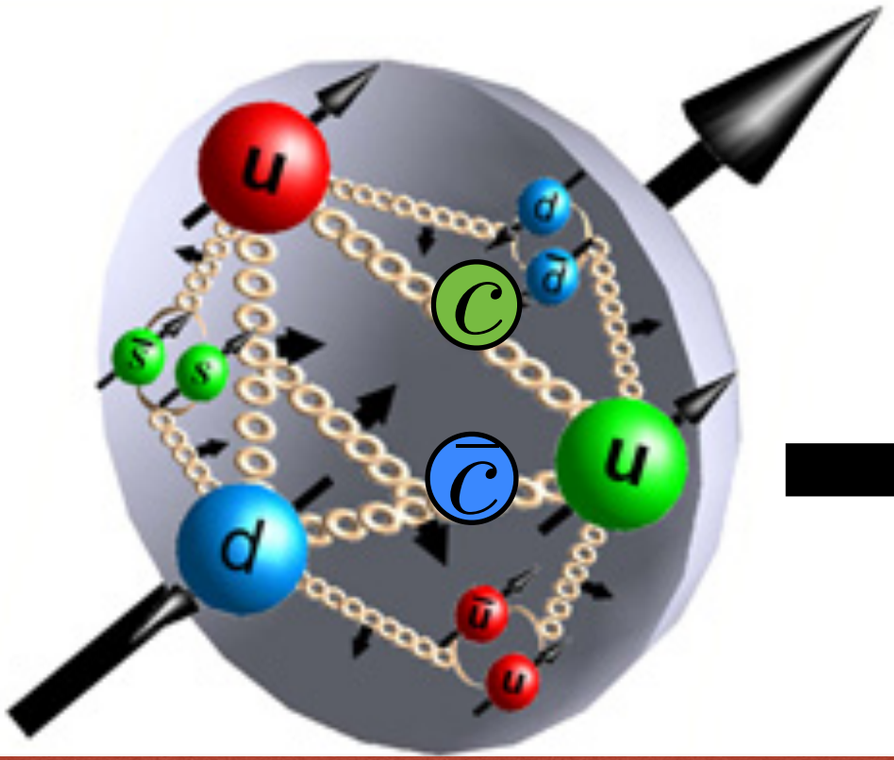
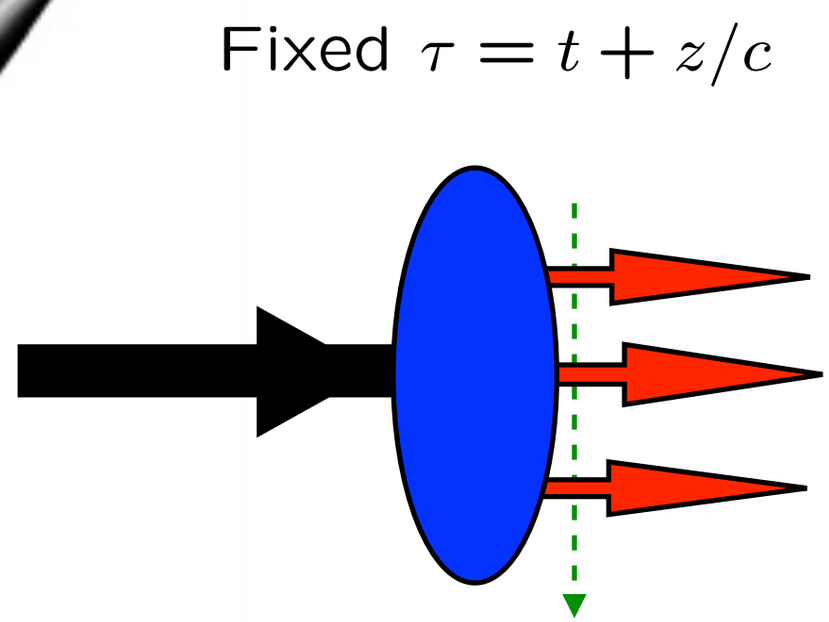
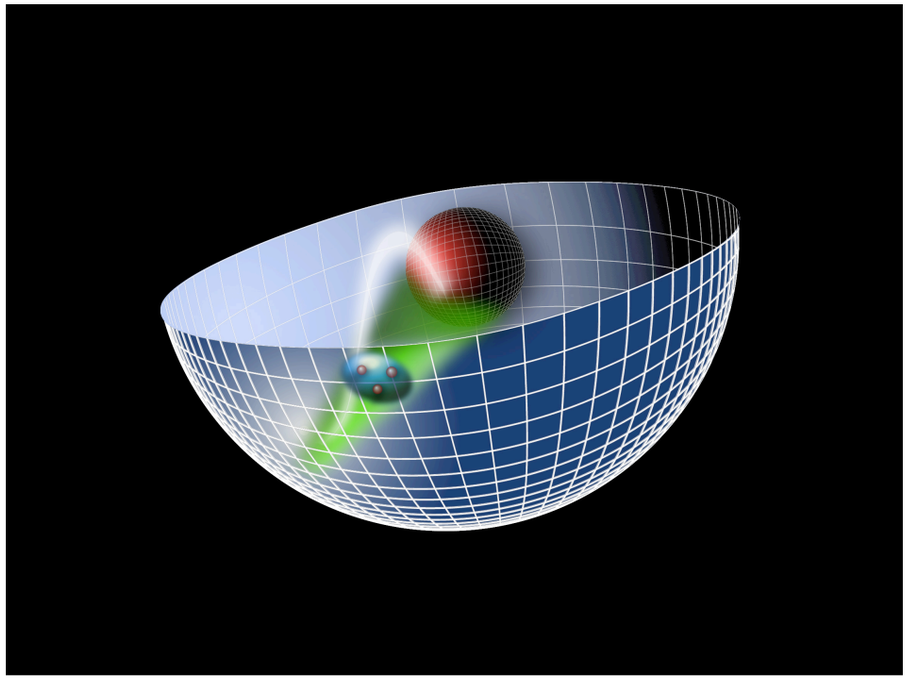
Dynamically-Generated Exotic Hadrons from Superconformal Algebra and Light-Front Holography







Exotic Hadrons and Flavor Physics May 28, 2018

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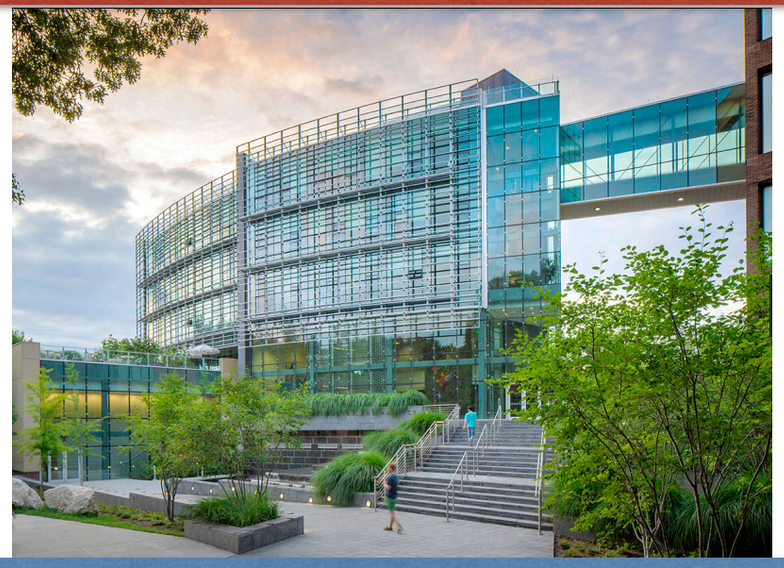


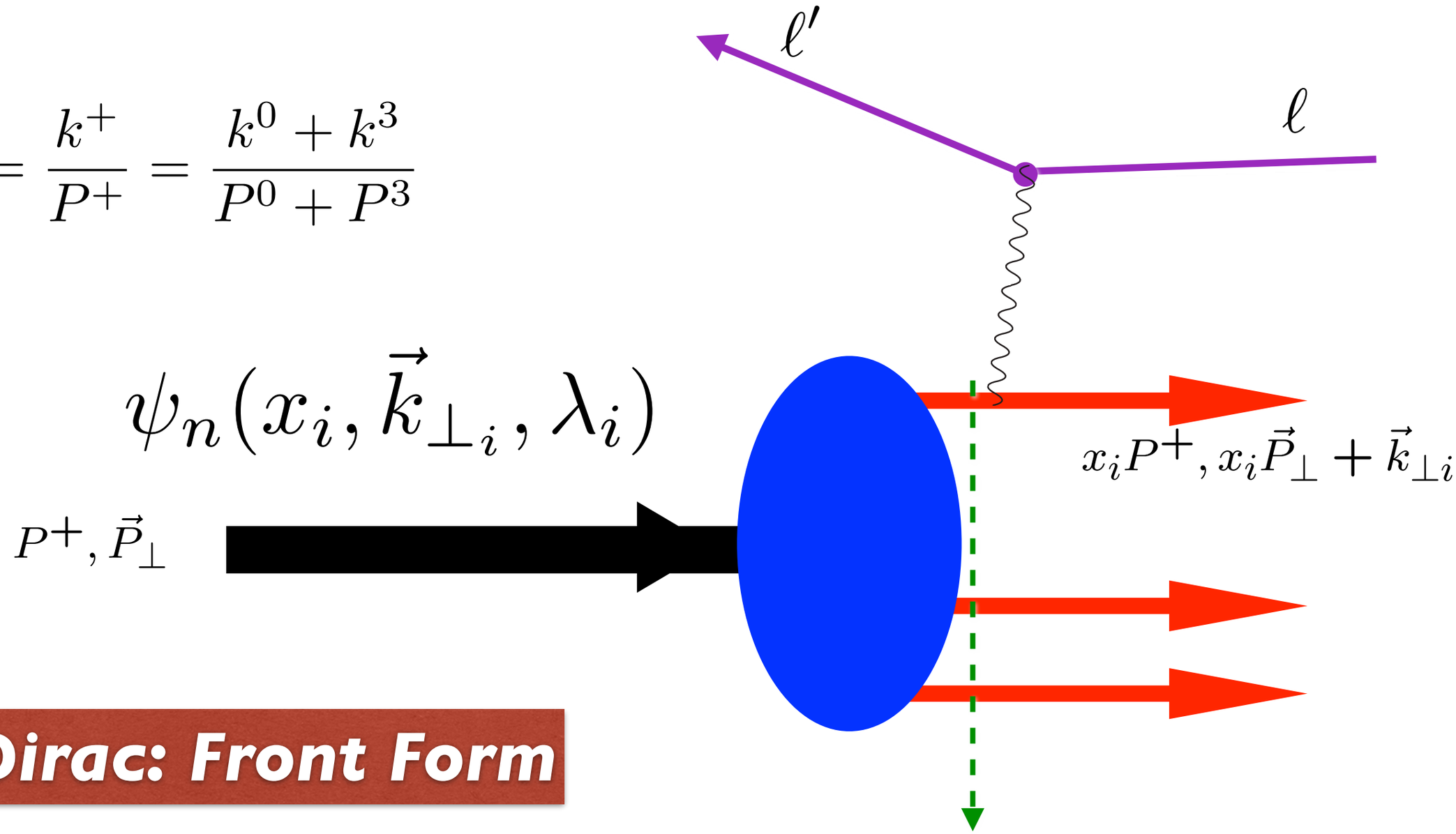
Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY

with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

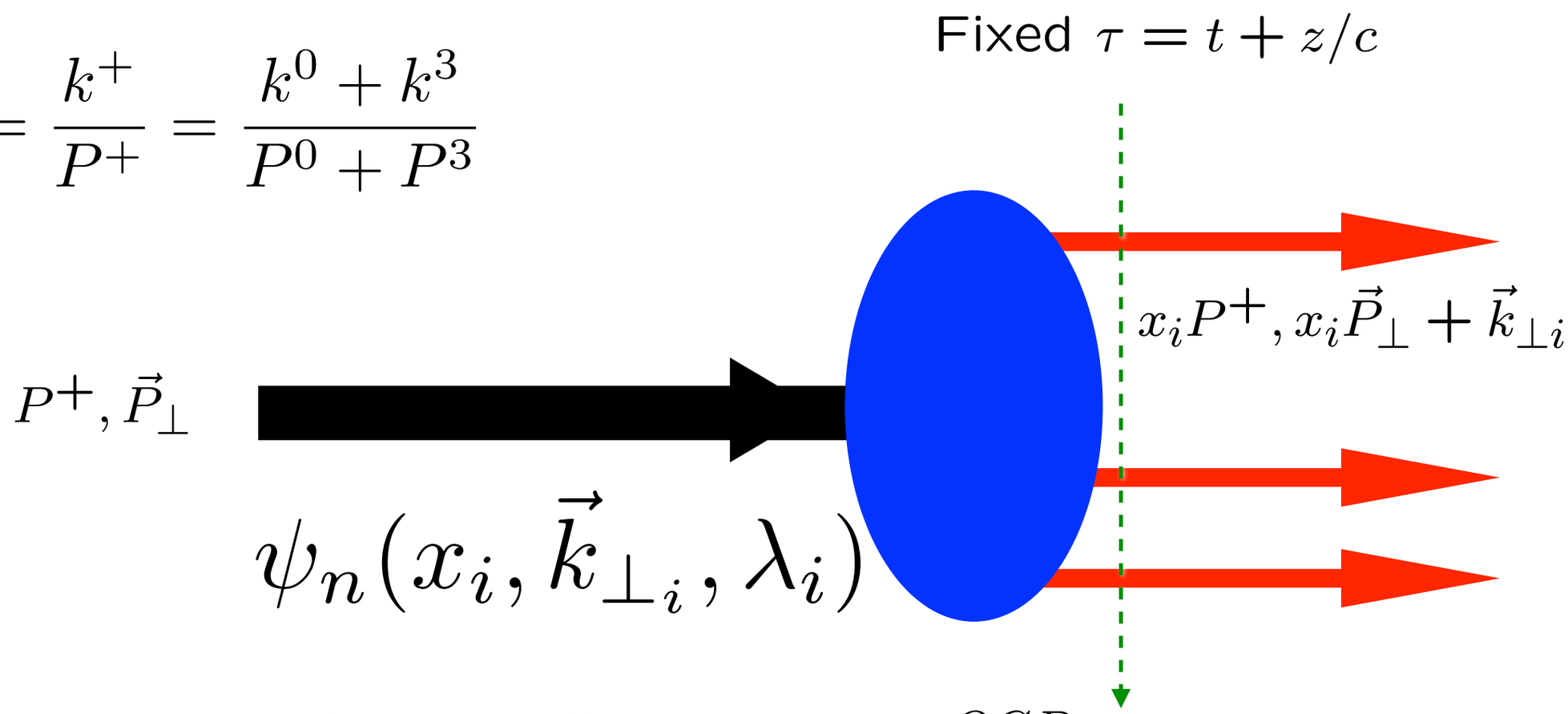
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$$

Eigenstate of LF Hamiltonian

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

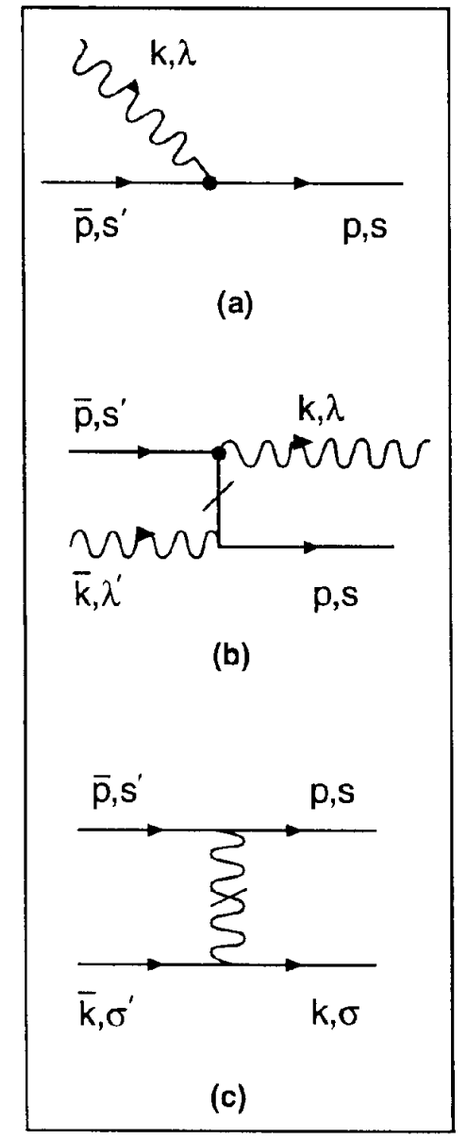
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

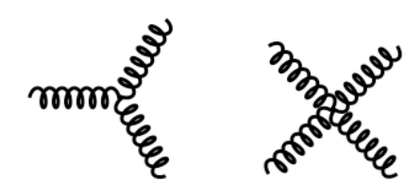
$$H^{QCD}_{LF} |\Psi_h >= \mathcal{M}^2_h |\Psi_h >$$

$$|p, J_z >= \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i >$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

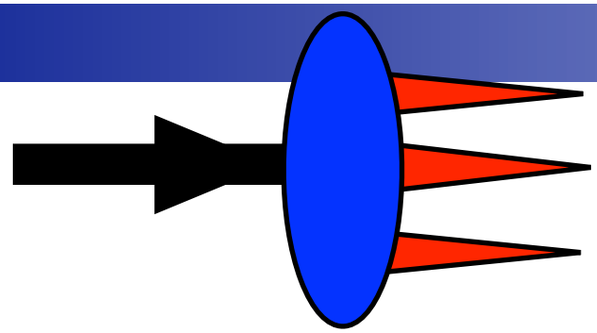
LFWFs: Off-shell in P- and invariant mass



H^{int}_{LF}

Light-Front Wavefunctions underly hadronic observables

Lorce, Pasquini



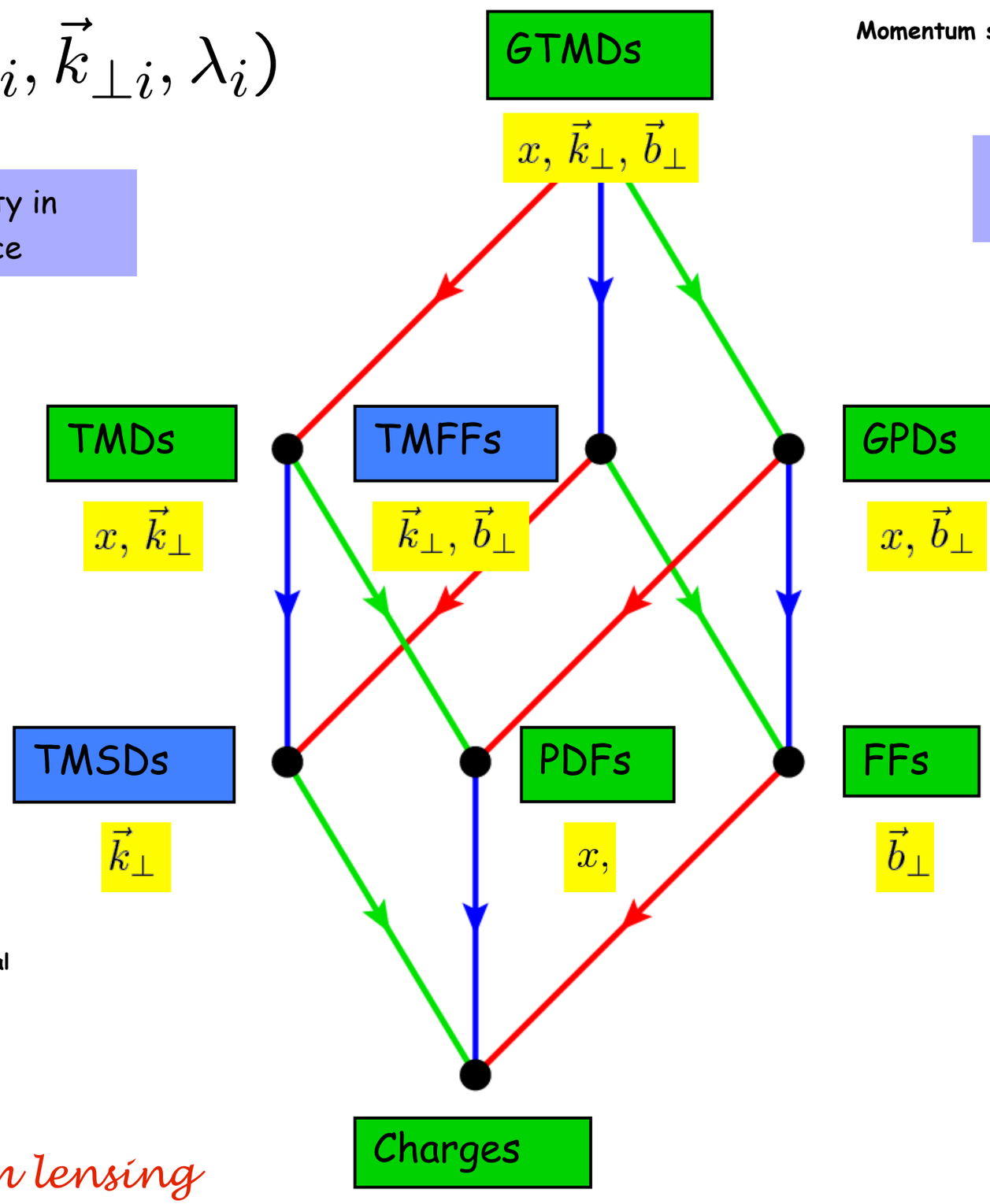
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

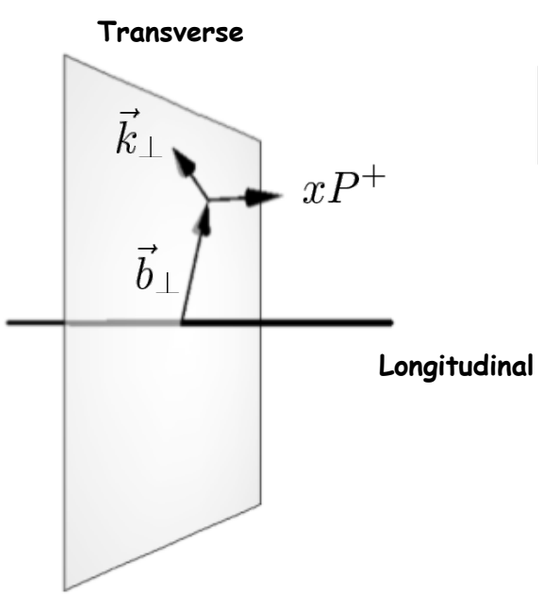
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in momentum space

Transverse density in position space



DGLAP, ERBL Evolution
Factorization Theorems



Sivers, T-odd from lensing

$\int \mathrm{d}^2 b_\perp$

$\int \mathrm{d}x$

$\int \mathrm{d}^2 k_\perp$

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\,|\Psi> = E\,|\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\;\psi(\vec{r}) = E\;\psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\;\psi(r) = E\;\psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

Bohr Spectrum

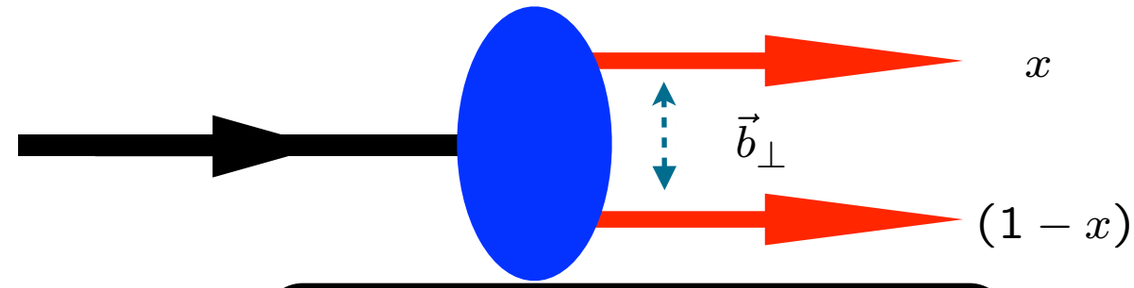
Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$



$$\zeta^2 = x(1-x)b_\perp^2$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x,\vec{k}_\perp) = M^2\ \psi_{LF}(x,\vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$$m_q = 0$$

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

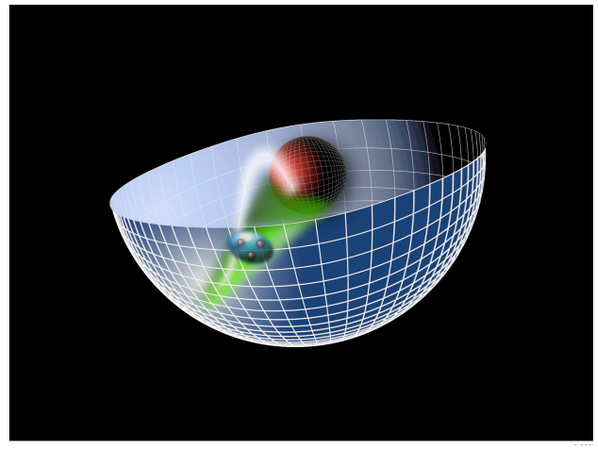
Sums an infinite # diagrams

Semiclassical first approximation to QCD

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$

Light-Front Holography

$$\Big[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable ζ

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

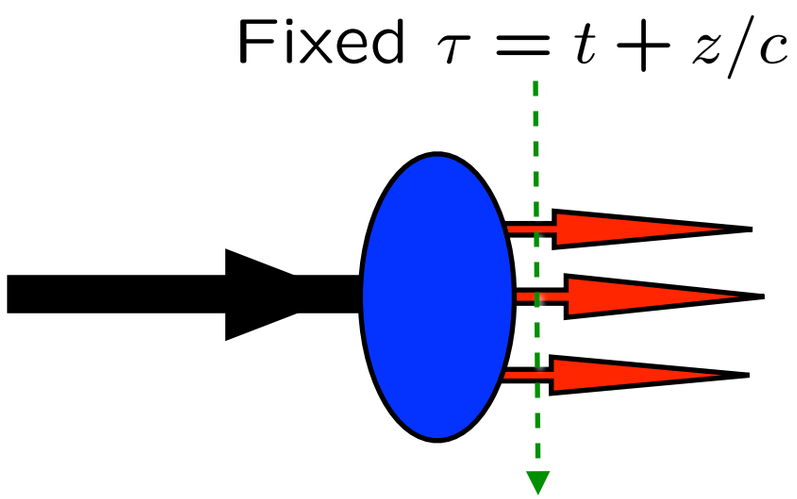
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\boxed{\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>}$$

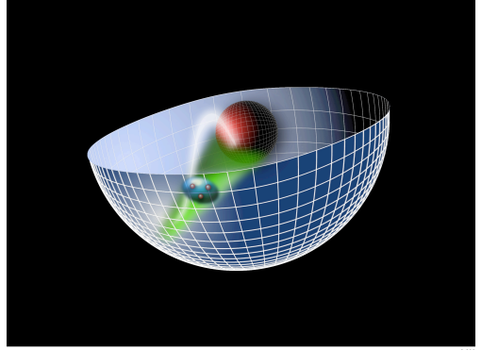
Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Maldacena

AdS_5



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu,\ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

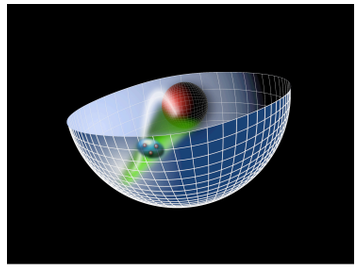
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

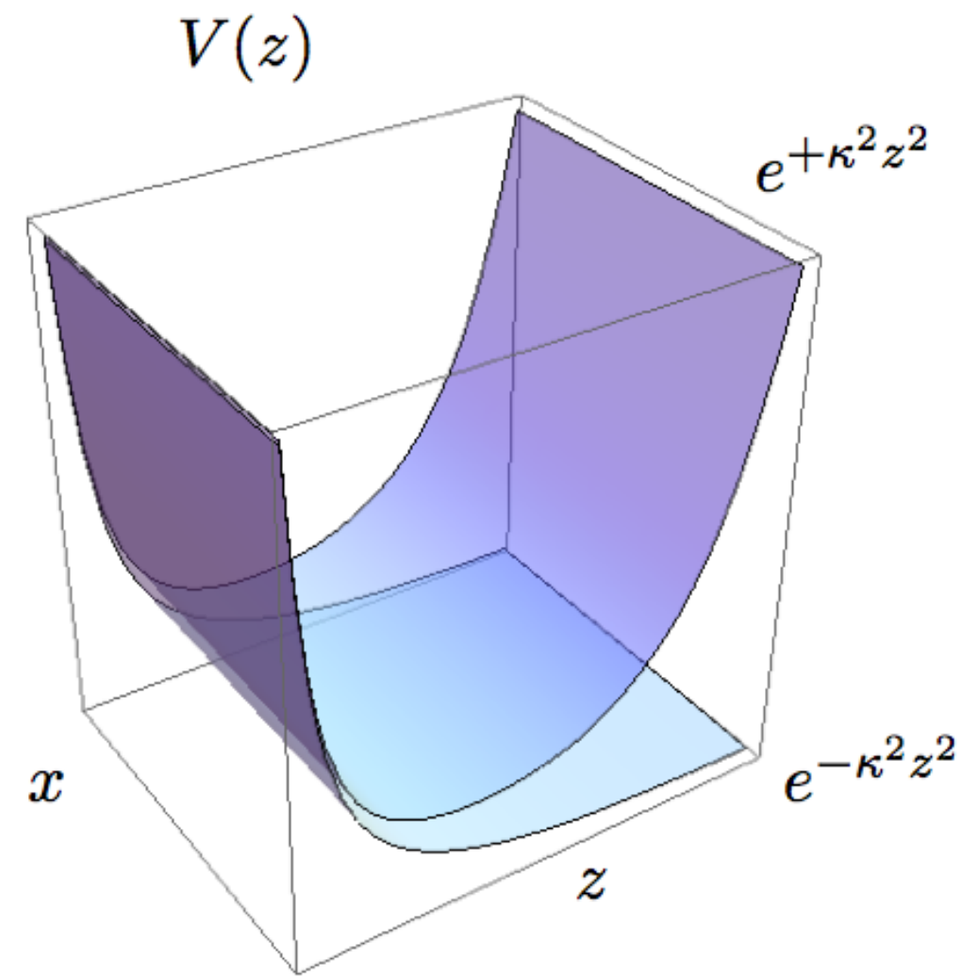
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS$_5$

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\Big[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\Big]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

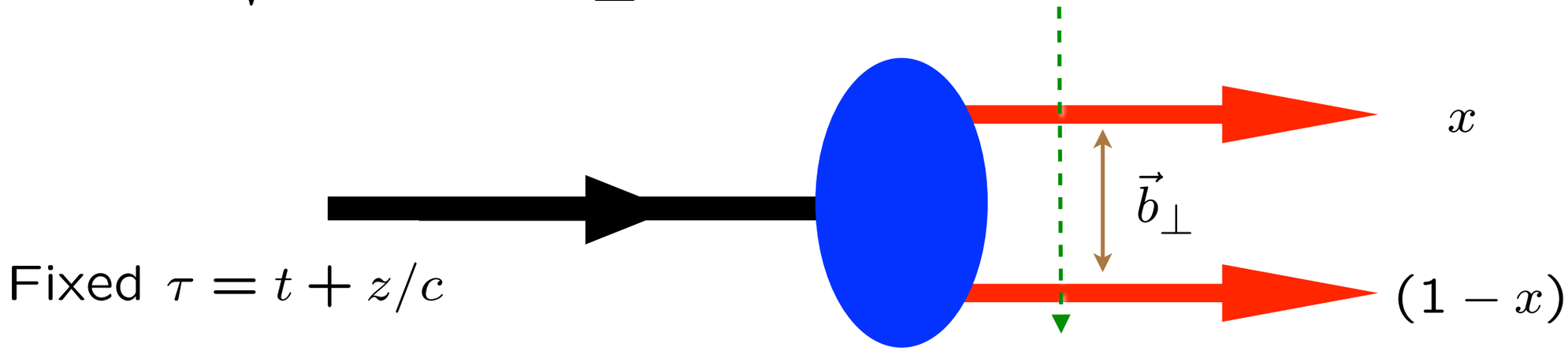
de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Meson Spectrum in Soft Wall Model

Massless pion!

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

$$\vec{\zeta}^2 = \vec{b}_\perp^2\, x(1-x)$$

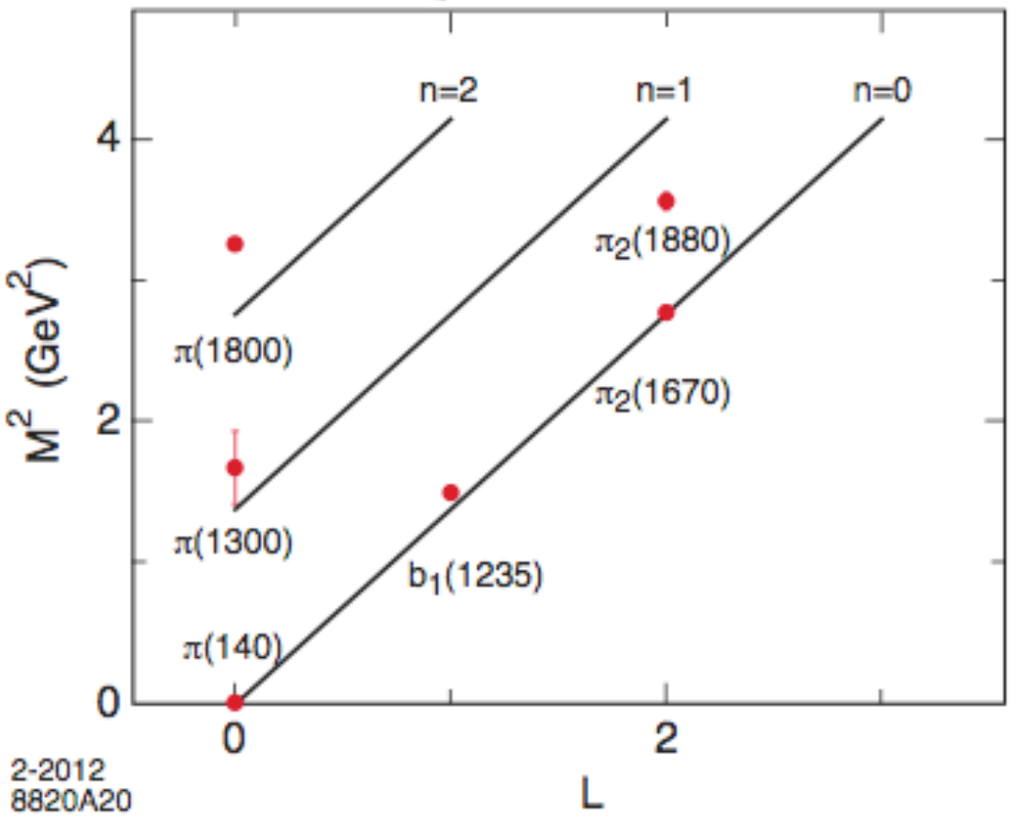
G. de Teramond, H. G. Dosch, sjb

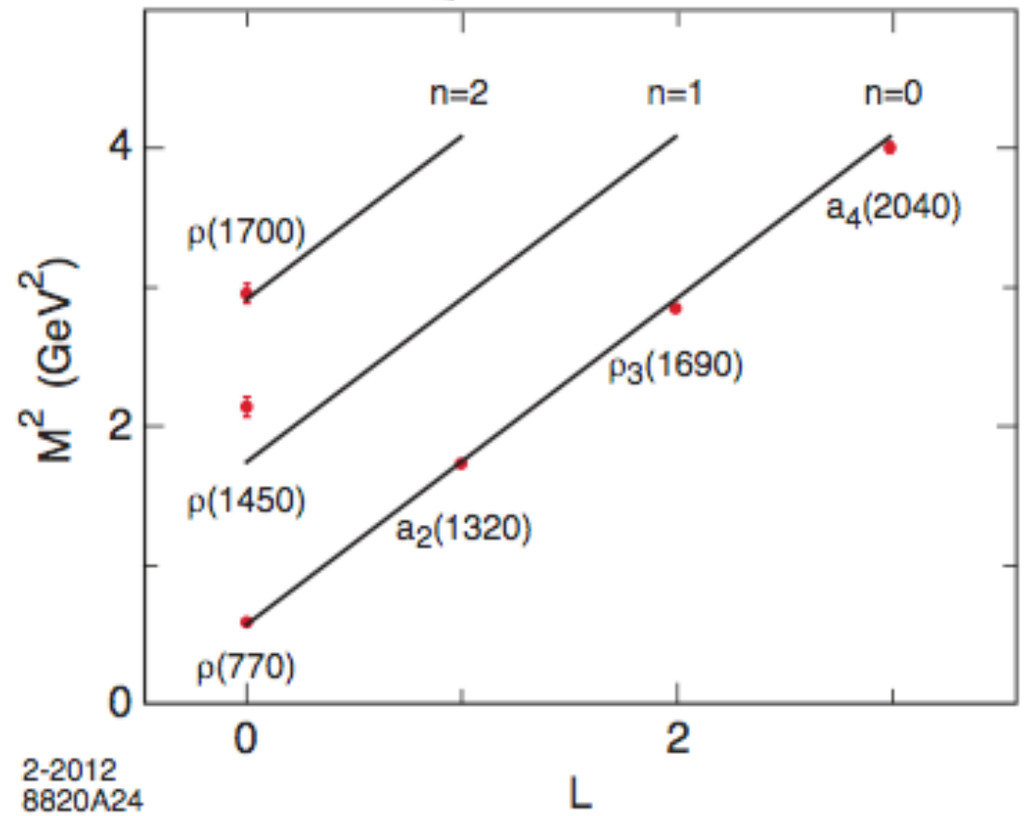
- $J = L + S$, $I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

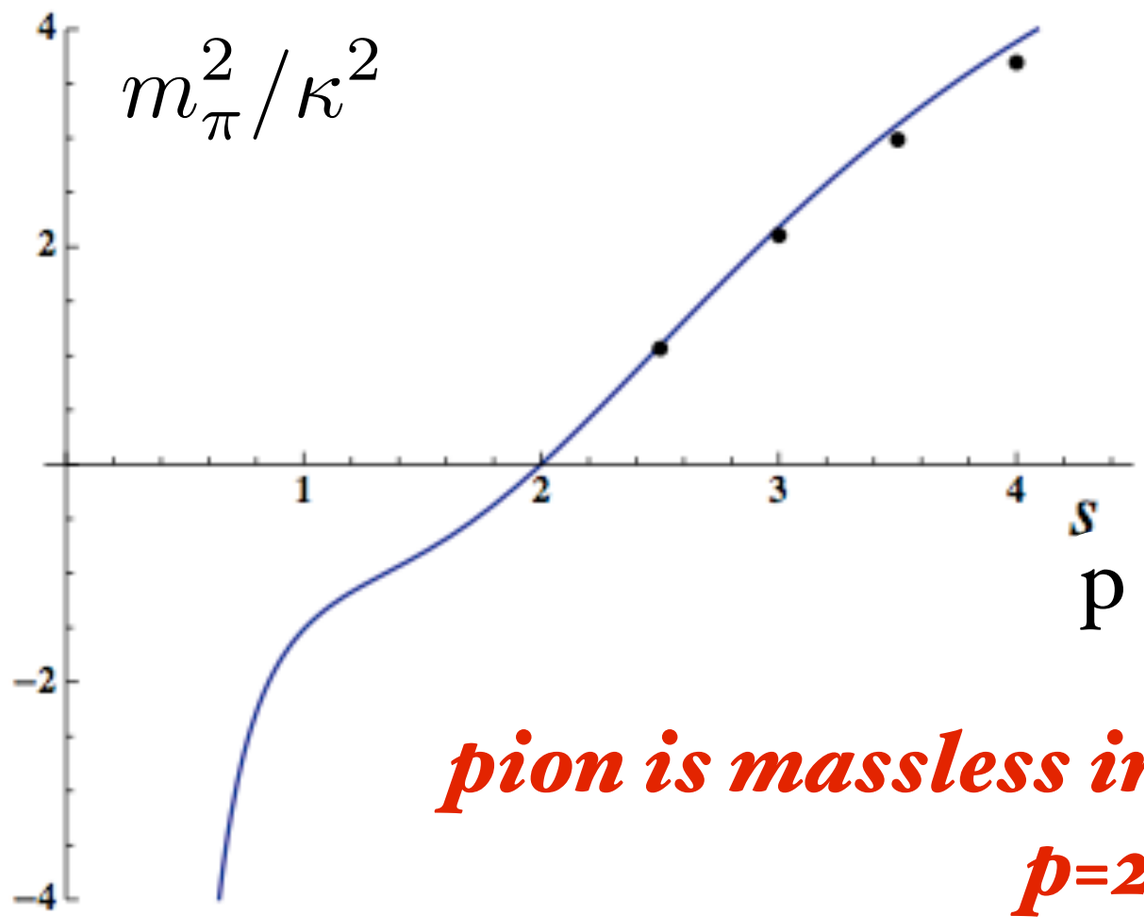
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

Uniqueness of Dilaton

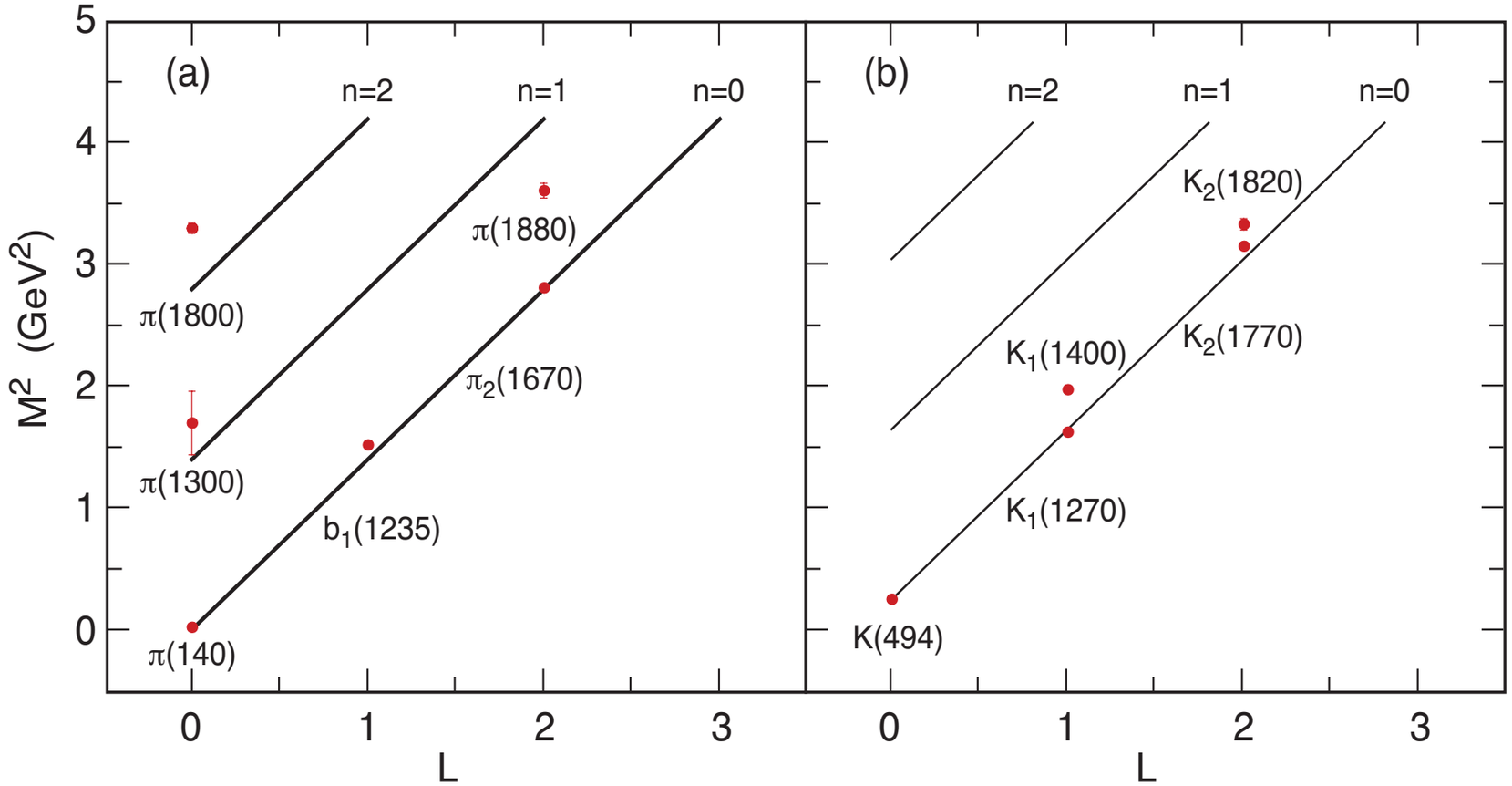
$$\varphi_p(z) = \kappa^p z^p$$

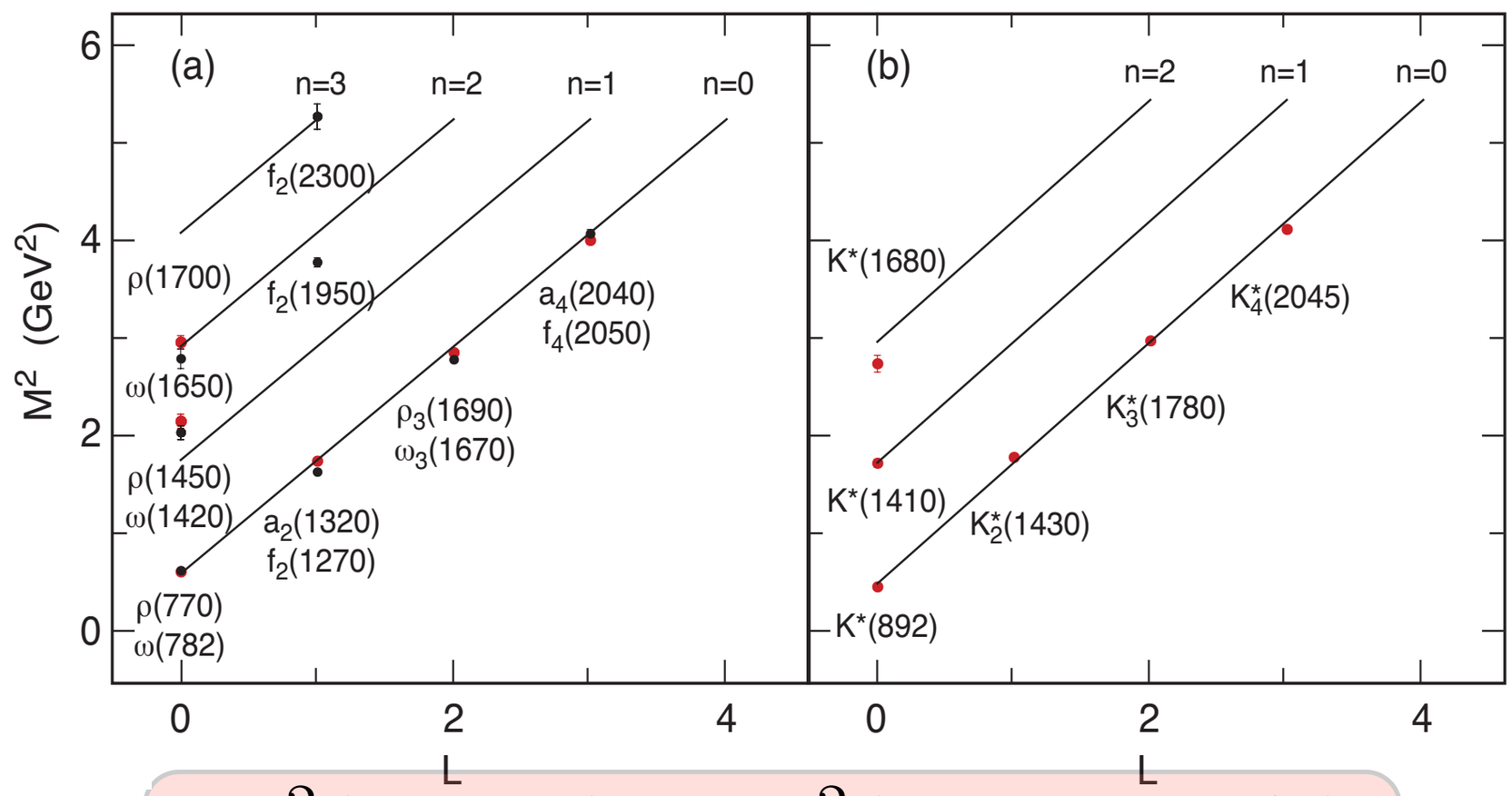


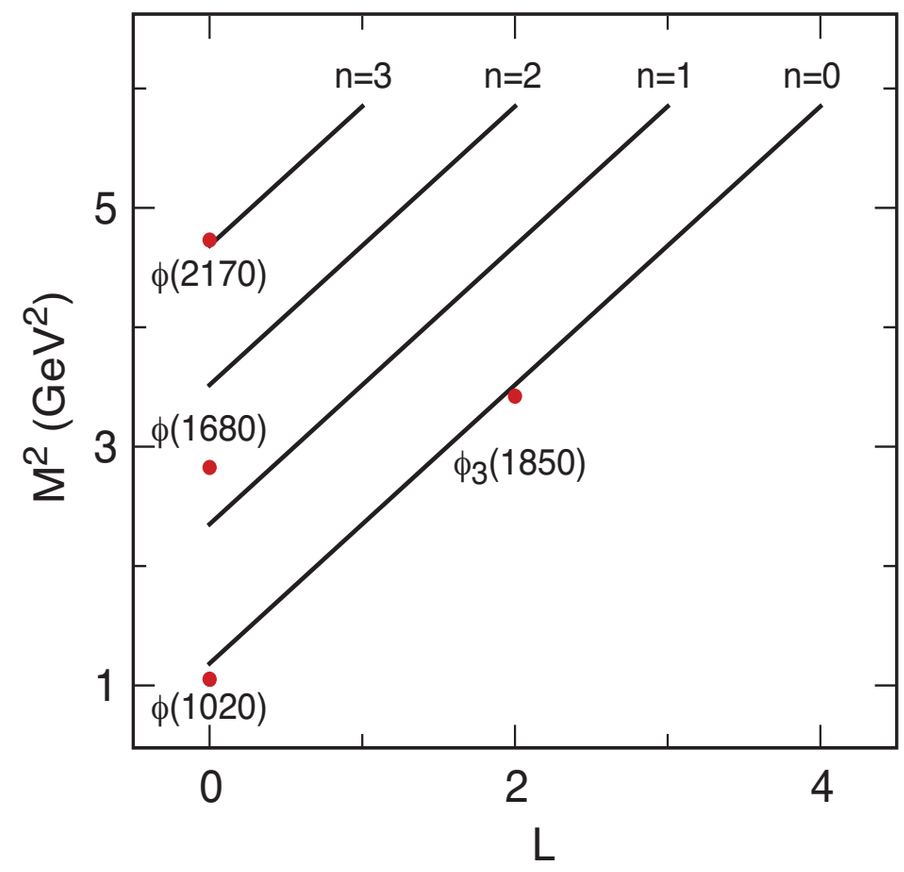
pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**







$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

Quark separation increases with L

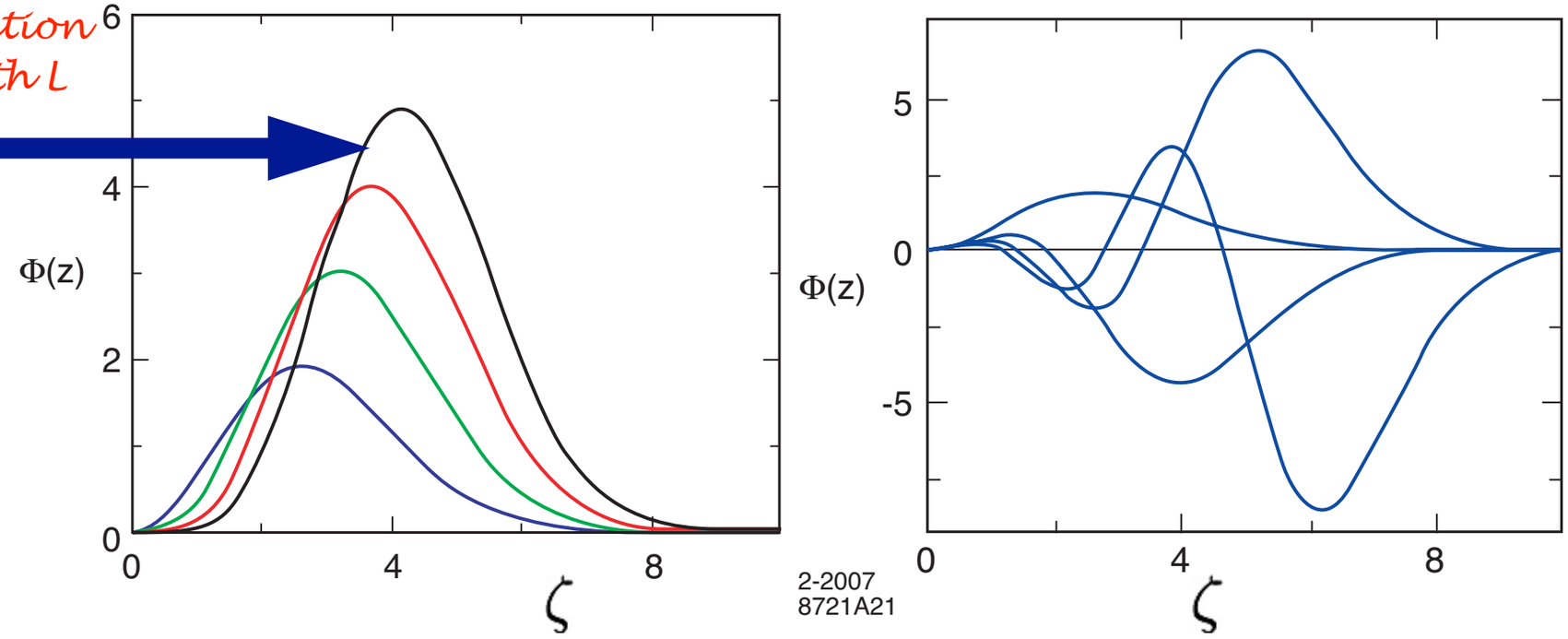
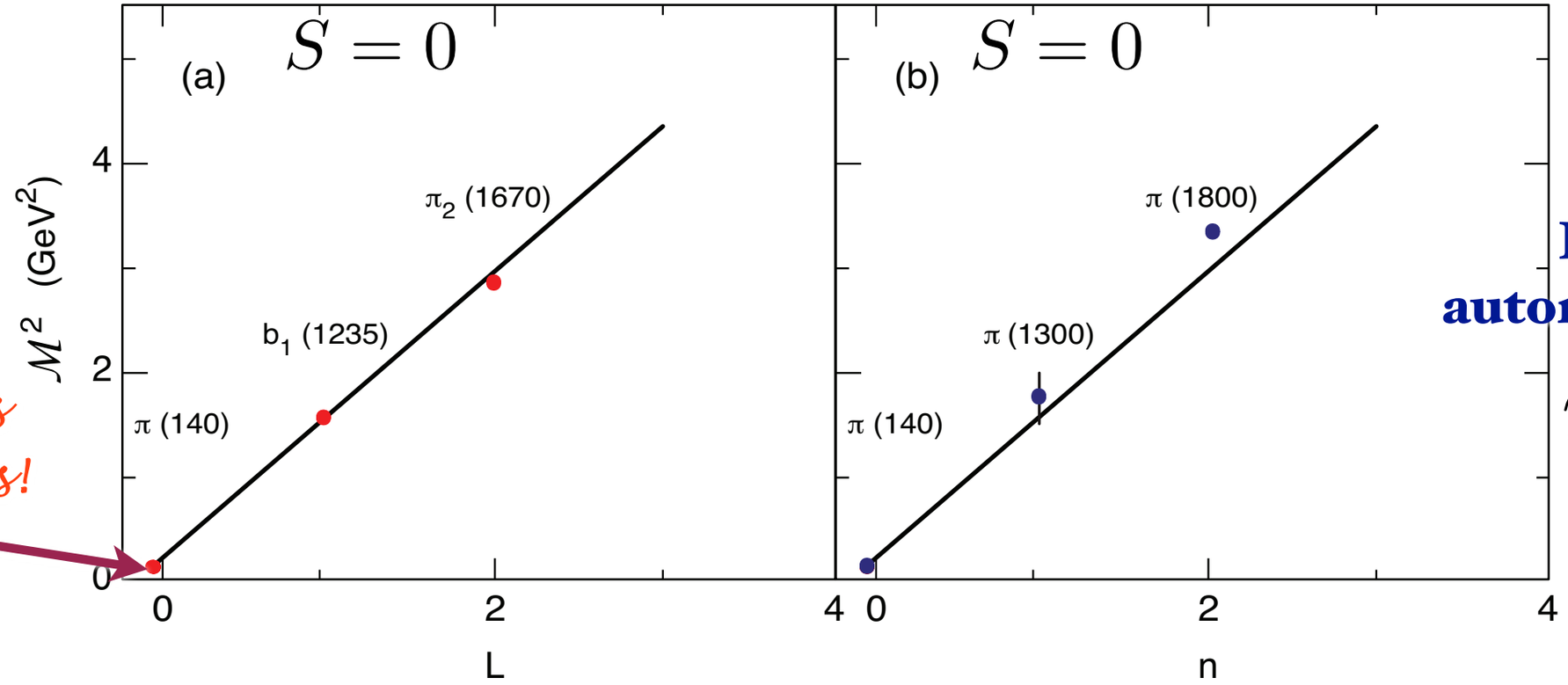


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

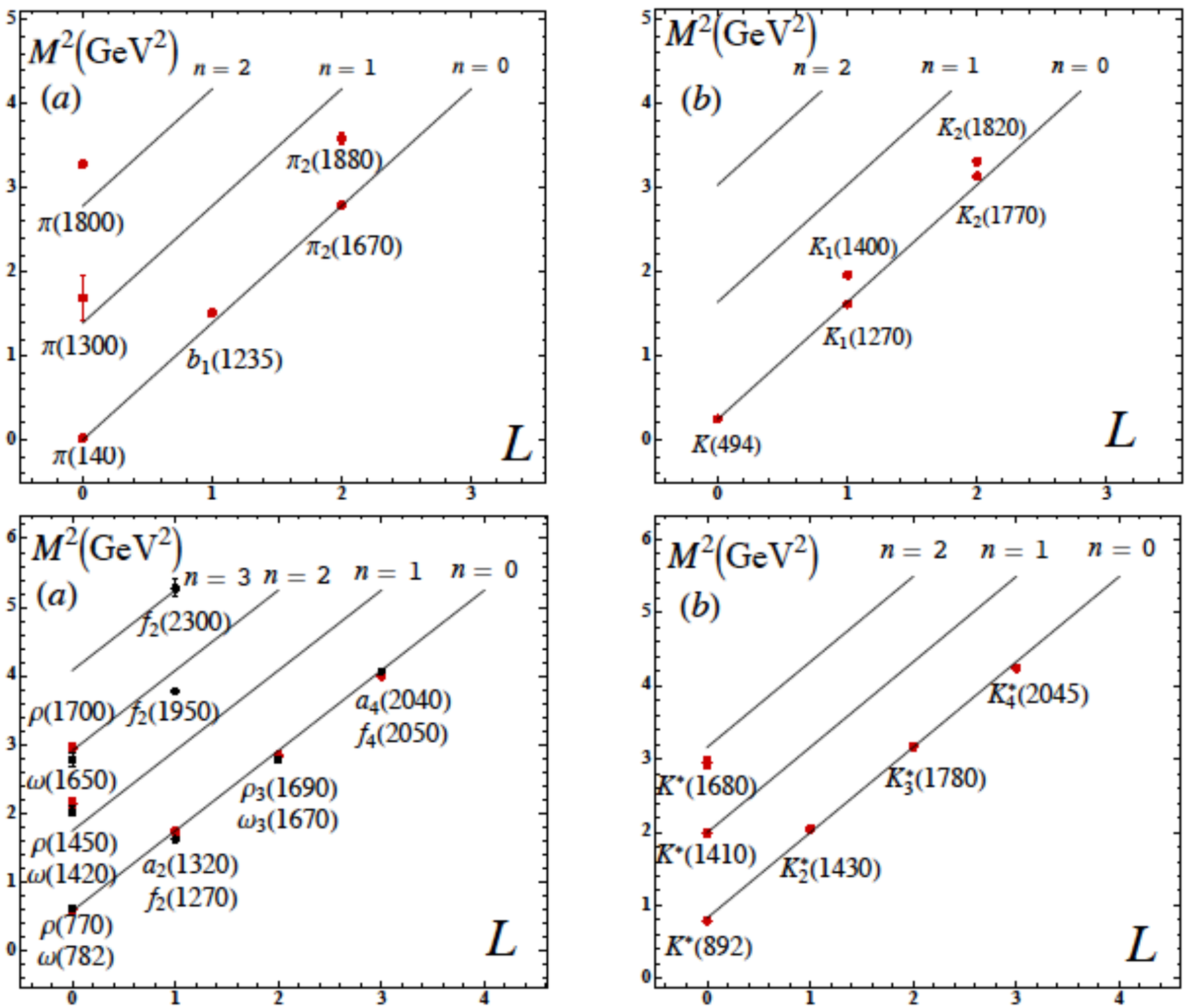
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

De Tèramond, Dosch, sjb

$$m_u = m_d = 46\ \mathrm{MeV}, \quad m_s = 357\ \mathrm{MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism



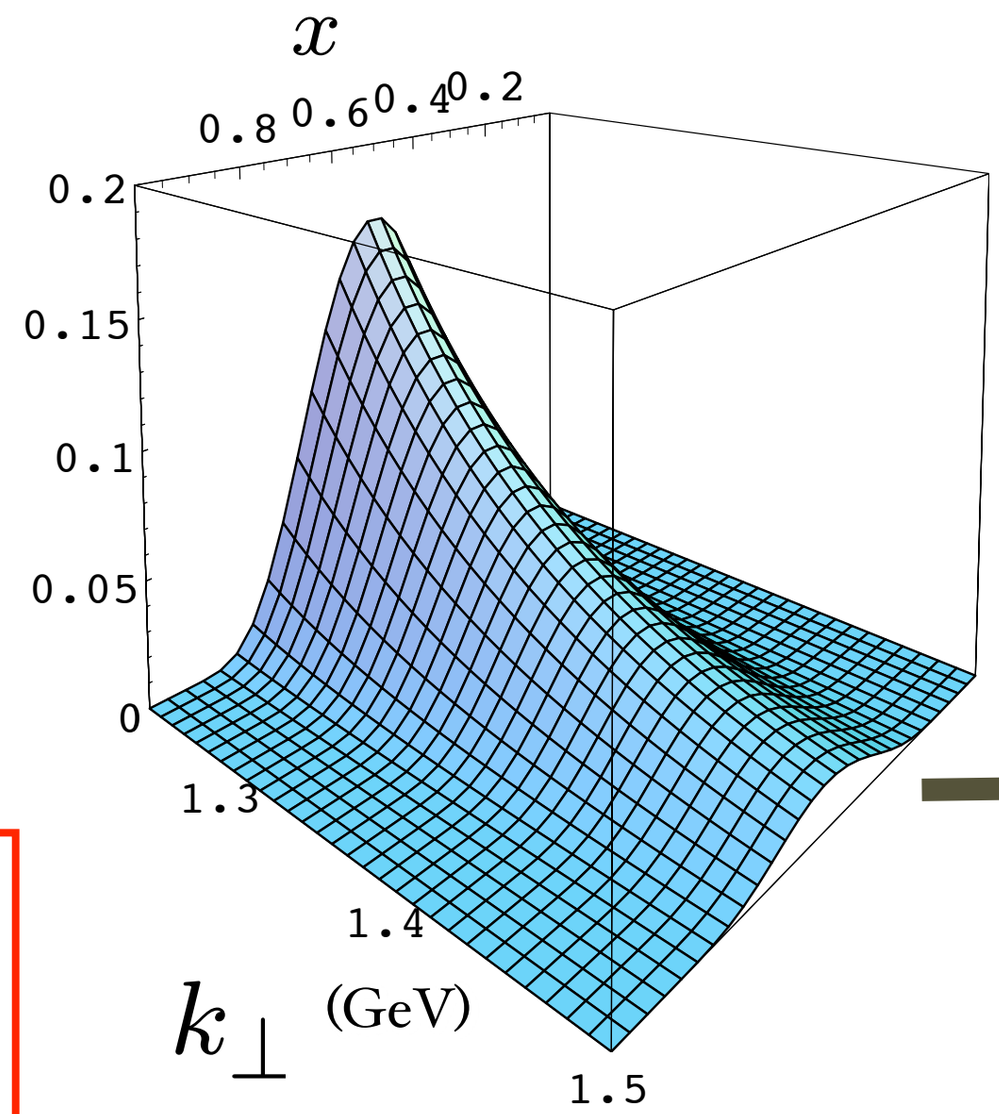
Effective mass from $m(p^2)$

Tandy, Roberts, et al

Prediction from AdS/QCD: Meson LFWF

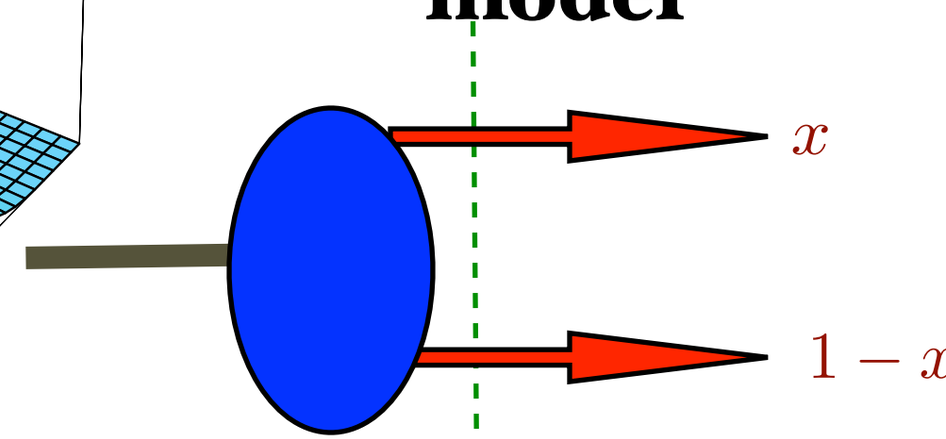
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

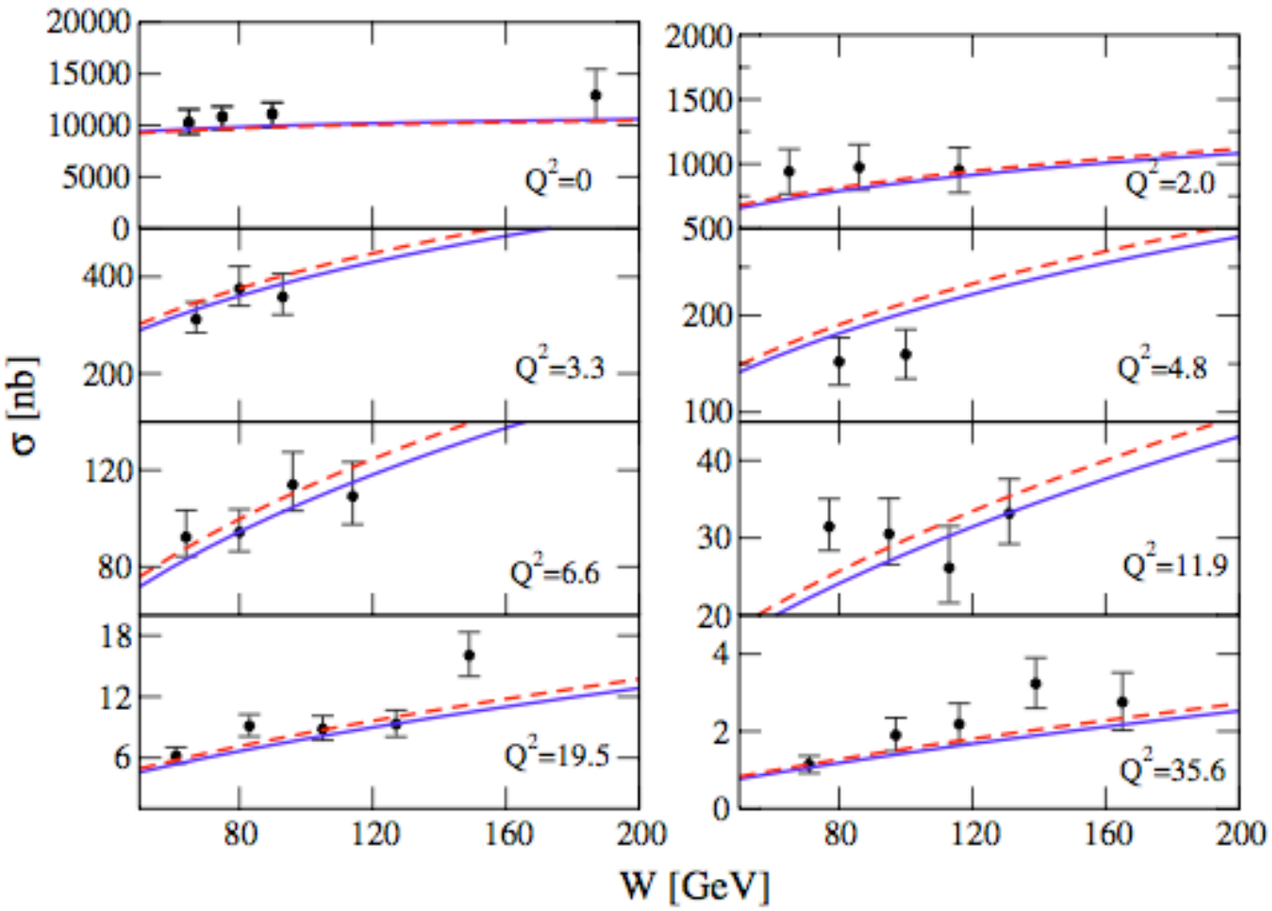
$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

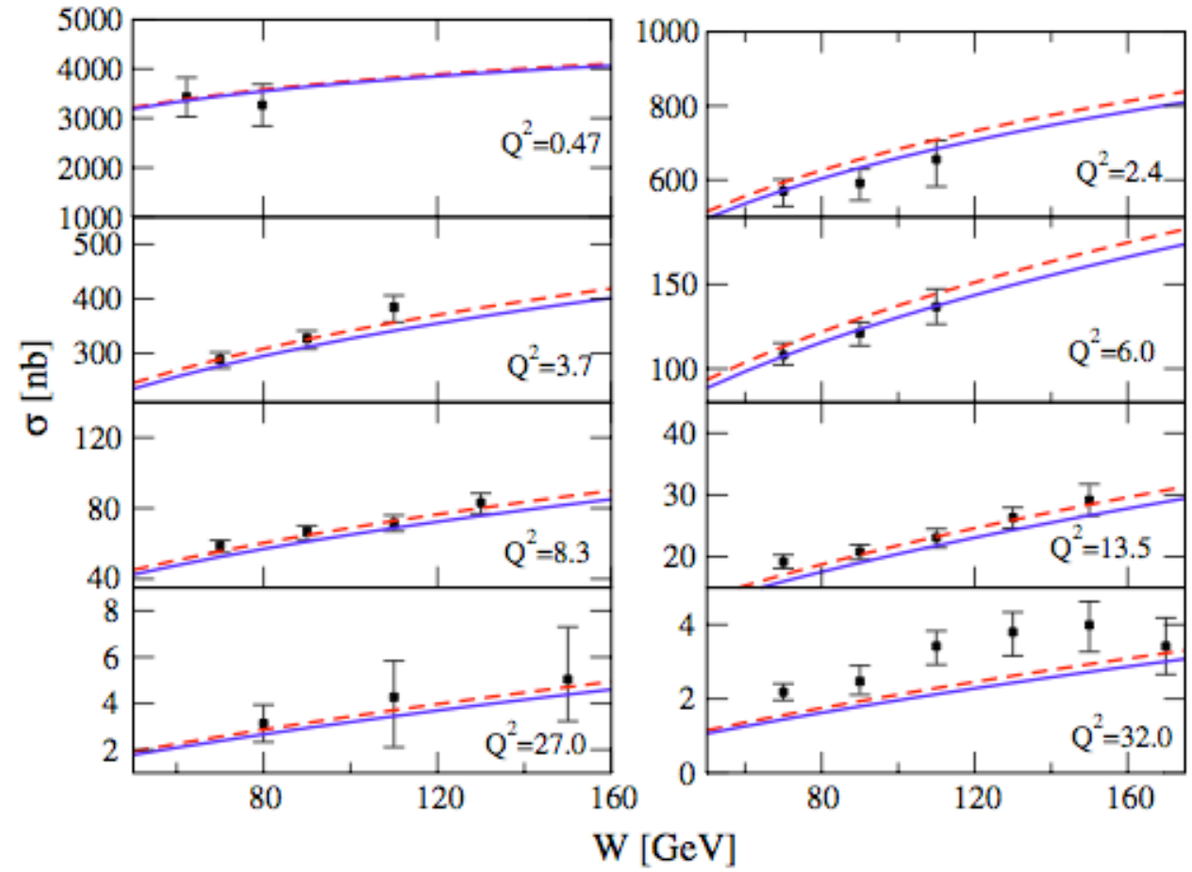
Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



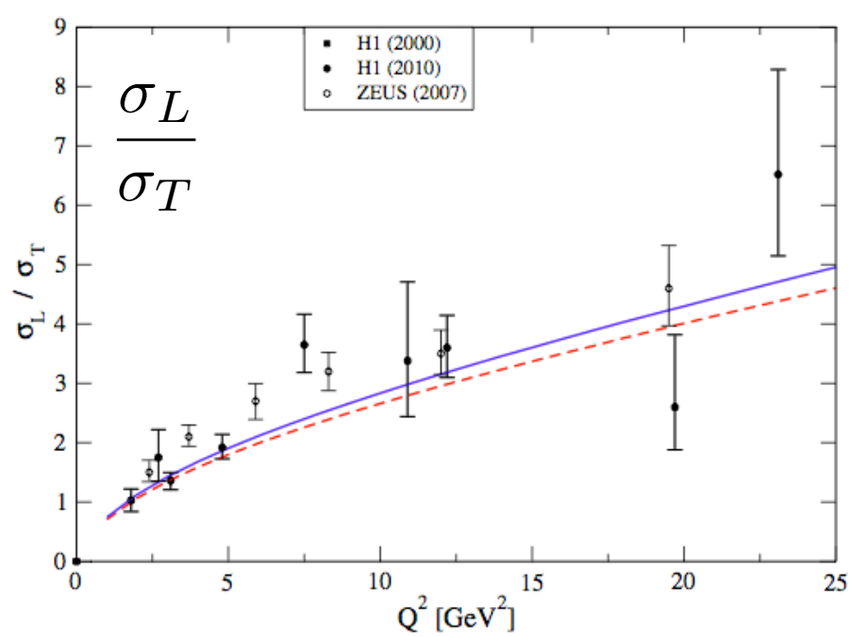
(a) H1



(b) ZEUS

J. R. Forshaw,
R. Sandapen

$$\gamma^* p \to \rho^0 p'$$



$$\frac{\sigma_L}{\sigma_T}$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

$\Updownarrow$

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

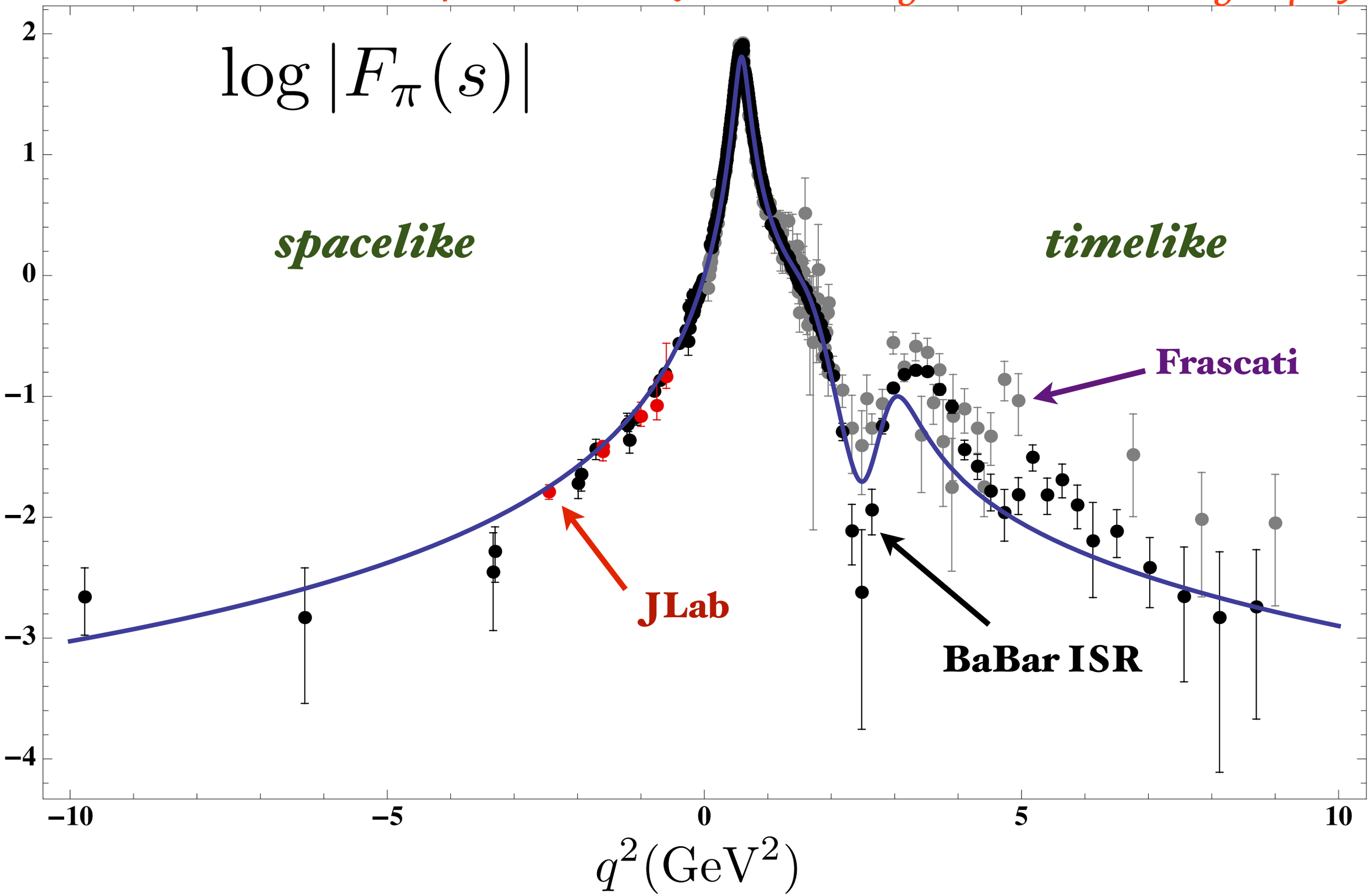
Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Pion Form Factor from AdS/QCD and Light-Front Holography



QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\Big(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\Big)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

- **Dosch, de Teramond, sjb**

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[\mathrm{H},\mathrm{D}] = \mathrm{i\ H}, \quad [\mathrm{H},\ \mathrm{K}] = 2\ \mathrm{i\ D}, \quad [\mathrm{K},\ \mathrm{D}] = -\ \mathrm{i\ K}$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Baryon Equation $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$ $\quad w$: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \left(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B \ , \quad w = \kappa^2$ $\qquad \lambda = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Superconformal Quantum Mechanics

Baryon Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

Meson Equation

$$\lambda = \kappa^2$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

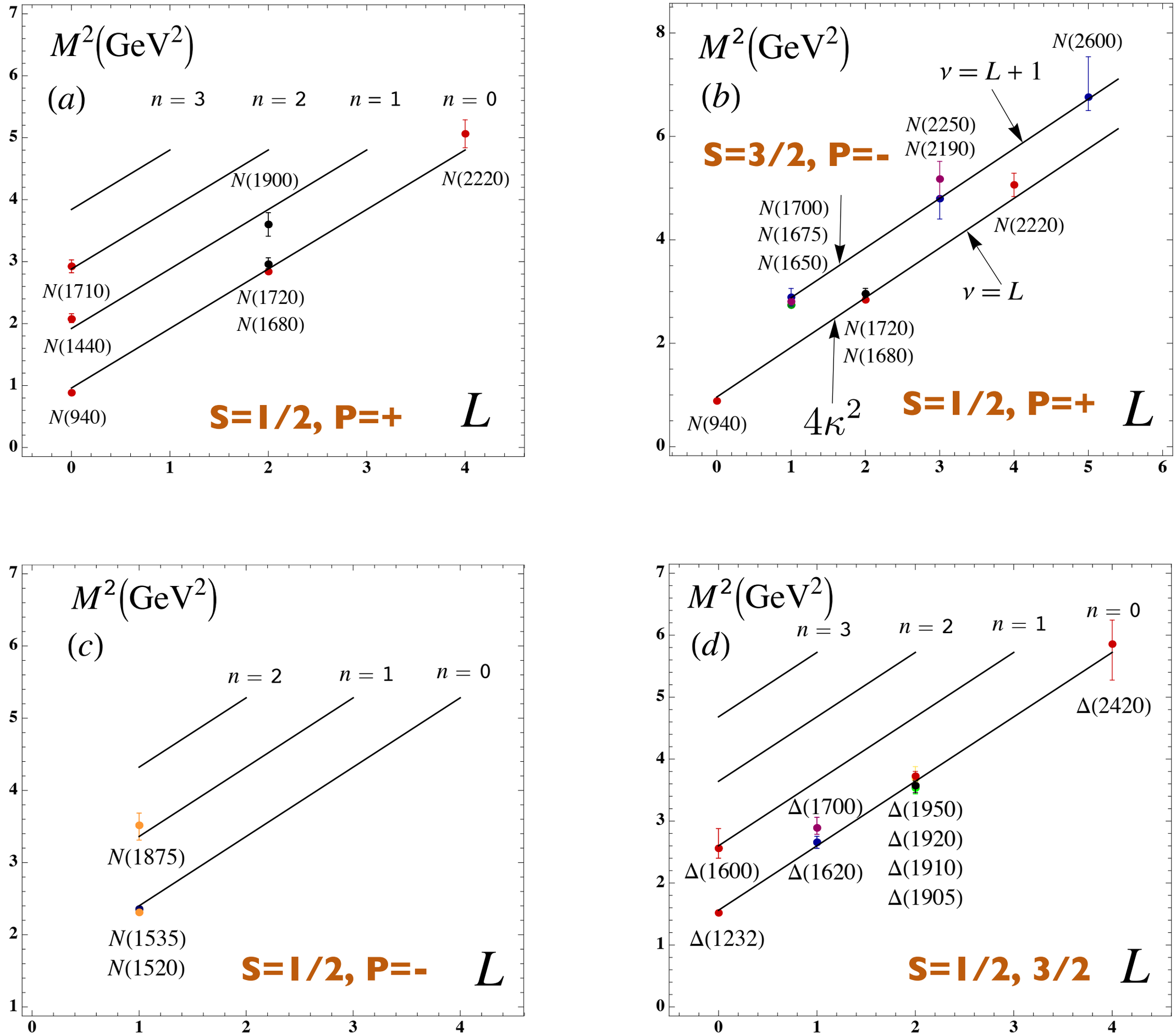
S=0, P=+

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ *!*

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$


$M^2(\mathrm{GeV}^2)$
(a)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
N(1900)
N(2220)
N(1710)
N(1720)
N(1680)
N(1440)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
$\nu = L + 1$
N(2600)
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
$n = 2$
$n = 1$
$n = 0$
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
$n = 3$
$n = 2$
$n = 1$
$n = 0$
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

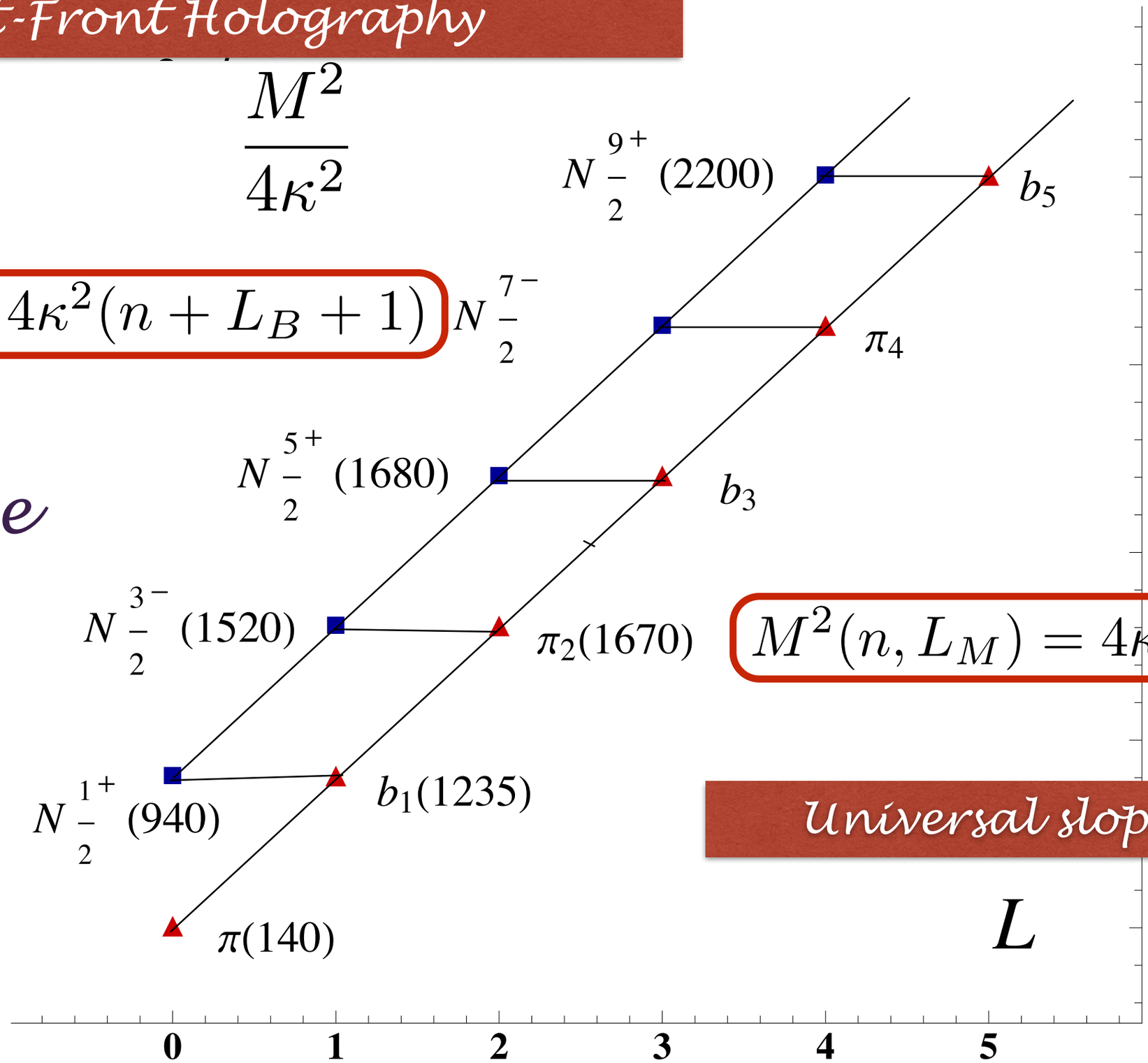
Superconformal Quantum Mechanics
Light-Front Holography

de Tèramond, Dosch, Lorcè, sjb

$$\frac{M^2}{4\kappa^2}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

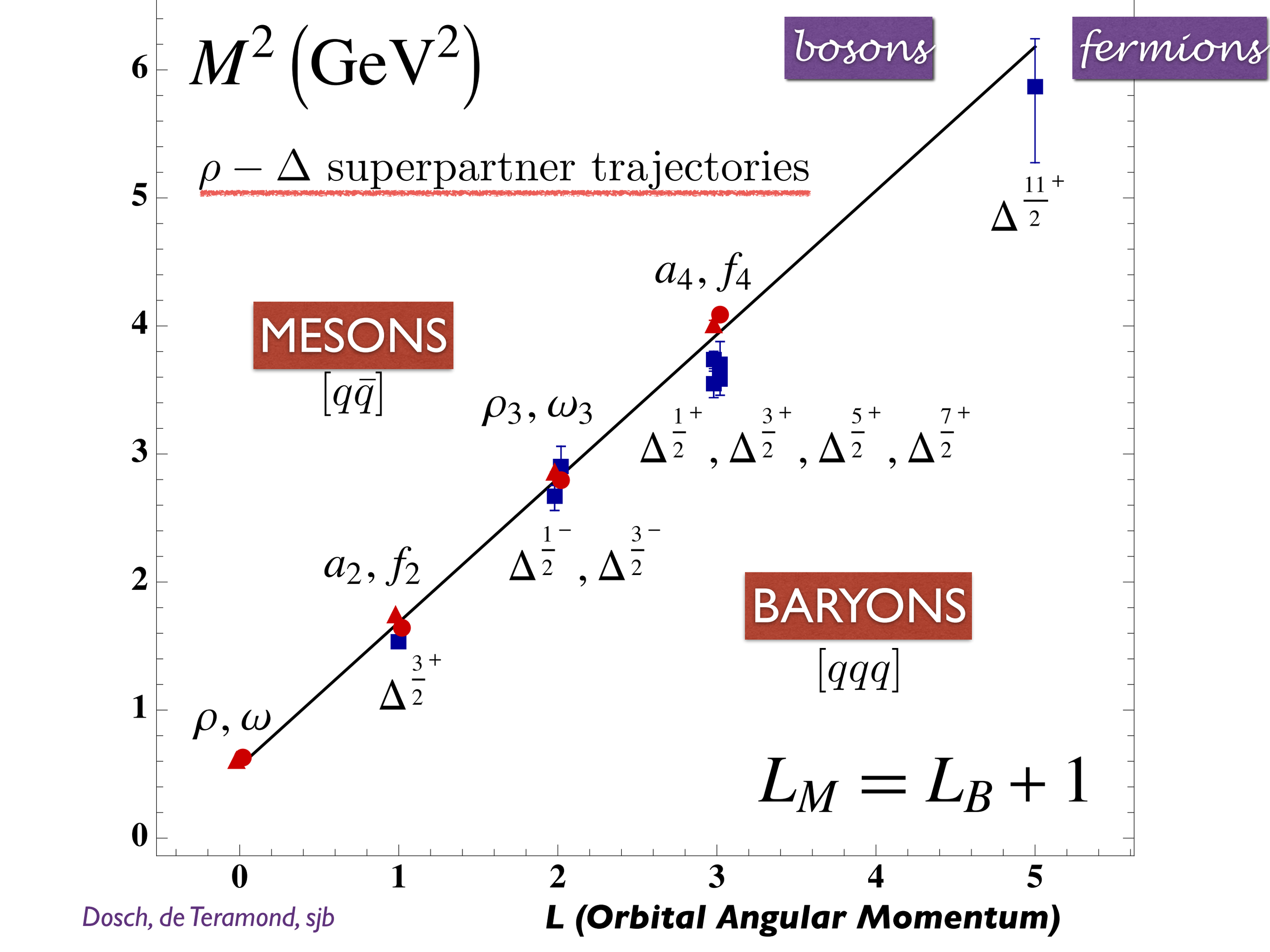


$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



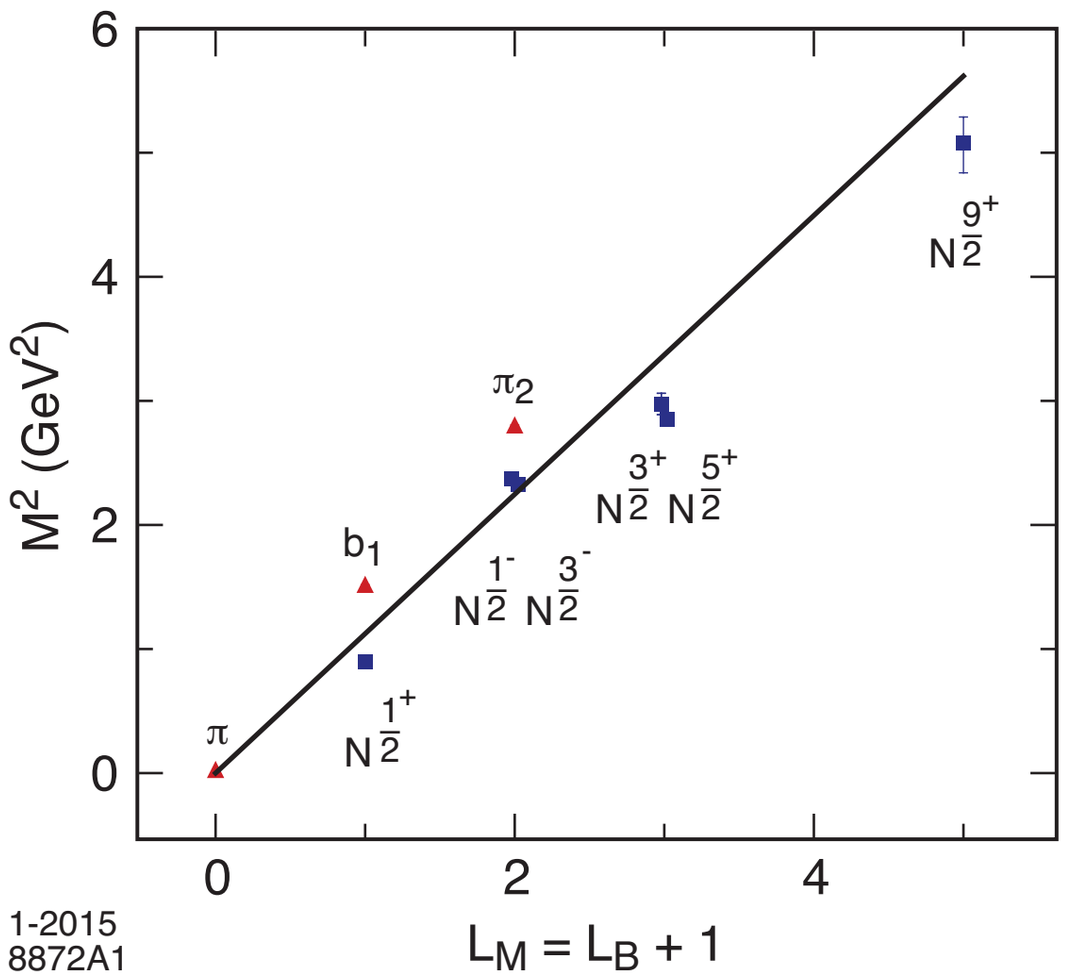
Dosch, de Teramond, sjb

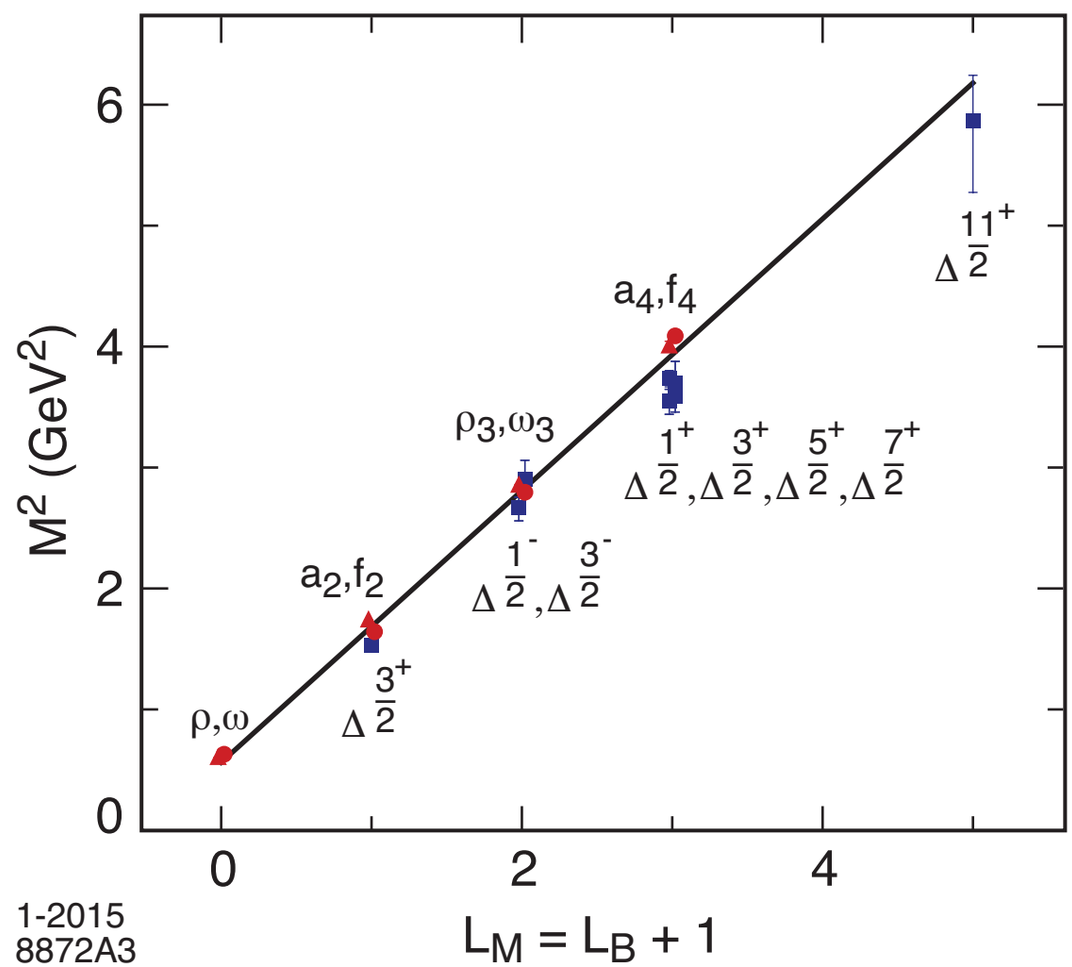
- Superconformal spin-dependent Hamiltonian to describe mesons and baryons (chiral limit)

  [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]

$$\boxed{G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S} \qquad\qquad S = 0, 1$$

$$\text{Mesons}: M^2 = 4\lambda\,(n + L_M) + 2\lambda S, \quad \text{Baryons}: M^2 = 4\lambda\,(n + L_B + 1) + 2\lambda S$$





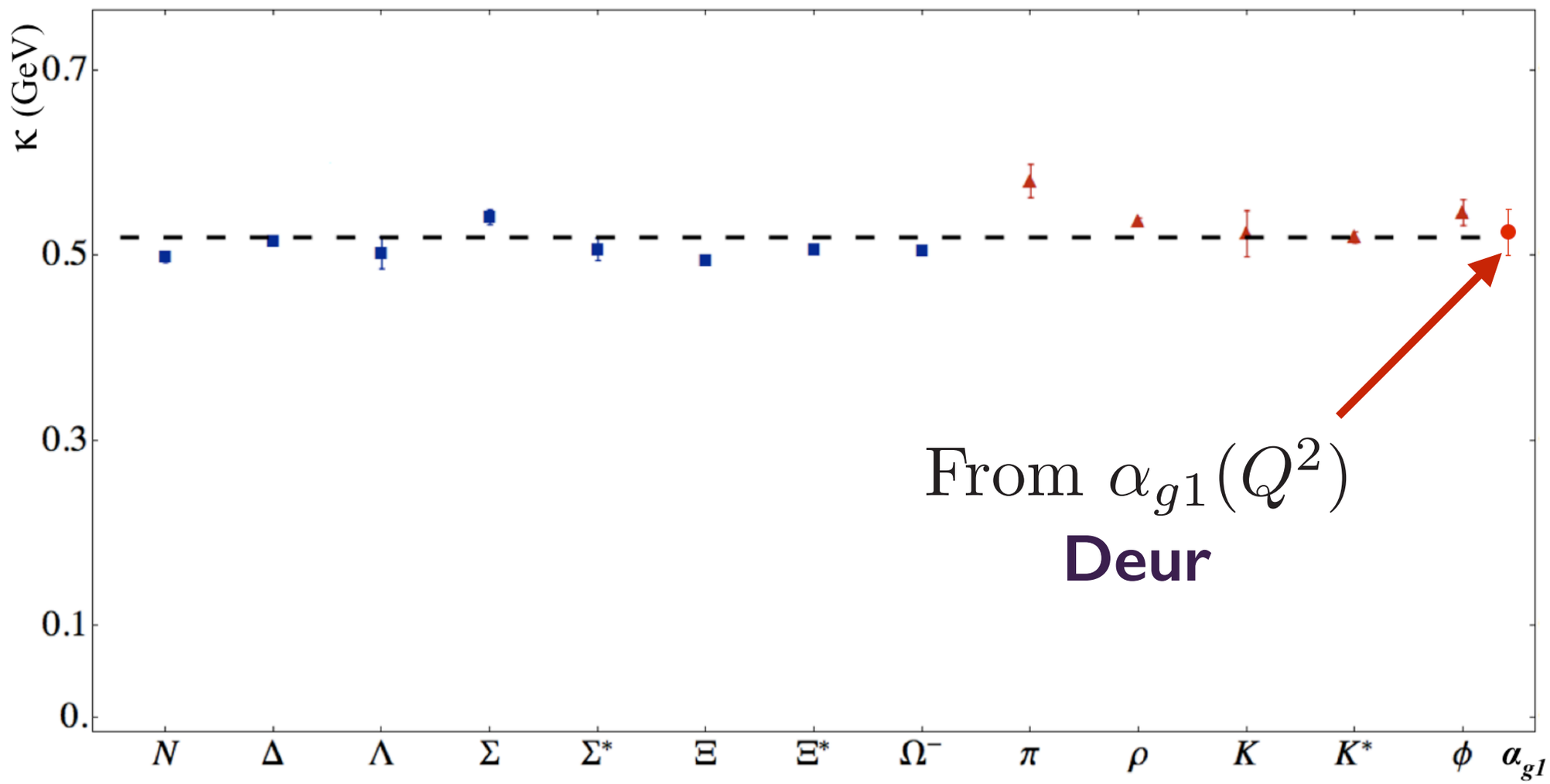
Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

$\lambda = \kappa^2$

de Tèramond, Dosch, Lorce', sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ



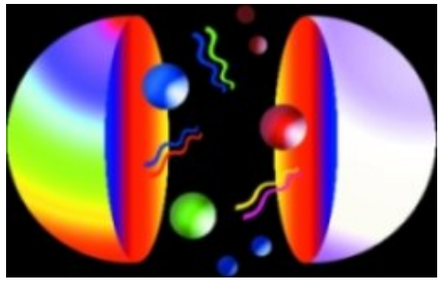
Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int_0^\infty d\zeta\int_0^1 dx\psi_+^2(\zeta^2,x) = \int_0^\infty d\zeta\int_0^1 dx\psi_-^2(\zeta^2,x) = \frac{1}{2}$$

Quark Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Nucleon: Equal Probability for L=0,1

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z)\Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

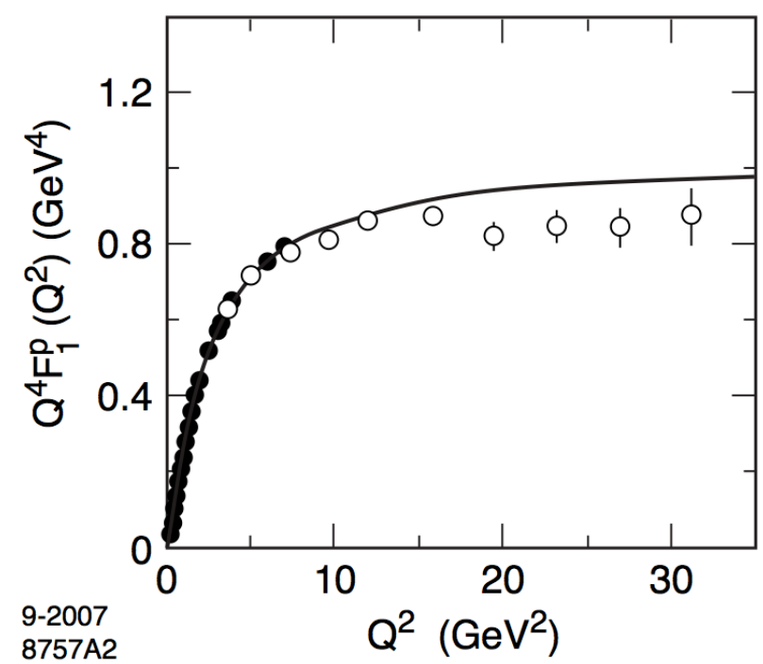
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n+1/2)$



Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta)|\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

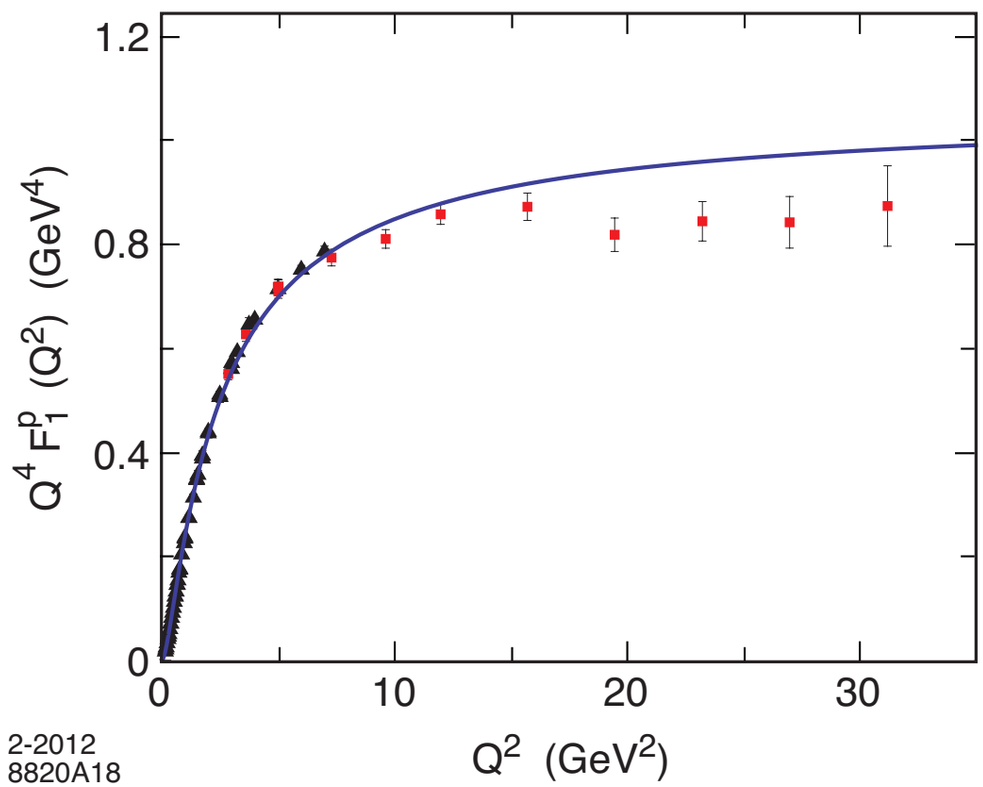
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

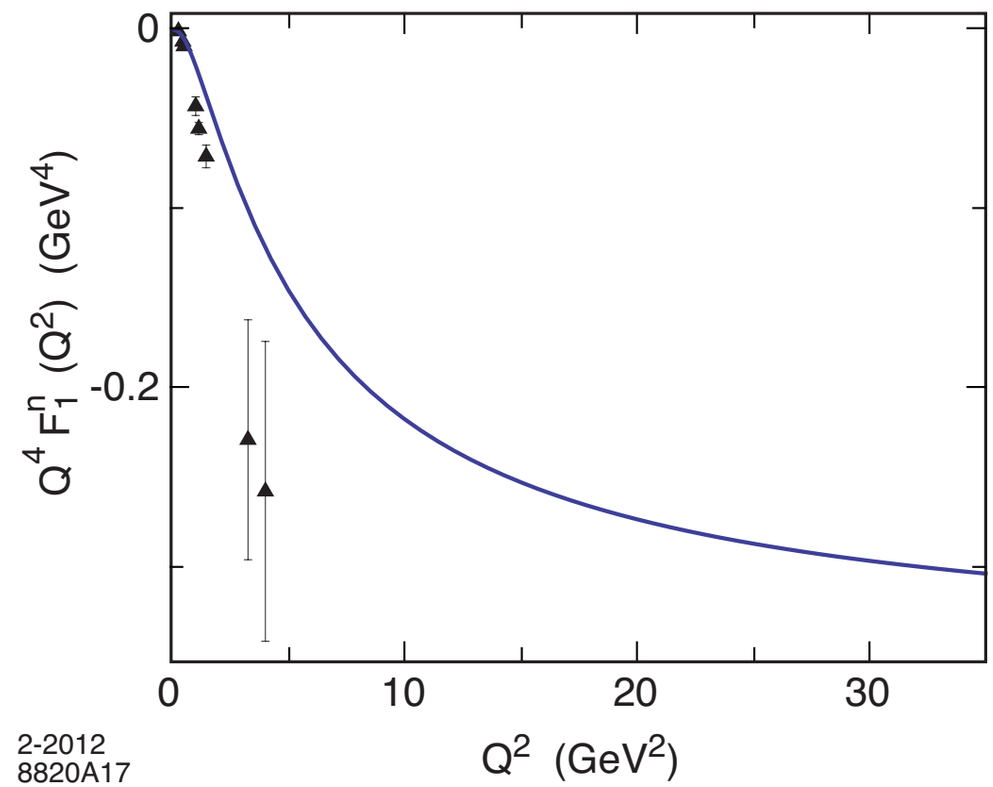
- For $SU(6)$ spin-flavor symmetry

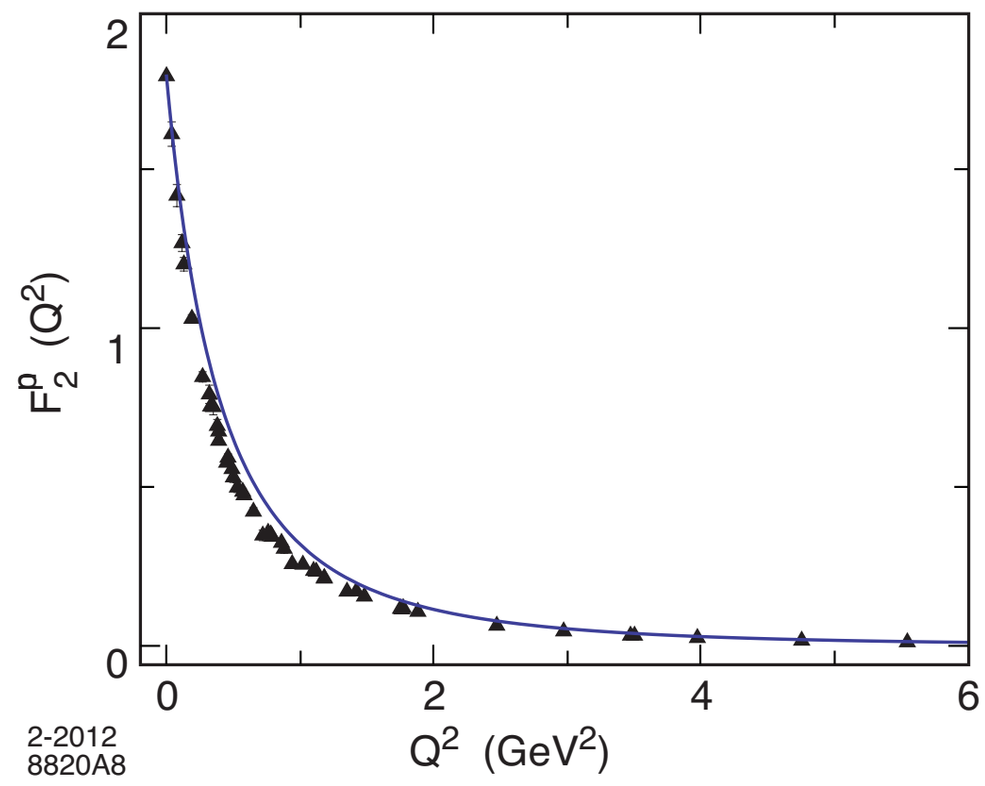
$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta)|\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3}\int d\zeta\, J(Q,\zeta)\left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

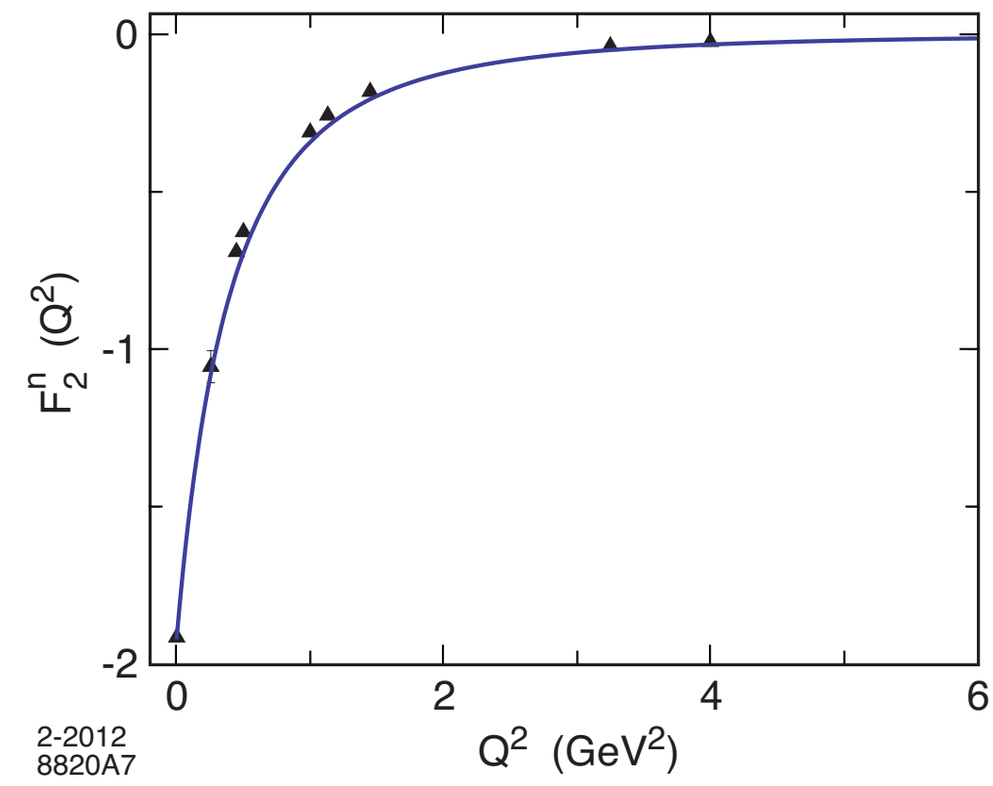
where $F_1^p(0) = 1,\ F_1^n(0) = 0.$

Using $SU(6)$ flavor symmetry and normalization to static quantities



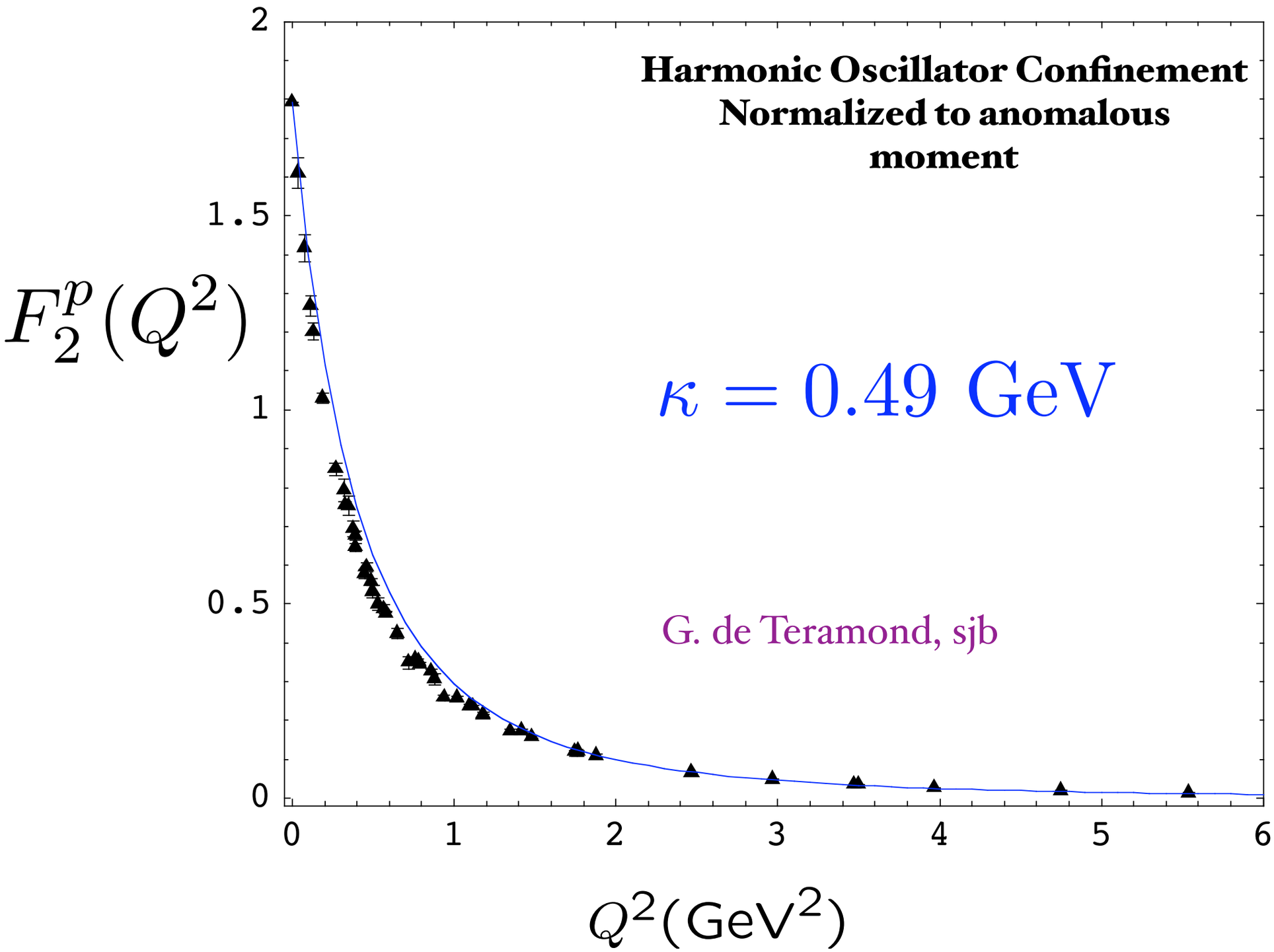






Spacelike Pauli Form Factor

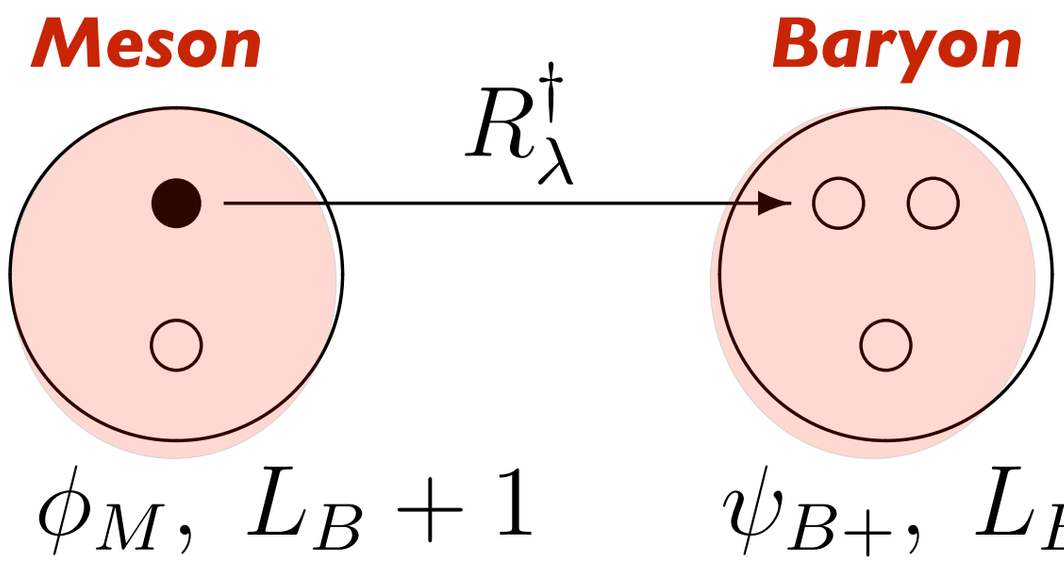
From overlap of $L = 1$ and $L = 0$ LFWFs



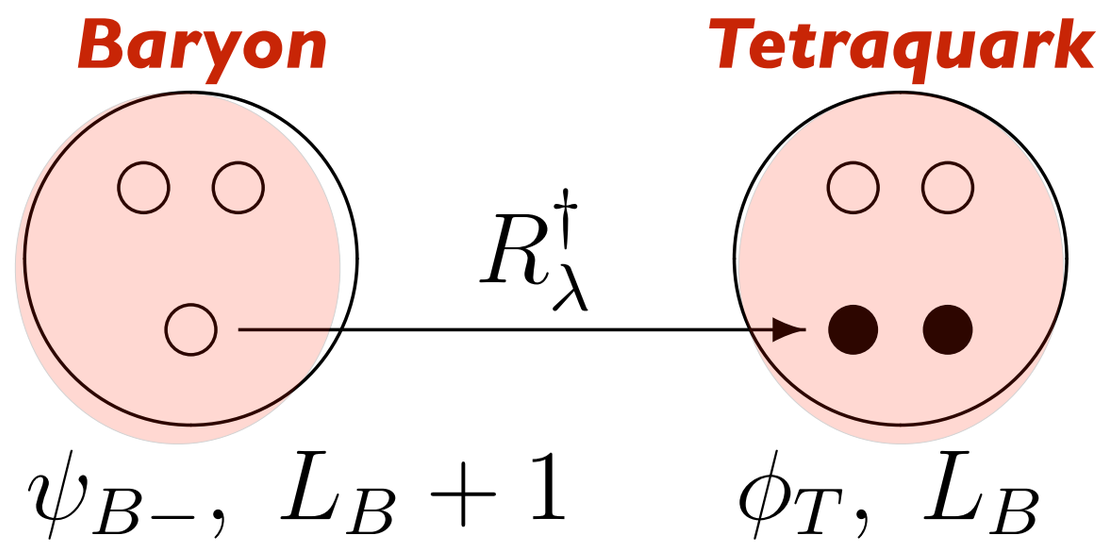
Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



$$R_\lambda^\dagger\ \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$



$$R_\lambda^\dagger\ q \to [\bar{q}\bar{q}]$$
$$3_C \to 3_C$$

Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}^-}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770)$, $\omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320)$, $f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690)$, $\omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}^-}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)$?
				$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040)$, $f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)$?
$\bar{q}s$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$K_1^*(\sim 1700)$?
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$
								$f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)$?
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)$?
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$
								$a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)$?
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)$?
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)$?

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

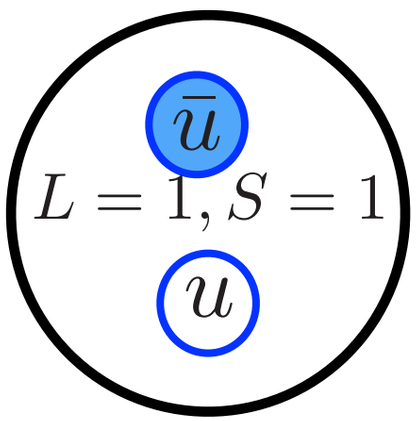
Superconformal Algebra 4-Plet

$R^\dagger_\lambda \quad \bar{q} \to (qq) \quad S = 1$

$\bar{3}_C \to \bar{3}_C$

Vector ()+ Scalar [] Diquarks

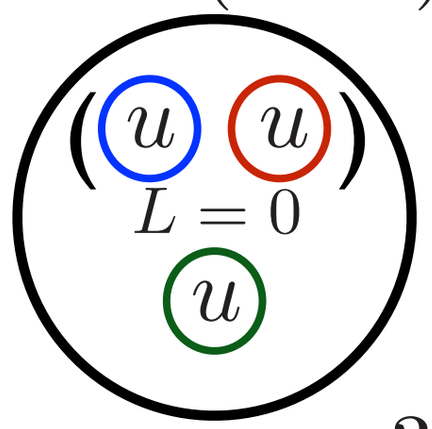
$f_2(1270)$



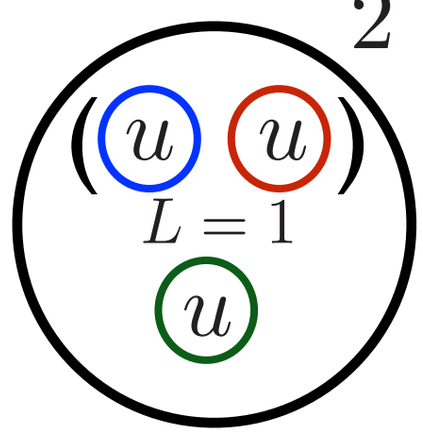
ω'

$J^{PC} = 2^{++}$

$\Delta^+(1232)$

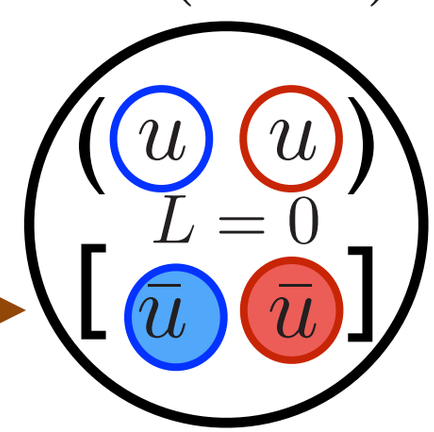


$J^P = \frac{3}{2}^+$



$J^{PC} = 1^{++}$

$A_1(1260)$



$R^\dagger_\lambda \; q \to [\bar{q}\bar{q}] \quad S = 0$

$3_C \to 3_C$

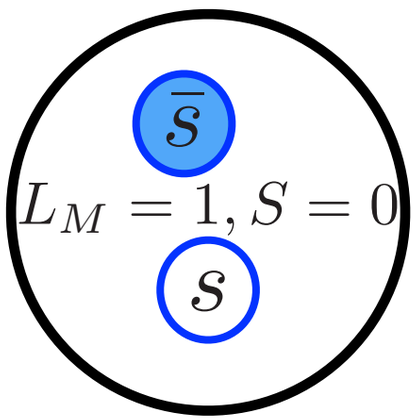
Double-Strange Baryon

$$S = 0$$

$$R_\lambda^\dagger \; \bar{q} \to [qq]$$

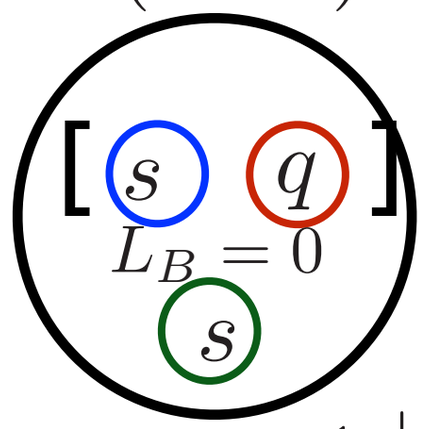
$$\bar{3}_C \to \bar{3}_C$$

$h_1(1170)$



η'

$\Xi(1320)$

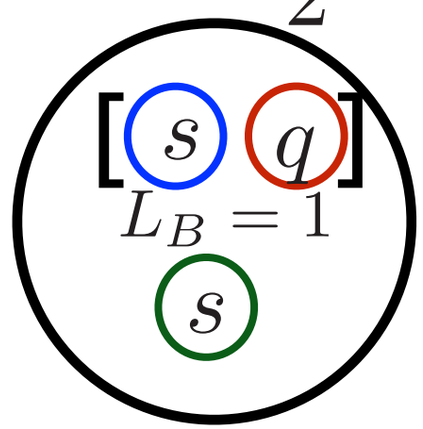


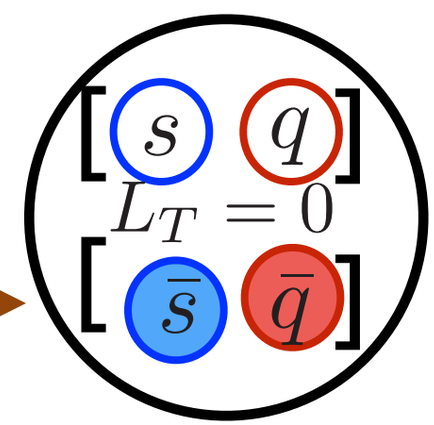
$$J^P = \frac{1}{2}^+$$

Scalar[] Diquarks

$f_0(1370)$

$$J^{PC} = 0^{++}$$



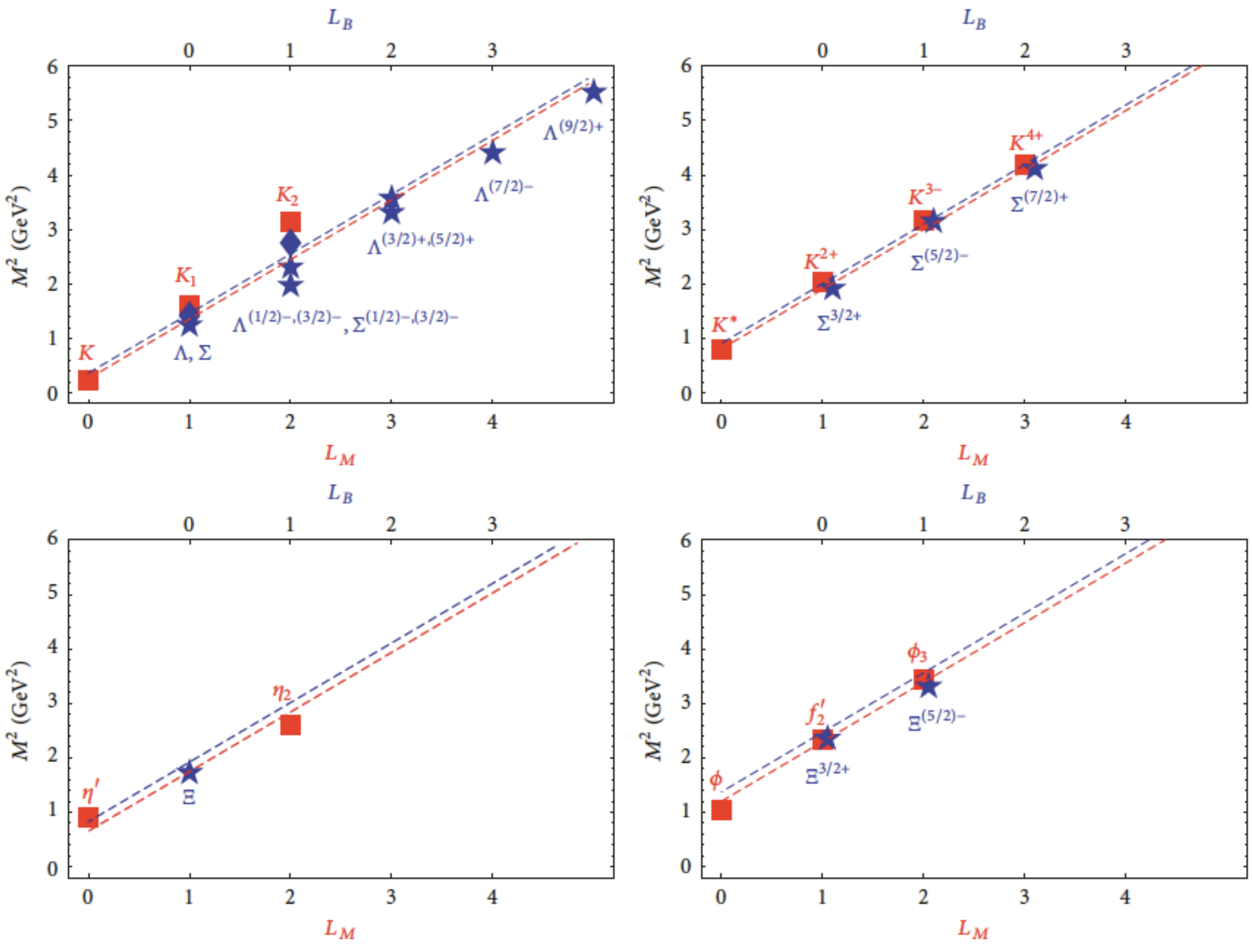


$$R_\lambda^\dagger \; q \to [\bar{q}\bar{q}] \quad S = 0$$

$$3_C \to 3_C$$

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)$?	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)$?	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)$?	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

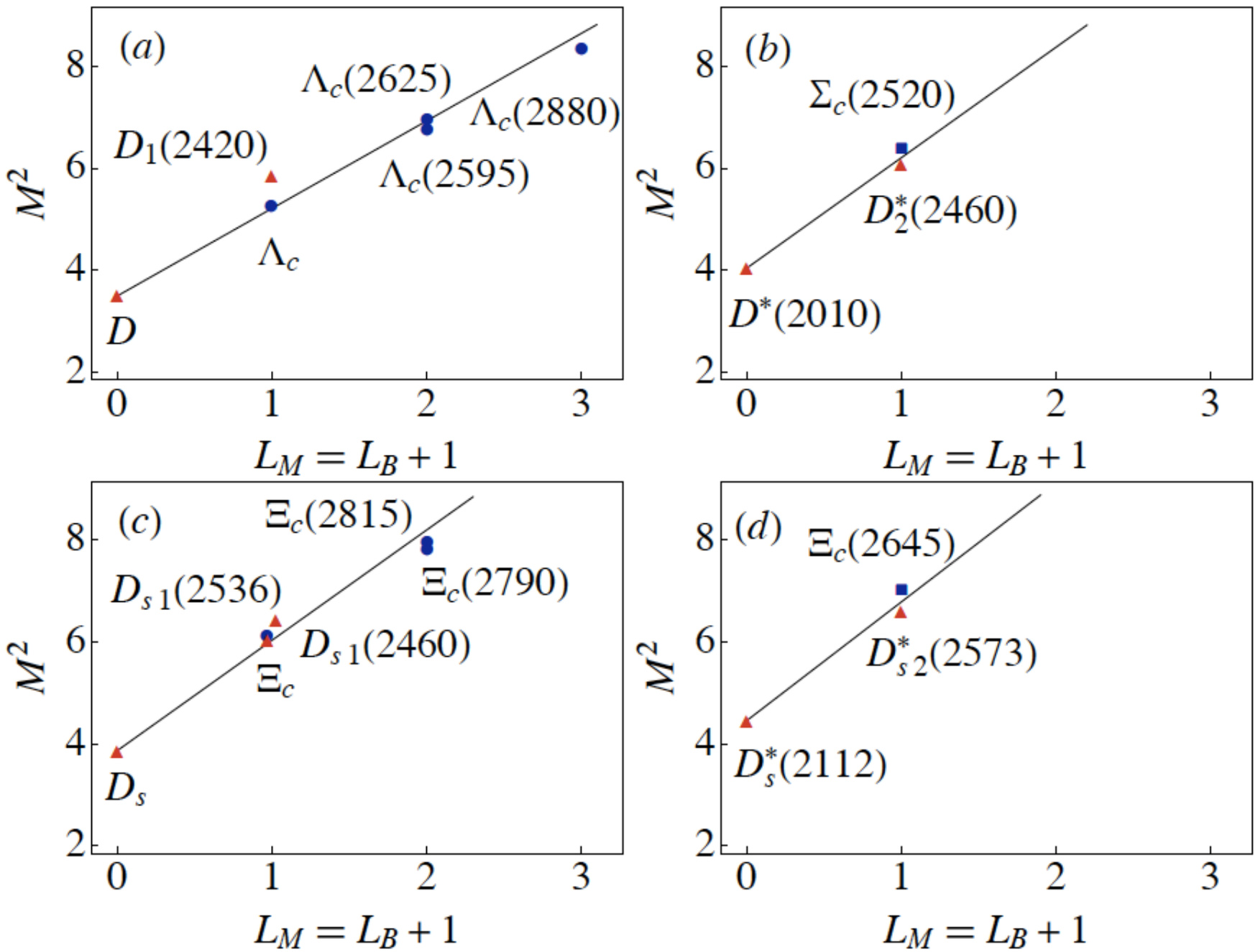
predictions

beautiful agreement!

M. Nielsen, sjb

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Nielsen, Navarra, Dosch, de Tèramond, Zou, sjb

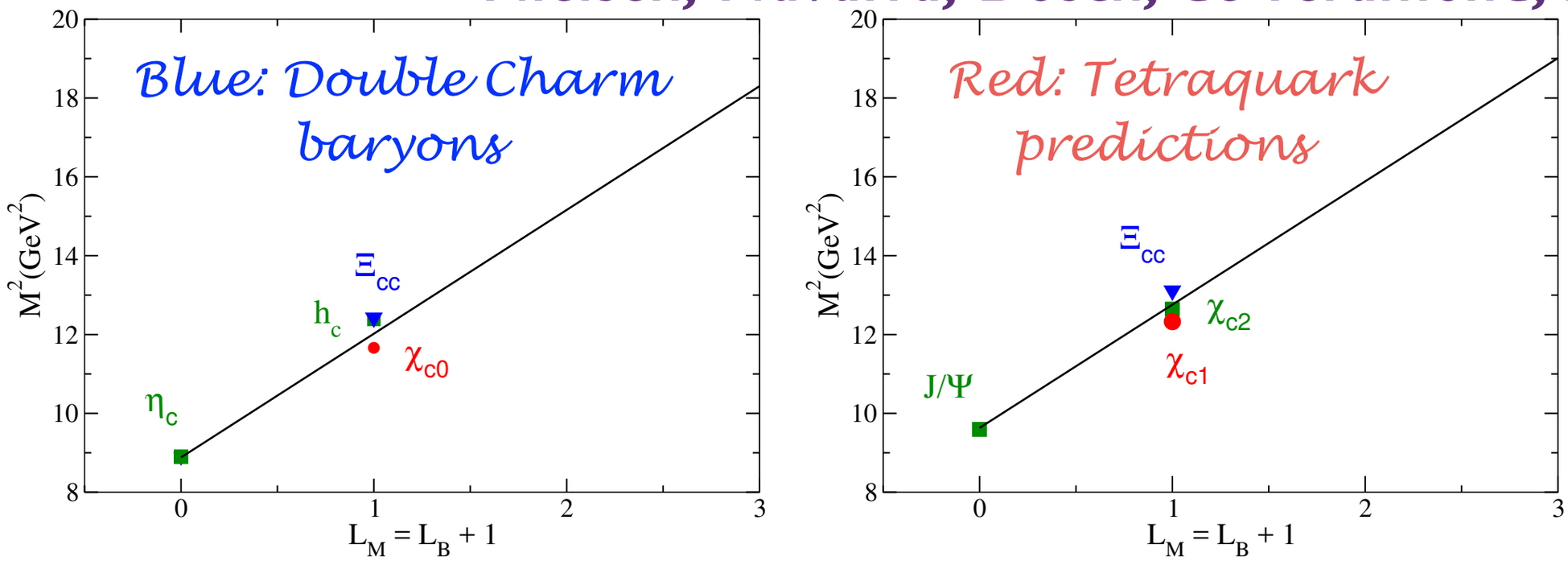


FIG. 1. Double charm mesons (shown as green squares) baryons (shown as blue triangles) and tetraquarks (shown as red circles). The solid lines are the trajectories fit from (13). Hadron masses are taken from PDG [27]. In the left hand side figure we show states with $S_M = S_D = S_T = 0$. In the right hand side figure we show states with $S_M = S_D = S_T = 1$.

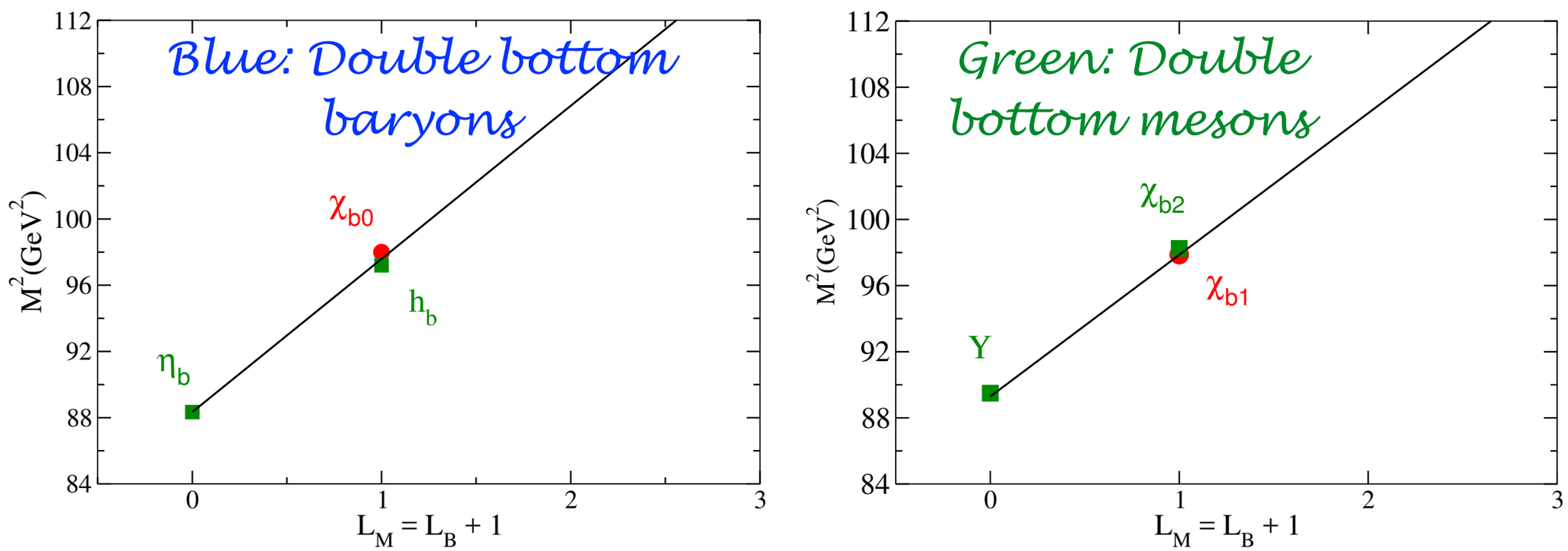


FIG. 2. Same as in Fig. 1 for double beauty hadrons.

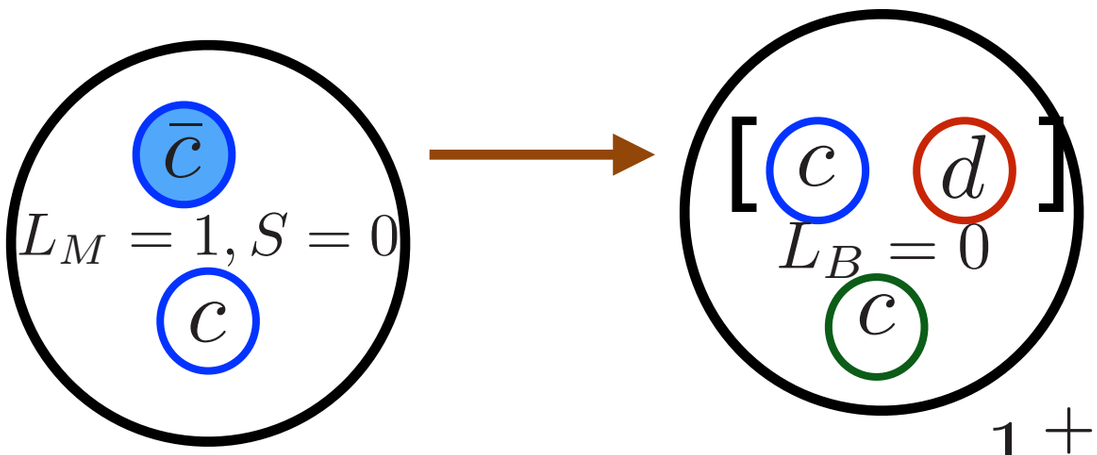
Double-Charm Baryon (SELEX)

$$S = 0$$

$$R^\dagger_\lambda \; \bar{q} \to [qq]$$

$$\bar{3}_C \to \bar{3}_C$$

$$h_c(3525) \qquad \Xi^+_{CC}(3520)$$



Predict Tetraquark $T_{c\bar{c}q\bar{q}}$

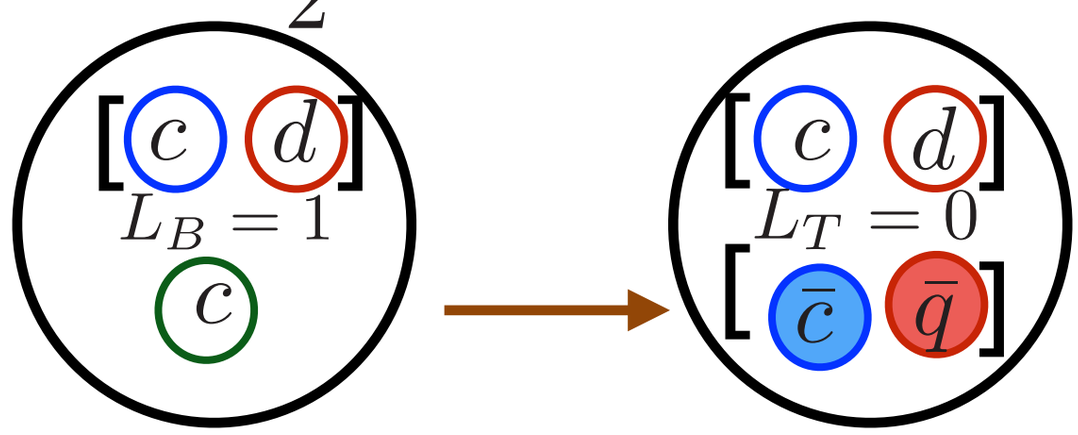
$M_T \sim 3520 \; MeV$

Scalar Diquarks

$$J^P = \frac{1}{2}^+ \qquad J^{PC} = 0^{++}$$

$$\eta_c'$$

$$J^{PC} = 1^{+-}$$



$$R^\dagger_\lambda \; q \to [\bar{q}\bar{q}] \qquad S = 0$$

$$3_C \to 3_C$$

SELEX $(3520 \pm 1\ MeV)$ $J^P = \frac{1}{2}^-$ $|[cd]c>$

Two decay channels: $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+, pD^+K^-$

SELEX Collaboration / Physics Letters B 628 (2005) 18–24

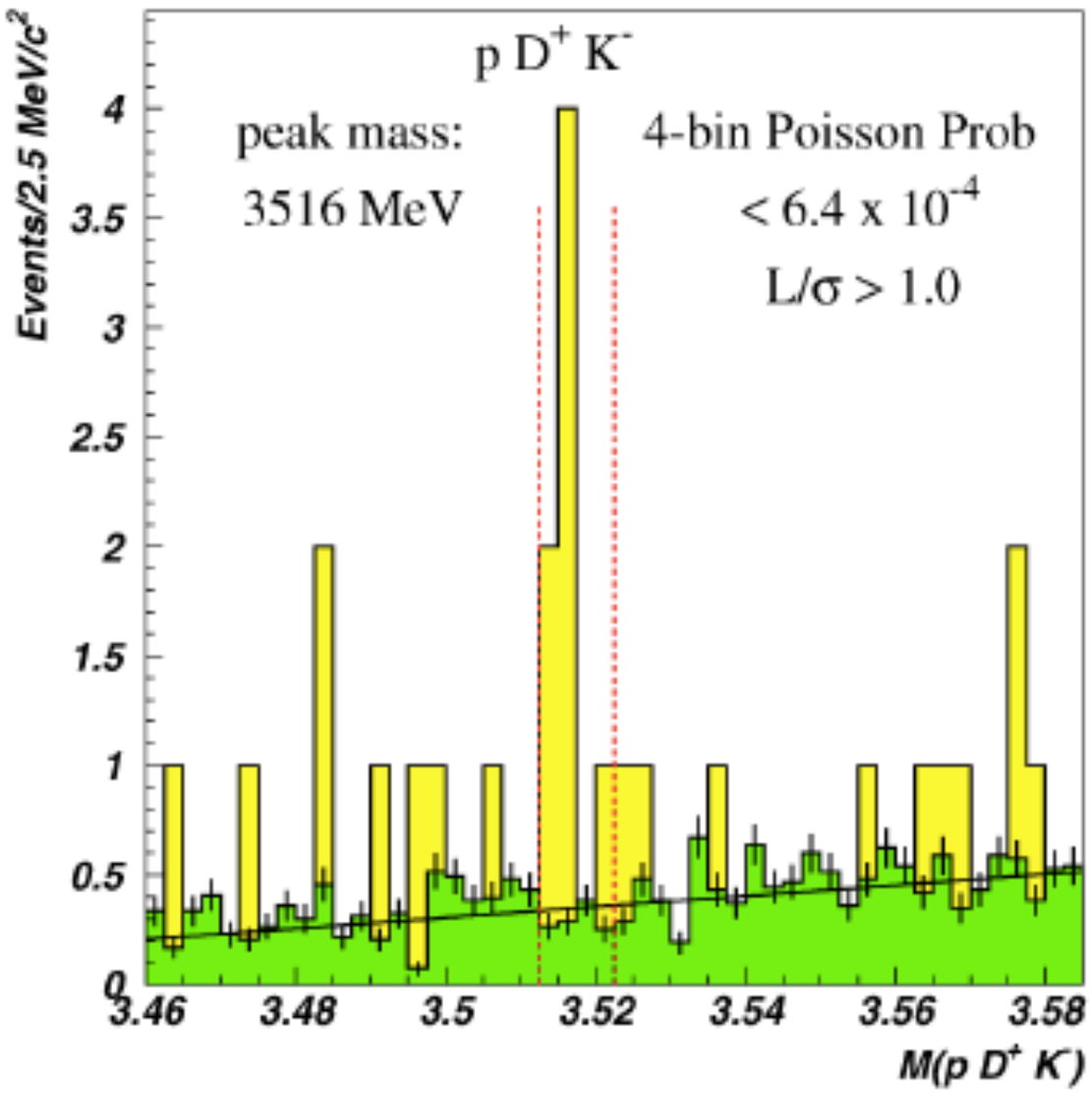


Fig. 3. $\Xi_{cc}^+ \to pD^+K^-$ mass distribution from Fig. 2(a) with high-statistics measurement of random combinatoric background computed from event-mixing.

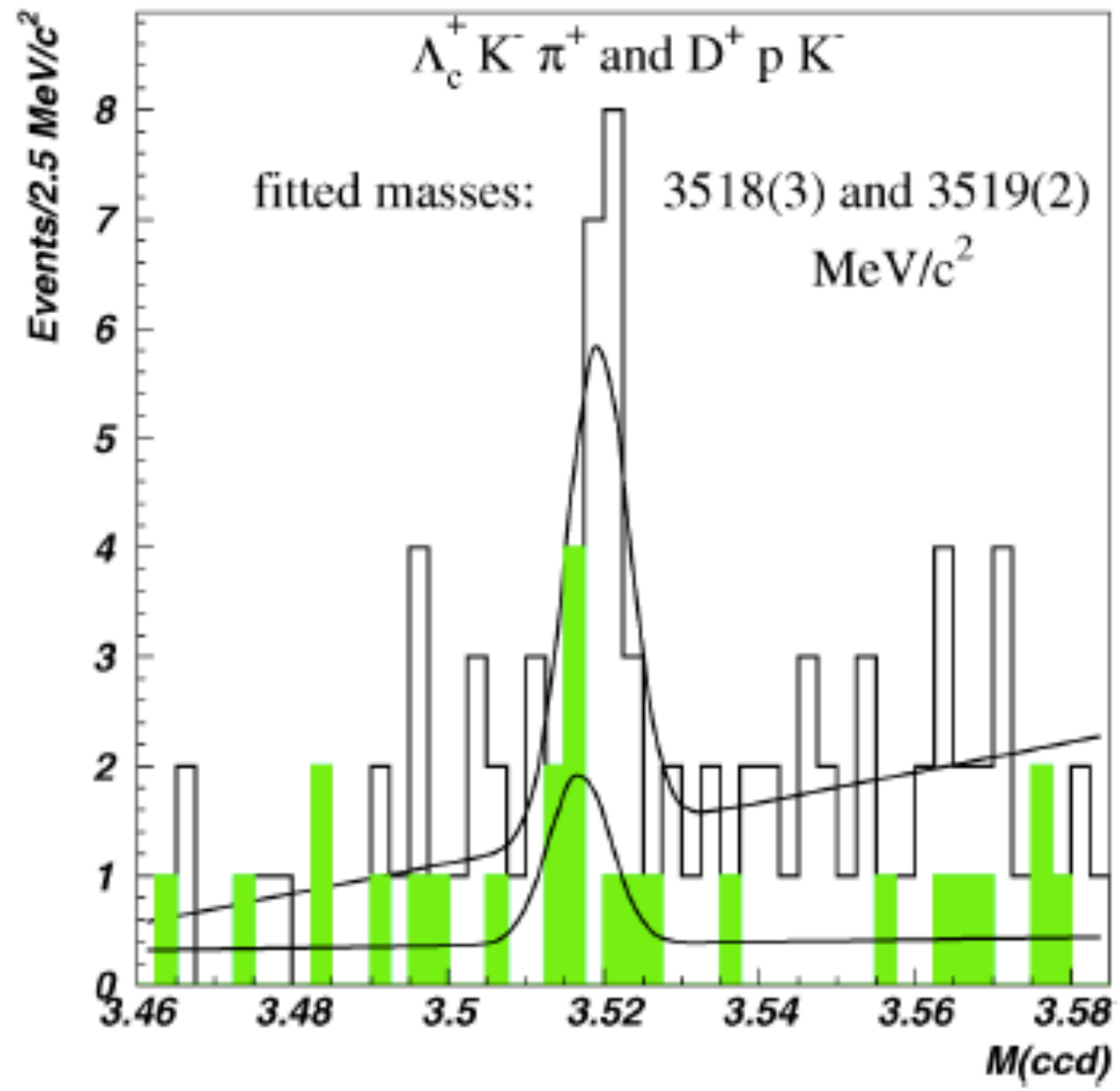
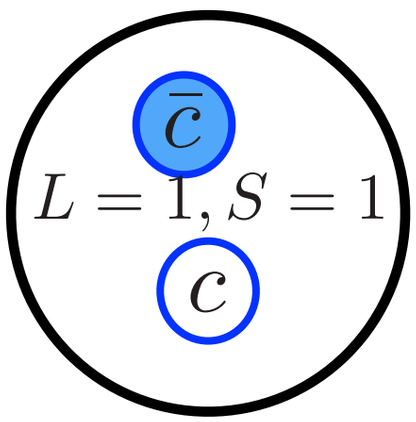


Fig. 4. Gaussian fits for $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+ \to pD^+K^-$ (shaded data) on same plot.

Double-Charm Baryon (LHCb)

$$R^\dagger_\lambda \quad \bar{q} \to (qq) \quad S = 1$$
$$\bar{3}_C \to \bar{3}_C$$

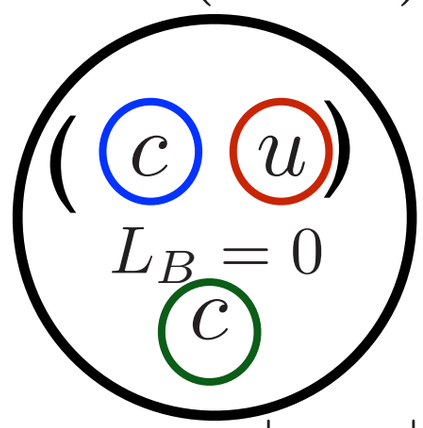
$\chi_{c2}(3556)$



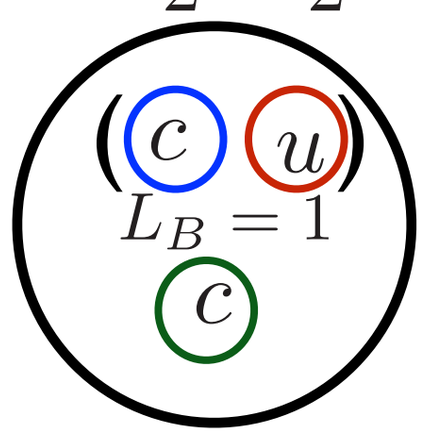
J/ψ'

$J^{PC} = 2^{++}$

$\Xi^{++}(3621)$



$J^P = \frac{1}{2}^+, \frac{3}{2}^+$

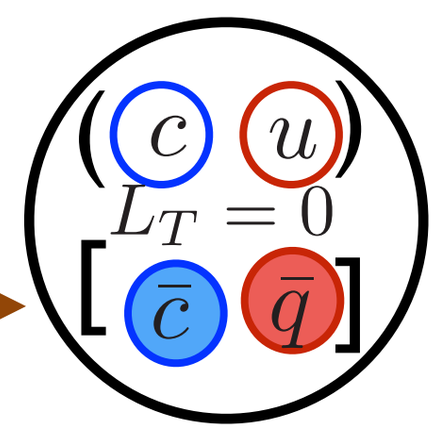


Predict Tetraquark $T_{c\bar{c}q\bar{q}}$

$M_T \sim 3621\ MeV$

Vector and Scalar Diquarks

$J^P = 1^+$



$$R^\dagger_\lambda \quad q \to [\bar{q}\bar{q}] \quad S = 0$$
$$3_C \to 3_C$$

$$\text{SELEX } (3520 \pm 1\ MeV) \quad J^P = \tfrac{1}{2}^- \quad |[cd]c>$$

Two decay channels: $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+, pD^+K^-$

$$\text{LHCb } (3621 \pm 1\ MeV) \quad J^P = \tfrac{1}{2}^- \text{ or } \tfrac{3}{2}^- \quad |(cu)c>$$

$$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$$

Groote, Koshkarev, sjb: SELEX& LHCb could both be correct!

Very different production kinematics:

LHCb (central region)

SELEX (Forward, High x_F) where Λ_c , Λ_b were discovered

NA3: Double J/ψ Hadroproduction measured at High x_F

Radiative Decay:

$\text{LHCb}(3621) \to \text{SELEX}(3520) + \gamma$

strongly suppressed: $[\frac{100\ MeV}{M_c}]^7$

Also: Different diquark structure possible for LHCb: $|(cc)u>$

Karliner and Rosner

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

Complete Regge spectrum in n, L

$$3_C \times 3_C = \bar{3}_C + 6_C$$

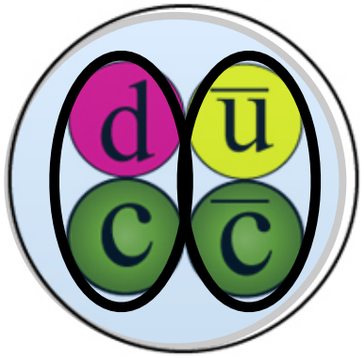
Bound!

- Diquark Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states
- Confinement Force Similar to quark-antiquark mesons $\bar{3}_C \times 3_C = 1_C$
- Isospin $I = 0, \pm 1, \pm 2$ Charge $Q = 0, \pm 1, \pm 2$

$u u \bar{d} \bar{d}$ $u u \bar{s} \bar{d}$ $u u \bar{s} \bar{s}$

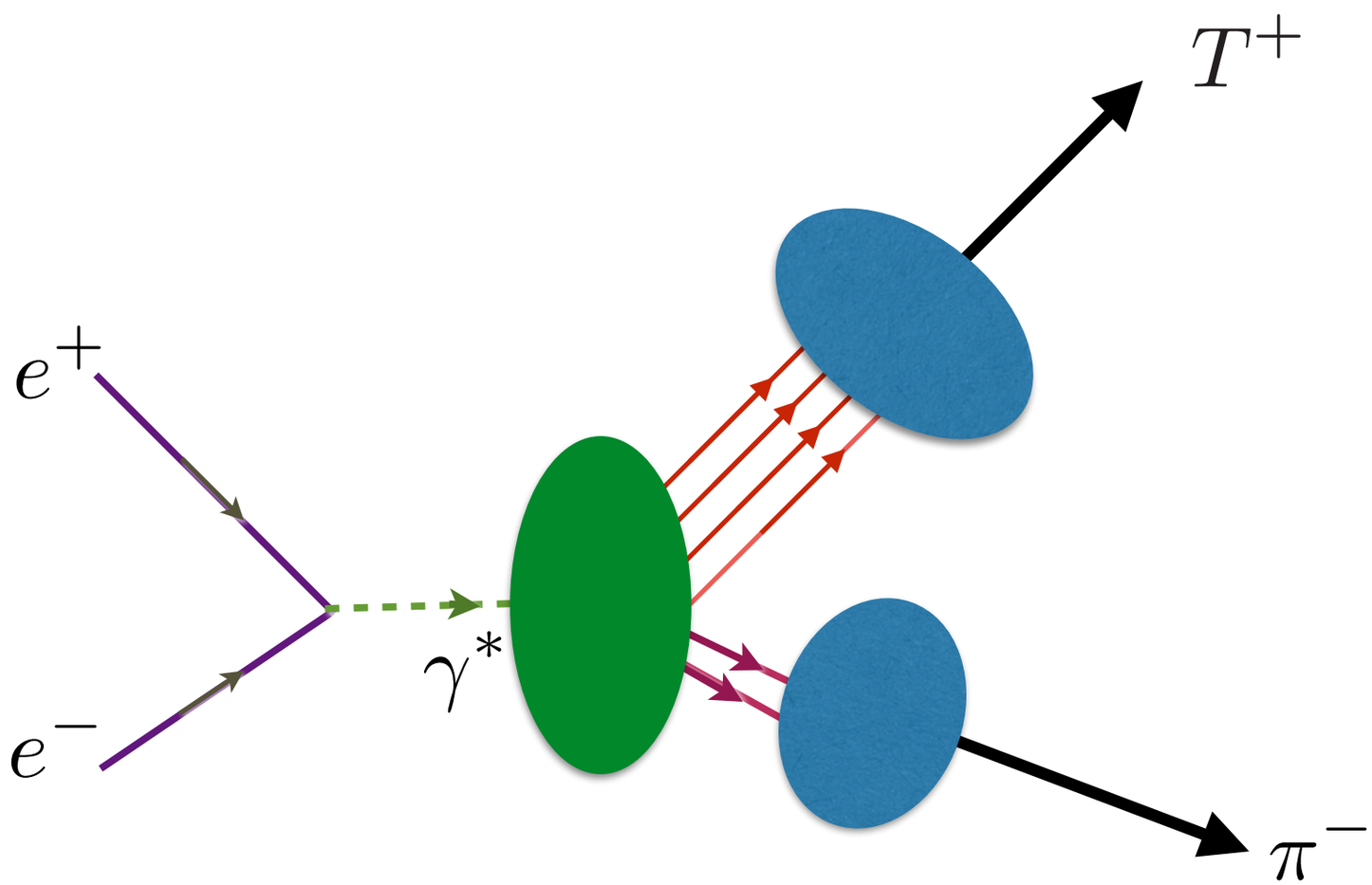
Q= +2

Z(4430)



Q= - 1

$$\sigma(e^+e^- \to MT) \propto \frac{1}{s^{N-1}} \quad N = 6$$



Use counting rules to identify composite structure

Lebed, sjb

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad\qquad S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2}\, g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

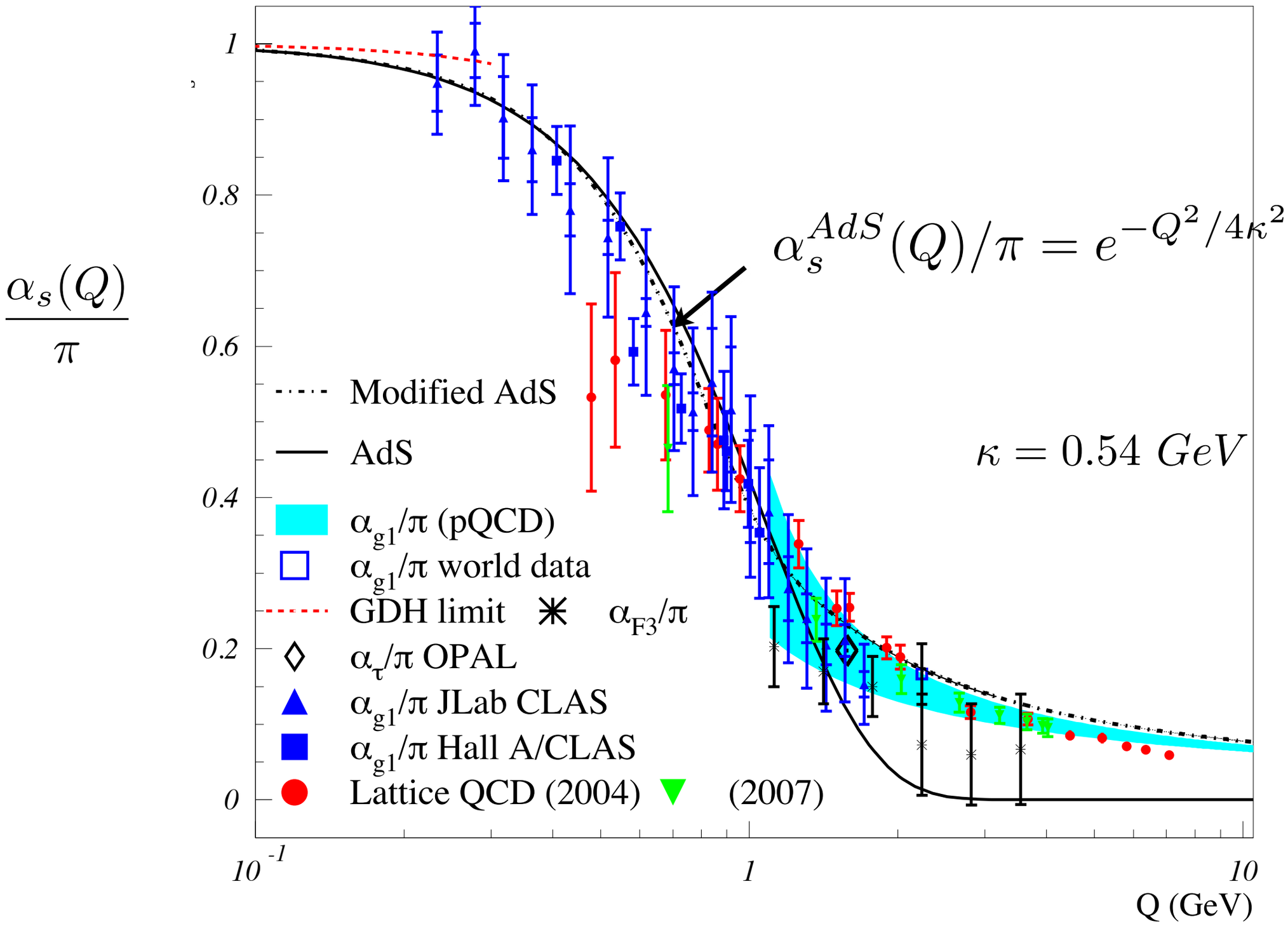
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal*** β_0, β_1

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

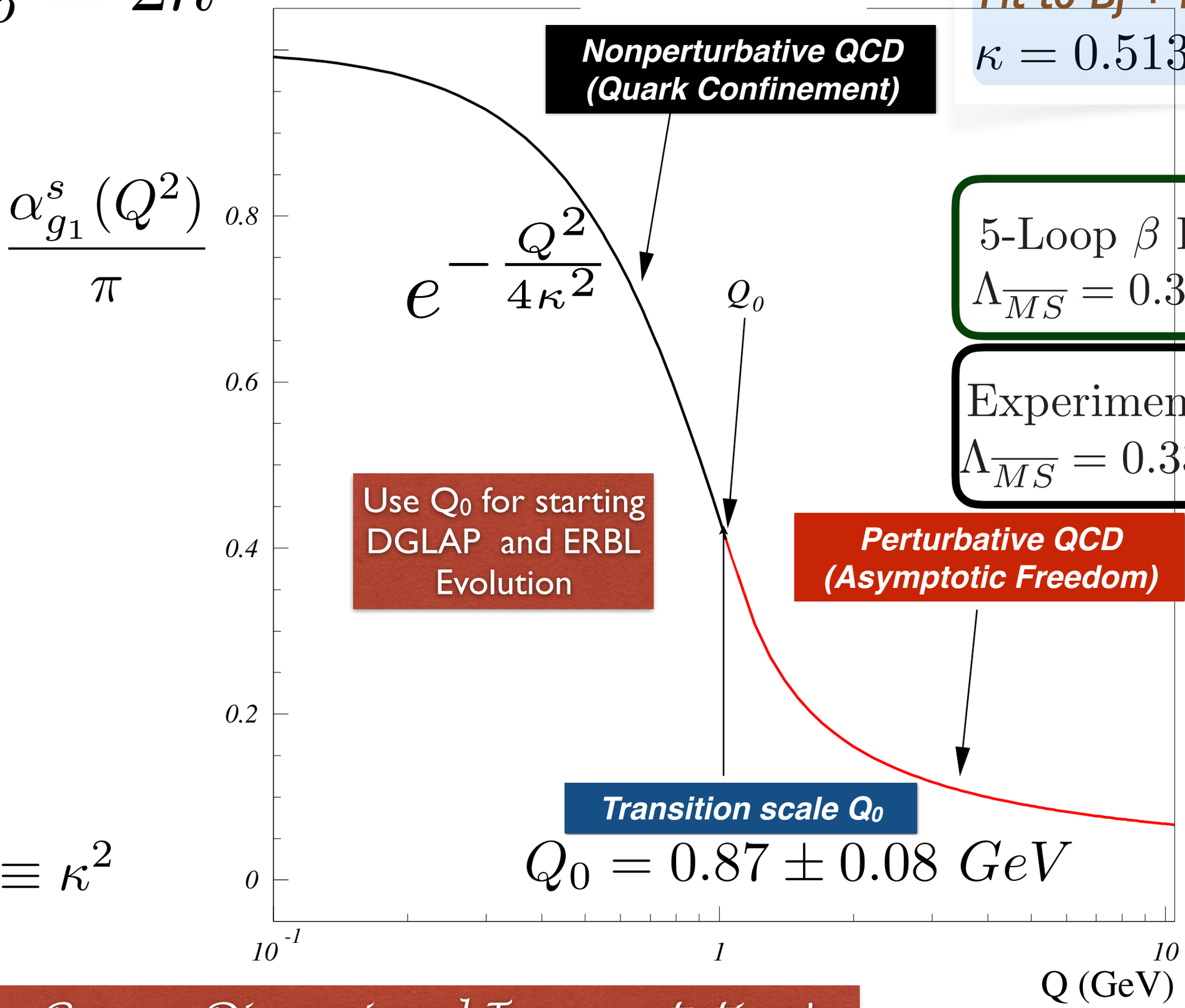
Deur, de Teramond, sjb

$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

All-Scale QCD Coupling

Fit to Bj + DHG Sum Rules:
$$\kappa = 0.513 \pm 0.007\ GeV$$



5-Loop β Prediction:
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$$

Experiment:
$$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$$

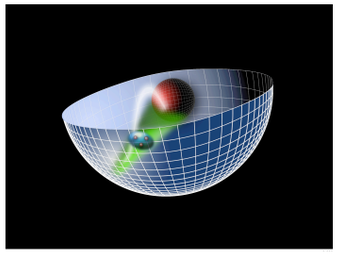
$$\lambda \equiv \kappa^2$$

Reverse Dimensional Transmutation!

$\overline{MS}$ *scheme*

Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time* τ
- *Causality: Information within causal horizon: Light-Front*
- *Light-Front Holography: AdS$_5$ = LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_\perp^2 x(1-x)$$

- *Introduce Mass Scale* κ *while retaining the Conformal Invariance of the Action (dAFF)*
- *Unique Dilaton in AdS$_5$:* $e^{+\kappa^2 z^2}$
- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

Features of LF Holographic QCD

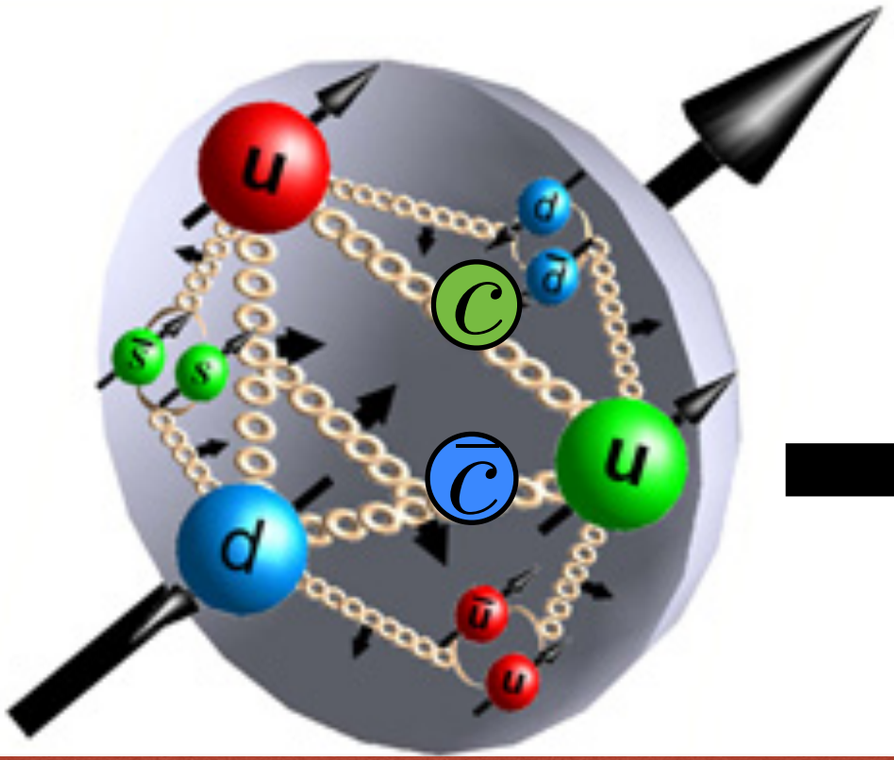
- Color Confinement, Analytic form of confinement potential
- Massless pion bound state in chiral limit
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincare' Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Analytic First Approximation to QCD
- Systematically improvable: Basis LF Quantization (BLFQ)

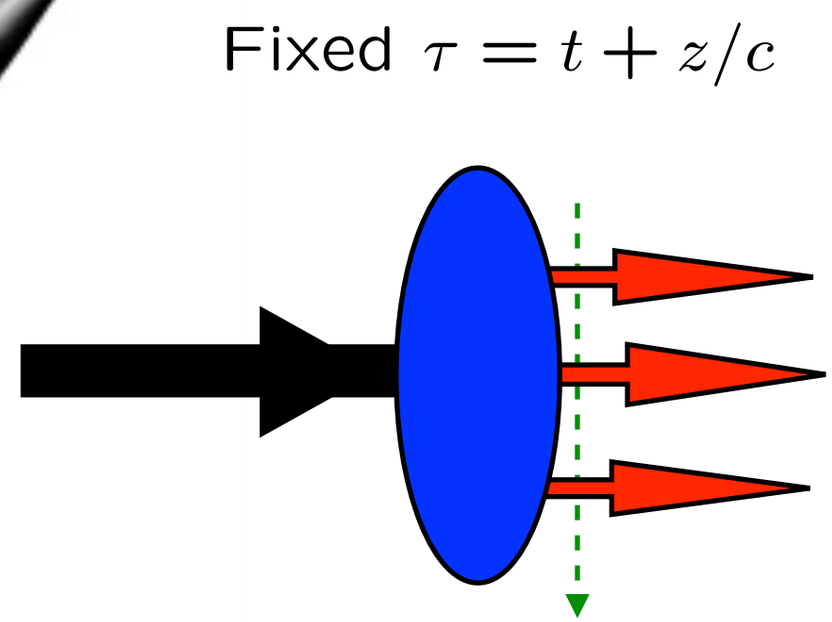
Many phenomenological tests

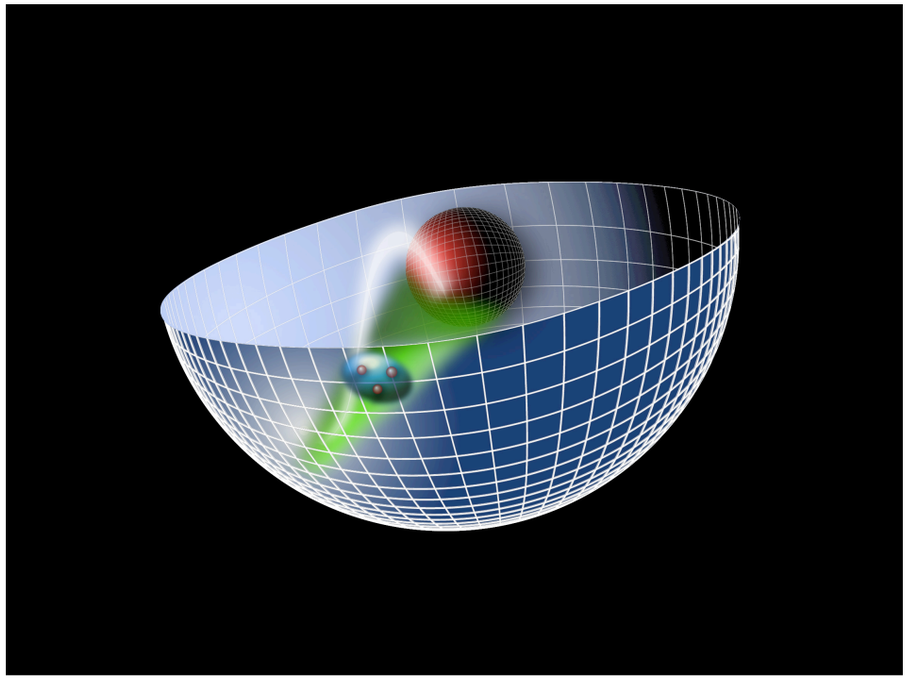
Ansatz:

- Gluons subsumed in the LF confining potential
- Glueballs and glue-quark hybrids absent
- pentaquarks: bound by QCD van der Waals interaction
- Nuclear-Bound Quarkonium allowed
- Krisch Effect:
  Anomalous R_{NN} observed in $\frac{d\sigma}{dt}(pp \to \bar{p}p)$
  large-θ_{CM} elastic scattering at $\sqrt{s} = 5\ GeV$
  due to "octoquark" $|uuduudc\bar{c}>$ production

Dynamically-Generated Exotic Hadrons from Superconformal Algebra and Light-Front Holography







Exotic Hadrons and Flavor Physics May 28, 2018

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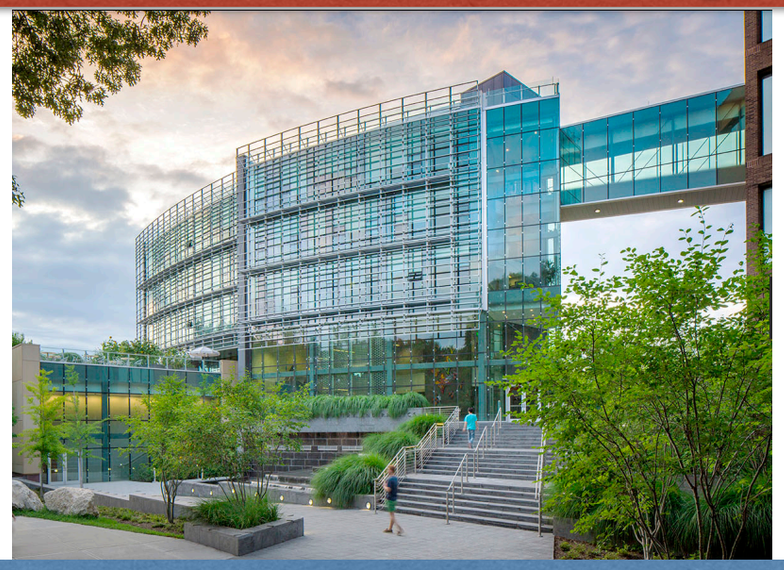

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with Guy de Tèramond, Hans Günter Dosch, Marina Nielsen, F. Navarra, Liping Zou, S. Groote, S. Koshkarev, C. Lorcè, R. S. Sufian, A. Deur