise (1992); Yan et al. (1992)

- this gives the [Weinberg–Tomozawa term](#) for  $P\phi$  scattering

- At the next-to-leading order  $\mathcal{O}(p^2)$ : FKG, Hanhart, Krewald, Meißner, PLB666(2008)251

$$\mathcal{L}_{\phi P}^{(2)} = P [-h_0 \langle \chi_+ \rangle - h_1 \chi_+ + h_2 \langle u_\mu u^\mu \rangle - h_3 u_\mu u^\mu] P^\dagger + D_\mu P [h_4 \langle u_\mu u^\nu \rangle - h_5 \{u^\mu, u^\nu\}] D_\nu P^\dagger,$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B_0 \text{diag}(m_u, m_d, m_s)$$

- LECs:  $h_{1,3,5} = \mathcal{O}(N_c^0)$ ,  $h_{2,4,6} = \mathcal{O}(N_c^{-1})$

$$M_{D_s} - M_D \Rightarrow h_1 = 0.42$$

$h_0$ : can be fixed from lattice results of charmed meson masses

$h_{2,3,4,5}$ : to be fixed from lattice results on scattering lengths

- Extensions to  $\mathcal{O}(p^3)$ , see Y.-R. Liu, X. Liu, S.-L. Zhu, PRD79(2009)094026; L.-S. Geng et al., PRD82(2010)054022; D.-L. Yao, M.-L. Du, FKG, U.-G. Meißner, JHEP1511(2015)058;

M.-L. Du, FKG, U.-G. Meißner, D.-L. Yao, EPJC77(2017)728

renormalization:

M.-L. Du, FKG, U.-G. Meißner, JPG44(2017)014001

PCB-term subtraction in EOMS scheme using path integral:

M.-L. Du, FKG, U.-G. Meißner, JHEP1610(2016)122

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## Energy levels in a finite volume

- **Goal:** predict **finite volume** (FV) energy levels for  $I = 1/2$ , and compare with recent lattice data by the Hadron Spectrum Col. in **JHEP1610(2016)011**  
 $\Rightarrow$  insights into  $D_0^*(2400)$
- In a FV, momentum gets quantized:  $\vec{q} = \frac{2\pi}{L}\vec{n}$ ,  $\vec{n} \in \mathbb{Z}^3$
- Loop integral  $G(s)$  gets modified:  $\int d^3\vec{q} \rightarrow \frac{1}{L^3} \sum_{\vec{q}}$ , and one gets

M. Döring, U.-G. Meißner, E. Oset, A. Rusetsky, EPJA47(2011)139

$$\tilde{G}(s, L) = G(s) + \lim_{\Lambda \rightarrow +\infty} \underbrace{\left[ \frac{1}{L^3} \sum_{\vec{n}}^{|\vec{q}| < \Lambda} I(\vec{q}) - \int_0^\Lambda \frac{q^2 dq}{2\pi^2} I(\vec{q}) \right]}_{\text{finite volume effect}}$$

$I(\vec{q})$ : loop integrand

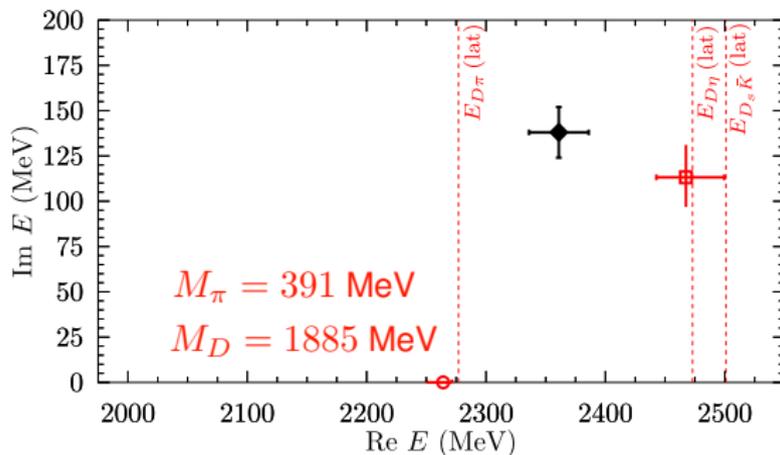
- FV energy levels obtained by as poles of  $\tilde{T}(s, L)$ :

$$\tilde{T}^{-1}(s, L) = V^{-1}(s) - \tilde{G}(s, L)$$

## Two $I = 1/2$ states!

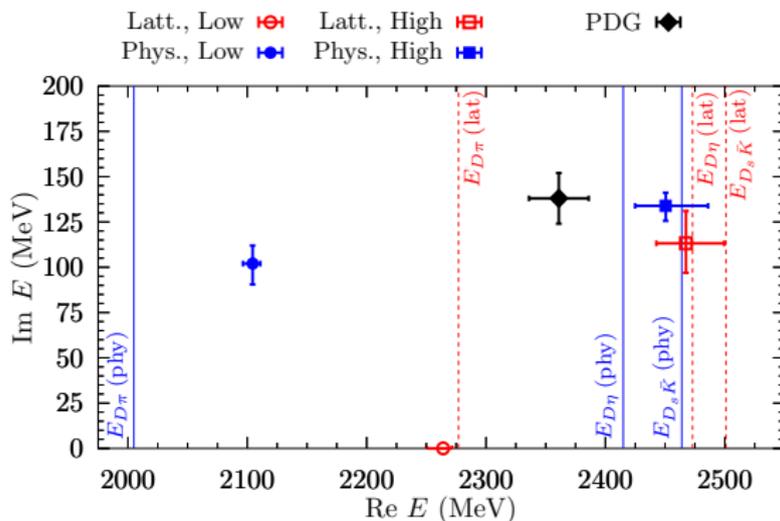
Masses	$M$ (MeV)	$\Gamma/2$ (MeV)	RS	$ g_{D\pi} $	$ g_{D\eta} $	$ g_{D_s \bar{K}} $
lattice	$2264^{+8}_{-14}$	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	$2468^{+32}_{-25}$	$113^{+18}_{-16}$	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$

Latt., Low  $\circ$  Latt., High  $\square$  PDG  $\blacklozenge$

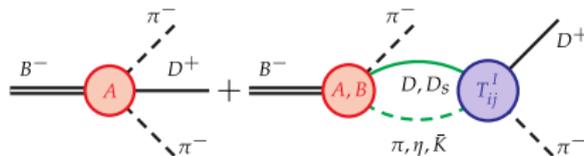


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physical	$2105^{+6}_{-8}$	$102^{+10}_{-11}$	(100)	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	$2451^{+36}_{-26}$	$134^{+7}_{-8}$	(110)	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$



- $B^- \rightarrow D^+ \pi^- \pi^-$  contains **coupled-channel**  $D\pi$  FSI
- consider  $S, P, D$  waves:  $\mathcal{A}(B^- \rightarrow D^+ \pi^- \pi^-) = \mathcal{A}_0(s) + \mathcal{A}_1(s) + \mathcal{A}_2(s)$ 
  - $P$ -wave:  $D^*, D^*(2680)$ ;  $D$ -wave:  $D_2(2460)$  as in the LHCb paper
  - $S$ -wave: use the coupled-channel (1:  $D\pi$ ; 2:  $D\eta$ ; 3:  $D_s \bar{K}$ ) amplitudes with **all parameters fixed before**

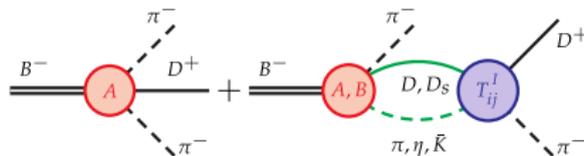


only 2 parameters in  $S$ -wave:  $C$  and a subtraction constant in  $G_i(s)$

$$\text{SU}(3)+\text{chiral} \Rightarrow \mathcal{A}_0(s) \propto E_\pi \left[ 2 + G_{D\pi}(s) \left( \frac{5}{3} T_{11}^{1/2}(s) + \frac{1}{3} T^{3/2}(s) \right) \right] \\ + \frac{1}{3} E_\eta G_{D\eta}(s) T_{21}^{1/2}(s) + \sqrt{\frac{2}{3}} E_{\bar{K}} G_{D_s \bar{K}}(s) T_{31}^{1/2}(s) \\ + C E_\eta G_{D\eta}(s) T_{21}^{1/2},$$

$$\text{Im } G_i(s) = -\rho_i(s) \Rightarrow \text{Unitarity: } \text{Im } \mathcal{A}_{0,i}(s) = -\sum_j T_{ij}^*(s) \rho_j(s) \mathcal{A}_{0,j}(s)$$

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