

# Compact Tetra- and Pentaquarks

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DESY, Hamburg

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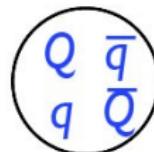
Workshop: Exotic Hadrons and Flavor Physics,  
Simons Center for Geometry and Physics

- Models for  $X, Y, Z$  Mesons
- The Diquark model of Tetraquarks
- Mass Spectrum of the low-lying  $S$  and  $P$  Wave Tetraquark States
- A New Look at the excited  $\Omega_c$  and the  $Y$  States in the Diquark Model
- The Pentaquarks  $\mathbb{P}^\pm(4380)$  and  $\mathbb{P}^\pm(4450)$  in the Diquark Model
- Doubly Heavy Tetraquarks - Prospects at a Tera-Z Factory and LHC
- Summary

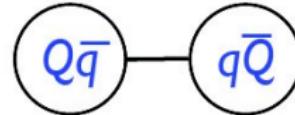
## Models for XYZ Mesons

### Quarkonium Tetraquarks

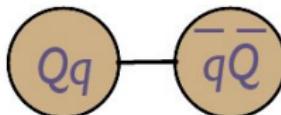
- compact tetraquark



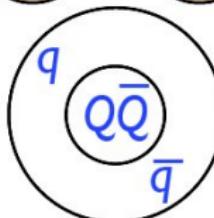
- meson molecule



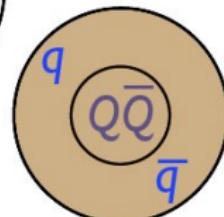
- diquark-onium



- hadro-quarkonium



- quarkonium adjoint meson

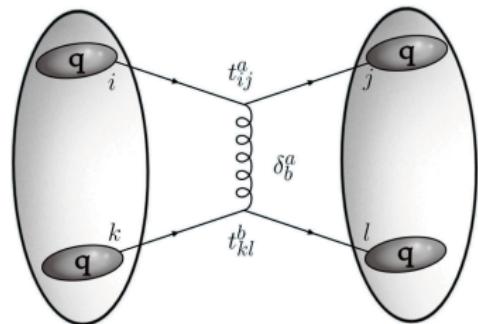


# Diquarks: Color Representation

- One gluon exchange model [Jaffe, Phys.Rept.(2005)]

→ Color factor determines binding:

Negative sign → Attractive



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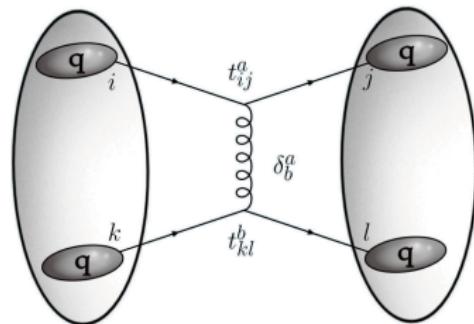
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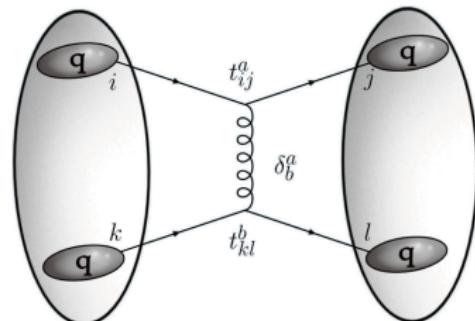
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- $qq$  bound state color factor:

$$t_{ij}^a t_{kl}^a = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{\mathbf{3}}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } \mathbf{6}}$$



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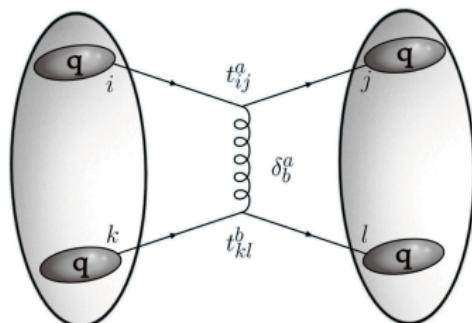
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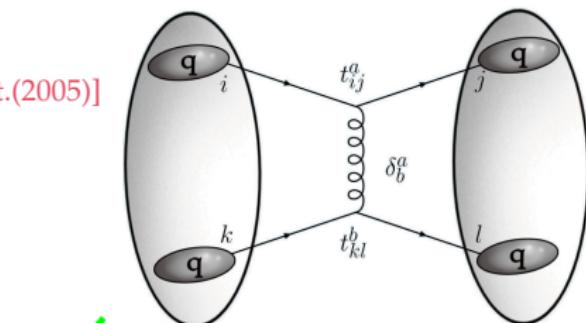
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✓ ✗

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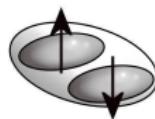
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# Diquarks: Spin representation

$s=1/2$



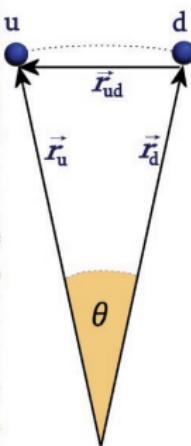
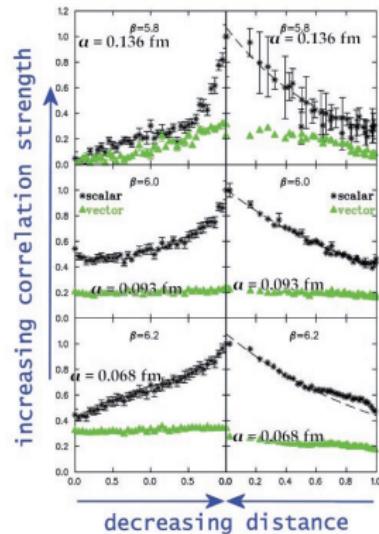
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$s=1$



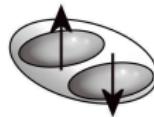
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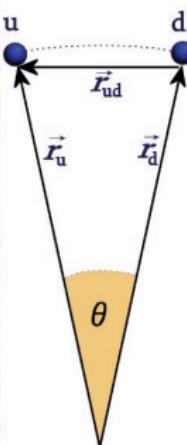
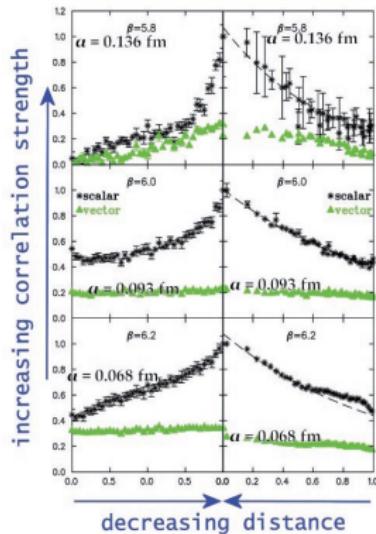
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Lattice simulations for light quarks  
[Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Calculation of 2 quark correlation strength
- Decreasing distance
- Increasing strength for "good" diquarks
- Diquark size  $\mathcal{O}(1\text{fm})$

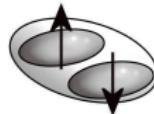
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Lattice simulations for light quarks

[Alexandrou, Forcrand, Lucini, PRL (2006)] :

- Binding for “good” spin 0 diquarks
- No binding for “bad” spin 1 diquarks

- Calculation of 2 quark correlation strength
- Decreasing distance ↗ Increasing strength for “good” diquarks
- Diquark size  $\mathcal{O}(1\text{fm})$

Spin decoupling in HQ-Limit

↗ “Bad” diquarks in  $b$ -sector might bind

## Diquark Model of Tetra- and Pentaquarks

Diquarks and Anti-diquarks are the building blocks of Tetraquarks

Color representation:  $\bar{3} \otimes 3 = \bar{3} \oplus 6$ ; only  $\bar{3}$  is attractive;  $C_{\bar{3}} = 1/2 C_3$

Interpolating diquark operators for the two spin-states of diquarks

$$\text{Scalar: } 0^+ \quad Q_{i\alpha} = \epsilon_{\alpha\beta\gamma} (\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{i_c}^\beta \gamma_5 c^\gamma) \quad \alpha, \beta, \gamma: SU(3)_C \text{ indices}$$

$$\text{Axial-Vector: } 1^+ \quad \vec{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma} (\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{i_c}^\beta \vec{\gamma} c^\gamma)$$

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NR limit: States parametrized by Pauli matrices :

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Diquark spin  $s_Q \rightarrow$  tetraquark total angular momentum  $J$ :

$$|Y_{[bq]}\rangle = |s_Q, s_{\bar{Q}}; J\rangle$$

→ Tetraquarks:  $|0_Q, 0_{\bar{Q}}; 0_J\rangle = \Gamma^0 \otimes \Gamma^0$

$$|1_Q, 1_{\bar{Q}}; 0_J\rangle = \frac{1}{\sqrt{3}} \Gamma^i \otimes \Gamma_i \dots$$

$$|0_Q, 1_{\bar{Q}}; 1_J\rangle = \Gamma^0 \otimes \Gamma^i$$

## NR Hamiltonian for Tetraquarks with hidden charm

Involves constituent diquark mass, spin-spin, spin-orbit, and tensor forces

$$H = 2m_Q + H_{SS}^{(qq)} + H_{SS}^{(q\bar{q})} + H_{SL} + H_{LL} + H_T$$

- In the following, assume  $\kappa_{q\bar{q}'} \simeq 0$

$$H_{\text{eff}}(X, Y, Z) = 2m_Q + \frac{B_Q}{2}L^2 + 2A_Q(L \cdot S) + 2\kappa_{qQ}[s_q \cdot s_Q + s_{\bar{q}} \cdot s_{\bar{Q}}] + b_Y \frac{S_{12}}{4}$$

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constituent mass

$$= b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})] ; \quad (\mathbf{n} = \text{unit vector})$$

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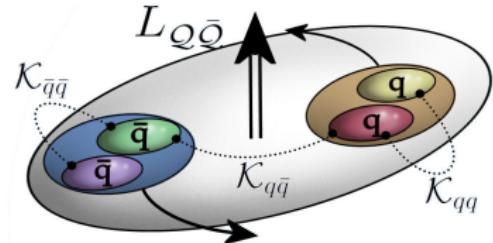
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$qq$  spin coupling

$$H_{SS}^{(qq)} = 2(\mathcal{K}_{cq})_{\bar{3}}[(\mathbf{S}_c \cdot \mathbf{S}_q) + (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{\bar{q}})]$$

$q\bar{q}$  spin coupling

$$\begin{aligned} H_{SS}^{(q\bar{q})} &= 2(\mathcal{K}_{c\bar{q}})(\mathbf{S}_c \cdot \mathbf{S}_{\bar{q}} + \mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) \\ &\quad + 2\mathcal{K}_{c\bar{c}}(\mathbf{S}_c \cdot \mathbf{S}_{\bar{c}}) + 2\mathcal{K}_{q\bar{q}}(\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}) \end{aligned}$$



$$= b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})] ; \quad (\mathbf{n} = \text{unit vector})$$

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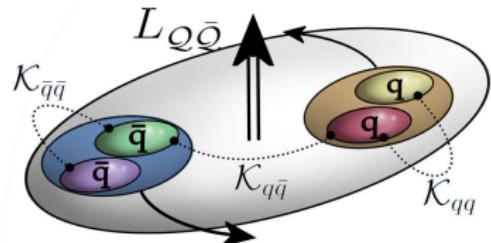
$$H_{SL} = 2A_Q(\mathbf{S}_Q \cdot \mathbf{L} + \mathbf{S}_{\bar{Q}} \cdot \mathbf{L})$$

$$H_{LL} = B_Q \frac{L_{Q\bar{Q}}(L_{Q\bar{Q}} + 1)}{2}$$

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$$H_{SL} = 2A_Q(\mathbf{S}_Q \cdot \mathbf{L} + \mathbf{S}_{\bar{Q}} \cdot \mathbf{L})$$

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$$H_T = b_Y \frac{S_{12}}{4} = b_Y [3(\mathbf{S}_Q \cdot \mathbf{n})(\mathbf{S}_{\bar{Q}} \cdot \mathbf{n}) - (\mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}})] ; \quad (\mathbf{n} = \text{unit vector})$$

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## Low-lying S-Wave Tetraquark States

- In the  $|s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$  and  $|s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$  bases, the positive parity S-wave tetraquarks are listed below;  $M_{00} = 2m_Q$

Label	$J^{PC}$	$ s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	$ s_{q\bar{q}}, s_{Q\bar{Q}}; S', L'\rangle_J$	Mass
$X_0$	$0^{++}$	$ 0, 0; 0, 0\rangle_0$	$( 0, 0; 0, 0\rangle_0 + \sqrt{3} 1, 1; 0, 0\rangle_0)/2$	$M_{00} - 3\kappa_{qQ}$
$X'_0$	$0^{++}$	$ 1, 1; 0, 0\rangle_0$	$(\sqrt{3} 0, 0; 0, 0\rangle_0 -  1, 1; 0, 0\rangle_0)/2$	$M_{00} + \kappa_{qQ}$
$X_1$	$1^{++}$	$( 1, 0; 1, 0\rangle_1 +  0, 1; 1, 0\rangle_1)/\sqrt{2}$	$ 1, 1; 1, 0\rangle_1$	$M_{00} - \kappa_{qQ}$
$Z$	$1^{+-}$	$( 1, 0; 1, 0\rangle_1 -  0, 1; 1, 0\rangle_1)/\sqrt{2}$	$( 1, 0; 1, 0\rangle_1 -  0, 1; 1, 0\rangle_1)/\sqrt{2}$	$M_{00} - \kappa_{qQ}$
$Z'$	$1^{+-}$	$ 1, 1; 1, 0\rangle_1$	$( 1, 0; 1, 0\rangle_1 +  0, 1; 1, 0\rangle_1)/\sqrt{2}$	$M_{00} + \kappa_{qQ}$
$X_2$	$2^{++}$	$ 1, 1; 2, 0\rangle_2$	$ 1, 1; 2, 0\rangle_2$	$M_{00} + \kappa_{qQ}$

- The spectrum of these states depends on just two parameters,  $M_{00}(Q)$  and  $\kappa_{qQ}$ ,  $Q = c, b$ , hence very predictive
- Some of the states, such as  $X_0, X'_0, X_2$ , still missing and are being searched for at the LHC

# Charmonium-like and Bottomonium-like Tetraquark Spectrum

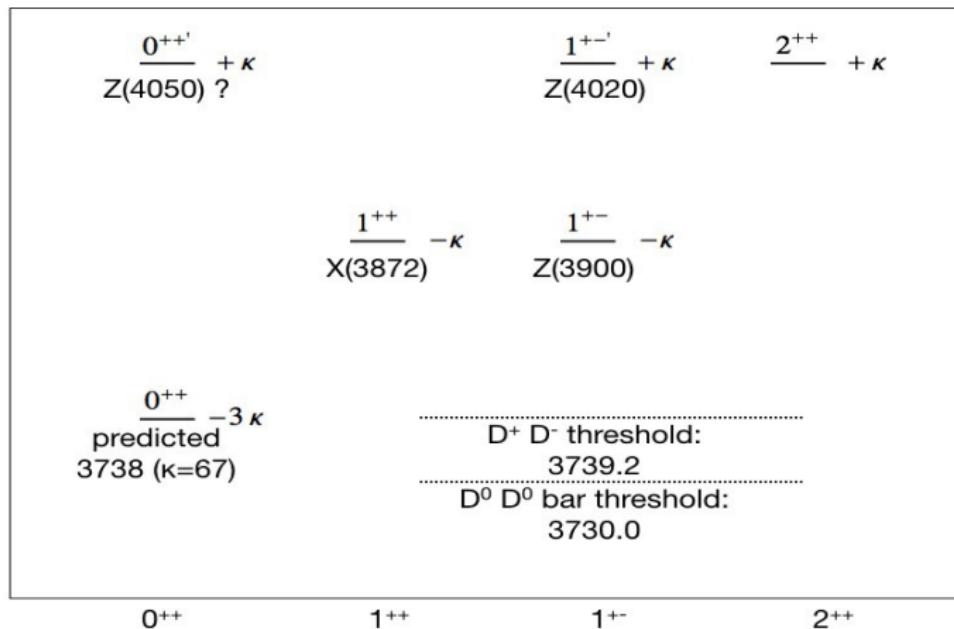
## Parameters in the Mass Formula

	charmonium-like	bottomonium-like
$M_{00}$ [MeV]	3957	10630
$\kappa_{qQ}$ [MeV]	67	22.5

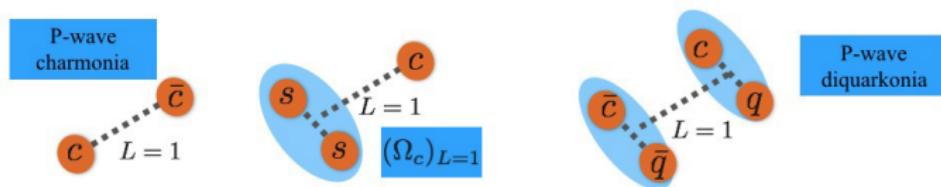
Label	$J^{PC}$	charmonium-like		bottomonium-like	
		State	Mass [MeV]	State	Mass [MeV]
$X_0$	$0^{++}$	—	3756	—	10562
$X'_0$	$0^{++}$	—	4024	—	10652
$X_1$	$1^{++}$	$X(3872)$	3890	—	10607
$Z$	$1^{+-}$	$Z_c^+(3900)$	3890	$Z_b^{+,0}(10610)$	10607
$Z'$	$1^{+-}$	$Z_c^+(4020)$	4024	$Z_b^+(10650)$	10652
$X_2$	$2^{++}$	—	4024	—	10652

# Predictions of the diquark model for the $S$ -wave teraquarks and data

[L. Maiani, A.D. Polosa, V. Riquer]



## $P$ -wave states in charmonia, heavy baryons, and tetraquarks



- Their mass spectra depend on tensor contributions, in addition to the spin-spin and spin-orbit energies

## Excited $\Omega_c$ states in the Diquark model

- Observation of 5 narrow excited  $\Omega_c$  baryons in  $\Omega_c \rightarrow \Xi_c^+ K^-$  [LHCb, PRL 118, 182001 (2017)]
- Masses measured recently [LHCb, Belle] ; Plausible  $J^P$  quantum numbers, assuming diquark model  $\Omega_c (= css) = c[ss]$  [M. Karliner, J.L. Rosner, PR D95, 114012 (2017)]
- For the  $P$  states, important to take into account the tensor couplings

$$H_{\text{eff}} = m_c + m_{[ss]} + \kappa_{ss} S_s \cdot S_s + \frac{B_Q}{2} L^2 + V_{SD},$$

$$V_{SD} = a_1 L \cdot S_{[ss]} + a_2 L \cdot S_c + b \frac{\langle S_{12} \rangle}{4} + c S_{[ss]} \cdot S_c$$

- $b\langle S_{12} \rangle / 4$  represents the matrix element of the tensor interaction

$$\frac{S_{12}}{4} = Q(S_1, S_2) = 3(S_1 \cdot n)(S_2 \cdot n) - (S_1 \cdot S_2)$$

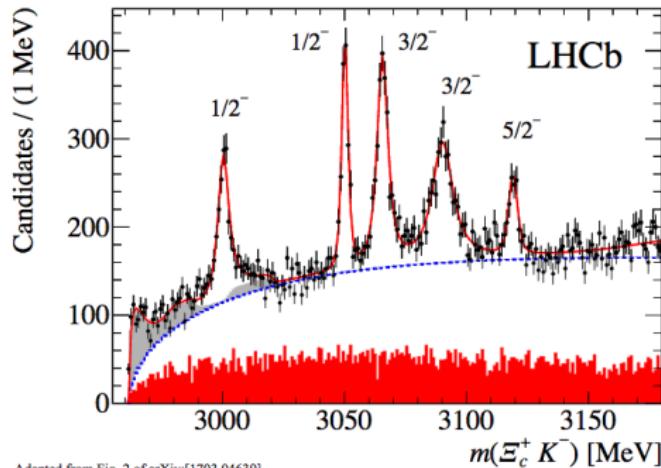
- $S_1 = S_{[ss]}$  and  $S_2 = S_c$  are the spins of the diquark and the charm quark, respectively,  $\vec{n} = \vec{r}/r$  is the unit vector along the radius vector
- $\rightarrow \langle Q(S_X, S_X) \rangle = -\frac{3}{5} \langle [2(L \cdot S_X)^2 + (L \cdot S_X) - \frac{4}{3}(S_X \cdot S_X)] \rangle$   
where  $S_X = S_{[ss]}, S_c, S = S_{[ss]} + S_c$

## Excited $\Omega_c$ states in the Diquark-Quark model- contd.

- Coeffs. determined from the masses of the  $J^P$  states (in MeV)

$a_1$	$a_2$	$b$	$c$	$M_0$
26.95	25.75	13.52	4.07	3079.94

$$M_0 \equiv m_c + m_{[ss]} + 2\kappa_{ss} + B_Q$$



Adapted from Fig. 2 of arXiv:[1703.04639]

## Analysis of the tetraquark Y states in the diquark model

$$\begin{aligned}
 H_{\text{eff}} &= 2m_Q + \frac{B_Q}{2}L^2 - 3\kappa_{cq} + 2a_Y L \cdot S + b_Y \frac{\langle S_{12} \rangle}{4} \\
 &+ \kappa_{cq} [2(S_q \cdot S_c + S_{\bar{q}} \cdot S_{\bar{c}}) + 3] \\
 \frac{1}{4}\langle S_{12} \rangle &= \begin{pmatrix} 0 & 2/\sqrt{5} \\ 2/\sqrt{5} & -7/5 \end{pmatrix}
 \end{aligned}$$

- There are four  $L = 1$  and one  $L = 3$  tetraquark states with  $J^{PC} = 1^{--}$
- Tensor couplings non-vanishing only for the states with  $S_Q = S_{\bar{Q}} = 1$

*P-wave ( $J^{PC} = 1^{--}$ ) states*

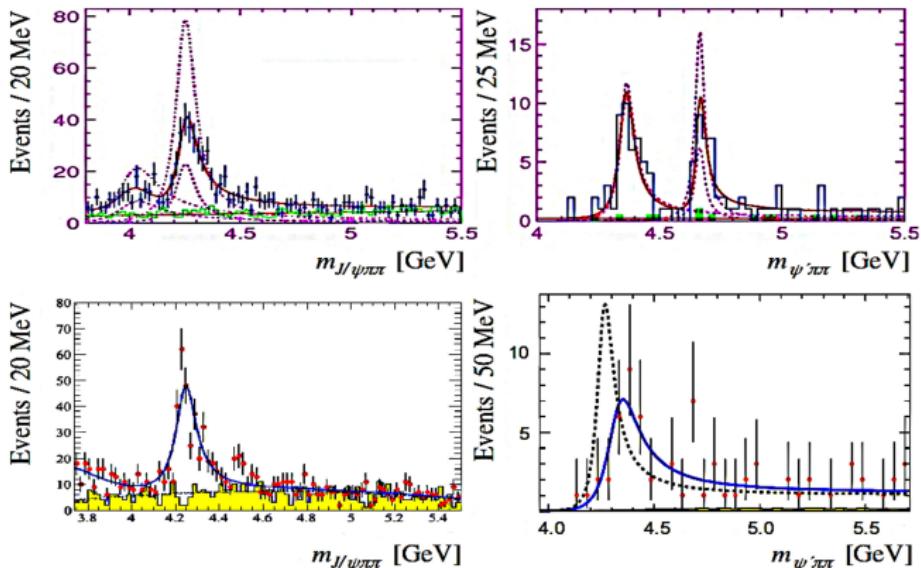
Label	$J^{PC}$	$ s_{qQ}, s_{\bar{q}\bar{Q}}; S, L\rangle_J$	Mass
$Y_1$	$1^{--}$	$ 0, 0; 0, 1\rangle_1$	$M_{00} - 3\kappa_{qQ} + B_Q \equiv \tilde{M}_{00}$
$Y_2$	$1^{--}$	$( 1, 0; 1, 1\rangle_1 +  0, 1; 1, 1\rangle_1)/\sqrt{2}$	$\tilde{M}_{00} + 2\kappa_{qQ} - 2A_Q$
$Y_3$	$1^{--}$	$ 1, 1; 0, 1\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_+$
$Y_4$	$1^{--}$	$ 1, 1; 2, 1\rangle_1$	$\tilde{M}_{00} + 4\kappa_{qQ} + E_-$
$Y_5$	$1^{--}$	$ 1, 1; 2, 3\rangle_1$	$M_{Y_2} + 2\kappa_{qQ} - 14A_Q + 5B_Q - 8/5b_Y$

$$E_{\pm} = \frac{1}{10} (-30A_Q - 7b_Y \mp \sqrt{3} \sqrt{300A_Q^2 + 140A_Qb_Y + 43b_Y^2})$$

## Experimental situation with the tetraquark Y states rather confusing

- Summary of the  $Y$  states observed in Initial State Radiation (ISR) processes in  $e^+e^-$  annihilation [BaBar, Belle, CLEO]

$$e^+e^- \rightarrow \gamma_{\text{ISR}} J/\psi \pi^+ \pi^-; \gamma_{\text{ISR}} \psi' \pi^+ \pi^-$$
$$\implies Y(4008), Y(4260), Y(4360), Y(4660)$$

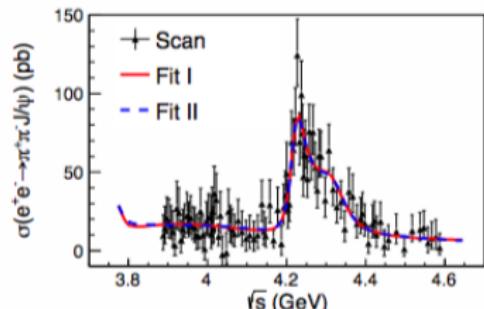
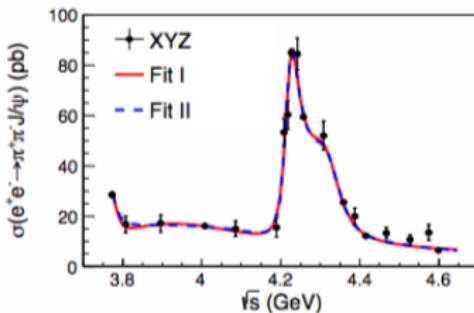


$e^+e^- \rightarrow J/\psi\pi^+\pi^-$  cross section at  $\sqrt{s} = (3.77 - 4.60)$  GeV

( BESIII, PRL 118, 092001 (2017) )

- $Y(4008)$  is not confirmed;  $Y(4260)$  is split into 2 resonances:  $Y(4220)$  and  $Y(4320)$ , with the  $Y(4220)$  probably the same as  $Y(4260)$

Parameters	Fit result
$M(R_1)$	$3812.6^{+61.9}_{-96.6} (\dots)$
$\Gamma_{\text{tot}}(R_1)$	$476.9^{+78.4}_{-64.8} (\dots)$
$M(R_2)$	$4222.0 \pm 3.1 (4220.9 \pm 2.9)$
$\Gamma_{\text{tot}}(R_2)$	$44.1 \pm 4.3 (44.1 \pm 3.8)$
$M(R_3)$	$4320.0 \pm 10.4 (4326.8 \pm 10.0)$
$\Gamma_{\text{tot}}(R_3)$	$101.4^{+25.3}_{-19.7} (98.2^{+25.4}_{-19.6})$

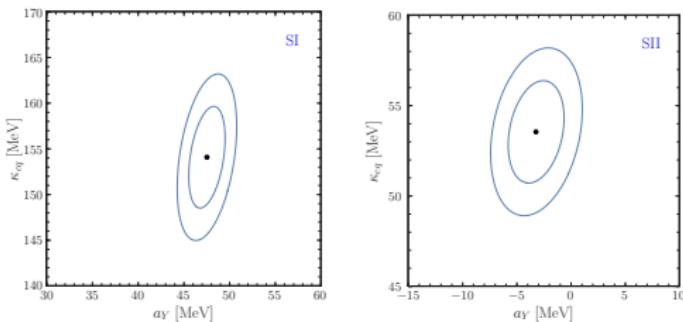


## Two Experimental Scenarios for the Y States

[AA, L. Maiani, A. Borisov, I. Ahmed, A. Rehman, M.J. Aslam, A. Parkhomenko, A.D. Polosa,  
EPJC (2018) 78:29]

- SI (Based on CLEO, BaBar, Belle):  $Y(4008)$ ,  $Y(4260)$ ,  $Y(4360)$ ,  $Y(4660)$
- SII (BESIII, PRL 118, 092001 (2017)):  $Y(4220)$ ,  $Y(4320)$ , with  $Y(4390)$ ,  
 $Y(4660)$  the same as in SI

### $a_Y - \kappa_{cq}$ Correlations



- SII (based on BESIII data) is favored, with  $a_Y$  and  $\kappa_{cq}$  values similar to the  $\Omega_c$  analysis

## Correlations (Contd.)

- Fixing  $\kappa_{cq} = 67 \text{ MeV}$  (from the  $S$  states); fitted the two scenarios  $\implies$  clear preference for SII, with the following parameters (in MeV)

Scenario	$M_{00}$	$a_Y$	$b_Y$	$\chi^2_{\min}/\text{n.d.f.}$
SI	$4321 \pm 79$	$2 \pm 41$	$-141 \pm 63$	12.8/1
SII	$4421 \pm 6$	$22 \pm 3$	$-136 \pm 6$	1.3/1

- SII:  $M_{00} \equiv 2m_Q + B_Q \implies B_Q = 442 \text{ MeV}$
- Comparable to the orbital angular momentum excitation energy in charmonia

$$B_Q(c\bar{c}) = M(h_c) - \frac{1}{4} [3M(J/\psi) + M(\eta_c)] = 457 \text{ MeV}$$

- $\kappa_{cq}$  and  $a_Y$  for  $Y$  states similar to the ones in  $(X, Z)$  and  $\Omega_c$
- Precise data on the  $Y$ -states is needed to confirm or refute the diquark picture

## Predictions for additional Y states based on SII

$J^{PC}$	$ S_Q, S_{\bar{Q}}; S, L\rangle_J$	$N_1$	$2\mathbf{L} \cdot \mathbf{S}$	$S_{12}/4$	Mass(MeV) best fit Tab. 10.2
$3^{--}$	$ 1, 1; 2, 1\rangle_3$	2	4	-2/5	4630
$2^{--}$	$ 1, 1; 2, 1\rangle_2$	2	-2	+7/5	4253
$2_a^{--}$	$ \frac{(1,0)+(0,1)}{\sqrt{2}}; 1, 1\rangle_2$	1	+2	0	4398
$2^{-+}$	$ 1, 1; 1, 1\rangle_2$	2	+2	-1/5	4559
$2_b^{-+}$	$ \frac{(1,0)-(0,1)}{\sqrt{2}}; 1, 1\rangle_2$	1	+2	0	4398
$1^{-+}$	$ 1, 1; 1, 1\rangle_1$	2	-2	+1	4308
$1_b^{-+}$	$ \frac{(1,0)-(0,1)}{\sqrt{2}}; 1, 1\rangle_1$	1	-2	0	4310
$0^{-+}$	$ 1, 1; 1, 1\rangle_0$	2	-4	-2	4672
$0_b^{-+}$	$ \frac{(1,0)-(0,1)}{\sqrt{2}}; 1, 1\rangle_0$	1	-4	0	4266
$0_a^{--}$	$ \frac{(1,0)+(0,1)}{\sqrt{2}}; 1, 1\rangle_0$	1	-4	0	4266

- $N_1$  is the number of spin-1 diquarks in the state

# The Pentaquarks $P_c^+(4380)$ and $P_c^+(4450)$ as resonant $J/\psi p$ states

- Discovery Channel (LHC;  $\sqrt{s} = 7 \& 8 \text{ TeV}$ ;  $\int L dt = 3 \text{ fb}^{-1}$ )

$$pp \rightarrow b\bar{b} \rightarrow \Lambda_b X; \quad \Lambda_b \rightarrow K^- J/\psi p$$

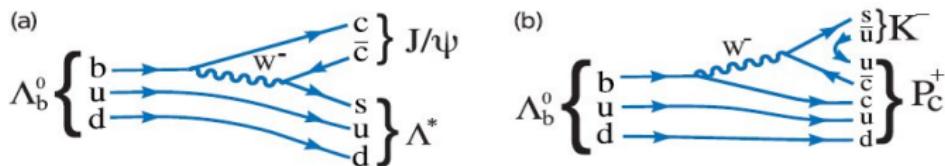


Figure 1: Feynman diagrams for (a)  $A_b^0 \rightarrow J/\psi \Lambda^*$  and (b)  $A_b^0 \rightarrow P_c^+ K^-$  decay.

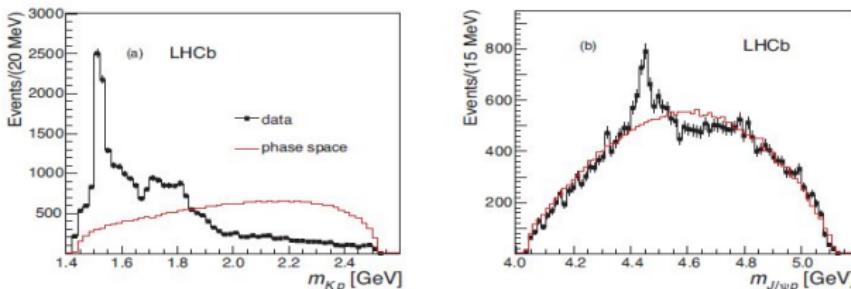
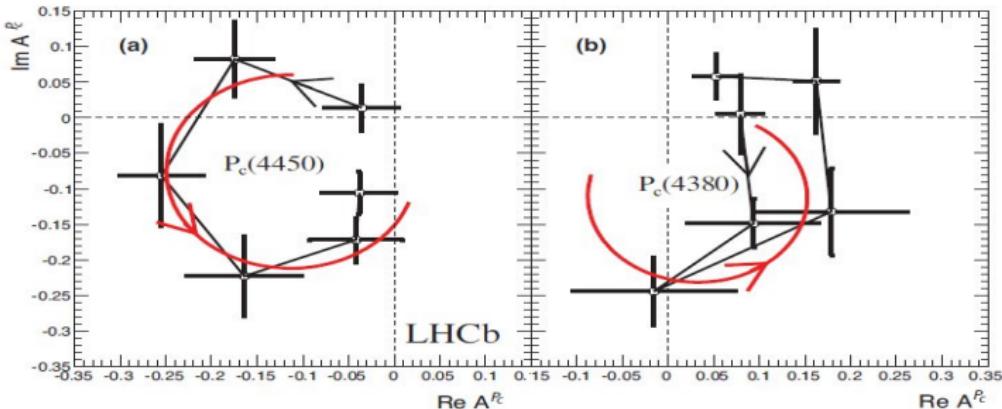


Figure 2: Invariant mass of (a)  $K^- p$  and (b)  $J/\psi p$  combinations from  $A_b^0 \rightarrow J/\psi K^- p$  decays. The solid (red) curve is the expectation from phase space. The background has been subtracted.

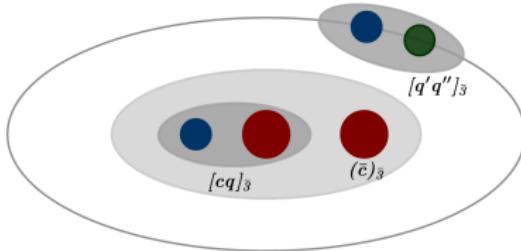
## Summary of the LHCb Pentaquark Measurements

- Higher mass state (statistical significance  $12\sigma$ )  
 $M = 4449.8 \pm 1.7 \pm 2.5$  MeV;  $\Gamma = 39 \pm 5 \pm 19$  MeV
- Lower mass state (statistical significance  $9\sigma$ )  
 $M = 4380 \pm 8 \pm 29$  MeV;  $\Gamma = 205 \pm 18 \pm 86$  MeV
- Fitted Values of the real and imaginary parts of the amplitudes



- For  $P_c^+(4450)$ , fit shows a phase change in amplitudes consistent with a resonance

## Pentaquarks in the diquark model



- Effective Hamiltonian for the  $S$ -wave pentaquark mass spectrum:

$$H^{(L=0)} = H_t + H_d$$

- $H_t$  is related with the colored triquark:

$$H_t = m_c + m_{[cq]} + 2(\mathcal{K}_{cq})_{\bar{3}} (\mathbf{S}_c \cdot \mathbf{S}_q) + 2\mathcal{K}_{\bar{c}c} (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_c) + 2\mathcal{K}_{\bar{c}q} (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q)$$

- $H_d$  describes the light diquark interaction with the color triquark:

$$H_d = m_{[q'q'']} + 2(\mathcal{K}_{q'q''})_{\bar{3}} (\mathbf{S}_{q'} \cdot \mathbf{S}_{q''}) + H_{SS}^{t-d}$$

- $H_{SS}^{t-d}$  involves the light diquark and heavy triquark spin-spin terms

$$\begin{aligned} H_{SS}^{t-d} = & 2(\mathcal{K}_{cq'})_{\bar{3}} (\mathbf{S}_c \cdot \mathbf{S}_{q'}) + 2(\mathcal{K}_{qq'})_{\bar{3}} (\mathbf{S}_q \cdot \mathbf{S}_{q'}) + 2(\mathcal{K}_{\bar{c}q'}) (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{q'}) \\ & + 2(\mathcal{K}_{cq''})_{\bar{3}} (\mathbf{S}_c \cdot \mathbf{S}_{q''}) + 2(\mathcal{K}_{qq''})_{\bar{3}} (\mathbf{S}_q \cdot \mathbf{S}_{q''}) + 2(\mathcal{K}_{\bar{c}q''}) (\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{q''}) \end{aligned}$$

## S-wave hidden $c\bar{c}$ Pentaquarks

- Complete orthogonal set of the basis vectors (with  $L_t = L_{[q'q'']} = L = 0$ ):

$$\left| S_{[cq]}, S_t, L_t; S_{[q'q'']}, L_{[q'q'']}; S, L \right\rangle_J$$

Table: Spin-parity  $J^P$  and state vectors of the S-wave pentaquarks.

$J^P$	$\left  S_{[cq]}, S_t, L_t; S_{[q'q'']}, L_{[q'q'']}; S, L \right\rangle_J$
$1/2^-$	$ 0, 1/2, 0; 0, 0; 1/2, 0\rangle_{1/2}$
$1/2^-$	$ 1, 1/2, 0; 0, 0; 1/2, 0\rangle_{1/2}$
$3/2^-$	$ 1, 3/2, 0; 0, 0; 3/2, 0\rangle_{3/2}$
$1/2^-$	$ 0, 1/2, 0; 1, 0; 1/2, 0\rangle_{1/2}$
$3/2^-$	$ 0, 1/2, 0; 1, 0; 3/2, 0\rangle_{3/2}$
$1/2^-$	$ 1, 1/2, 0; 1, 0; 1/2, 0\rangle_{1/2}$
$3/2^-$	$ 1, 1/2, 0; 1, 0; 3/2, 0\rangle_{3/2}$
$1/2^-$	$ 1, 3/2, 0; 1, 0; 1/2, 0\rangle_{1/2}$
$3/2^-$	$ 1, 3/2, 0; 1, 0; 3/2, 0\rangle_{3/2}$
$5/2^-$	$ 1, 3/2, 0; 1, 0; 5/2, 0\rangle_{5/2}$

- There are 10 states with a fixed light-quark flavour content, 5 of which have  $J^P = 1/2^-$ , 4 have  $J^P = 3/2^-$  and one is with  $J^P = 5/2^-$

## *P*-wave hidden $c\bar{c}$ Pentaquarks (25 States)

Table: Spin-parity  $J^P$  and state vectors of the *P*-wave pentaquarks

$J^P$	$\left  S_{[cq]}, S_t, L_t; \textcolor{magenta}{S}_{[q'q'']}, L_{[q'q'']}; S, L \right\rangle_J$
$1/2^+$	$ 0, 1/2, 0; \textcolor{red}{0}, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$ 0, 1/2, 0; \textcolor{red}{0}, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$ 1, 1/2, 0; \textcolor{red}{0}, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$ 1, 1/2, 0; \textcolor{red}{0}, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$ 1, 3/2, 0; \textcolor{red}{0}, 1; 3/2, 1\rangle_{1/2}$
$3/2^+$	$ 1, 3/2, 0; \textcolor{red}{0}, 1; 3/2, 1\rangle_{3/2}$
$5/2^+$	$ 1, 3/2, 0; \textcolor{red}{0}, 1; 3/2, 1\rangle_{5/2}$
$1/2^+$	$ 0, 1/2, 0; 1, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$ 0, 1/2, 0; 1, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$ 0, 1/2, 0; 1, 1; 3/2, 1\rangle_{1/2}$
$3/2^+$	$ 0, 1/2, 0; 1, 1; 3/2, 1\rangle_{3/2}$
$5/2^+$	$ 0, 1/2, 0; 1, 1; 3/2, 1\rangle_{5/2}$
$1/2^+$	$ 1, 1/2, 0; 1, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$ 1, 1/2, 0; 1, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$ 1, 1/2, 0; 1, 1; 3/2, 1\rangle_{1/2}$
$3/2^+$	$ 1, 1/2, 0; 1, 1; 3/2, 1\rangle_{3/2}$
$5/2^+$	$ 1, 1/2, 0; 1, 1; 3/2, 1\rangle_{5/2}$
$1/2^+$	$ 1, 3/2, 0; 1, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$ 1, 3/2, 0; 1, 1; 1/2, 1\rangle_{3/2}$
$1/2^+$	$ 1, 3/2, 0; 1, 1; 3/2, 1\rangle_{1/2}$
$3/2^+$	$ 1, 3/2, 0; 1, 1; 3/2, 1\rangle_{3/2}$
$5/2^+$	$ 1, 3/2, 0; 1, 1; 3/2, 1\rangle_{5/2}$
$3/2^+$	$ 1, 3/2, 0; 1, 1; 5/2, 1\rangle_{3/2}$
$5/2^+$	$ 1, 3/2, 0; 1, 1; 5/2, 1\rangle_{5/2}$
$7/2^+$	$ 1, 3/2, 0; 1, 1; 5/2, 1\rangle_{7/2}$

# Pentaquarks in the diquark model

[L. Maiani, A.D. Polosa, V. Riquer, Phys.Lett. B749, 289 (2015)]

- $\Lambda_b(bud) \rightarrow \mathbb{P}^+ K^-$  decaying according to  $\mathbb{P}^+ \rightarrow J/\Psi + p$
- $\mathbb{P}^+$  carry a unit of baryonic number and have the valence quarks

$$\mathbb{P}^+ = \bar{c} c u u d$$

- Assume the assignments

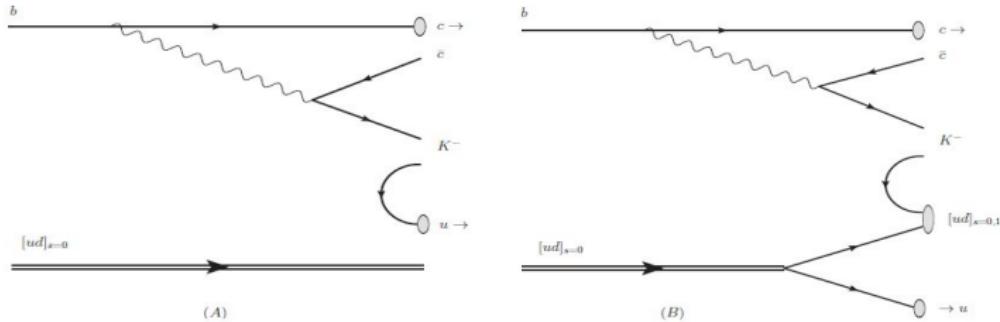
$$\mathbb{P}^+(3/2^-) = \{\bar{c} [cq]_{s=1} [q'q'']_{s=1}, L=0\}$$

$$\mathbb{P}^+(5/2^+) = \{\bar{c} [cq]_{s=1} [q'q'']_{s=0}, L=1\}$$

- Mass difference:
  - Level spacing for  $\Delta L = 1$  in light baryons;  $\Lambda(1405) - \Lambda(1116) \sim 290$  MeV
  - Light-light diquark mass difference for  $\Delta S = 1$ :  
 $[qq']_{s=1} - [qq']_{s=0} = \Sigma_c(2455) - \Lambda_c(2286) \simeq 170$  MeV
- Orbital gap  $\mathbb{P}^+(3/2^-) - \mathbb{P}^+(5/2^+)$  is thereby reduced to 120 MeV, more or less in agreement with data, 70 MeV

## Pentaquark production mechanisms in $\Lambda_b^0 \rightarrow K^- J/\psi p$

- Two possible mechanisms are proposed by Maiani et al.
  - In the first,  $b$ -quark spin is shared between the  $K^-$ , and the  $\bar{c}$  and  $[cu]$  components, the final  $[ud]$  diquark has spin-0, Fig. A
  - In the second, the  $[ud]$  diquark is formed from the original  $d$  quark, and the  $u$  quark from the vacuum  $u\bar{u}$ ; angular momentum is shared among all components, and the diquark  $[ud]$  may have both spins,  $s = 0, 1$ , Fig. B
- Which of the two diagrams dominate is a dynamical question; semileptonic decays of  $\Lambda_b$  hint that the mechanism in Fig. B is dynamically suppressed



# Heavy quark symmetry and observed pentaquarks

[Ahmed, Rehman, Aslam, AA, Phys. Rev. D94, 054001 (2016)]

Selection rules from the data on  $b \rightarrow c$  baryonic decays and HQS

$$P_c^+(4450) = \{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_P = 1, J^P = \frac{5}{2}^+\} \quad \text{Favored}$$

$$P_c^+(4380) = \{\bar{c}[cu]_{s=1}[ud]_{s=1}; L_P = 0, J^P = \frac{3}{2}^-\} \quad \text{Disfavored}$$

$\implies \frac{3}{2}^-$  state may require a different interpretation.

$m[\Lambda_c^+(2625); J^P = \frac{3}{2}^-] - m[\Lambda_c^+(2286); J^P = \frac{1}{2}^+] \simeq 341 \text{ MeV} \implies$  the mass of  $J^P = 3/2^-$  state to be about 4110 MeV.

In diquark-diquark-antiquark spectrum,  $\frac{3}{2}^-$  state is favored by HQS,

$$\{\bar{c}[cu]_{s=1}[ud]_{s=0}; L_P = 0, J^P = \frac{3}{2}^-\},$$

Third state anticipated in 4110-4130 MeV range. A renewed fit of the LHCb data by allowing a third resonance is called for.

## Weak decays of the $b$ -baryons into pentaquark states

$$\mathcal{A} = \langle \mathcal{PM} | H_{\text{eff}}^W | \mathcal{B} \rangle, \text{ with } H_{\text{eff}}^W = \frac{4G_F}{\sqrt{2}} \left[ V_{cb} V_{cq}^* (c_1 O_1^{(q)} + c_2 O_2^{(q)}) \right]$$

$H_{\text{eff}}^W$  inducing the Cabibbo-allowed  $\Delta I = 0, \Delta S = -1$  transition  $b \rightarrow c\bar{c}s$ , and the Cabibbo-suppressed  $\Delta S = 0$  transition  $b \rightarrow c\bar{c}d$ .

$$O_1^{(q)} = (\bar{q}_\alpha c_\beta)_{V-A} (\bar{c}_\alpha b_\beta)_{V-A} \text{ and } O_2^{(q)} = (\bar{q}_\alpha c_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A}$$

$$\mathcal{B}_{ij}(\bar{\mathbf{3}}) = \Lambda_b^0(usb), \Xi_b^0(usb), \Xi_b^-(dsb), \quad \mathcal{C}_{ij}(\mathbf{6}) = \Sigma_b^-(ddb), \Sigma_b^0(usb), \Sigma_b^+(uub), \Xi_b'(dsb), \Xi_b'^0(usb), \Omega_b^-(ssb)$$

$$\mathcal{M}_i^j = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \quad \mathcal{P}_i^j(J^P) = \begin{pmatrix} \frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_{\Sigma^+} & P_p \\ P_{\Sigma^-} & -\frac{P_{\Sigma^0}}{\sqrt{2}} + \frac{P_\Lambda}{\sqrt{6}} & P_n \\ P_{\Xi^-} & P_{\Xi^0} & -\frac{P_\Lambda}{\sqrt{6}} \end{pmatrix}.$$

**A decuplet  $\mathcal{P}_{ijk}$ :**  $\mathcal{P}_{111} = P_{\Delta_{10}^{++}}$ ,  $\mathcal{P}_{112} = P_{\Delta_{10}^+}/\sqrt{3}$ ,  $\mathcal{P}_{122} = P_{\Delta_{10}^0}/\sqrt{3}$ ,  $\mathcal{P}_{222} = P_{\Delta_{10}^-}$ ,  $\mathcal{P}_{113} = P_{\Sigma_{10}^+}/\sqrt{3}$ ,  $\mathcal{P}_{123} = P_{\Sigma_{10}^0}/\sqrt{6}$ ,  $\mathcal{P}_{223} = P_{\Sigma_{10}^-}/\sqrt{3}$ ,  $\mathcal{P}_{133} = P_{\Xi_{10}^0}/\sqrt{3}$ ,  $\mathcal{P}_{233} = P_{\Xi_{10}^-}/\sqrt{3}$  and  $\mathcal{P}_{333} = P_{\Omega_{10}^-}$ .

- ◊ Calculating the decay amplitudes is a formidable challenge.
- ◊  $SU(3)_F$  symmetry relations provided useful guide for pentaquark searches, Li *et al.* [JHEP 1512 (2015) 128]; Jing Wu *et al.* [PRD 95 (2017) 034002]; Rui Chen *et al.* [Chin. Phys. C41 (2017) 103105]

## $SU(3)$ based analysis of $\Lambda_b \rightarrow \mathbb{P}^+ K^- \rightarrow (J/\psi p) K^-$

- With respect to flavor  $SU(3)$ ,  $\Lambda_b$  (*bud*)  $\sim \bar{\mathbf{3}}$ , and is isosinglet  $I = 0$
- The weak non-leptonic Hamiltonian for  $b \rightarrow c\bar{c}s$  decays is:

$$H_W^{(3)}(\Delta I = 0, \Delta S = -1)$$

- With  $M$  a nonet of  $SU(3)$  light mesons,  $\langle \mathbb{P}, M | H_W | \Lambda_b \rangle$  requires  $\mathbb{P} + M$  to be in  $\mathbf{8} \oplus \mathbf{1}$  representation
- Recalling the  $SU(3)$  group multiplication rule

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{\bar{10}} \oplus \mathbf{27}$$

$$\mathbf{8} \otimes \mathbf{10} = \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35}$$

the decay  $\langle \mathbb{P}, M | H_W | \Lambda_b \rangle$  can be realized with  $\mathbb{P}$  in either an octet (8) or a decuplet (10)

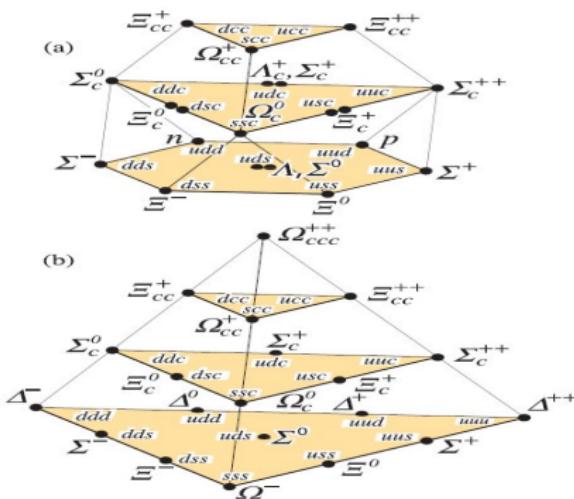
- The discovery channel  $\Lambda_b \rightarrow \mathbb{P}^+ K^- \rightarrow J/\psi p K^-$  corresponds to  $\mathbb{P}$  in an octet (8)

## Weak decays with $\mathbb{P}$ in Decuplet representation

- Decays involving the decuplet (10) pentaquarks may also occur, if the light diquark pair having spin-0  $[ud]_{s=0}$  in  $\Lambda_b$  gets broken to produce a spin-1 light diquark  $[ud]_{s=1}$

$$\Lambda_b \rightarrow \pi \mathbb{P}_{10}^{(S=-1)} \rightarrow \pi(J/\psi \Sigma(1385))$$

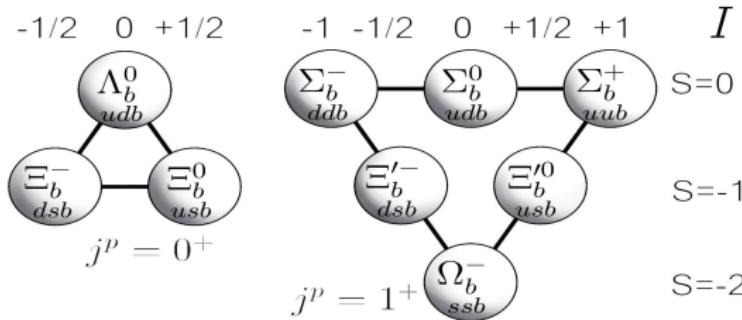
$$\Lambda_b \rightarrow K^+ \mathbb{P}_{10}^{(S=-2)} \rightarrow K^+ (J/\psi \Xi^-(1530))$$



**Figure 15.4:** SU(4) multiplets of baryons made of  $u$ ,  $d$ ,  $s$ , and  $c$  quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

## Weak decays with $\mathbb{P}$ in Decuplet representation - Contd.

- Apart from  $\Lambda_b(bud)$ , several  $b$ -baryons, such as  $\Xi_b^0(usb)$ ,  $\Xi_b^-(dsb)$  and  $\Omega_b^-(ssb)$  undergo weak decays



- Examples of bottom-strange b-baryon in various charge combinations, respecting  $\Delta I = 0$ ,  $\Delta S = -1$  are:

$$\Xi_b^0(5794) \rightarrow K(J/\psi\Sigma(1385))$$

which corresponds to the formation of the pentaquarks with the spin configuration ( $q, q' = u, d$ )

$$\mathbb{P}_{10}(\bar{c} [cq]_{s=0,1} [q's]_{s=0,1})$$

## Weak decays with $\mathbb{P}$ in Decuplet representation - Contd.

- The  $s\bar{s}$  pair in  $\Omega_b$  is in the symmetric (6) representation of flavor  $SU(3)$  with spin 1; expected to produce decuplet Pentaquarks in association with a  $\phi$  or a Kaon

$$\begin{aligned}\Omega_b(6049) &\rightarrow \phi(J/\psi \Omega^-(1672)) \\ \Omega_b(6049) &\rightarrow K(J/\psi \Xi(1387))\end{aligned}$$

- These correspond, respectively, to the formation of the following pentaquarks ( $q = u, d$ )

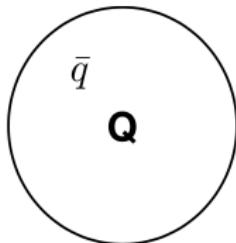
$$\mathbb{P}_{10}^-(\bar{c} [cs]_{s=0,1} [ss]_{s=1})$$

$$\mathbb{P}_{10}(\bar{c} [cq]_{s=0,1} [ss]_{s=1})$$

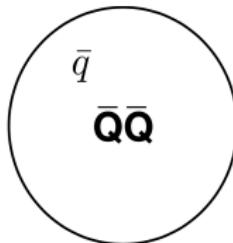
- These transitions are on firmer theoretical footings, as the initial  $[ss]$  diquark in  $\Omega_b$  is left unbroken; more transitions can be found relaxing this condition
- At a mega-Z factory, such as CEPC, the entire  $b$ -baryon multiplet will be measured through the process  $Z \rightarrow b\bar{b}; b \rightarrow (\Lambda_b, \Xi_b, \Omega_b, \dots)$ , allowing to reconstruct a lot of pentaquarks in  $b$ -baryon decays.

# HQ Symmetry relations involving heavy Mesons, Baryons and Tetraquarks

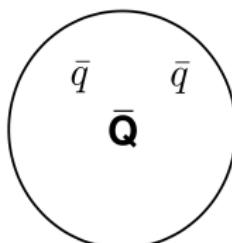
[See Eichten, Karliner, Maltman, Quigg, Voloshin @ this workshop]



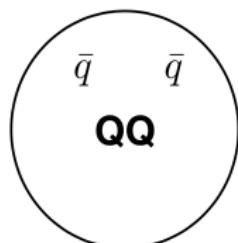
Singly Heavy Meson



Doubly Heavy anti-Baryon



Singly Heavy anti-Baryon



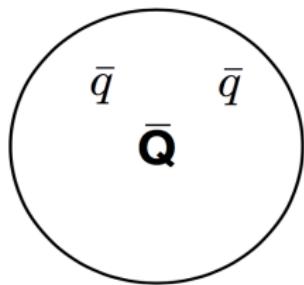
Doubly Heavy Tetraquark

- Heavy quark symmetry relates a singly heavy meson  $Q\bar{q}$  and a doubly heavy antibaryon  $\bar{Q}\bar{Q}\bar{q}$
- Likewise, it relates a singly heavy antibaryon  $\bar{Q}\bar{q}\bar{q}$  and a doubly heavy tetraquark  $Q\bar{Q}\bar{q}\bar{q}$

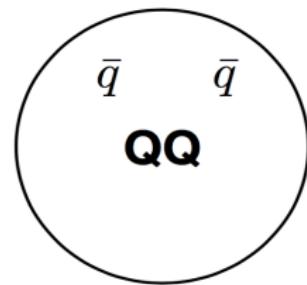
Stable Heavy Tetraquarks  $T(QQ\bar{q}\bar{q}')$ , ( $QQ = cc, cb, bb$ )

### Heavy Quark-Diquark Symmetry (HQDQS)

$m_Q \rightarrow \infty$  **QQ is compact object in color  $\bar{3}$**



Singly Heavy anti-Baryon

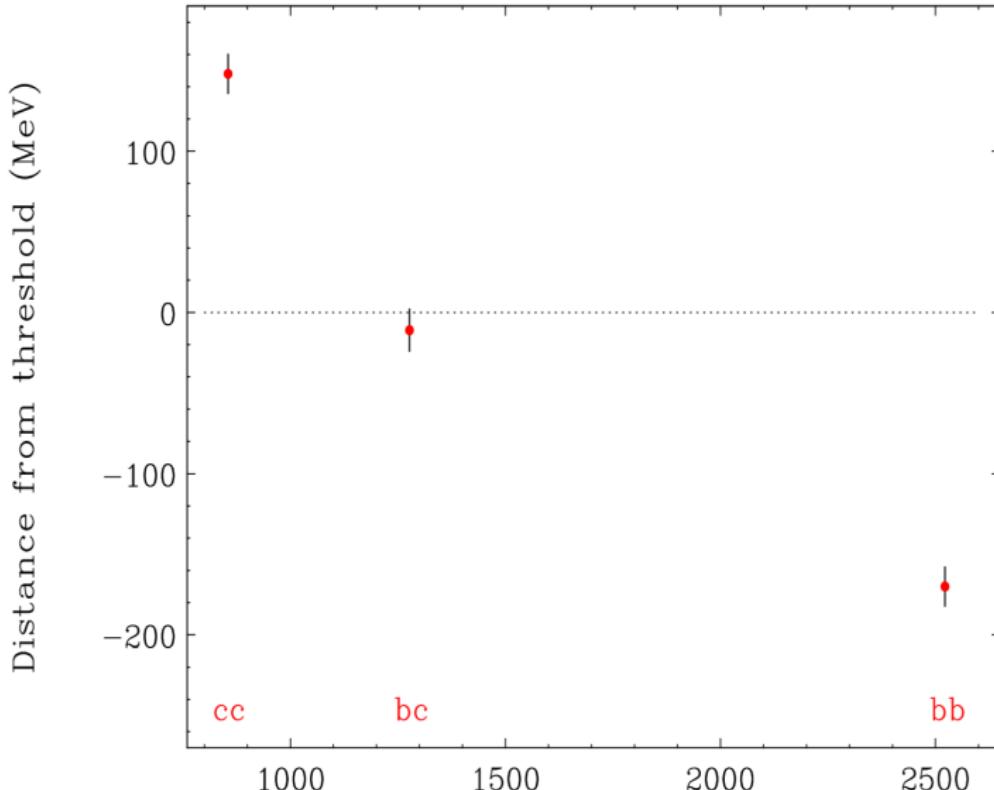


Doubly Heavy Tetraquark

I.d.o.f are the same in these hadrons

# Distance from Thresholds in MeV for $T(QQ\bar{q}\bar{q}')$ , ( $QQ = cc, cb, bb$ )

[M. Karliner, J. Rosner, PRL 119, 202001 (2017)]



# Stable Heavy Tetraquarks $T(QQ\bar{q}\bar{q}')$ , ( $QQ = cc, cb, bb$ )

[E. Eichten, C. Quigg, PRL 119, 202002 (2017)]

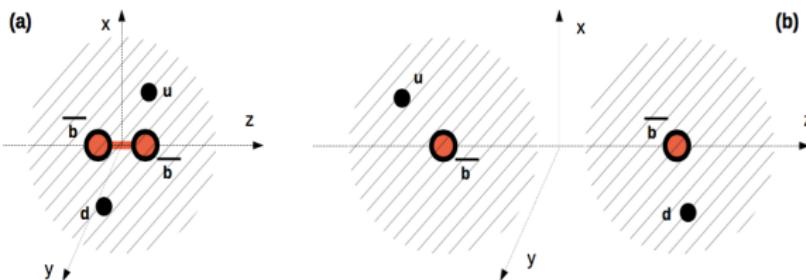
- $T(bb[\bar{u}\bar{d}])$  and  $T(bb[\bar{q}\bar{s}])$  are stable against strong decay
- Will decay weakly by charged current interactions

## Expectations for ground-state tetraquark masses

State	$J^P$	$m(Q_i Q_j \bar{q}_k \bar{q}_l)$	Decay Channel	$\mathcal{Q}$ [MeV]
$\{cc\}[\bar{u}\bar{d}]$	$1^+$	3978	$D^+ D^{*0}$	3876
$\{cc\}[\bar{q}_k \bar{s}]$	$1^+$	4156	$D^+ D_s^{*-}$	3977
$\{cc\}[\bar{q}_k \bar{q}_l]$	$0^+, 1^+, 2^+$	4146, 4167, 4210	$D^+ D^0, D^+ D^{*0}$	3734, 3876
$[bc][\bar{u}\bar{d}]$	$0^+$	7229	$B^- D^+/B^0 D^0$	7146
$[bc][\bar{q}_k \bar{s}]$	$0^+$	7406	$B_s D$	7236
$[bc]\{\bar{q}_k \bar{q}_l\}$	$1^+$	7439	$B^* D / BD^*$	7190/7290
$[bc][\bar{u}\bar{d}]$	$1^+$	7272	$B^* D / BD^*$	7190/7290
$[bc][\bar{q}_k \bar{s}]$	$1^+$	7445	$DB_s^*$	7282
$[bc]\{\bar{q}_k \bar{q}_l\}$	$0^+, 1^+, 2^+$	7461, 7472, 7493	$BD / B^* D$	7146/7190
$\{bb\}[\bar{u}\bar{d}]$	$1^+$	10482	$B^- \bar{B}^{*0}$	10603
$\{bb\}[\bar{q}_k \bar{s}]$	$1^+$	10643	$\bar{B} \bar{B}_s^*/\bar{B}_s \bar{B}^*$	10695/10691
$\{bb\}[\bar{q}_k \bar{q}_l]$	$0^+, 1^+, 2^+$	10674, 10681, 10695	$B^- B^0, B^- B^{*0}$	10559, 10603
				115, 78, 136
				-121
				-48

# QCD dynamics of a doubly heavy tetraquarks $T(QQ\bar{q}\bar{q}')$ , ( $QQ = cc, cb, bb$ )

[P. Bicudo et al., Phys.Rev. D95, 142001 (2017)]

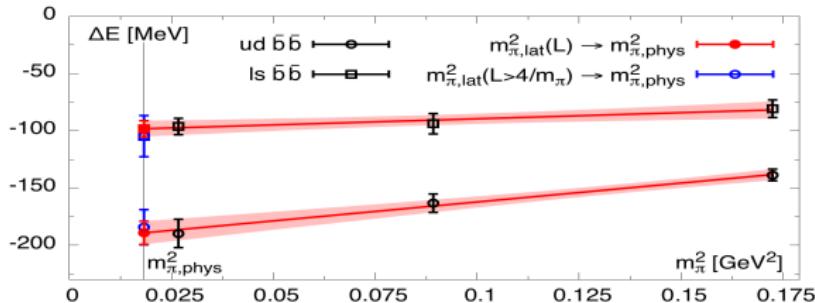


- At very short  $\bar{b}\bar{b}$  distances, the interaction is Coulomb-like, given by one-gluon exchange (a)
- At large  $\bar{b}\bar{b}$  separations, the light quarks  $ud$  screen the interaction, and the four quarks form two rather weakly interacting  $B B^*$  mesons (b)
- Using this (Born-Oppenheimer) potential, a coupled-channel Schrödinger equation is solved, leading to a bound state, whose mass is estimated as  $M(\mathcal{T}_{[\bar{u}\bar{d}]}^{bb})^- = 10545^{+38}_{-30} \text{ MeV.}$

# Lattice QCD estimates of $bb\bar{u}\bar{d}$ tetraquark mass using NRQCD

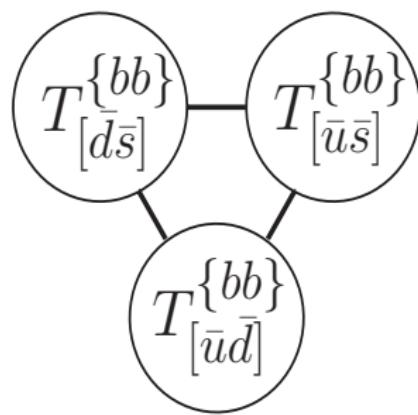
[A. Francis et al., PRL 118, 142001 (2017)]

- Chiral extrapolations of the  $ud\bar{b}\bar{b}$  and  $qs\bar{b}\bar{b}$  binding energies
- $M(\mathcal{T}_{[\bar{u}\bar{d}]}^{\{bb\}-}) = 10415 \pm 10 \text{ MeV}$
- $M(\mathcal{T}_{[\bar{q}\bar{s}]}^{\{bb\}-}) = 10549 \pm 8 \text{ MeV}$
- Both lie below their respective thresholds



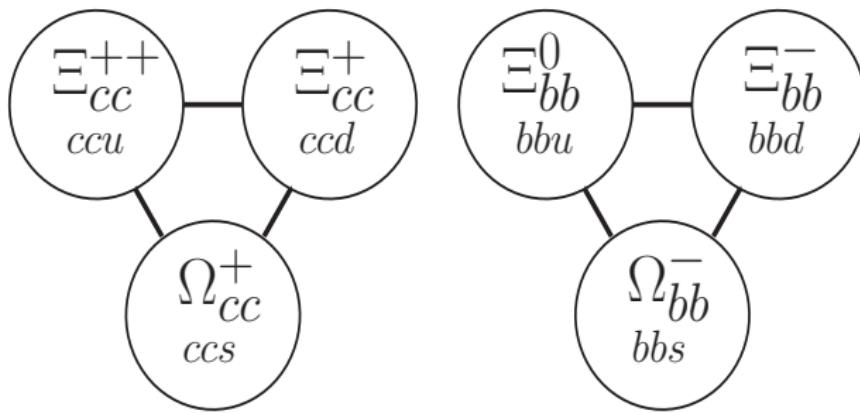
$SU(3)_F$ -triplet of stable double-bottom tetraquarks  $T(bb\bar{q}\bar{q}')$

$$S_{\{bb\}} = 1, S_{[\bar{q}\bar{q}']} = 0, J^P = 1^+$$



$SU(3)_F$ -triplet of double charm & double-bottom baryons

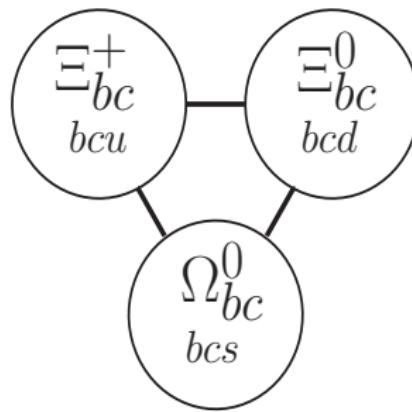
Of these,  $\Xi_{cc}^{++} = (ccu)$  has been discovered at the LHCb



Great discovery potential at the LHC & Tera-Z factory!

## $SU(3)_F$ -triplet of bottom-charmed baryons

So far, the only known bottom-charmed hadrons are  $B_c$  and  $B_c^*$



Likewise, Great discovery potential at the LHC & Tera-Z factory!

# Prospects of observing Stable Tetraquarks at a Tera-Z Factory

[AA, A. Parkhomenko, Qin Qin, Wei Wang, arXiv:1805.02535; to appear in PLB]

- The partonic process at the Z-factory is:

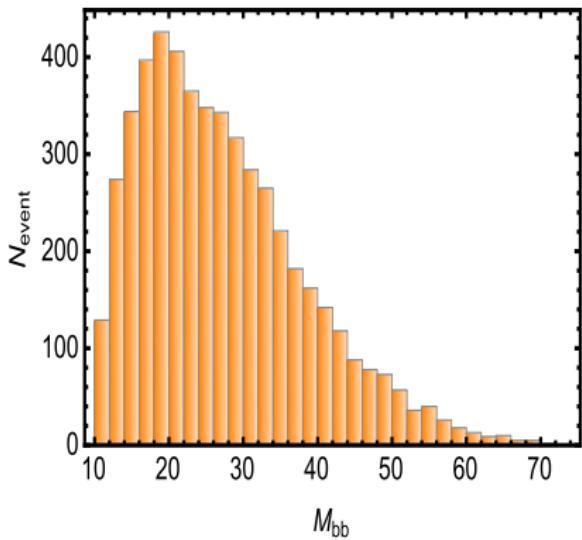
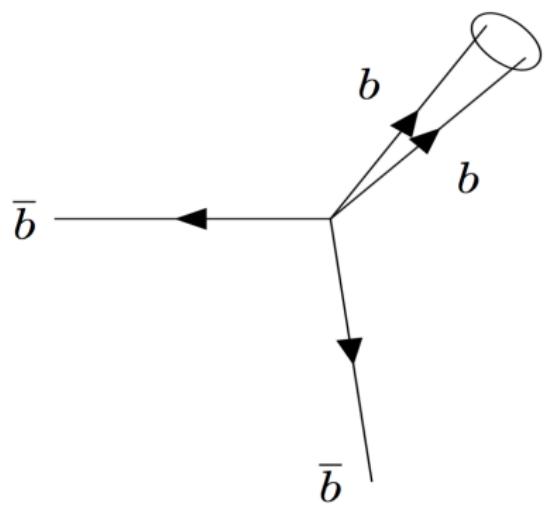
$$e^+ e^- \rightarrow Z \rightarrow b\bar{b}b\bar{b}$$

- Measured at LEP:  $\mathcal{B}(Z \rightarrow b\bar{b}b\bar{b}) = (3.6 \pm 1.3) \times 10^{-4}$
- $Z \rightarrow (B, \Lambda_b, \Sigma_b) + X$  result from the fragmentation of a single- $b$  quark; tetraquarks and baryons with double- $b$  quarks require the topology in which two  $b$ -quarks have to be in a jet:

$$e^+ e^- \rightarrow Z \rightarrow (bb)_{\text{Jet}} + \bar{b}\bar{b}$$

- The  $(bb)_{\text{Jet}}$  requires a jet-definition, such as the invariant mass  $M(bb)_{\text{Jet}}^2 = (p(b_1) + p(b_2))^2$ , with Jet-resolution parameter:  
$$M(T_{[\bar{u}\bar{d}]}^{\{bb\}})^2 \leq M(bb)_{\text{Jet}}^2 \leq (2m_b + \Delta M)^2$$
- For  $M(bb)_{\text{Jet}}^2 \gg 4M_B^2$ , independent fragmentation of the 2  $b$  quarks into two  $B$ -hadrons,  $(bb)_{\text{Jet}} \rightarrow (BB, BB^*, 2\Lambda_b + \dots) + X$  dominates, and the probability of a double- $b$  hadron production becomes insignificant,  $\mathcal{P}(bb \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + X) \ll 1$

## Typical topology of $Z \rightarrow (bb)_{\text{Jet}} + \bar{b}\bar{b}$



- Double- $b$ -hadrons, such as the tetraquark  $T_{[\bar{u}\bar{d}]}^{\{bb\}}$  and double- $b$  baryons  $\Xi_{bb}^q$ , are the fragmentation products of the  $(bb)_{\text{Jet}}$
- They are anticipated to populate low- $M_{bb}$  invariant mass region

## Estimates of the $(bb)_{\text{Jet}}$ -parameter $\Delta M$

[AA, A. Parkhomenko, Qin Qin, Wei Wang, arXiv:1805.02535; to appear in PLB]

- We model  $\Delta M$  by using a similar process which involves the fusion  $b\bar{c} \rightarrow B_c$  from  $e^+e^- \rightarrow Z \rightarrow b\bar{b}c\bar{c}$ , leading to  $e^+e^- \rightarrow Z \rightarrow B_c + \bar{b} + c$ , using NRQCD [Z. Yang *et al.*, Phys. Rev. D 85, 094015 (2012)] and the Monte Carlo generator MadGraph [J. Alwall *et al.*, JHEP 1407, 079 (2014).]
- Parametric uncertainty due to input quark masses (GeV):  
 $(m_b, m_c) = (4.9, 1.5)$  (central);  $(m_b, m_c) = (5.3, 1.2)$  (upper),  
 $(m_b, m_c) = (4.8, 1.5)$

Quark masses	$\sigma(B_c b\bar{c})$ [pb]	$\sigma(b\bar{b}c\bar{c})$ [pb]	$f(\bar{b}c \rightarrow B_c)$	$\Delta M$ [GeV]
central	5.19	64.50	8.05%	2.7
upper	11.41	76.79	14.86%	4.0
lower	2.77	56.75	4.88%	2.2

- This gives an estimate of  $\Delta M$ , which we use for simulating  $e^+e^- \rightarrow Z \rightarrow (bb)_{\text{Jet}} + \bar{b}\bar{b}$

## Estimates of the $(bb)_{\text{Jet}}$ -parameter $\Delta M$

[AA, A. Parkhomenko, Qin Qin, Wei Wang, arXiv:1805.02535; to appear in PLB]

- Processes simulated:  $e^+e^- \rightarrow Z \rightarrow b\bar{b}b\bar{b}$  (inclusive 4b-production) &  $e^+e^- \rightarrow Z \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}}\bar{b}\bar{b} + \dots$  (semi-inclusive),, with the invariant mass cut  $M(T_{[\bar{u}\bar{d}]}^{\{bb\}})^2 \leq M(bb)_{\text{Jet}}^2 \leq (2m_b + \Delta M)^2$
- MadGraph (NLO) yields:  $\mathcal{B}(Z \rightarrow b\bar{b}b\bar{b}) = 3.23 \times 10^{-4}$  for  $m_b = 4.9$  GeV, consistent with the LEP measurements  $(3.6 \pm 1.3) \times 10^{-4}$
- This yields the fraction  $f((bb)_{\text{Jet}}(\Delta M) \rightarrow H_{bb} + X) = (5.1^{+3.9}_{-1.2})\%$
- We assume that the double- $b$  hadron  $H_{bb}$  consists of  $T_{[\bar{q}\bar{q}']}^{\{bb\}}$  (tetraquark) or  $\Xi(bbq)$  (double- $b$  baryon)
- Using heavy quark symmetry, anticipate  
$$\frac{f(bb \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + X)}{f(bb \rightarrow \Xi_{bb} + X)} = \frac{f_{\Lambda_b}}{f_{B_u} + f_{B_d}} (\text{LEP}) \simeq 0.11 \pm 0.02$$
- Estimate:  $\mathcal{B}(Z \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + X) = (1.2^{+1.0}_{-0.3}) \times 10^{-6}$

# Prospects of observing Stable Tetraquarks at the LHC

[AA, Qin Qin, Wei Wang, work in progress]

- Using the CT14NNLO PDF, MadGraph, and Pythia, at  $\sqrt{s} = 13$  TeV:

$$\sigma(pp \rightarrow Z \rightarrow b\bar{b}b\bar{b}) + \sigma(pp \rightarrow Z b\bar{b} \rightarrow b\bar{b}b\bar{b}) \simeq 100 \text{ pb}$$

- With the branching fraction

$$\mathcal{B}(Z \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + b\bar{b} + X) = (1.4^{+1.1}_{-0.5}) \times 10^{-6}$$

- the Z-induced X-section at 13 TeV is:

$$\sigma(pp \rightarrow Z \rightarrow T_{[\bar{u}\bar{d}]}^{\{bb\}} + X) \sim 340 \text{ fb}$$

- The corresponding gluon-induced X-section, using  $\sigma(pp \rightarrow B_c + X) \times BR$  from LHCb, NRQCD, and  $\frac{f_{\Lambda_b}}{f_{B_u} + f_{B_d}}$  from the Tevatron & LHC

$$\sigma(pp \rightarrow T_{\bar{u}\bar{d}}^{\{bb\}} + X) \sim (0.5 - 1.0) \text{ nb}$$

# Estimates of the lifetime for $T_{[\bar{u}\bar{d}]}^{\{bb\}}$ using heavy quark expansion

[AA, A. Parkhomenko, Qin Qin, Wei Wang, arXiv:1805.02535; to appear in PLB]

- HQE simplifies the inclusive decay widths. Up to dimension 6 :

$$\mathcal{T} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{CKM}|^2 \left[ c_{3,b} \bar{b}b + \frac{c_{5,b}}{m_b^2} \bar{b}g_s \sigma_{\mu\nu} G^{\mu\nu} b + 2 \frac{c_{6,b}}{m_b^3} (\bar{b}q)_\Gamma (\bar{q}b)_\Gamma + \dots \right]$$

- At leading order in  $1/m_b$ , only the  $\bar{b}b$  operator contributes:

$$\Gamma(T_{[\bar{q}\bar{q}']}^{\{bb\}}) = \frac{G_F m_b^5}{192\pi^3} |V_{CKM}|^2 c_{3,b} \frac{\langle T_{[\bar{q}\bar{q}']}^{\{bb\}} | \bar{b}b | T_{[\bar{q}\bar{q}']}^{\{bb\}} \rangle}{2m_{T_{[\bar{q}\bar{q}']}^{\{bb\}}}}$$

- $\frac{\langle T_{[\bar{q}\bar{q}']}^{\{bb\}} | \bar{b}b | T_{[\bar{q}\bar{q}']}^{\{bb\}} \rangle}{2m_{T_{[\bar{q}\bar{q}']}^{\{bb\}}}}$  corresponds to the bottom-quark number in  $T_{[\bar{q}\bar{q}']}^{\{bb\}}$ , and is twice the matrix element for  $B$  meson and  $\Lambda_b$  baryon

- Hence, expect  $\tau(T_{[\bar{u}\bar{d}]}^{\{bb\}}) \simeq 1/2\tau(B)$ :

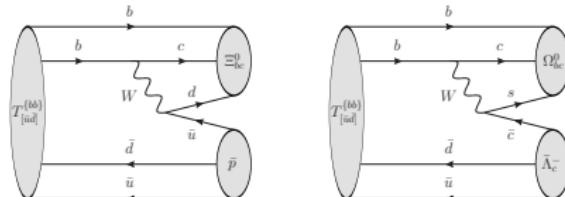
$$\tau(T_{[\bar{q}\bar{q}']}^{\{bb\}}) \sim \frac{1}{2} \times 1.6 \times 10^{-12} s = 800 \times 10^{-15} s$$

## Weak Decays of $T_{[\bar{u}\bar{d}]}^{\{bb\}}$

### ■ Effective Weak Hamiltonian

$$\begin{aligned}
 \mathcal{H}_{\text{eff}}^{(cc)} = & \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left\{ C_1 [\bar{c}_\alpha \gamma_\mu P_L b^\alpha] [\bar{d}_\beta \gamma^\mu P_L u^\beta] \right. \\
 & \left. + C_2 [\bar{c}_\beta \gamma_\mu P_L b^\alpha] [\bar{d}_\alpha \gamma^\mu P_L u^\beta] \right\} \\
 + & \frac{4G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left\{ C_1 [\bar{c}_\alpha \gamma_\mu P_L b^\alpha] [\bar{s}_\beta \gamma^\mu P_L c^\beta] \right. \\
 & \left. + C_2 [\bar{c}_\beta \gamma_\mu P_L b^\alpha] [\bar{s}_\alpha \gamma^\mu P_L c^\beta] \right\} + \text{h. c.}
 \end{aligned}$$

### ■ Two-Body Baryonic Decays from $b \rightarrow c + d + \bar{u}$ (left panel) and $b \rightarrow c + s + \bar{c}$ (right panel)



## An order of magnitude estimate

- Involve non-factorizable Amplitudes . For the  $J^P = 1^+$  tetraquark, the general form of the decay amplitude is:

$$\begin{aligned}\mathcal{M}(T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{p}) = & \bar{v}(p_p) \left[ f_1^{\Xi_{bc}\bar{p}} q_\mu + f_2^{\Xi_{bc}\bar{p}} \gamma_\mu \right. \\ & + f_3^{\Xi_{bc}\bar{p}} \sigma_{\mu\nu} \frac{q^\nu}{M_T} + g_1^{\Xi_{bc}\bar{p}} \gamma_5 q_\mu + g_2^{\Xi_{bc}\bar{p}} \gamma_\mu \gamma_5 \\ & \left. + g_3^{\Xi_{bc}\bar{p}} \sigma_{\mu\nu} \gamma_5 \frac{q^\nu}{M_T} \right] u(p_{\Xi_{bc}}) \epsilon_T^\mu(p_T)\end{aligned}$$

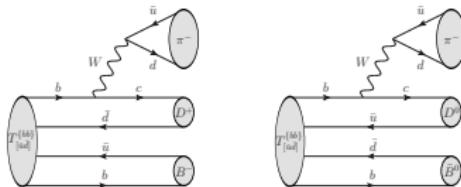
- Inspired by the  $B$  meson decay data

$$\begin{aligned}\mathcal{B}(\bar{B}^0 \rightarrow D^+ \pi^-) &= (2.52 \pm 0.13) \times 10^{-3} \\ \mathcal{B}(\bar{B}^0 \rightarrow D^+ D_s^-) &= (7.2 \pm 0.8) \times 10^{-3}\end{aligned}$$

- Infer that  $\mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow \Xi_{bc}^0 \bar{p})$  and  $\mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow \Omega_{bc}^0 \bar{\Lambda}_c^-)$  are of  $O(10^{-3})$
- Needs reconstructing the doubly heavy baryons  $\Xi_{bc}^0$  and  $\Omega_{bc}^0$ , such as through  $\Xi_{bc}^0 \rightarrow \Lambda_b K^- \pi^+$ , expect the two-body baryonic decay modes of  $T_{[\bar{u}\bar{d}]}^{\{bb\}-}$  can have branching fractions of order  $10^{-6}$

## Three-body Mesonic Decay Modes of $T_{[\bar{u}\bar{d}]}^{\{bb\}-}$

- Feynman diagrams due to the  $b$ -quark decay  $b \rightarrow c + d + \bar{u}$



- The factorizable amplitudes of these decays can be written as:

$$\begin{aligned} \mathcal{M}(T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow B^- D^+ \pi^-) &= i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1^{\text{eff}} f_\pi p_\pi^\mu \\ &\times \langle (BD)_0^0 (p_{BD}) | \bar{d} \gamma_\mu (1 - \gamma_5) u | T_{[\bar{u}\bar{d}]}^{\{bb\}-} (p_T) \rangle, \\ \mathcal{M}(T_{[\bar{u}\bar{d}]}^{\{bb\}-} \rightarrow \bar{B}^0 D^0 \pi^-) &= i \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1^{\text{eff}} f_\pi p_\pi^\mu \\ &\times \langle (BD)_0^0 (p_{BD}) | \bar{s} \gamma_\mu (1 - \gamma_5) c | T_{[\bar{u}\bar{d}]}^{\{bb\}-} (p_T) \rangle \end{aligned}$$

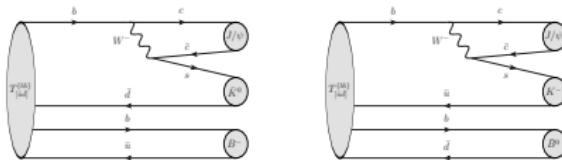
$$\mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}} \rightarrow B^- D^+ \pi^-) = \mathcal{B}(T_{[\bar{u}\bar{d}]}^{\{bb\}} \rightarrow \bar{B}^0 D^0 \pi^-) \sim 0.5 \times 10^{-3}$$

## Hidden-Charm final states in $T_{[\bar{u}\bar{d}]}^{\{bb\}-}$ decays

- In some decays hidden-charm mesons, such as  $J/\psi, \psi'$ , can be produced

$$T_{[\bar{u}\bar{d}]}^{\{bb\}} \rightarrow J/\psi \bar{K}^0 B^-,$$

$$T_{[\bar{u}\bar{d}]}^{\{bb\}} \rightarrow J/\psi K^- \bar{B}^0$$



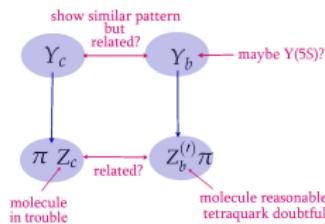
- Their decay branching ratios can be comparable with the  $\mathcal{B}(B \rightarrow J/\psi K)$ :

$$\mathcal{B}(\bar{B}^0 \rightarrow J/\psi \bar{K}^0) = (8.73 \pm 0.32) \times 10^{-4}$$

- Expect that the product branching ratios to measure the mass of  $T_{[\bar{u}\bar{d}]}^{\{bb\}-}$  are at most of  $O(10^{-5})$

# Summary

- A new facet of QCD is opened by the discovery of the exotic states  $X, Y, Z, \bar{P}(4380), \bar{P}(4450)$
- Important puzzles remain in the complex:



- What is the nature of  $Y_c(4260)$ ? A tetraquark? or a  $c\bar{c}g$  hybrid? Is  $Y_c(4260)$  split? How many  $P$  states are there? We do expect a tower of radial and orbital excited states in the diquark picture!
- A very rich spectrum of tetraquark and pentaquark states is anticipated, including the ones with a single  $c$ , or a single  $b$  quark, as well as those with multiple heavy quarks
- A  $Z$  factory, as well as HL-LHC will advance the multiquark sector of QCD decisively, in particular, the entire pentaquark spectrum with hidden charm discussed here can be measured from the production and decays of the  $b$ -baryons
- They also have great potential to discover doubly heavy tetraquarks, establishing diquarks as a building block in QCD
- Look forward to decisive experimental results from BESIII, Belle-II, LHC, and  $Z$  factory