中国科学院高能物理研究所

Institute of High Energy Physics

中國科學院

CHINESE ACADEMY OF SCIENCES

Threshold phenomena, triangle singularity and exotic hadrons

– a long story about the pseudoscalar glueball

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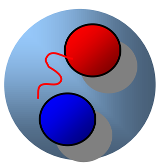
Outline

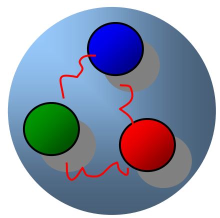
1. Hadrons beyond the conventional quark model and three types of exotics signals
2. Do not forget the nearby S-wave thresholds, and the presence of the “triangle singularity”
3. Story of the pseudoscalar glueball puzzle
4. Observables sensitive to the underlying dynamics
5. Brief summary

1. Hadrons beyond the conventional QM and…

Exotic hadrons

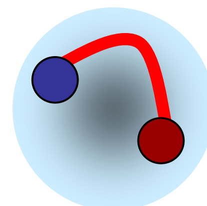
convential hadron



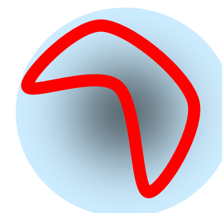


$(q\bar{q})$ (qqq)

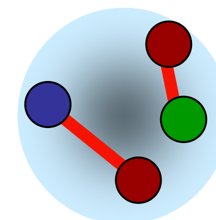
Hybrid



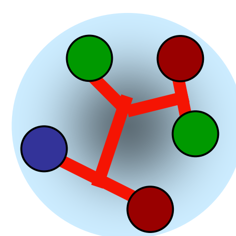
Glueball



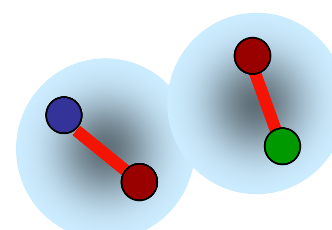
Tetraquark



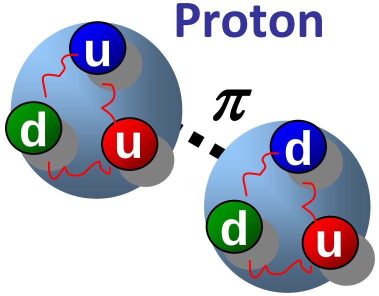
Pentaquark



Hadronic molecule



Proton



Neutron

Deuteron: p-n molecule

Evidence for QCD exotic states is a missing piece of knowledge about the Nature of strong QCD.

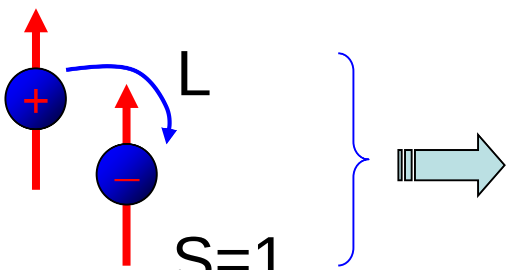
Exotics of Type-I:
J^{PC} are not allowed by $Q\bar{Q}$ configurations

States in **natural spin-parity**: if $P=(-1)^{L+1}=(-1)^{J}$, then $S=1$ and hence

$CP=(-1)^{(L+S)+(L+1)}=+1$.

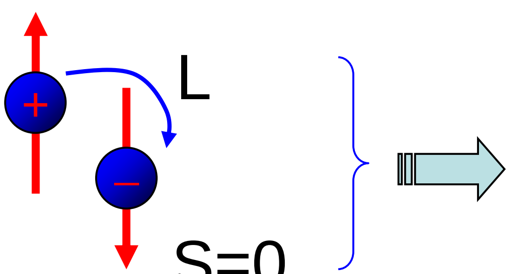
→ Mesons with **natural spin-parity** but $CP=-1$ will be forbidden:

0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, …



Natural: 0^{++}, 1^{--}, 2^{++}, 3^{--}, …

Unnatural: (0^{--}), 1^{++}, 2^{--},3^{++}, …

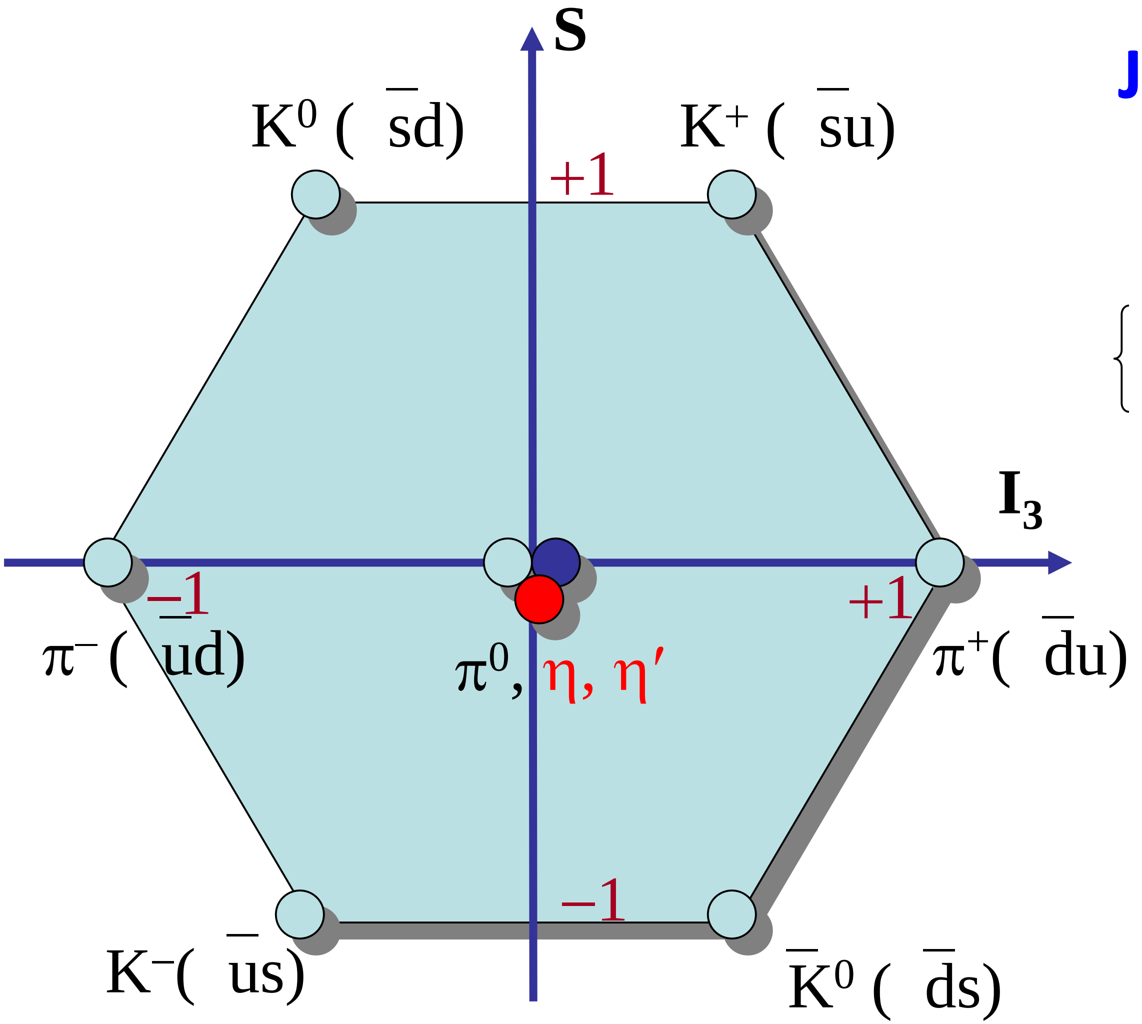


Unnatural: 0^{-+}, 1^{+-}, 2^{-+}, 3^{+-}, …

Exotics of Type-II:

J^{PC} are the same as $Q\bar{Q}$ configurations

$\bar{q}q$ SU(3) flavor nonet: $\bar{3}\otimes 3 = 1\oplus 8$



$J^{PC} = 0^{-+}$

$$\begin{cases} \eta &= \cos\alpha_P |n\bar{n}\rangle - \sin\alpha_P |s\bar{s}\rangle \\ \eta' &= \sin\alpha_P |n\bar{n}\rangle + \cos\alpha_P |s\bar{s}\rangle \end{cases}$$

$$n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$$

$$\alpha_P = \theta_P + \arctan\sqrt{2}$$

Light hadrons: $\bar{q}q$ SU(3) flavor nonet: $\bar{3}\otimes 3 = 1\oplus 8$

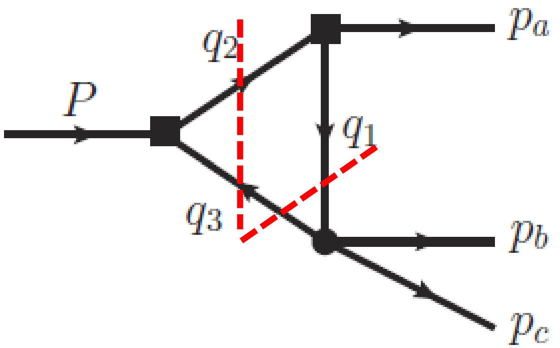
$n\,^{2s+1}\ell_J$	J^{PC}	$I=1$ $u\bar{d},\ \bar{u}d,\ \frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	$I=\frac{1}{2}$ $u\bar{s},\ d\bar{s};\ \bar{d}s,\ -\bar{u}s$	$I=0$ f'	$I=0$ f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.5	-24.6
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$	$f_0(1500)$?	
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$	$a_1(1420)$?	
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$	$\eta(1405)$?	
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		

Additional states beyond the QM flavor symm. pattern imply "exotic" signals!

Exotics of Type-III:
Peak structures caused by kinematic effects, in particular, by triangle singularity.

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{i(2\pi)^4}\int \frac{d^4q_1}{(q_1^2 - m_1^2 + i\epsilon)(q_2^2 - m_2^2 + i\epsilon)(q_3^2 - m_3^2 + i\epsilon)}$$
$$= \frac{-1}{16\pi^2}\int_0^1\int_0^1\int_0^1 da_1\, da_2\, da_3 \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon},$$

$$D \equiv \sum_{i,j=1}^{3} a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2}\left[m_i^2 + m_j^2 - (q_i - q_j)^2\right]$$



The TS occurs when all the three internal particles can approach their on-shell condition simultaneously:

$\partial D/\partial a_j = 0$ for all j=1,2,3. ⇨ $\det[Y_{ij}] = 0$

L. D. Landau, Nucl. Phys. 13, 181 (1959);
J.J. Wu, X.-H. Liu, Q. Zhao, B.-S. Zou, Phys. Rev. Lett. 108, 081003 (2012);
Q. Wang, C. Hanhart, Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013); Phys. Lett. B 725, 106 (2013)
X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016);
F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, arXiv:1705.00141[hep-ph], Rev. Mod. Phys. 90, 015004 (2018)

Big progresses in experiment in the past 15 years:

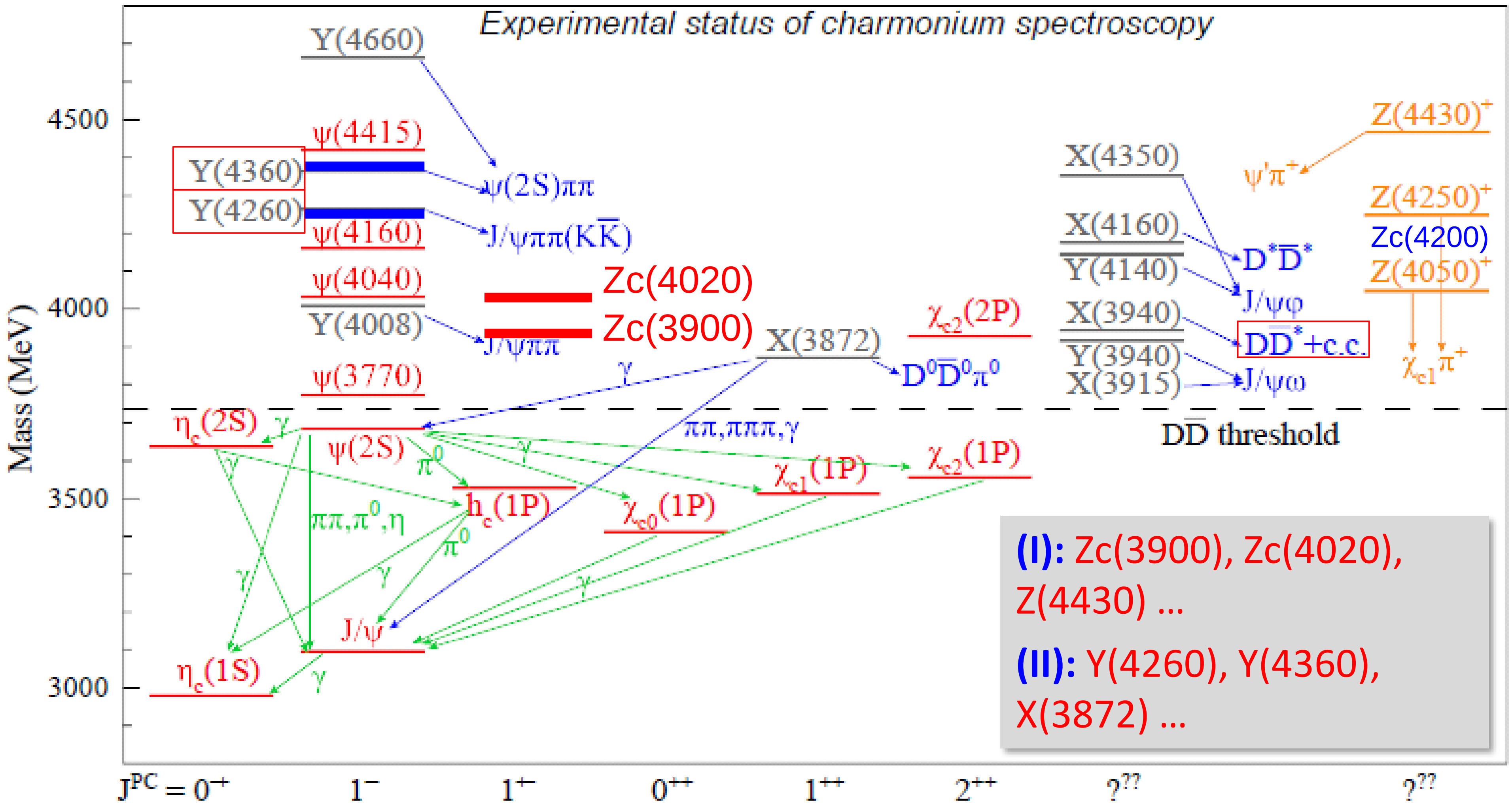
- Most of the ground states and states below open-flavor threshold are well established.

- A large number of excited states, i.e. XYZ states, cannot be accommodated by the conventional quark model, while a large fraction of these states appear to be correlated with the nearby S-wave open thresholds.

- Lattice QCD are still unable to provide a full quantitative description of the hadron spectroscopy.

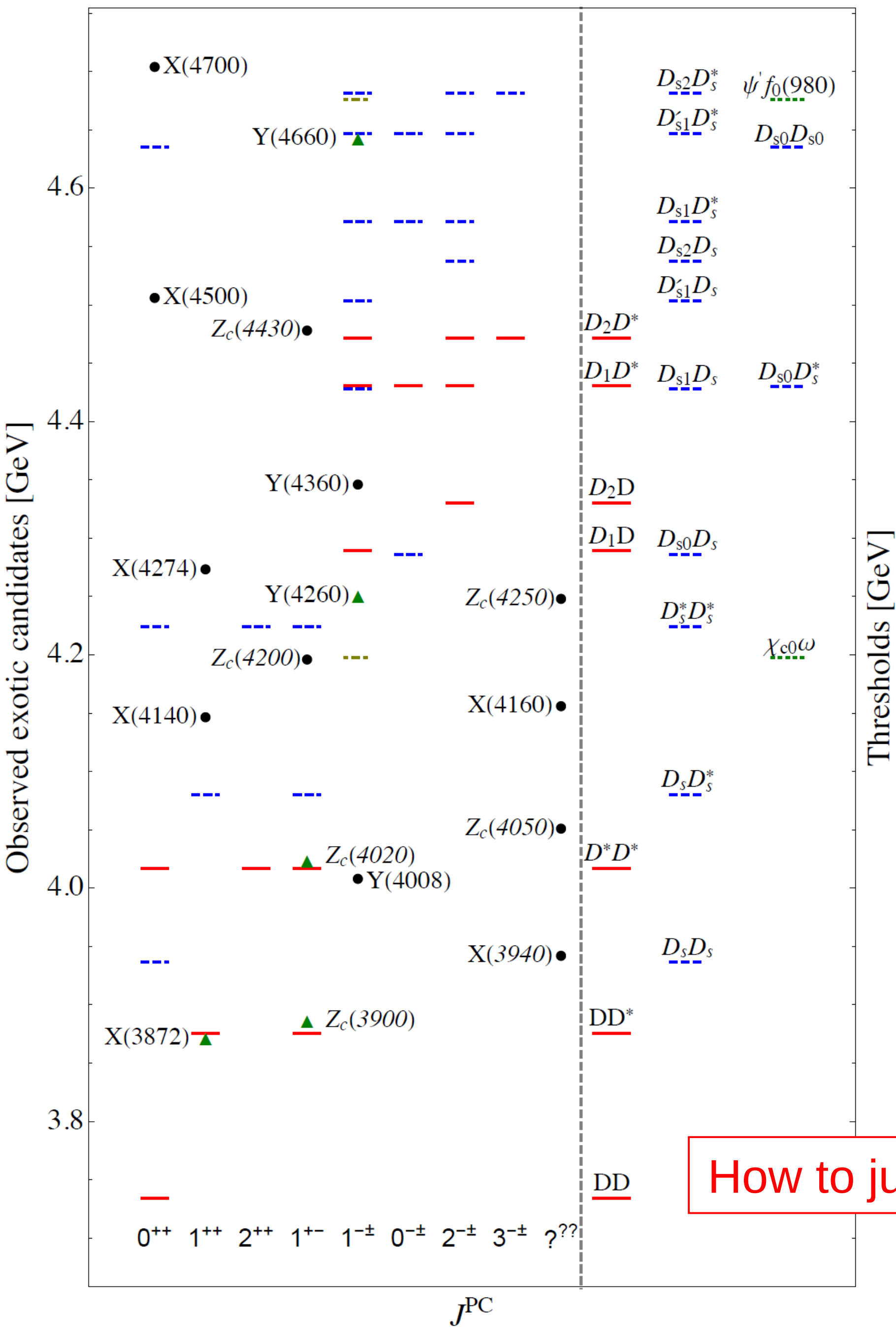
2. Expected and unexpected: Do not forget the nearby S-wave thresholds!

Experimental progress:
Charmonia and charmonium-like states, i.e. X, Y, Z's.



(I): Zc(3900), Zc(4020), Z(4430) …

(II): Y(4260), Y(4360), X(3872) …



S wave thresholds and effects on the lineshapes

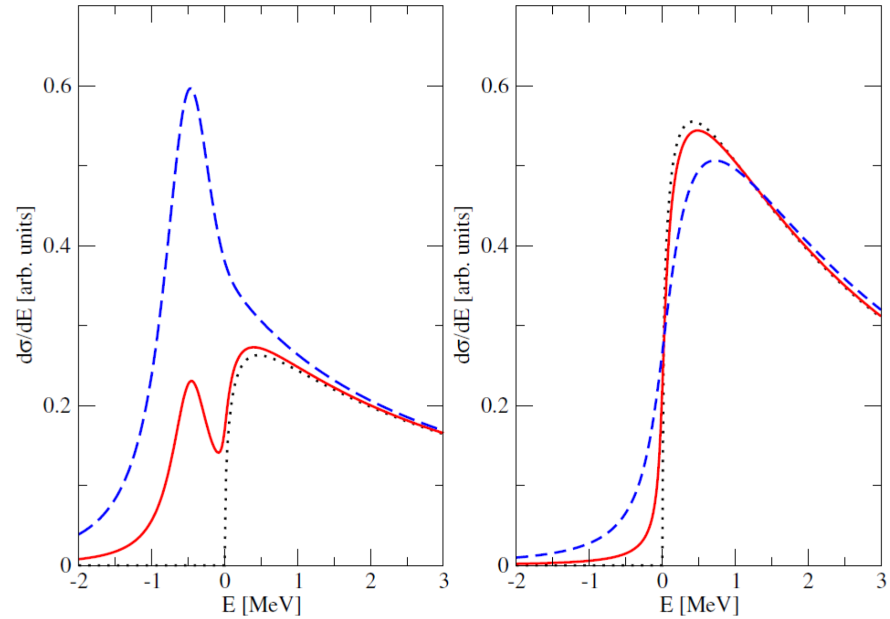
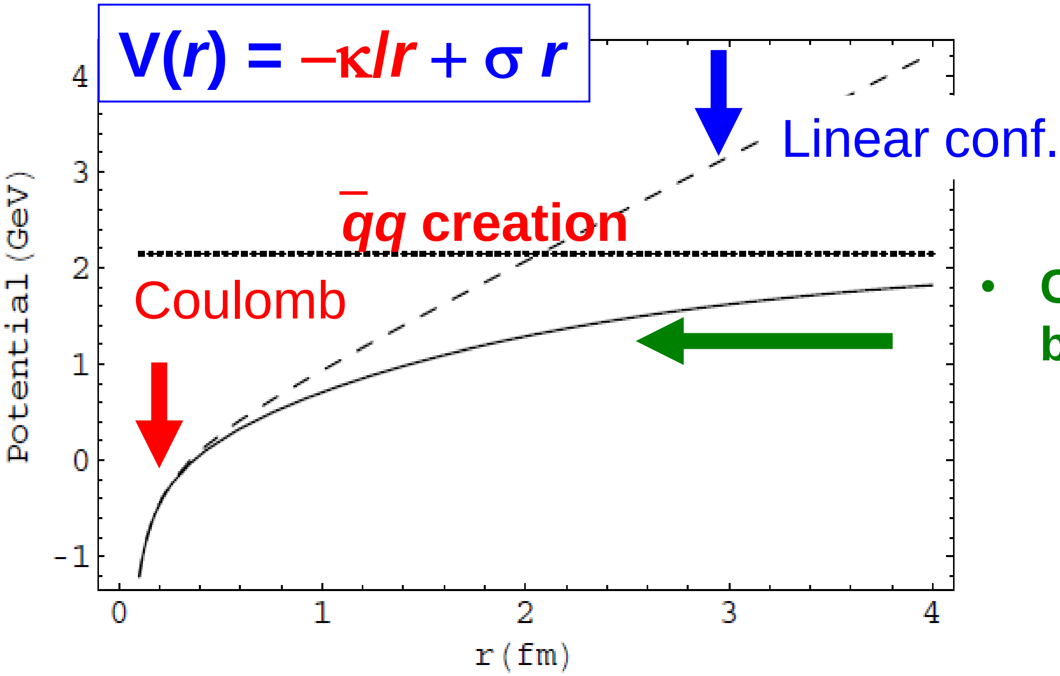


FIG. 10 Line shapes that emerge for a bound state (left panel) and for a virtual state (right panel) once one of the constituents is unstable. The dotted, solid and dashed line show the results for $\Gamma = 0, 0.1$ and 1 MeV, respectively. The other parameters of the calculation are given in Eq. (36).

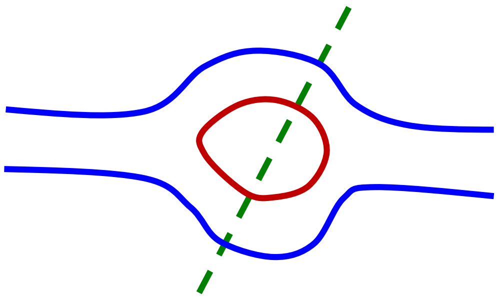
How to judge the nature of threshold enhancement?

F.-K. Guo, C. Hanhart, U.-G. Meissner, Q. Wang, Q. Zhao, B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)

How the potential QM is broken down



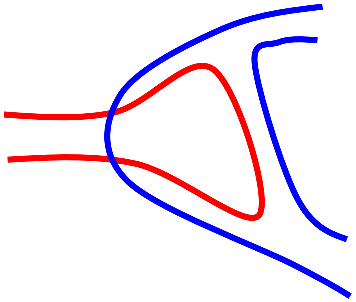
- Color screening effects? String breaking effects?



- The effect of vacuum polarization due to dynamical quark pair creation may be manifested by the strong coupling to open thresholds and compensated by that of the hadron loops, i.e. coupled-channel effects.

E. Eichten et al., PRD17, 3090 (1987)
B.-Q. Li and K.-T. Chao, Phys. Rev. D79, 094004 (2009);
T. Barnes and E. Swanson, Phys.Rev. C77, 055206 (2008)

Typical processes where the open threshold coupled channels can play a role



$\psi(3770) \to nonD\bar{D}$

"ρπ puzzle"

$\chi_{c1} \to VV, \ \chi_{c2} \to VP$

$\eta_c(\eta_c') \to VV$

Y.J. Zhang et al, PRL(2009);

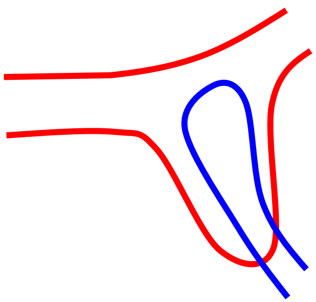
X. Liu, B. Zhang, X.Q. Li, PLB(2009)

Q. Wang et al. PRD(2012), PLB(2012)

X.-H. Liu et al, PRD81, 014017(2010);

X. Liu et al, PRD81, 074006(2010)

Q. Wang et al, PRD2012



$\psi' \to J/\psi\pi^0, \psi' \to J/\psi\eta$

$\psi' \to \gamma\eta_c, J/\psi \to \gamma\eta_c$

Z_c(3900), **Zc(4020)** prod.

Lineshape of $e^+e^- \to \psi'\pi^+\pi^-$

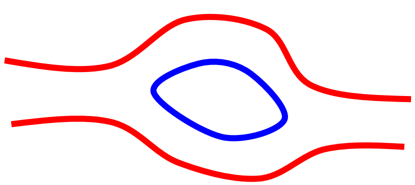
G. Li and Q. Zhao, PRD(2011)074005

F.K. Guo, C. Hanhart, G. Li, U.-G. Meiβner and Q. Zhao, PRD82, 034025 (2010); PRD83, 034013 (2011)

F.K. Guo and Ulf-G Meiβner, PRL108(2012)112002

Q. Wang, C. Hanhart, QZ, PRL111, 132003 (2013); PLB(2013)

Z. Cao et al. to appear



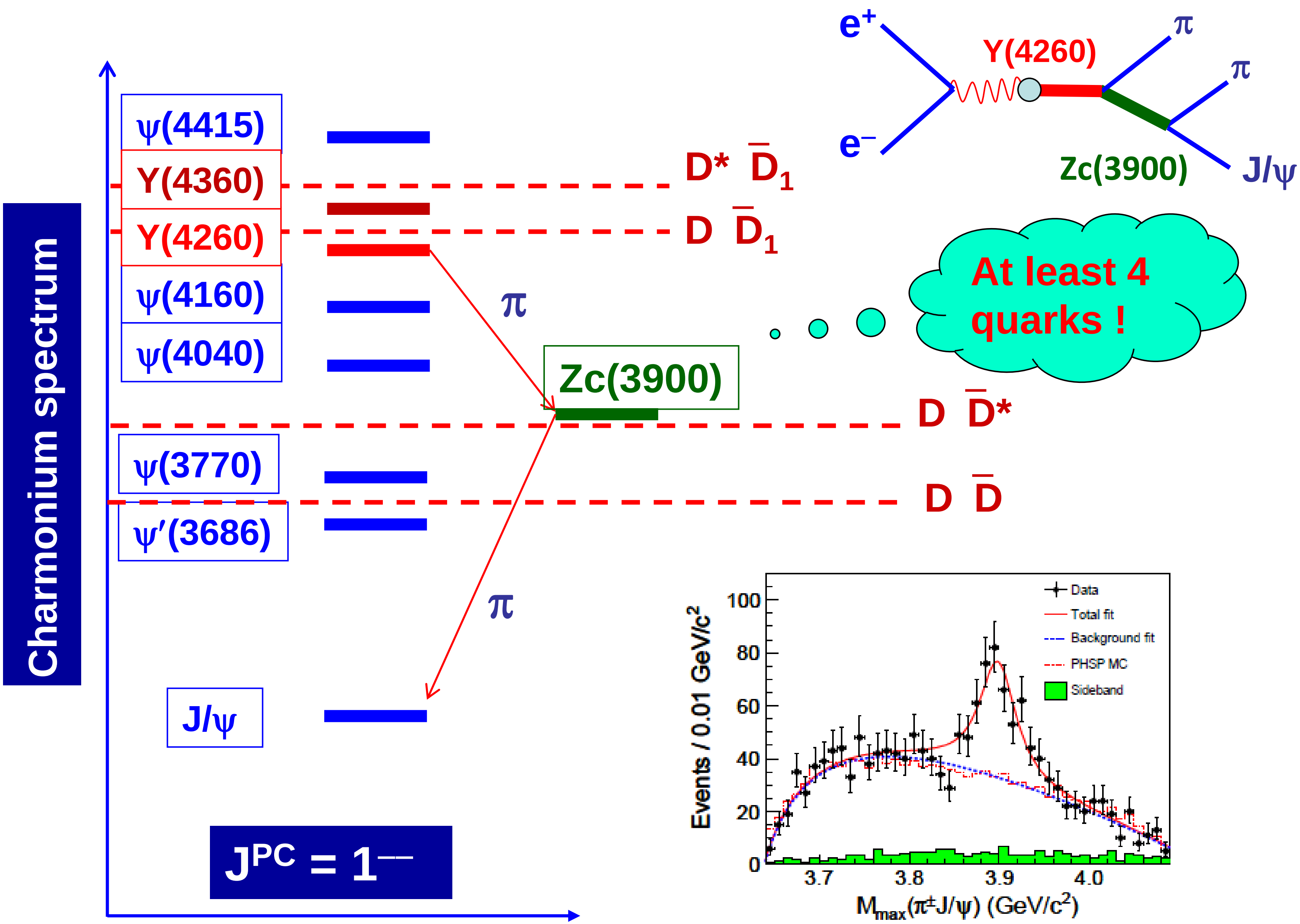
$D_{s1}(2460) - D_{s1}(2536)$

The mass shift in charmonia and charmed mesons,

E.Eichten et al., PRD17(1987)3090

X.-G. Wu and Q. Zhao, PRD85, 034040 (2012)

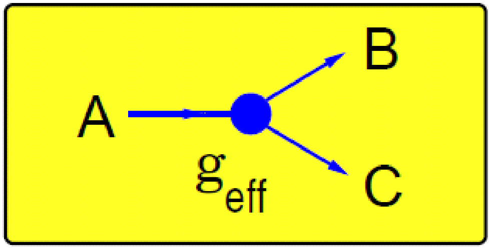
The first S-wave open charm threshold in vector channel



Weinberg's Compositeness Theorem

Weinberg (1963); Morgan et al. (1992); Baru, Hanhart et al. (2003); G.-Y. Chen, W.-S. Huo, Q. Zhao (2013) ...

- Consider S-wave decay $A \to BC$
  with a coupling constant $g_{\rm eff}$
  and $m_A = m_B + m_C - \varepsilon$



$$\Rightarrow \quad \frac{g_{\rm eff}^2}{4\pi} = 4(m_B + m_C)^{5/2}\,\lambda^2 \sqrt{\frac{2\varepsilon}{m_B m_C}}$$

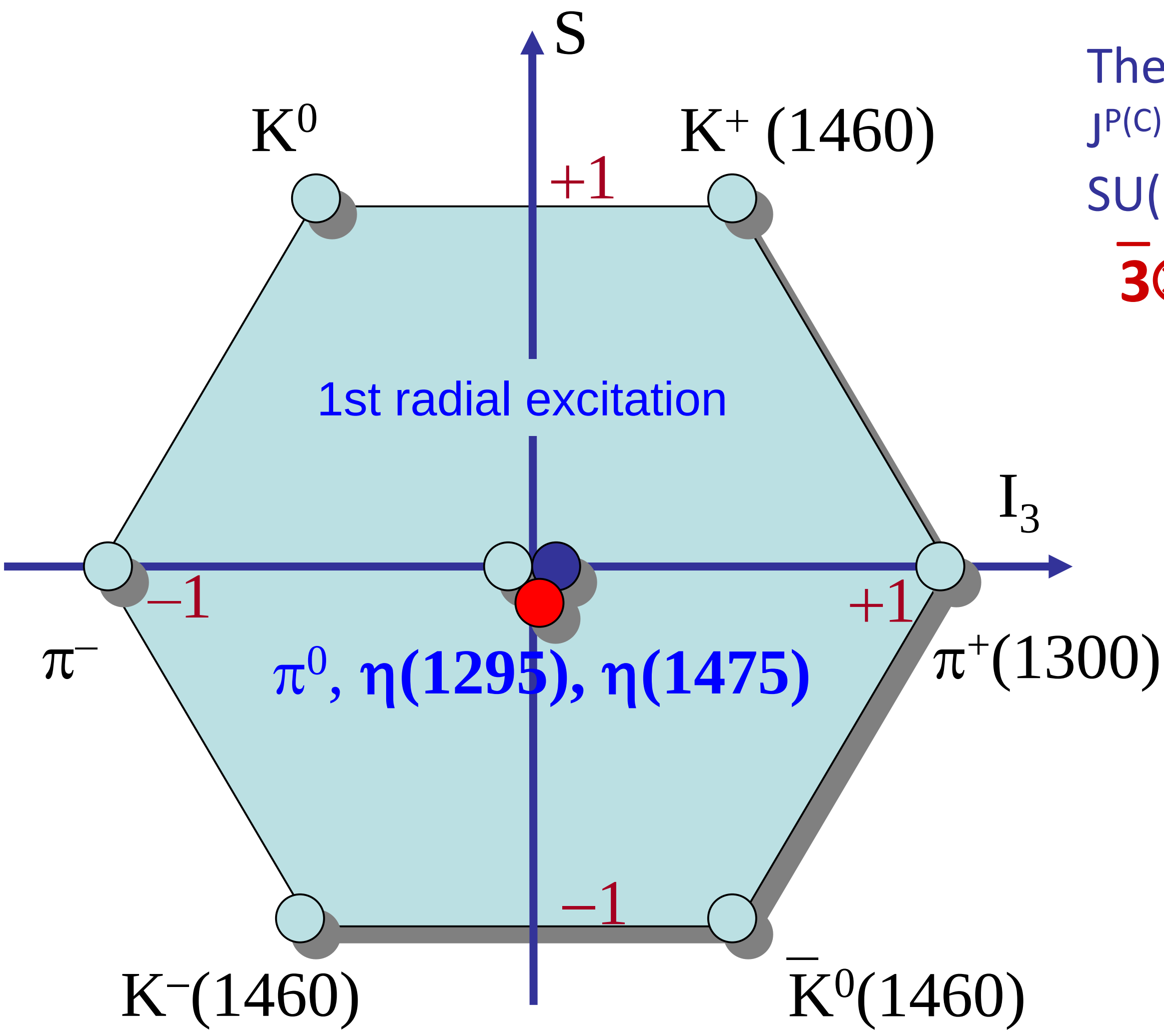
$$\leq \quad 4(m_B + m_C)^{5/2} \sqrt{\frac{2\varepsilon}{m_B m_C}}$$

- λ^2 = **Probability to find the hadronic molecule component in the physical state A**

The effective coupling $g_{\rm eff}$ encodes the structure information and can be extracted model-independently from experiment.

3. Story of the "η(1405/1475) puzzle"

-- also known as "E-ι meson puzzle"

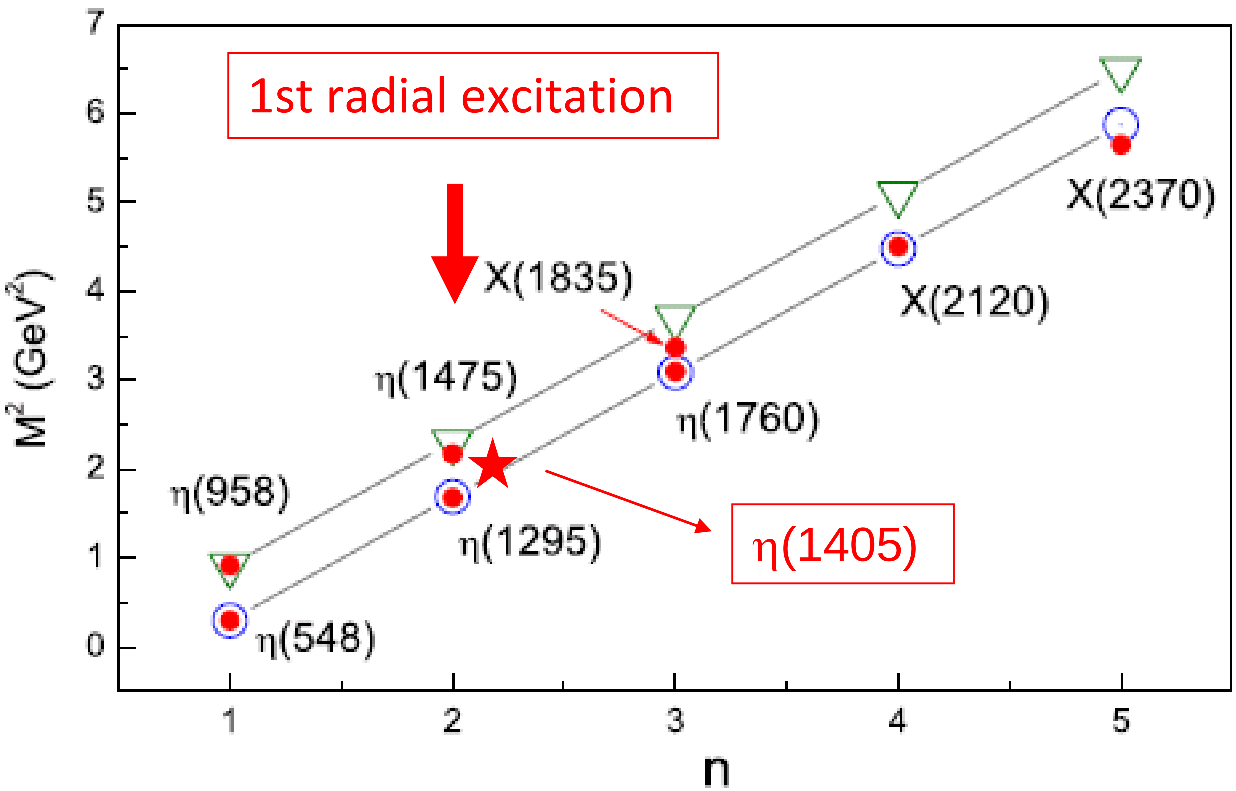


The first radial excitation of $J^{P(C)}=0^{-(+)}$ states also make a $\bar{q}q$ SU(3) flavor nonet:

$\bar{\mathbf{3}}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{8}$

Three η states have been listed by Particle Data Group around 1.2 ~ 1.5 GeV: **η(1295)**, **η(1405)**, and **η(1475)**

• Regge trajectory for the η/η′ mass spectrum



$$M^2 = M_0^2 + (n-1)\mu^2 \;\; (\mu^2 = 1.39 \text{ GeV})$$

• How to understand the presnece of η(1405) ?

J.S. Yu, Z.F. Sun, X. Liu, and Q. Z., PRD83, 114007 (2011)

E meson was first observed in 1965 in $p\bar{p} \to (K\bar{K}\pi)\,\pi^+\pi^-$.

Observation of ι(1440) at Mark II (left, 1980) and Crystal Ball (right, 1982)

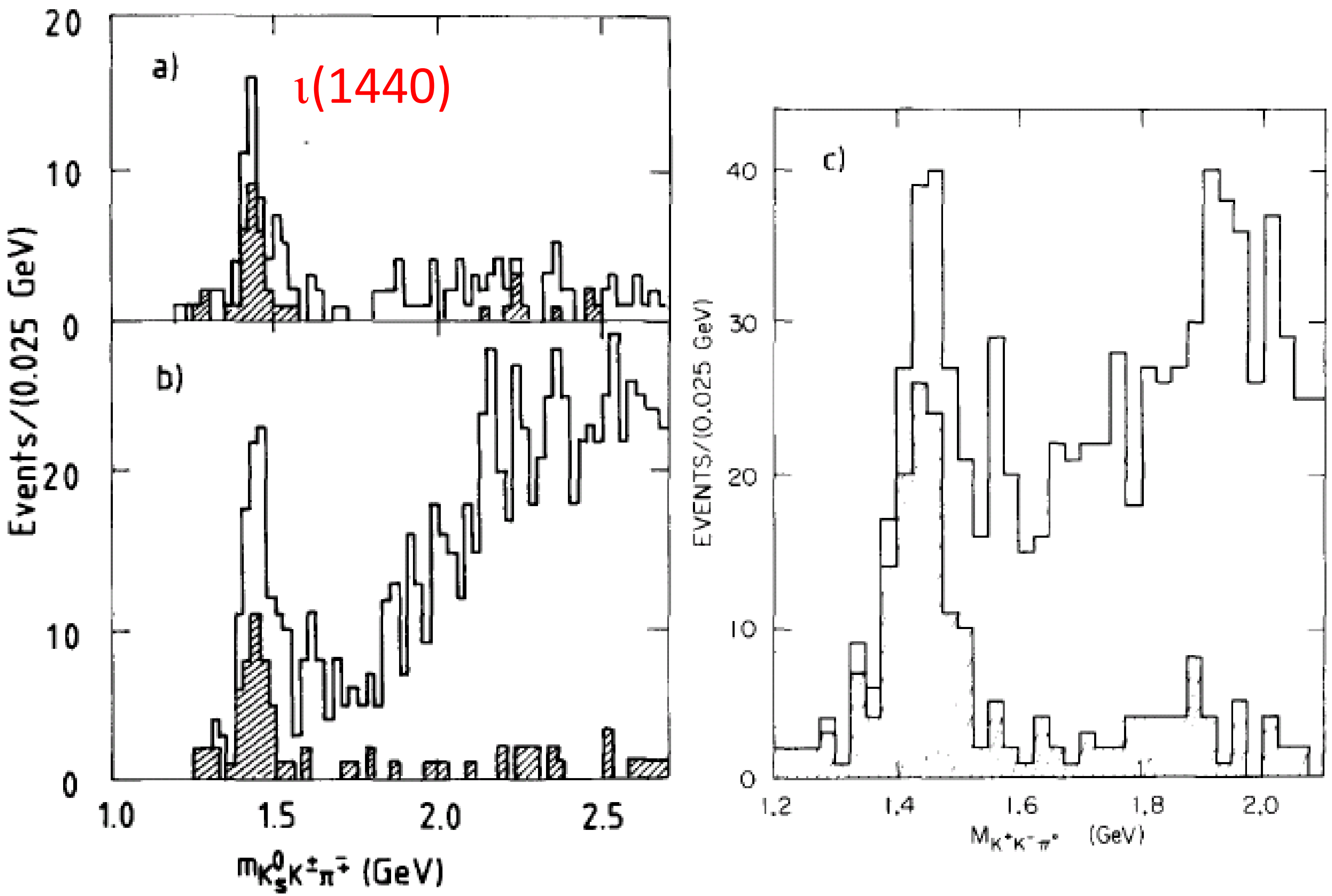
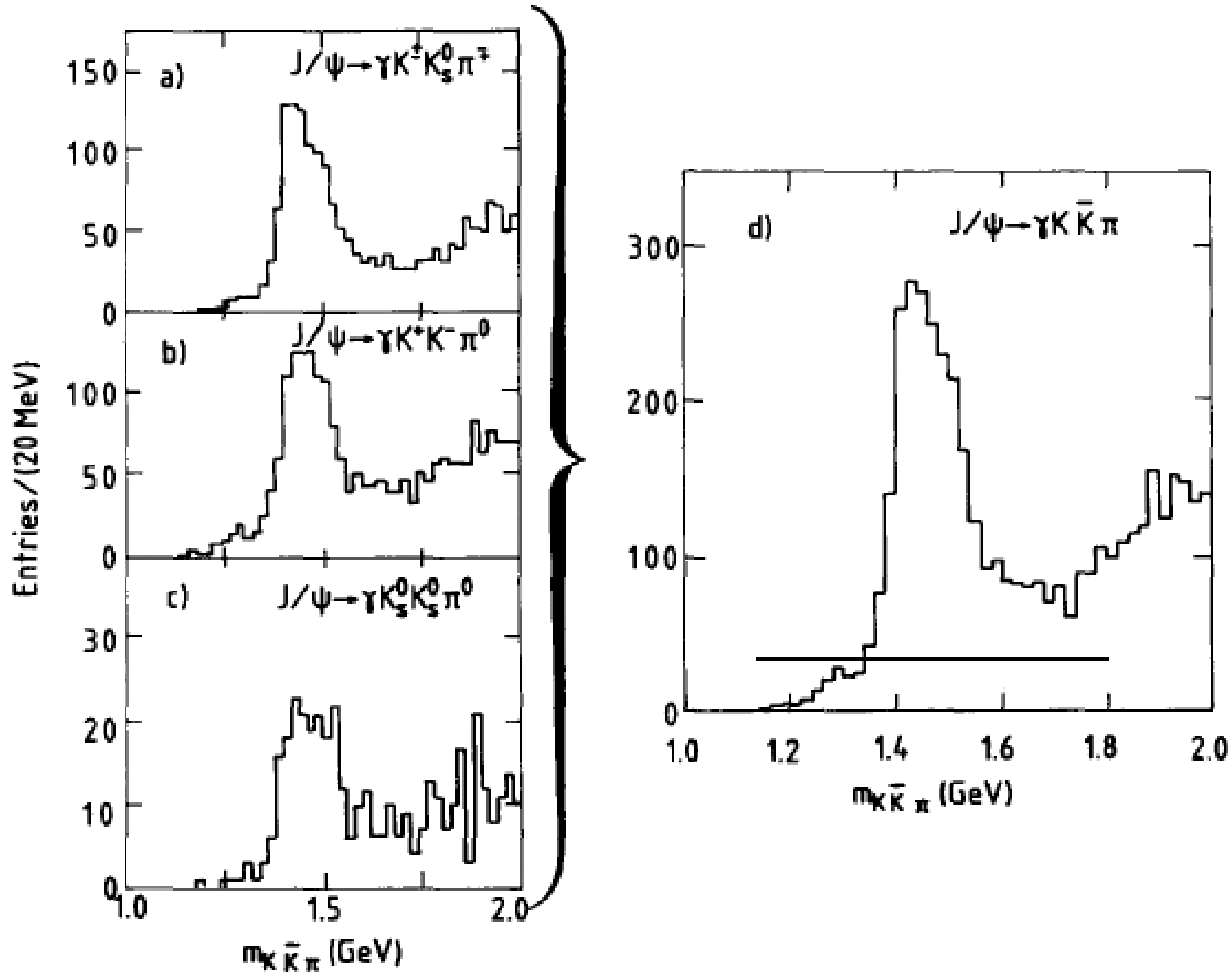
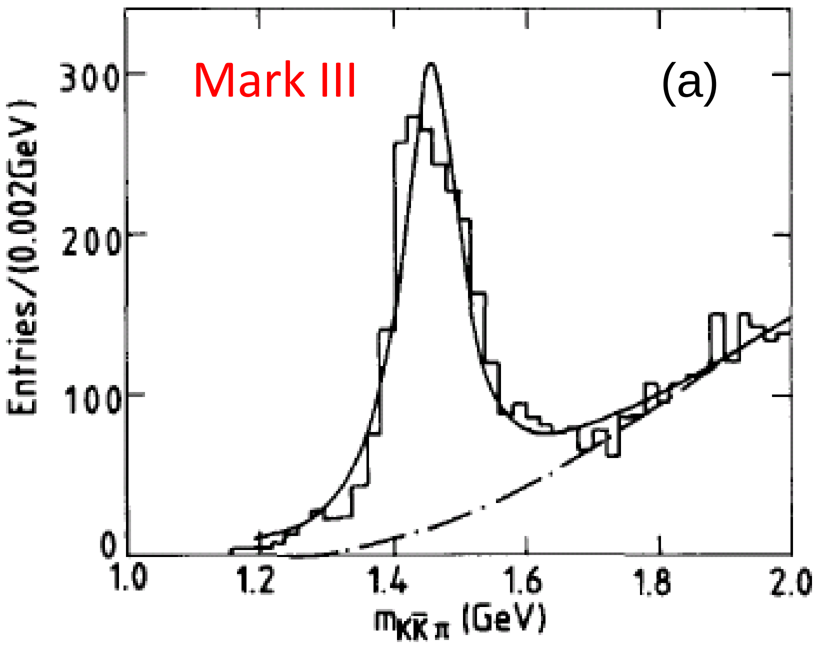
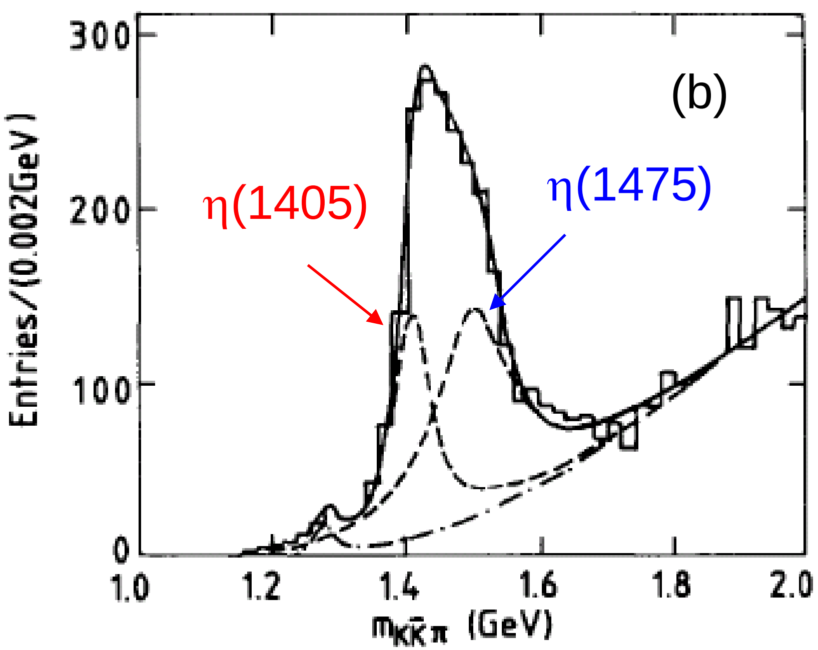


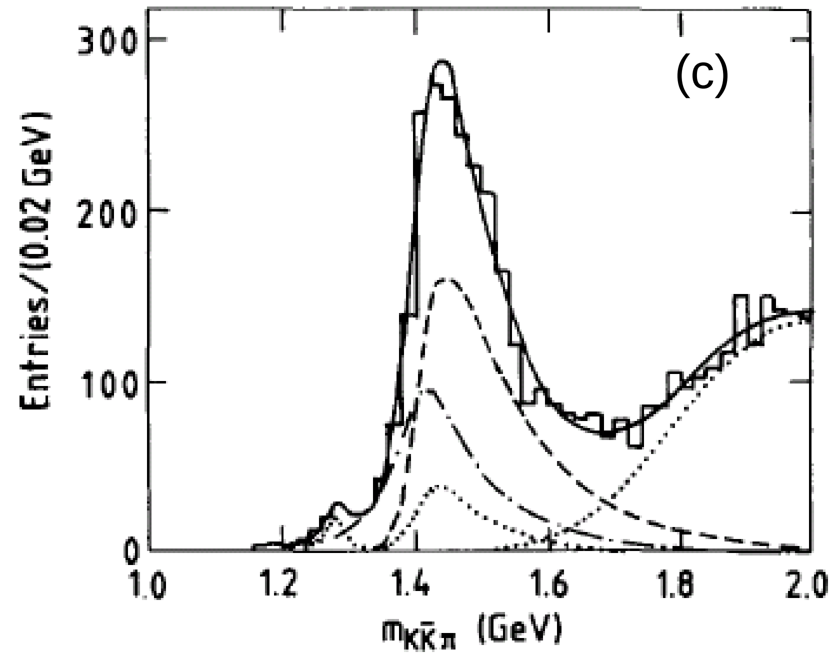
Fig. 69. Observation of the η(1440) by Mark II and Crystal Ball. (a) Mark II, radiative photon detection required, (b) Mark II, photon detection not required. The events in the shaded region have $m_{K\bar{K}} < 1.05$ GeV ("delta cut"). (c) Crystal Ball, events in the shaded region have $m_{K\bar{K}} < 1.125$ GeV.

Observation of η(1440) at Mark III in 1987









(a) A single Breit-Wigner fit

(b) Two interfering B-W fit

(c) Coupled channel B-W fit

$$M = 1416 \pm 8^{+7}_{-5};\ \Gamma = 91^{+67}_{-31-38}{}^{+15}\ \mathrm{MeV}/c^2$$

$$M = 1490^{+14+3}_{-8-6};\ \Gamma = 54^{+37+13}_{-21-24}\ \mathrm{MeV}/c^2$$

Also “confirmed” by Obelix collaboration

- Mark III, Obelix, Crystal Ball: Two-state solution (since 2002)

$\eta(1405) \rightarrow a0(980)\,\pi \rightarrow \eta\,\pi\,\pi$, with M=1405±5 MeV, and Γ = 56±6 MeV

$\eta(1475) \rightarrow K^*\,\bar{K} + \bar{K}^*K \rightarrow K\,\bar{K}\,\pi$, with M=1475±5 MeV, and Γ=81±11 MeV

- **The abundance of 0^{-+} (I=0) states implies a glueball candidate?**

Positive: Flux tube model favors $M_G \cong 1.4$ GeV [1]

Caveat: LQCD (quenched) favors $M_G \cong 2.4$ GeV [2,3]

Keep in mind: More problems arising from such a scenario!

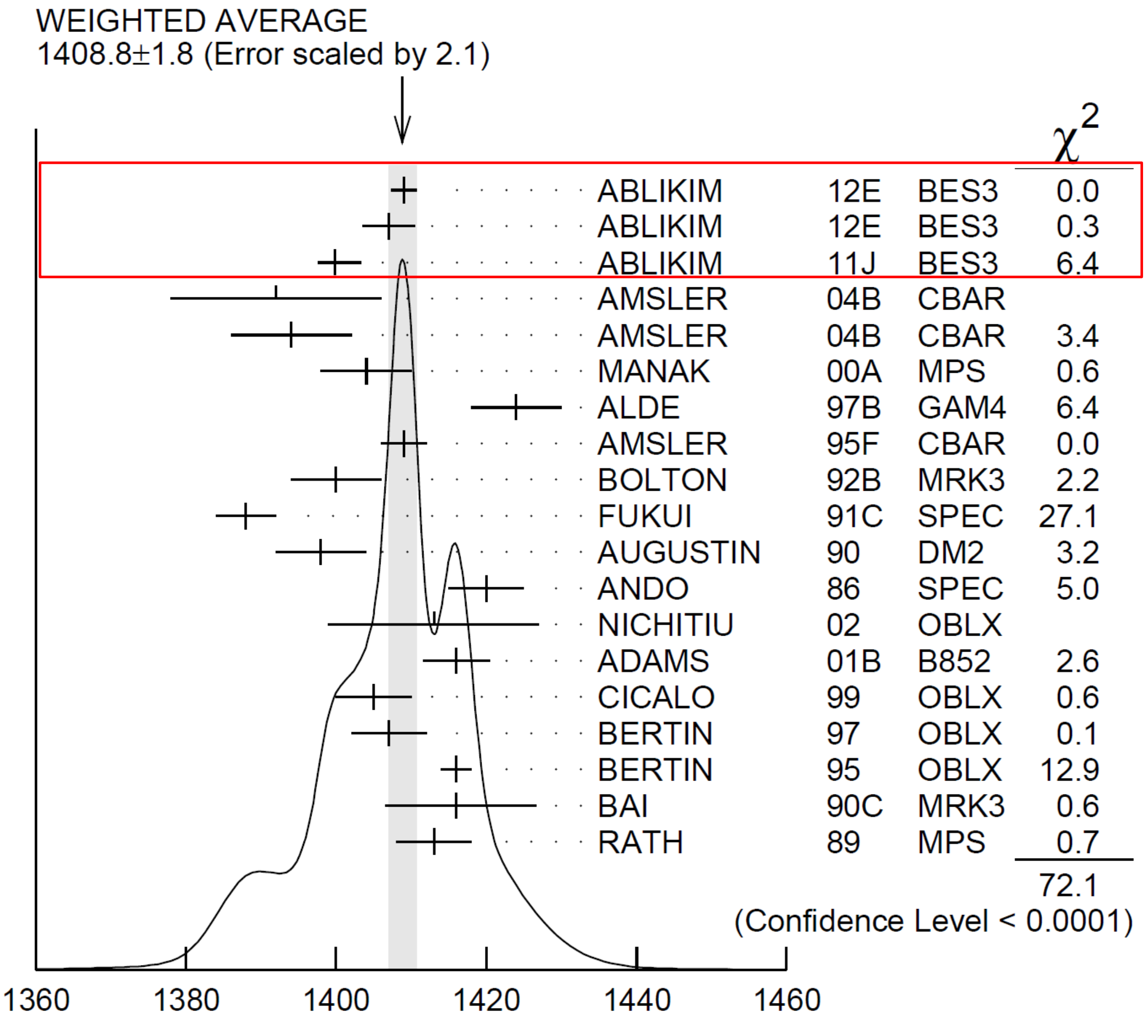
- Contradicting with new high-precision data from BESIII
- Contradicting with updated LQCD calculations

[1] Faddeev, Niemi, and Wiedner, PRD70, 114033 (2004)
[2] Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)
[3] Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update

$\eta(1405)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$



$\eta(1405)$ mass (MeV)

$\eta(1405)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	seen
Γ_2	$\eta\pi\pi$	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\eta(\pi\pi)_{S\text{-wave}}$	seen
Γ_5	$f_0(980)\eta$	seen
Γ_6	4π	seen
Γ_7	$\rho\rho$	<58 %
Γ_8	$\gamma\gamma$	
Γ_9	$\rho^0\gamma$	seen
Γ_{10}	$\phi\gamma$	
Γ_{11}	$K^*(892)K$	seen

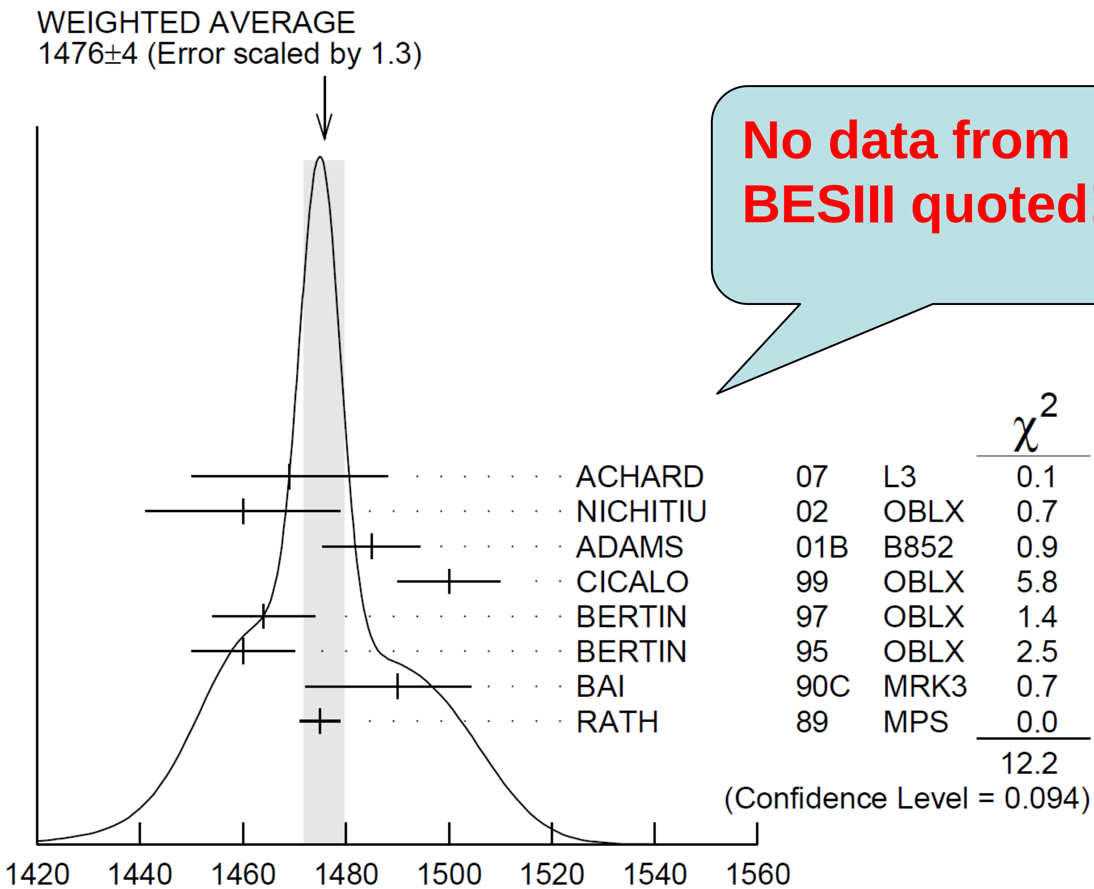
Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update

$\eta(1475)$

$$I^G(J^{PC}) = 0^+(0^{-+})$$

Apparent inconsistency between the analyses for η(1405) and η(1475)

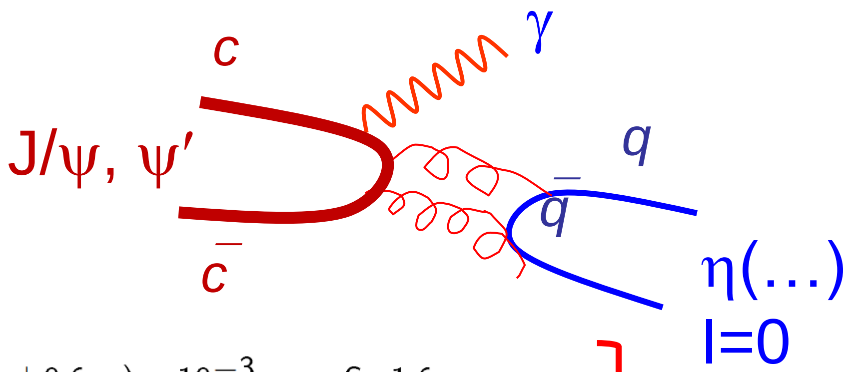
No data from BESIII quoted!!!



	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}\pi$	dominant
Γ_2	$K\overline{K}^*(892)+$ c.c.	seen
Γ_3	$a_0(980)\pi$	seen
Γ_4	$\gamma\gamma$	seen
Γ_5	$K^0_S K^0_S \eta$	possibly seen

BESIII measurements of η(…) states in J/ψ and ψ′ decays

Only a single state is observed in the J/ψ and ψ′ decays!



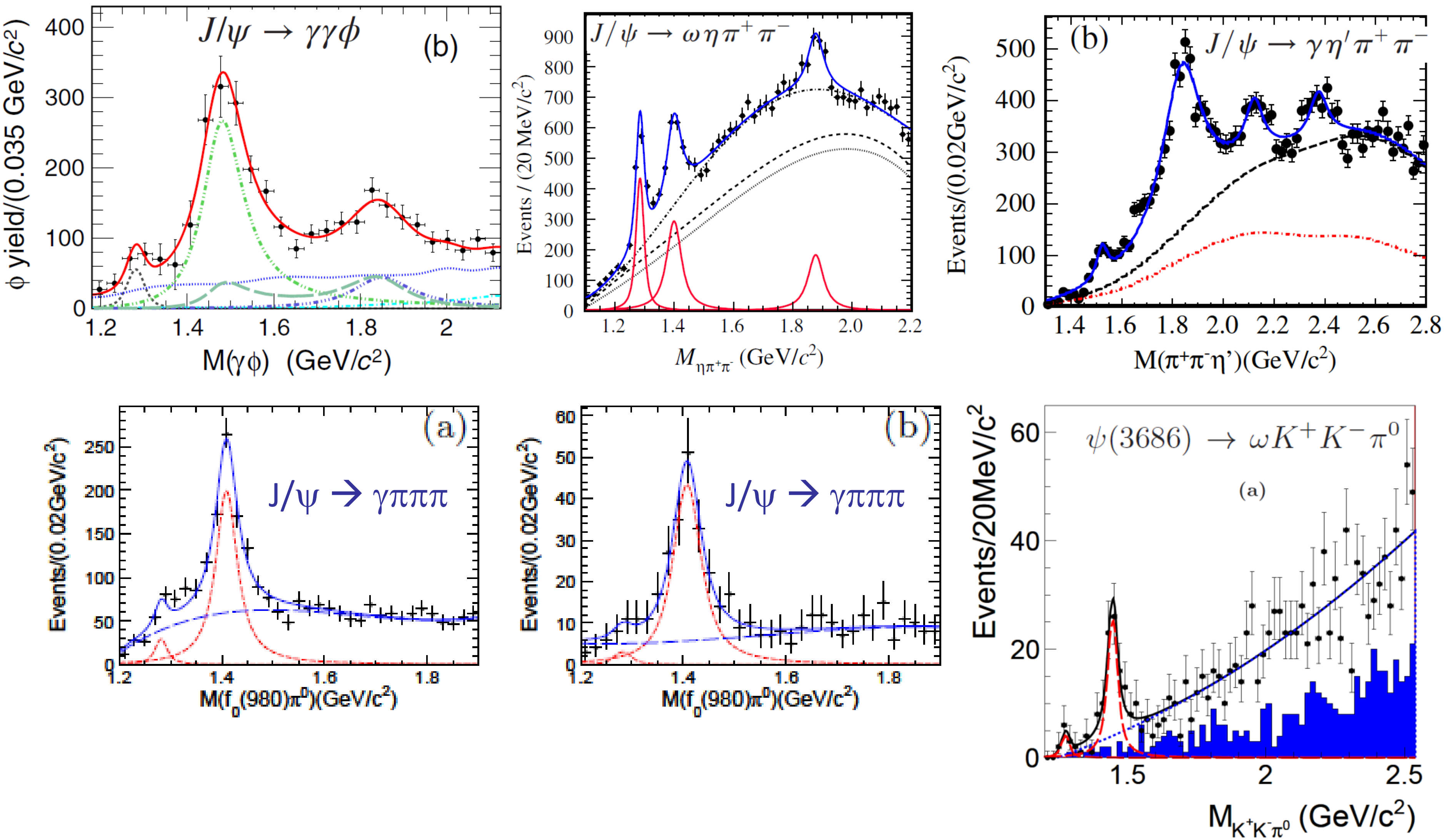
$J/\psi(1S)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

Γ_{151}	$\gamma\eta(1405/1475) \to \gamma K\overline{K}\pi$	[d]	$(2.8 \pm 0.6) \times 10^{-3}$	S=1.6	BES-III
Γ_{152}	$\gamma\eta(1405/1475) \to \gamma\gamma\rho^0$		$(7.8 \pm 2.0) \times 10^{-5}$	S=1.8	BES-III
Γ_{153}	$\gamma\eta(1405/1475) \to \gamma\eta\pi^+\pi^-$		$(3.0 \pm 0.5) \times 10^{-4}$		BES-III
Γ_{154}	$\gamma\eta(1405/1475) \to \gamma\gamma\phi$		$< 8.2 \times 10^{-5}$	CL=95%	BES-III
Γ_{165}	$\gamma\eta(1405/1475) \to \gamma\rho^0\rho^0$		$(1.7 \pm 0.4) \times 10^{-3}$	S=1.3	BES-III
Γ_{87}	$\phi\eta(1405) \to \phi\eta\pi^+\pi^-$		$(2.0 \pm 1.0) \times 10^{-5}$		BES-III

$\psi(2S)$

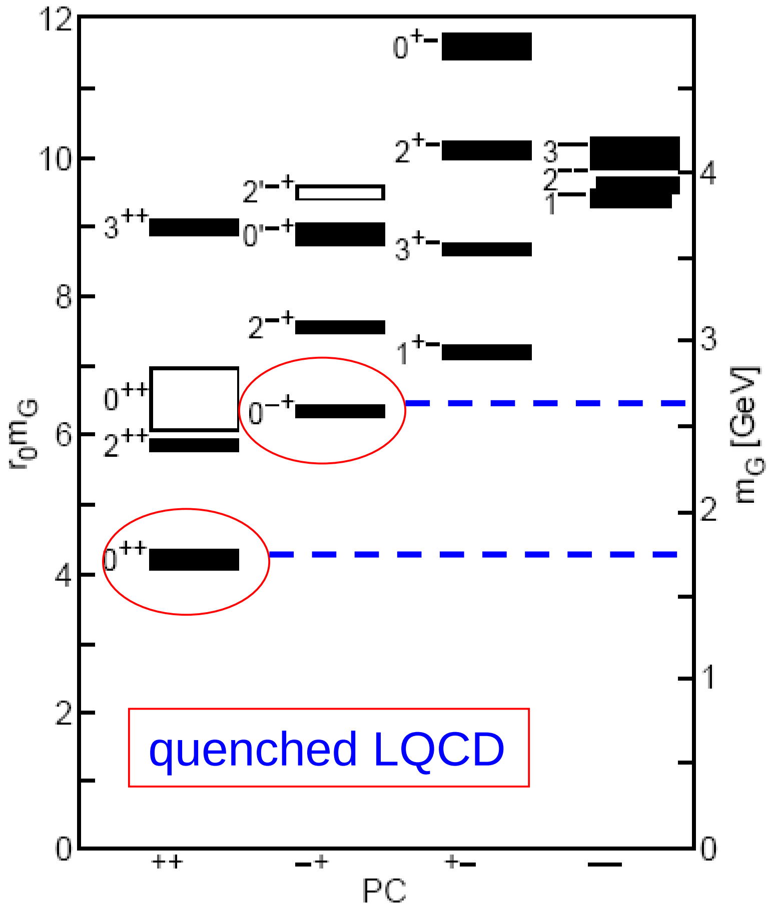
Γ_{94}	$\omega X(1440) \to \omega K^0_S K^-\pi^+ +$ c.c.	$(1.6 \pm 0.4) \times 10^{-5}$		BES-III
Γ_{95}	$\omega X(1440) \to \omega K^+ K^-\pi^0$	$(1.09 \pm 0.26) \times 10^{-5}$		BES-III
Γ_{155}	$\gamma\eta(1405)$			BES-II
Γ_{156}	$\gamma\eta(1405) \to \gamma K\overline{K}\pi$	$< 9 \times 10^{-5}$	CL=90%	BES-II
Γ_{157}	$\gamma\eta(1405) \to \eta\pi^+\pi^-$	$(3.6 \pm 2.5) \times 10^{-5}$		BES-II
Γ_{158}	$\gamma\eta(1475)$			BES-II
Γ_{159}	$\gamma\eta(1475) \to K\overline{K}\pi$	$< 1.4 \times 10^{-4}$	CL=90%	BES-II
Γ_{160}	$\gamma\eta(1475) \to \eta\pi^+\pi^-$	$< 8.8 \times 10^{-5}$	CL=90%	BES-II

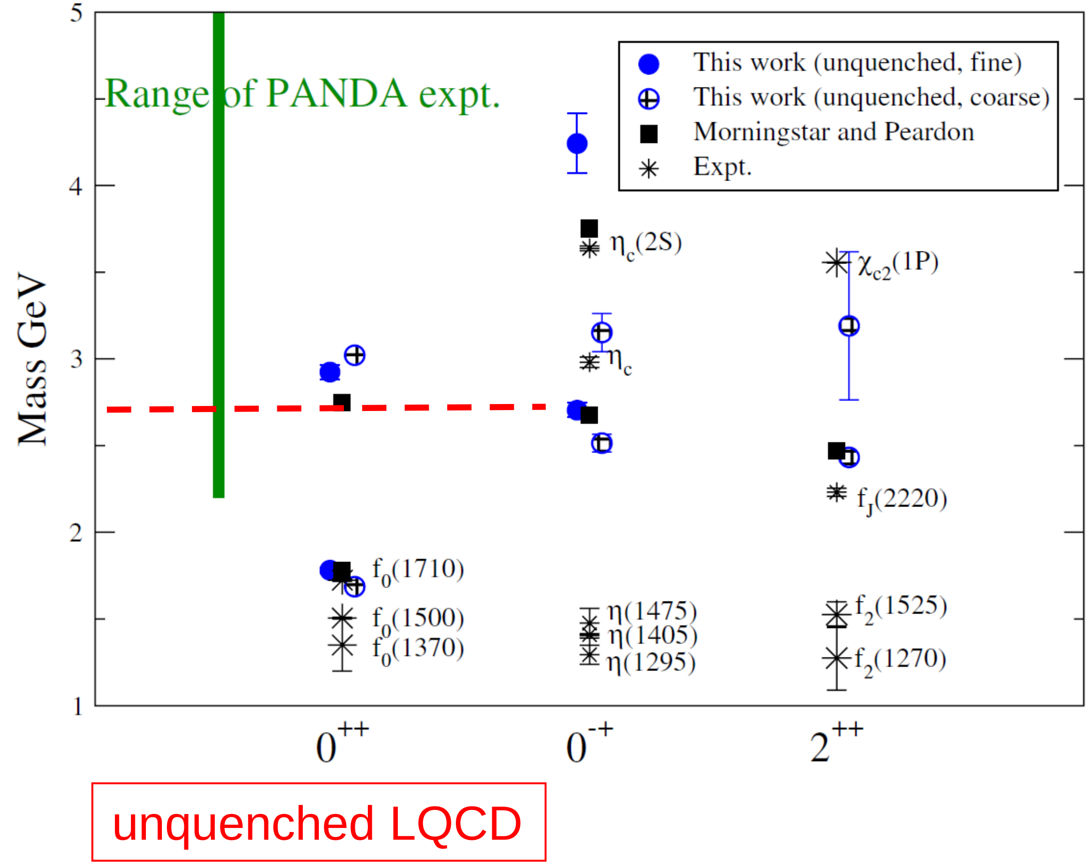
Invariant mass spectra measured at BES-III



No evidence for η(1405) and η(1475) to be present in the same decay channel

Lattice QCD results for the pseudoscalar glueball mass

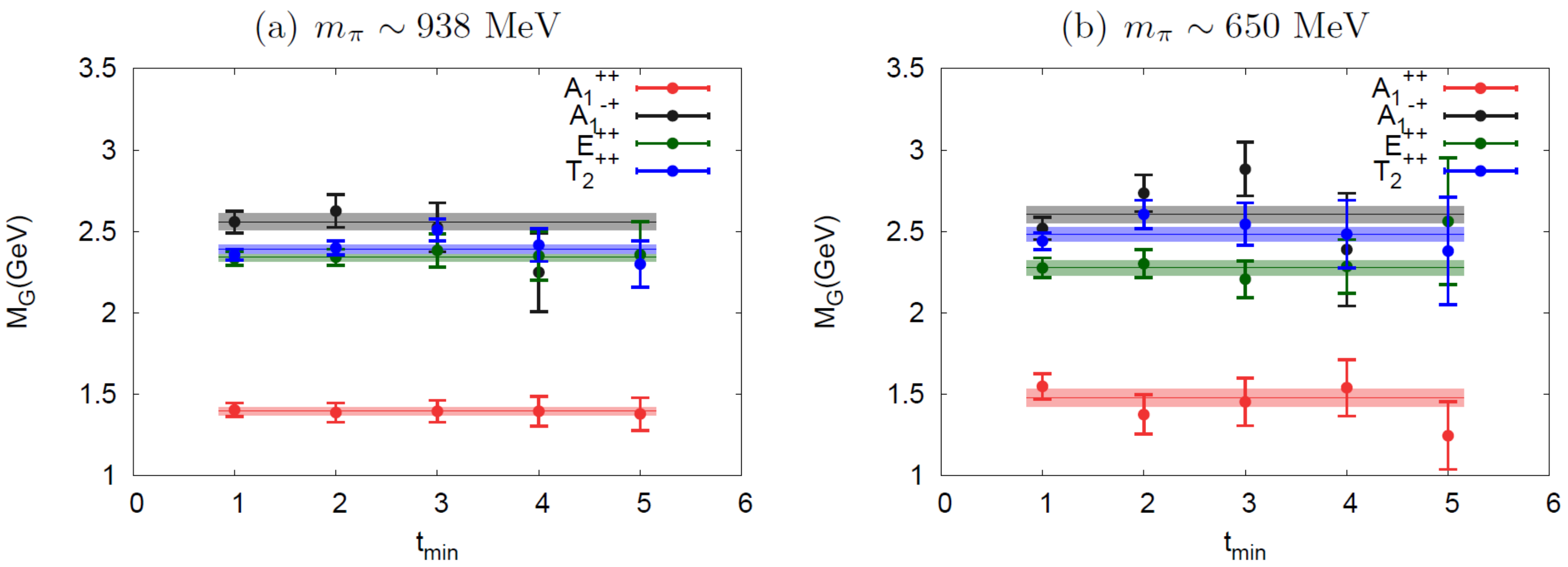




Morningstar and Peardon, PRD60, 034509 (1999); Y. Chen et al., PRD73, 014516(2006)

Richards, Irving, Gregory, and McNeile (UKQCD), PRD82, 034501 (2010)

N_f = 2 LQCD study on anisotropic lattices



	m_π (MeV)	$m_{0^{++}}$ (MeV)	$m_{2^{++}}$ (MeV)	$m_{0^{-+}}$ (MeV)
$N_f = 2$	938	1397(25)	2367(35)	2559(50)
	650	1480(52)	2380(61)	2605(52)
$N_f = 2+1$ [13]	360	1795(60)	2620(50)	—
quenched [8]	—	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched [9]	—	1730(50)(80)	2400(25)(120)	2590(40)(130)

W. Sun et al. [CLQCD], arXiv:1702.08174[hep-lat]

Given a low mass pseudoscalar glueball candidate $\eta(1405)$, Phenomenological studies have been focused on three aspects:

- Whether there are **mixings among the ground states η and η', and the pseudoscalar glueball**? How to disentangle their internal structures? What are the consequences from such state mixings?
- What causes the **low mass** of the pseudoscalar glueball compared with the LQCD calculations?
- What is the relation between $\eta(1405)$ and $\eta(1475)$? (What is the role played by the **triangle singularity mechanism**?)

Can all these three aspects be understood self-consistently?

A brief status review: Qin, QZ, and Zhong, PRD 97, 096002 (2018)

How to understand the mixing?

• η(1295) and η(1475) are the 1st radial excitation between the flavor singlet and octet with I=0.

$$\begin{cases} \eta(1295) &= \cos\alpha n\bar{n} - \sin\alpha s\bar{s} \\ \eta(1440) &= \sin\alpha n\bar{n} + \cos\alpha s\bar{s} \end{cases}$$

• η(1405) is a pseudoscalar glueball candidate which favors to mix with the ground states η(547) and η′(958).

• **Caution:** Lattice QCD gives the pseudoscalar glueball mass of ~2.4 GeV.

$$\begin{pmatrix} \eta \\ \eta' \\ \eta'' \end{pmatrix} = U \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{pmatrix} \begin{pmatrix} n\bar{n} \\ s\bar{s} \\ G \end{pmatrix}$$

• G. Li, Q. Zhao, C.H. Chang, JPG35, 055002 (2008); hep-ph/0701020
• C. Thomas, JHEP 0710:026, 2007
• R. Escribano, EPJC65, 467 (2010)
• H.Y. Cheng, H.n. Li and K.F. Liu, PRD79, 014024 (2009)
• … …

- **One can even include η_c ($\bar{c}c$) in the mixing scheme.**

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \\ |\eta_c\rangle \end{pmatrix} = U_{34}(\theta)U_{14}(\phi_G)U_{12}(\phi_Q)\begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix},$$

$M_G \cong 2.4$ GeV
$M\eta_c = 2.98$ GeV

$$U_{34}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad U_{14}(\phi_G) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi_G & \sin\phi_G & 0 \\ 0 & -\sin\phi_G & \cos\phi_G & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_{12}(\phi_Q) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi_Q & \sin\phi_Q \\ 0 & 0 & -\sin\phi_Q & \cos\phi_Q \end{pmatrix}. \qquad \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix} = U_{34}(\theta_i)\begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |g\rangle \\ |\eta_Q\rangle \end{pmatrix}$$

Constraints on the η and η′, but not strongly on a glueball candidate!

Y.-D. Tsai, H.-n. Li and Q.Z., PRD85, 034002 (2011)
Re-investigated in Qin, QZ, and Zhong, PRD 97, 096002 (2018)

Assuming that the decay constants in the flavor basis follow the same mixing pattern of the particle states, we have

$$\begin{pmatrix} f_\eta^q & f_\eta^s & f_\eta^c \\ f_{\eta'}^q & f_{\eta'}^s & f_{\eta'}^c \\ f_G^q & f_G^s & f_G^c \\ f_{\eta_c}^q & f_{\eta_c}^s & f_{\eta_c}^c \end{pmatrix} = U \begin{pmatrix} f_q & 0 & 0 \\ 0 & f_s & 0 \\ 0 & 0 & 0 \\ 0 & 0 & f_c \end{pmatrix}$$

where

$$\begin{aligned} U(\theta, \phi_G, \phi_Q) &= U_{34}(\theta) U_{14}(\phi_G) U_{12}(\phi_Q) U_{34}(\theta_i), \\ &= \begin{pmatrix} c\theta c\theta_i - s\theta c\phi_G s\theta_i & -c\theta s\theta_i - s\theta c\phi_G c\theta_i & -s\theta s\phi_G c\phi_Q & -s\theta s\phi_G s\phi_Q \\ s\theta c\theta_i + c\theta c\phi_G s\theta_i & -s\theta s\theta_i + c\theta c\phi_G c\theta_i & c\theta s\phi_G c\phi_Q & c\theta s\phi_G s\phi_Q \\ -s\phi_G s\theta_i & -s\phi_G c\theta_i & c\phi_G c\phi_Q & c\phi_G s\phi_Q \\ 0 & 0 & -s\phi_Q & c\phi_Q \end{pmatrix} \end{aligned}$$

The axial vector anomaly is given by the $U_A(1)$ Ward identity:

$$\partial^\mu J_{\mu 5}^j = \partial^\mu (\bar{j} \gamma_\mu \gamma_5 j) = 2 m_j (\bar{j} i \gamma_5 j) + \frac{\alpha_s}{4\pi} G \tilde{G}$$

The axial vector anomaly can then relate the pseudoscalar meson masses to the flavor singlet pseudoscalar densities and the topological charge density:

$$\langle 0|\partial^{\mu} J^{j}_{\mu 5}|P\rangle = M^2_P f^j_P$$

where $$M^2_P \equiv \begin{pmatrix} M^2_{\eta} & 0 & 0 & 0 \\ 0 & M^2_{\eta'} & 0 & 0 \\ 0 & 0 & M^2_G & 0 \\ 0 & 0 & 0 & M^2_{\eta_c} \end{pmatrix}$$

And $$\mathcal{M}_{qsgc} = U^{\dagger} M^2_P U$$ **--- (A)**

Meanwhile, the axial vector anomaly gives:

$$\tilde{\mathcal{M}}_{qsgc} = \begin{pmatrix} m^2_{qq} + \sqrt{2}G_q/f_q & m^2_{sq} + G_q/f_s & m^2_{cq} + G_q/f_c \\ m^2_{qs} + \sqrt{2}G_s/f_q & m^2_{ss} + G_s/f_s & m^2_{cs} + G_s/f_c \\ m^2_{qg} + \sqrt{2}G_g/f_q & m^2_{sg} + G_g/f_s & m^2_{cg} + G_g/f_c \\ m^2_{qc} + \sqrt{2}G_c/f_q & m^2_{sc} + G_c/f_s & m^2_{cc} + G_c/f_c \end{pmatrix}$$ **--- (B)**

The equivalence of Eqs. **(A)** and **(B)** gives:

$$U^\dagger \begin{pmatrix} M_\eta^2 & 0 & 0 & 0 \\ 0 & M_{\eta'}^2 & 0 & 0 \\ 0 & 0 & M_G^2 & 0 \\ 0 & 0 & 0 & M_{\eta_c}^2 \end{pmatrix} U \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} m_{qq}^2 + \sqrt{2}G_q/f_q & m_{sq}^2 + G_q/f_s & m_{cq}^2 + G_q/f_c \\ m_{qs}^2 + \sqrt{2}G_s/f_q & m_{ss}^2 + G_s/f_s & m_{cs}^2 + G_s/f_c \\ m_{qg}^2 + \sqrt{2}G_g/f_q & m_{sg}^2 + G_g/f_s & m_{cg}^2 + G_g/f_c \\ m_{qc}^2 + \sqrt{2}G_c/f_q & m_{sc}^2 + G_c/f_s & m_{cc}^2 + G_c/f_c \end{pmatrix}$$

with

$$\begin{cases} m_{qq,qs,qg,qc}^2 \equiv \dfrac{\sqrt{2}}{f_q}\langle 0|m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d|\eta_q, \eta_s, g, \eta_Q\rangle \\ m_{sq,ss,sg,sc}^2 \equiv \dfrac{2}{f_s}\langle 0|m_s\bar{s}i\gamma_5 s|\eta_q, \eta_s, g, \eta_Q\rangle, \\ m_{cq,cs,cg,cc}^2 \equiv \dfrac{2}{f_c}\langle 0|m_c\bar{c}i\gamma_5 c|\eta_q, \eta_s, g, \eta_Q\rangle, \\ G_{q,s,g,c} \equiv \dfrac{\alpha_s}{4\pi}\langle 0|G\tilde{G}|\eta_q, \eta_s, g, \eta_Q\rangle. \end{cases}$$

This allows a relation for the physical glueball mass and the topological charge density in association with the other constrained parameters:

$$\begin{aligned}\tilde{\mathcal{M}}^{31}_{qsgc} &= m^2_{qg} + \sqrt{2}G_g/f_q \\ &= -M^2_\eta(c\theta c\theta_i - s\theta c\phi_G s\theta_i)s\theta s\phi_G c\phi_Q + M^2_{\eta'}(s\theta c\theta_i + c\theta c\phi_G s\theta_i)c\theta s\phi_G c\phi_Q - M^2_G c\phi_G s\phi_G s\theta_i c\phi_Q,\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{M}}^{32}_{qsgc} &= m^2_{sg} + G_g/f_s \\ &= M^2_\eta(c\theta s\theta_i + s\theta c\phi_G c\theta_i)s\theta s\phi_G c\phi_Q + M^2_{\eta'}(-s\theta s\theta_i + c\theta c\phi_G c\theta_i)c\theta s\phi_G c\phi_Q - M^2_G c\phi_G s\phi_G c\theta_i c\phi_Q.\end{aligned}$$

$$\hat{R}_{31/32} \equiv \frac{\tilde{\mathcal{M}}^{31}_{qsgc}}{\tilde{\mathcal{M}}^{32}_{qsgc}} = \frac{m^2_{qg} + \sqrt{2}G_g/f_q}{m^2_{sg} + G_g/f_s}$$

$$\begin{aligned}M^2_G = &-\frac{1}{\cos\phi_G \sin\theta_i \cos\phi_Q}\left\{\frac{\sqrt{2}G_g/f_q}{\sin\phi_G} - [-M^2_\eta(\cos\theta\cos\theta_i - \sin\theta\cos\phi_G\sin\theta_i)\sin\theta\cos\phi_Q\right. \\ &\left. + M^2_{\eta'}(\sin\theta\cos\theta_i + \cos\theta\cos\phi_G\sin\theta_i)\cos\theta\cos\phi_Q]\right\}. \\ \approx &-\frac{1}{\sin\theta_i}\left\{\frac{\sqrt{2}G_g/f_q}{\sin\phi_G} - M^2_{\eta'}\sin\theta_i - (M^2_{\eta'} - M^2_\eta)\sin\theta\cos(\theta+\theta_i)\right\}\end{aligned}$$

With the LQCD results for the topological charge density, we can fit the parameters:

TABLE I. The numerical values of all the parameters with $G_g = -0.054$ GeV^3 and $\phi_G = 12°$ fixed. The two quantities, m_{qc}^{2*} and m_{sc}^{2*} involve more complicated issues and are sensitive to m_{cc}^2 and ϕ_G. Further detailed discussions can be found in the context.

f_s/f_q	M_G (GeV)	m_{qq}^2 (GeV)^2	m_{ss}^2	m_{sg}^2	m_{cg}^2	m_{qc}^{2*}	m_{sc}^{2*}	m_{cq}^2	m_{cs}^2	G_q (GeV)^3	G_s	G_c
1.2	2.1	0.055	0.45	−0.041	−0.81	0.87	0.50	−0.24	−0.15	0.060	0.035	−0.092
1.3	2.1	0.0012	0.47	−0.067	−0.81	0.87	0.46	−0.25	−0.15	0.065	0.035	−0.092

where we have applied the condition: $m_{qs,sq}^2 \ll m_{qg}^2 \ll m_{qq}^2$

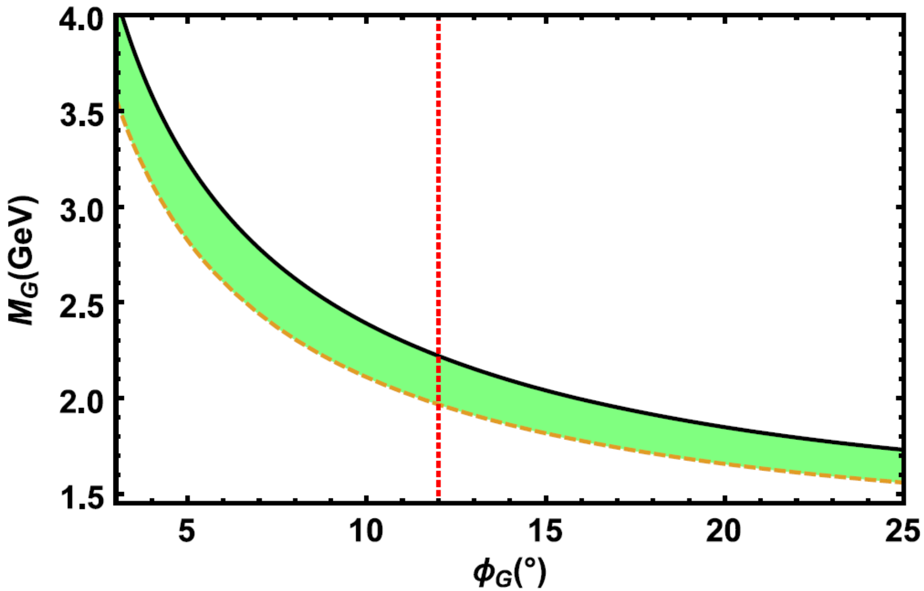
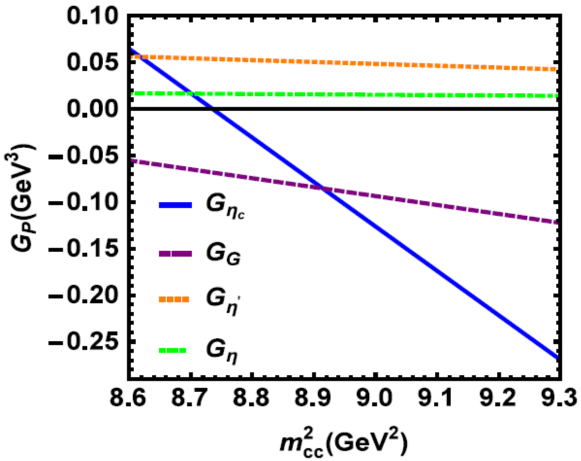


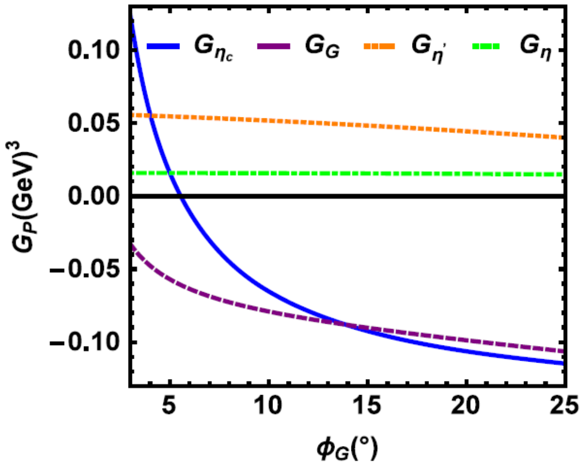
FIG. 1. The physical glueball mass M_G varies with $\phi_G \in (3\text{–}25)°$, with $\theta = -11°$, $\phi_Q = 11.6°$, and $f_q = 131$ MeV.

The dependence of G_P on m_{cc}^2, ϕ_G, and ϕ_Q

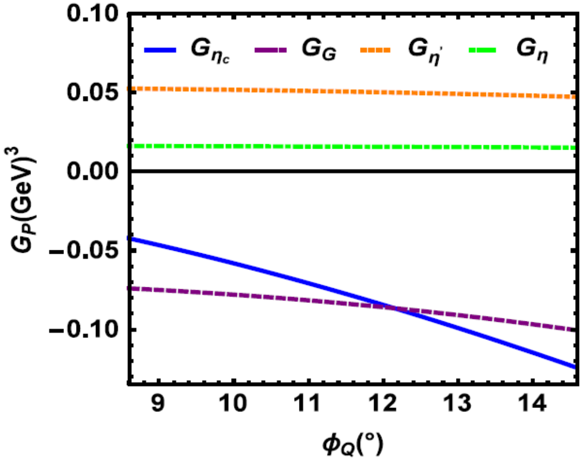
$\phi_G = 12^\circ$ and $\phi_Q = 11.6^\circ$



$m_{cc}^2 = M_{\eta_c}^2$, and $\phi_Q = 11.6^\circ$



$\phi_G = 12^\circ$ and $m_{cc}^2 = M_{\eta_c}^2$



The topological susceptibility can be extracted for the pseudoscalar mesons:

$$\left\{\begin{aligned}
\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle &= 0.016\ \mathrm{GeV}^3,\\
\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle &= 0.051\ \mathrm{GeV}^3,\\
\langle 0|\alpha_s G\tilde{G}/(4\pi)|G\rangle &= -0.084\ \mathrm{GeV}^3,\\
\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta_c\rangle &= -0.079\ \mathrm{GeV}^3,
\end{aligned}\right.$$

LQCD results:

$$\left\{\begin{aligned}
&\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta\rangle \approx 0.021\ \mathrm{GeV}^3\\
&\langle 0|\alpha_s G\tilde{G}/(4\pi)|\eta'\rangle \approx 0.035\ \mathrm{GeV}^3\\
&\boxed{G_g = -(0.054 \pm 0.008)\ \mathrm{GeV}^3}
\end{aligned}\right.$$

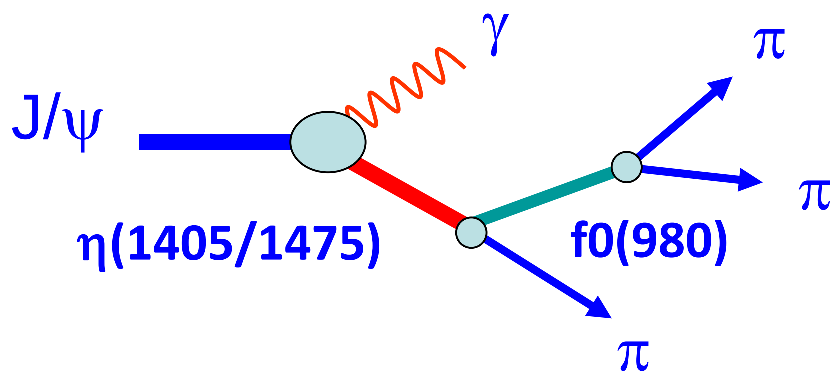
Low mass pseudoscalar glueball is unlikely to be favored!

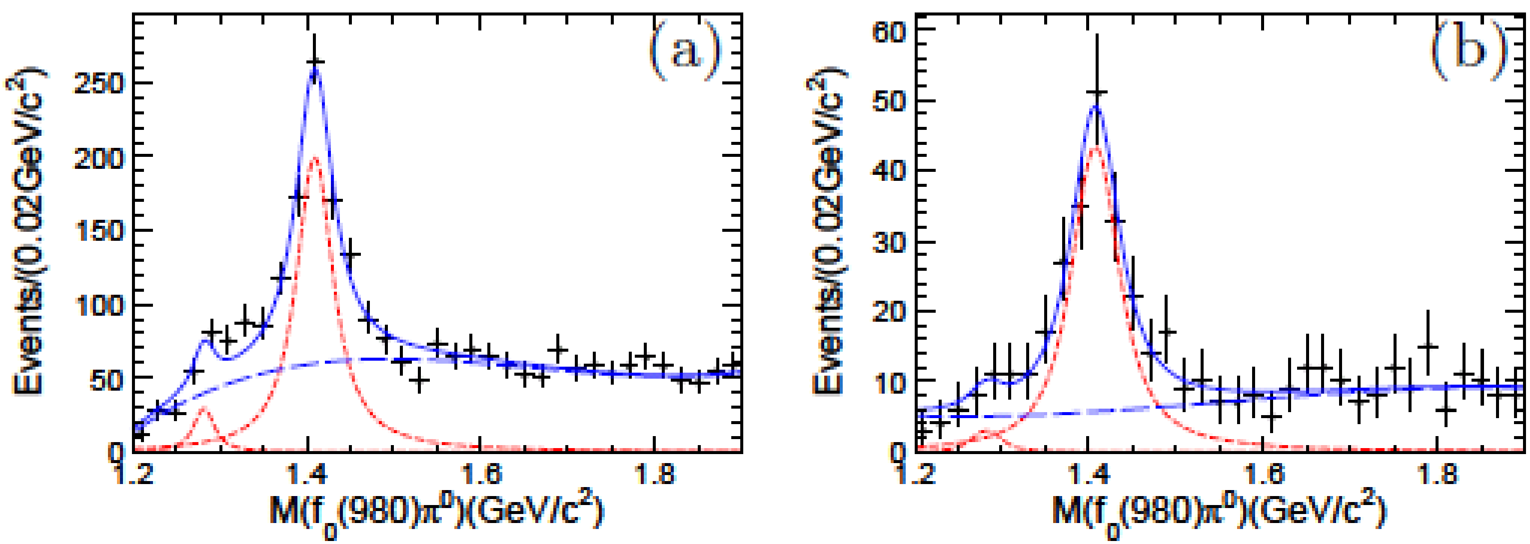
How to understand different masses and lineshapes for η(1405) and η(1475) in different channels?

PRL 108, 182001 (2012) PHYSICAL REVIEW LETTERS week ending 4 MAY 2012

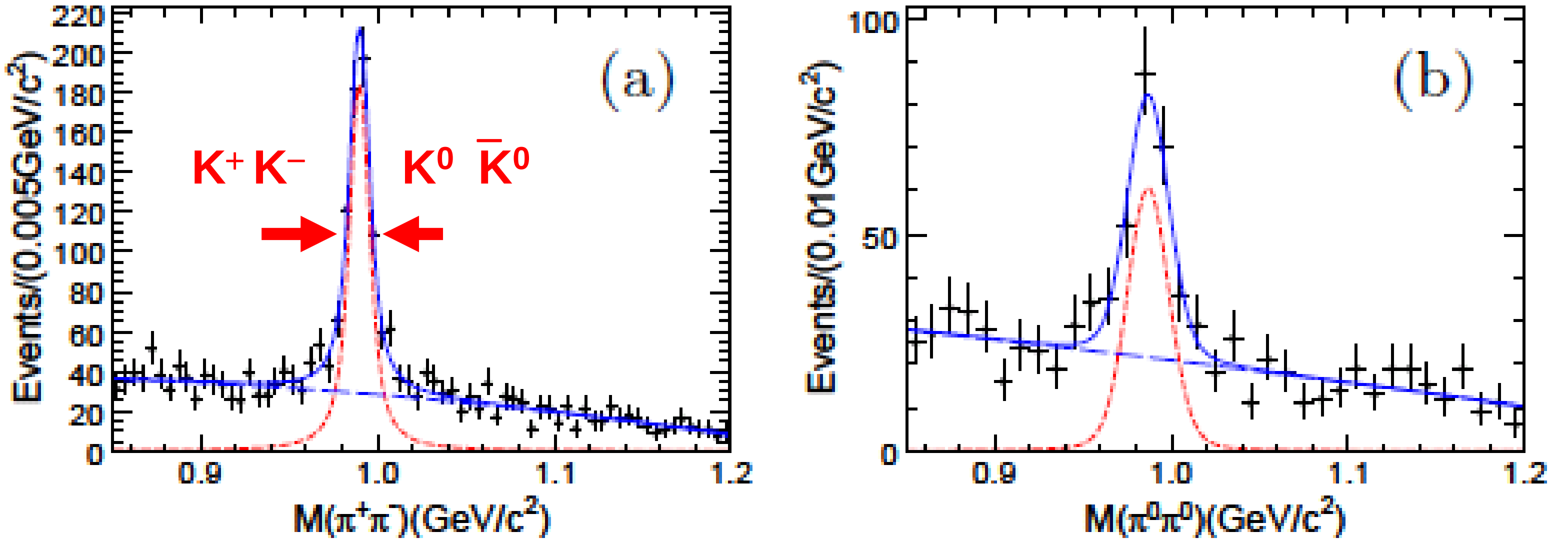
First Observation of $\eta(1405)$ Decays into $f_0(980)\pi^0$

Isospin-violating decay of J/ψ → γπππ





BES-III Collaboration, Phys. Rev. Lett. 108, 182001 (2012)



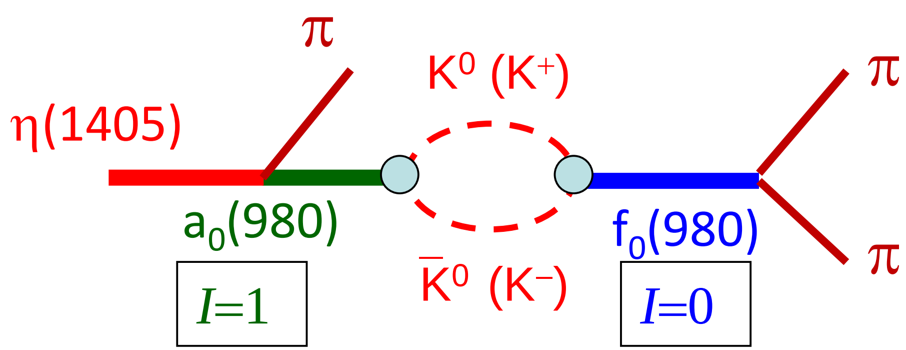
- $f_0(980)$ is extremely narrow: $\Gamma \cong 10$ MeV !

PDG: $\Gamma \cong 40\sim100$ MeV.

- Anomalously large isospin violation!

$$\frac{Br(\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)}{Br(\eta(1405) \to a_0^0(980)\pi^0 \to \eta\pi^0\pi^0)} \cong (17.9 \pm 4.2)\%$$

"$a_0(980)$-$f_0(980)$ mixing" gives only 1% isospin violation effects !

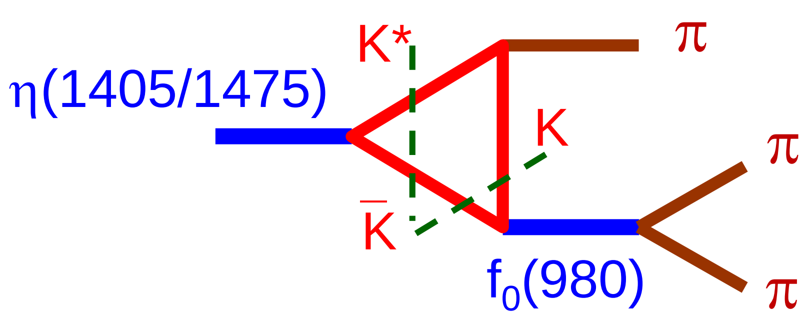


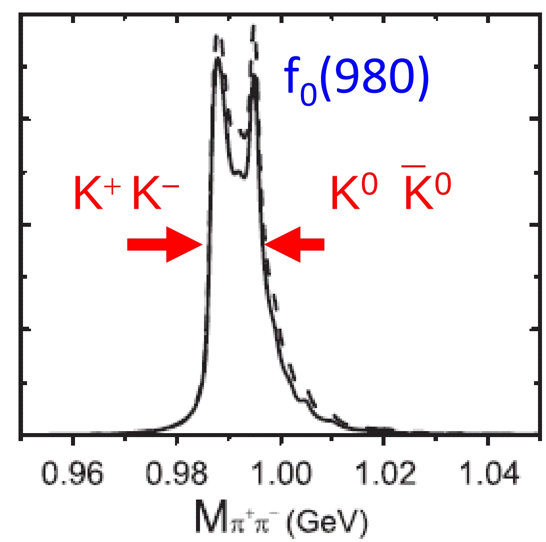
$$g(a_0K^+K^-)\, g(f_0K^+K^-) = -g(a_0K^0\bar{K}^0)\, g(f_0K^0\bar{K}^0)$$

$$M(K^0)-M(K^\pm) = m_d - m_u$$

"Triangle singularity"

Internal $\bar{K}K^*(K)$ approach the on-shell condition simultaneously!

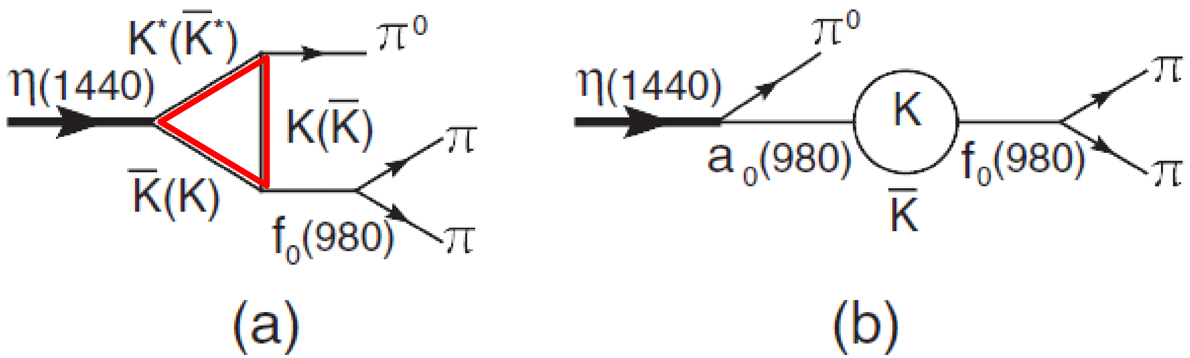


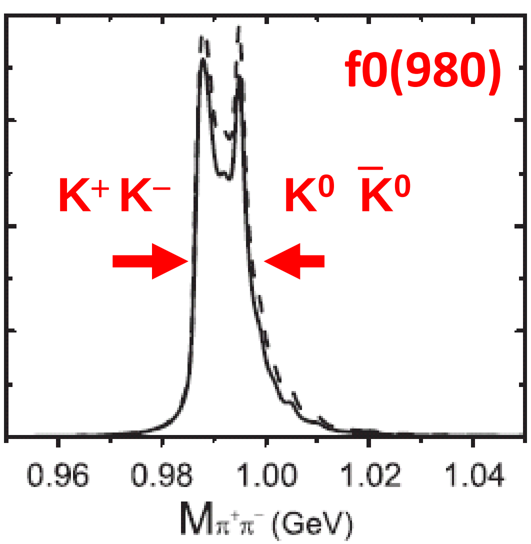


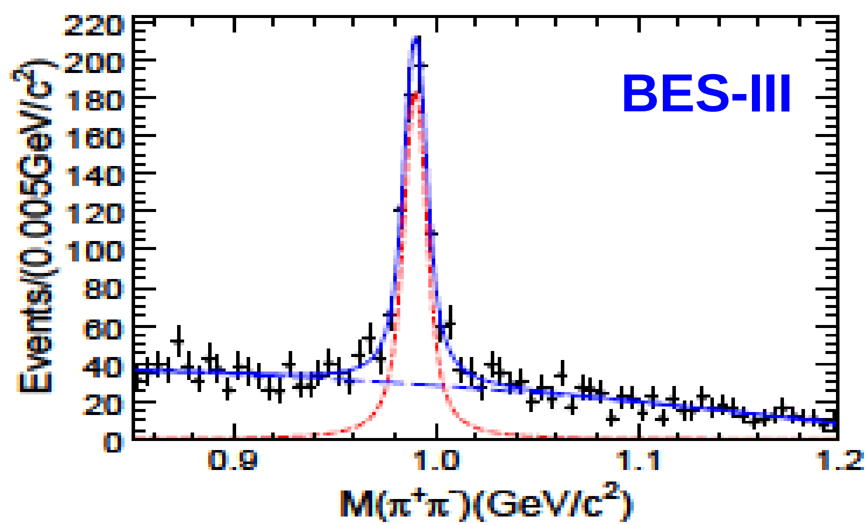
A novel isospin breaking mechanism!

Puzzle of Anomalously Large Isospin Violations in $\eta(1405/1475) \to 3\pi$

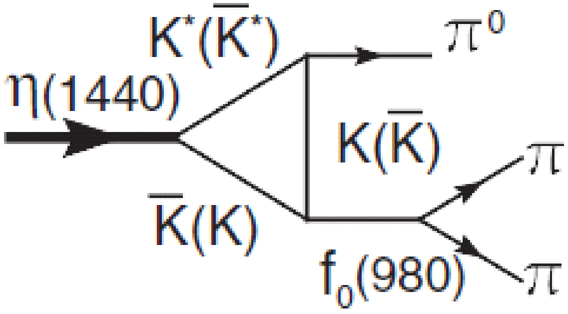
"Triangle Singularity" mechanism is dominant over the a0(980)-f0(980) mixing in the isospin-violating channel.



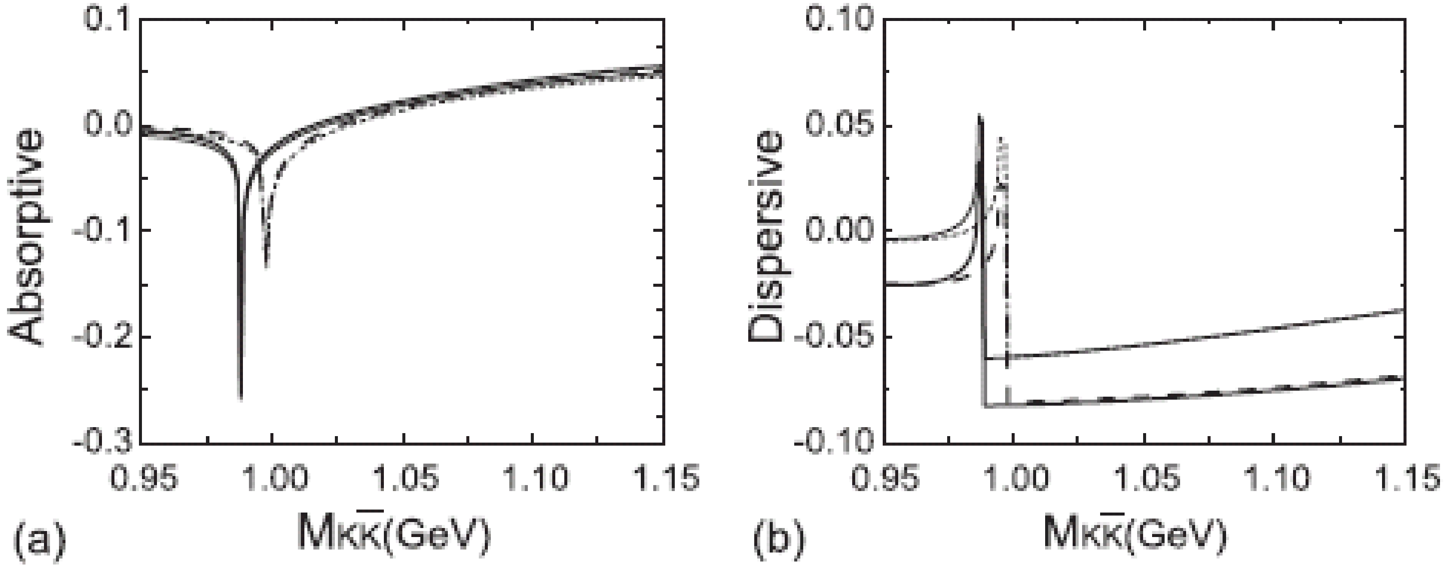




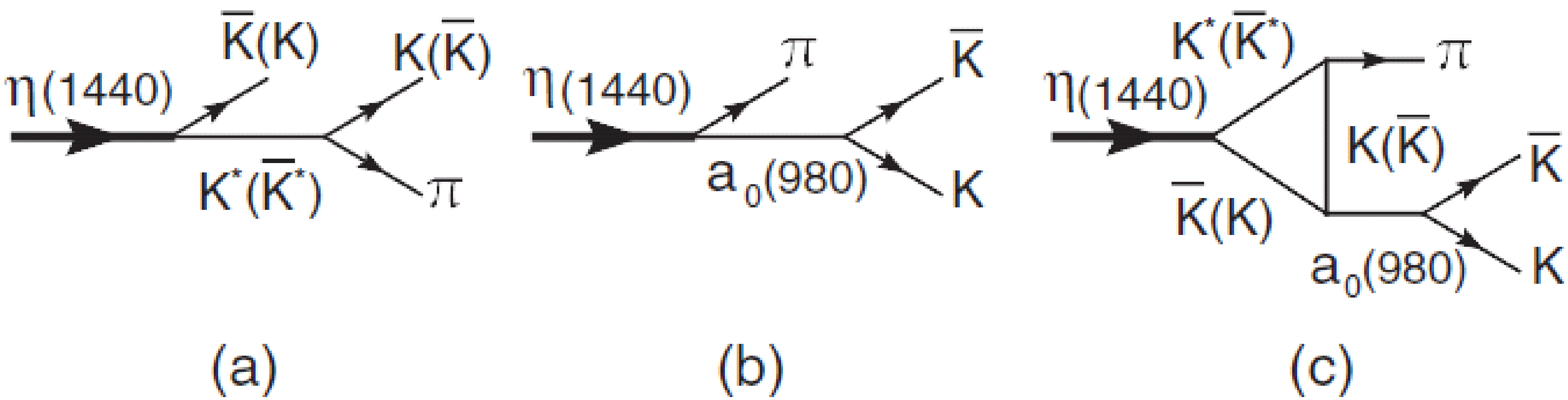
Triangle loop amplitudes:



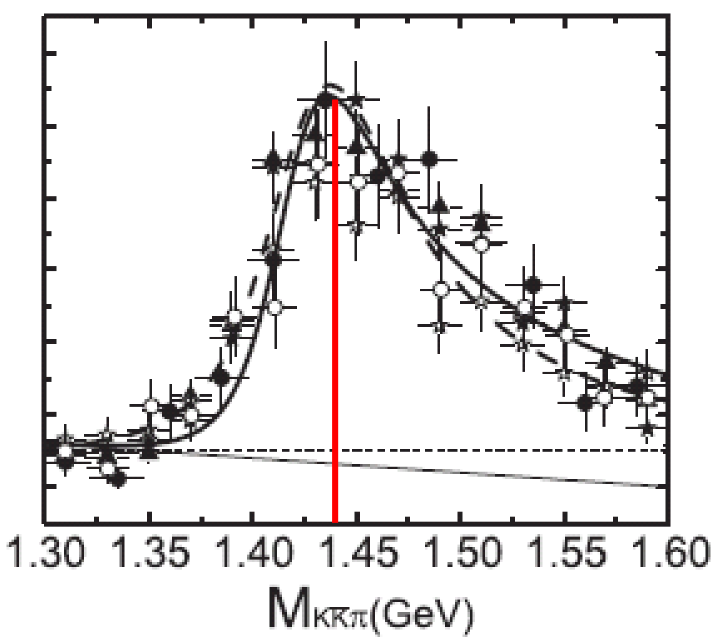
Absorptive amplitudes **Dispersive amplitudes**



$\eta(1440) \to K\ \bar{K}\ \pi$ decay mechanism:



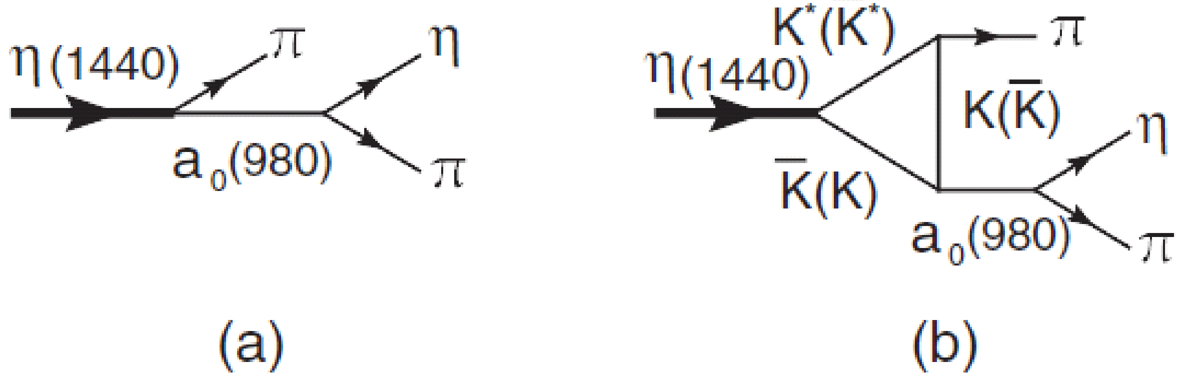
Data from Mark III, BES-I, and DM2



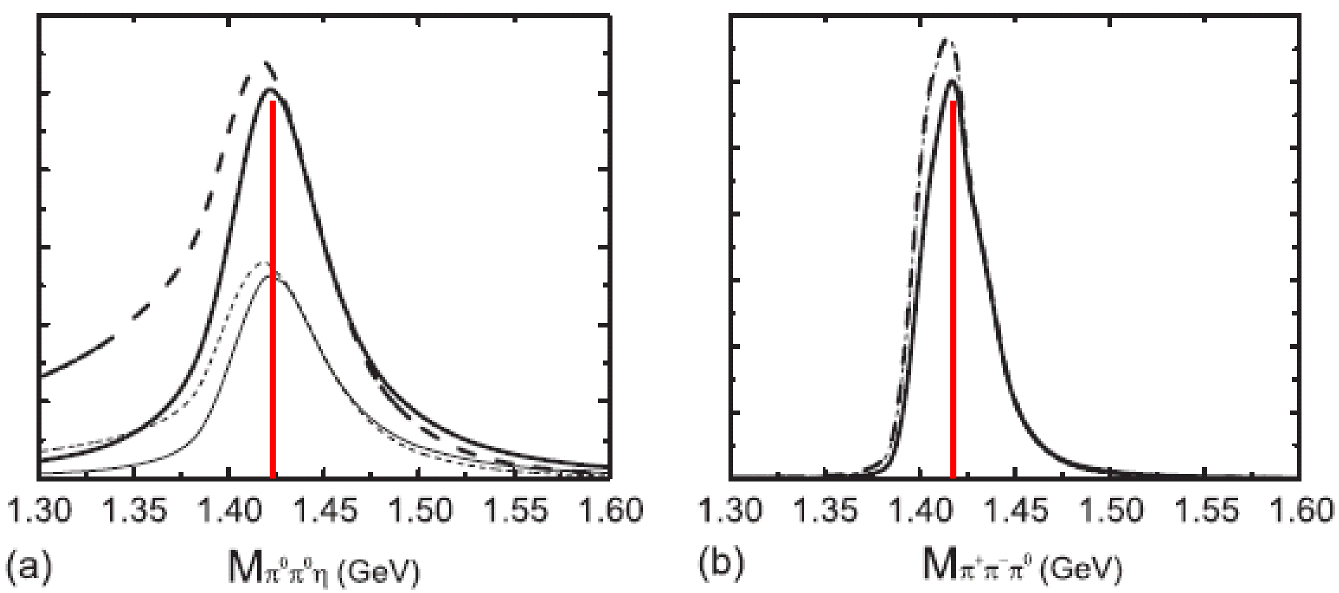
$$\frac{d\Gamma_{J/\psi\to\gamma\eta(1440)\to\gamma ABC}}{d\sqrt{s_0}} = \frac{2s_0}{\pi}\frac{\Gamma_{J/\psi\to\gamma\eta(1440)}(s_0)\Gamma_{\eta(1440)\to ABC}(s_0)}{(s_0-m^2_{\eta(1440)})^2+\Gamma^2_{\eta(1440)}m^2_{\eta(1440)}},$$

J.J. Wu, X.H. Liu, Q.Z. and B.S. Zou, PRL(2012)

$\eta(1440) \rightarrow \eta\pi\pi$ decay mechanism:



Invariant mass spectra of $\eta(1440) \rightarrow \eta\pi\pi$ and 3π, respectively. They have different lineshapes, i.e. drastically different widths.

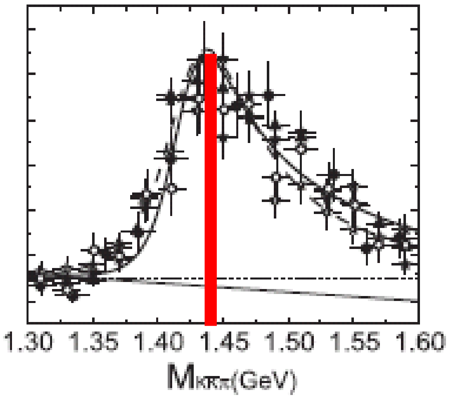


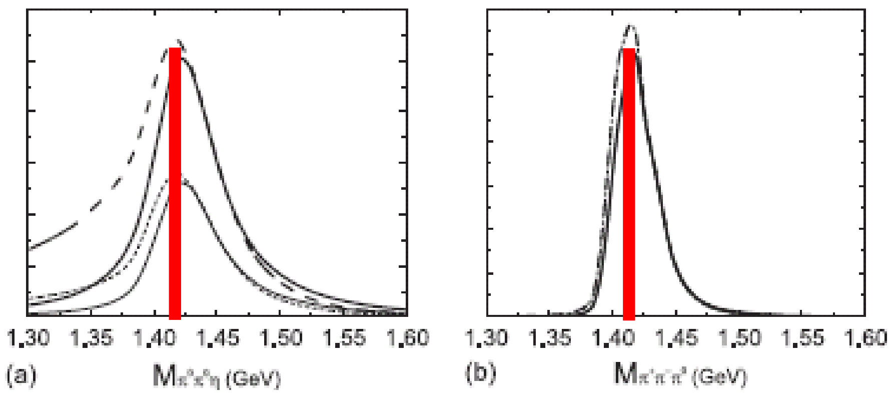
•The contributions from the "Triangle Singularity" mechanism can shift the peak positions in different channels.

•It leads to about 30~40 MeV mass shift between $K\bar{K}\pi$ and $\eta\pi\pi$ decay channels.

•The $\eta(1440)$ mass spectrum shapes are totally different in those three channels, i.e. $K\bar{K}\pi$, $\eta\pi\pi$, and 3π.

•There is no obvious need for two states, $\eta(1405)$ and $\eta(1475)$!





- **Radiative decay patterns are out of intuition**

Immediate crucial questions:

i) If $\eta(1440)$ is assigned as the ($s\bar{s}$) partner of $\eta(1295)$, can we understand that $\eta(1440) \to \phi\,(s\bar{s})\,\gamma$ is much smaller than $\eta(1440) \to \rho^0\,(n\bar{n})\,\gamma$?

$$
\begin{aligned}
\eta(1295) &= \cos\alpha n\bar{n} - \sin\alpha s\bar{s} \\
\eta(1440) &= \sin\alpha n\bar{n} + \cos\alpha s\bar{s}
\end{aligned}
$$

ii) Why $J/\psi \to \gamma\eta(1440)$ is so much stronger than $J/\psi \to \gamma\eta(1295)$?

Particle Data Group 2012:

$BR(J/\psi \to \gamma\eta(1295))/BR(J/\psi \to \gamma\eta(1440)) \leq 0.1$

Answer to question (i):

By assigning $\eta(1295)$ and $\eta(1440)$ as the first radial excitation of η and η', we can organize them as the following mixtures between $n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$ and $s\bar{s}$:

$$\begin{aligned} \eta(1295) &= \cos\alpha\, n\bar{n} - \sin\alpha\, s\bar{s} \\ \eta(1440) &= \sin\alpha\, n\bar{n} + \cos\alpha\, s\bar{s} \ , \end{aligned} \tag{1}$$

where α is the mixing angle.

In the J/ψ radiative decays, it is a good approximation that the photon is radiated by the charm (anti-)quark, and the light $q\bar{q}$ of 0^{-+} is produced by the gluon radiation. By defining the production strength for the $q\bar{q}$ of 0^{-+} as the following:

$$g_0 \equiv \langle q\bar{q}|\hat{H}|J/\psi, \gamma\rangle \ , \tag{2}$$

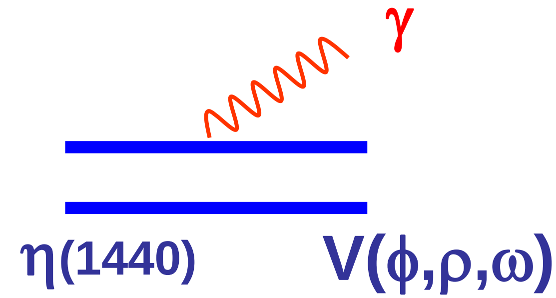
one can express the production amplitudes for $\eta(1295)$ and $\eta(1440)$ as

$$\begin{aligned} \mathcal{M}(\eta(1295)) &= (\sqrt{2}\cos\alpha - R\sin\alpha)g_0 \ , \\ \mathcal{M}(\eta(1440)) &= (\sqrt{2}\sin\alpha + R\cos\alpha)g_0 \ , \end{aligned} \tag{3}$$

$$\frac{B.R.(J/\psi \to \gamma\eta(1440))}{B.R.(J/\psi \to \gamma\eta(1295))} = \left(\frac{q_{\eta(1440)}}{q_{\eta(1295)}}\right)^3 \left(\frac{\sqrt{2}\sin\alpha + R\cos\alpha}{\sqrt{2}\cos\alpha - R\sin\alpha}\right)^2 \simeq 10$$

with $R \equiv 1$, one has $\alpha \simeq 38^\circ$

Answer to question (ii):



Magnetic dipole transition operator:

$$\hat{H}_{em} \equiv \langle \phi_A \chi_S | \sum_i^2 e_i \mu_i \overrightarrow{\sigma}_i \cdot \overrightarrow{\epsilon}_\gamma | \phi_S \chi_A \rangle$$

The flavor and spin wavefunction for the pseudoscalar:

$$\begin{aligned}
\phi_S(s\bar{s}) &\equiv (s\bar{s} + \bar{s}s)/\sqrt{2} \, , \\
\phi_S(n\bar{n}) &\equiv (n\bar{n} + \bar{n}n)/\sqrt{2} \, , \\
\chi_A &\equiv (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}
\end{aligned}$$

The flavor and spin wavefunction for the vector:

$$\begin{aligned}
\phi_A(\phi) &\equiv (s\bar{s} - \bar{s}s)/\sqrt{2} \, , \\
\phi_A(\rho^0) &\equiv ((u\bar{u} - \bar{u}u) - (d\bar{d} - \bar{d}d))/2 \, , \\
\phi_A(\omega) &\equiv ((u\bar{u} - \bar{u}u) + (d\bar{d} - \bar{d}d))/2 \, , \\
\chi_S &\equiv \uparrow\uparrow, \ \downarrow\downarrow, \ (\uparrow\downarrow + \downarrow\uparrow)/\sqrt{2} \, ,
\end{aligned}$$

The M1 transition amplitudes for η(1440) → γ V:

$$h_{\phi\gamma} = -\frac{e}{3m_s}\cos\alpha \ ,$$
$$h_{\rho^0\gamma} = \frac{e}{2m_q}\sin\alpha \ ,$$
$$h_{\omega\gamma} = \frac{e}{6m_q}\sin\alpha \ ,$$

where $m_q = m_u = m_d$ and $m_s \simeq 5m_q/3$.

$$B.R.(\gamma\phi) : B.R.(\gamma\rho^0) : B.R.(\gamma\omega) \simeq \frac{\cos^2\alpha}{25} : \frac{\sin^2\alpha}{4} : \frac{\sin^2\alpha}{36} \ .$$
$$\simeq 1 : 3.8 : 0.42.$$

with $\alpha \simeq 38°$

So far, there is no obvious difficulty for having only one η(1440) to cope with the existing observables!

5. Brief summary

- 1965-1980, Mark-II & Crystal Ball: E-meson / $\iota(1440) \rightarrow \eta(1440)$

- 1987, Mark-III: $\eta(1440) \rightarrow \eta(1405) + \eta(1475)$

- 2012: $\eta(1405) + \eta(1475) \rightarrow \eta(1440)$?

We have to alter our view of the pseudoscalar spectrum dramatically even for the 1st radial excitation!

- Where is the pseudoscalar glueball candidate located?

- We should look for the pseudoscalar glueball state at higher mass region! For instance, X(1835), X(2120), X(2370) …

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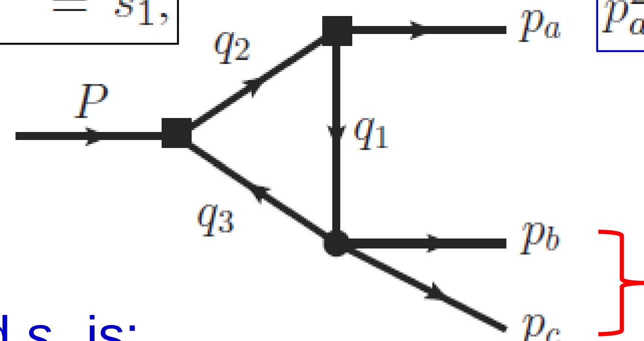
Thanks for your attention!

Kinematics :

$\boxed{P^2 = s_1,}$ $\boxed{p_a^2 = s_3}$



$\boxed{(p_b + p_c)^2 = s_2}$

The ATS condition for fixed s_1, m_j, and s_3 is:

$$s_2^{\pm} = (m_1 + m_3)^2 + \frac{1}{2m_2^2}[(m_1^2 + m_2^2 - s_3)(s_1 - m_2^2 - m_3^2) - 4m_2^2 m_1 m_3$$
$$\pm \lambda^{1/2}(s_1, m_2^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)],$$

Or for fixed s_2, m_j, and s_3 :

$$s_1^{\pm} = (m_2 + m_3)^2 + \frac{1}{2m_1^2}[(m_1^2 + m_2^2 - s_3)(s_2 - m_1^2 - m_3^2) - 4m_1^2 m_2 m_3$$
$$\pm \lambda^{1/2}(s_2, m_1^2, m_3^2)\lambda^{1/2}(s_3, m_1^2, m_2^2)].$$

with $\lambda(x, y, z) \equiv (x - y - z)^2 - 4yz.$

X.-H. Liu, M. Oka and Q. Zhao, PLB753, 297(2016); arXiv:1507.01674 [hep-ph]

Single dispersion relation in s_2 in the complex plane of $s_2{}'$:

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int\limits_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon}\, \sigma(s_1, s'_2, s_3)$$

The spectral function $\sigma(s_1, s_2, s_3)$ can be obtained by means of the Cutkosky's rules (absorptive part of the loop amplitude):

$$\sigma(s_1, s_2, s_3) \;=\; \frac{-1}{16\pi} \int_0^1 \int_0^1 \int_0^1 da_1\, da_2\, da_3\, \delta(1 - a_1 - a_2 - a_3)\delta(D).$$

which reads

$$\begin{aligned}
\sigma(s_1, s_2, s_3) &= \sigma_+ - \sigma_-, \\
\sigma_\pm(s_1, s_2, s_3) &= \frac{-1}{16\pi\lambda^{1/2}(s_1, s_2, s_3)} \log[-s_2(s_1 + s_3 - s_2 + m_1^2 + m_3^2 - 2m_2^2) \\
&- (s_1 - s_3)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_1, s_2, s_3)\lambda^{1/2}(s_2, m_1^2, m_3^2)].
\end{aligned}$$

For fixed s_1, s_3 and m_i, the spectral function $\sigma(s_1, s_2, s_3)$ has logarithmic branch points $s^{\pm}_2$, which correspond to the anomalous thresholds by solving the Landau equation.

How the logarithmic branch points $s^{\pm}_2$ move as s_1 increases from the threshold of $(m_2 + m_3)^2$, with s_3 and m_i fixed?

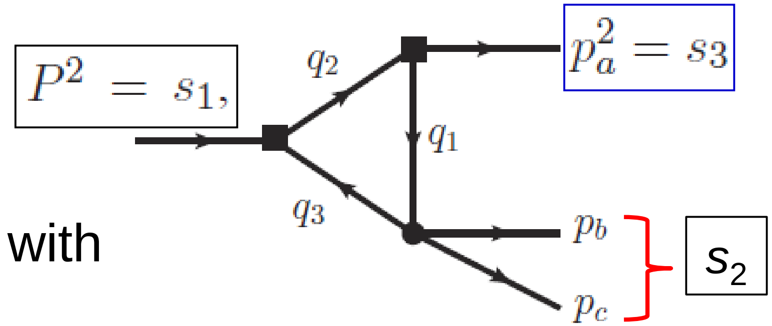
Substituting $s_1 \to s_1 + i\varepsilon$, $s^{\pm}_2$ in the s'-plane are then located at

$$s_2^{\pm}(s_1 + i\epsilon) = s_2^{\pm}(s_1) + i\epsilon \frac{\partial s_2^{\pm}}{\partial s_1},$$

With $\partial s_2^{\pm}/\partial s_1 = 0$ $(\partial s_1^{\pm}/\partial s_2 = 0)$

the normal and critical thresholds for s_1 and s_2 can be determined:

$$s_{1N} = (m_2 + m_3)^2, \; s_{1C} = (m_2 + m_3)^2 + \frac{m_3}{m_1}[(m_2 - m_1)^2 - s_3],$$
$$s_{2N} = (m_1 + m_3)^2, \; s_{2C} = (m_1 + m_3)^2 + \frac{m_3}{m_2}[(m_2 - m_1)^2 - s_3],$$

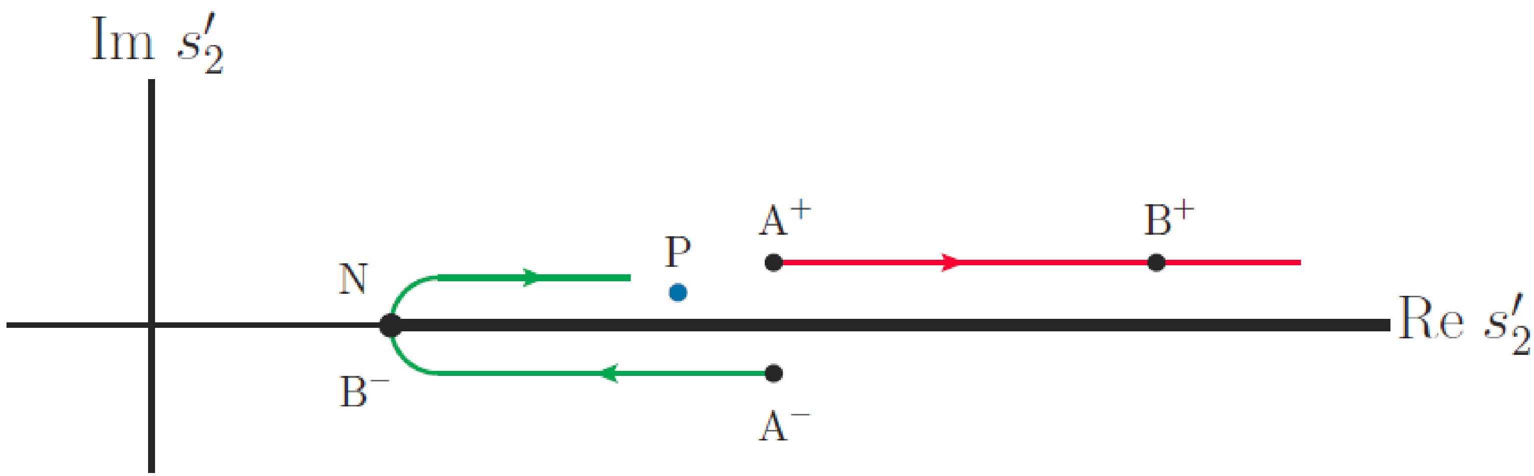


Trajectories of $s^{\pm}_2$ in the complex s'_2-plane with s_1 increasing from $s_{1N} \rightarrow \infty$:

A⁺ :$(s_1{=}s_{1N},\ s_2^+{=}s_{2C}+i\varepsilon)$ → **B⁺** :$(s_1{=}s_{1C},\ s_2^+{=}s_{2N}+m_3\,\lambda(s_3,m_1^2,m_2^2)/(m_1 m_2)+i\varepsilon)$

A⁻ :$(s_1{=}s_{1N},\ s_2^-{=}s_{2C}-i\varepsilon)$ → **B⁻** :$(s_1{=}s_{1C},\ s_2^-{=}s_{2N})$

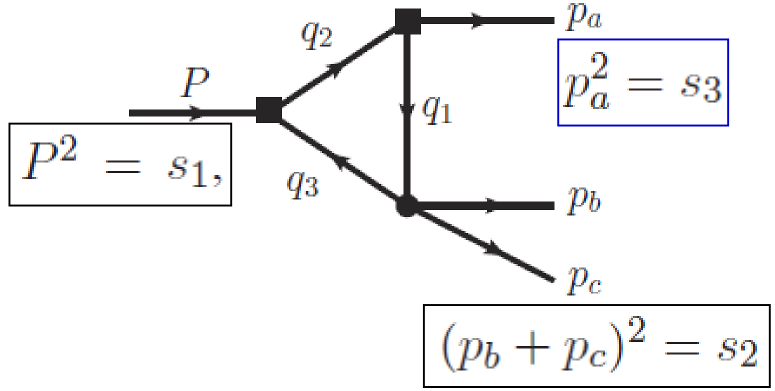
P : $s_2 + i\epsilon.$



$$\boxed{\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon}\, \sigma(s_1, s'_2, s_3)}$$

The difference between the normal and anomalous thresholds decides the kinematic range of the ATS effects:

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$
$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$



When $s_2=s_{2N}$ ($s_1=s_{1N}$), we will obtain the maximum value of Δs_1 (Δs_2),

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2+m_3)}[(m_2-m_1)^2 - s_3],$$
$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1+m_3)}[(m_2-m_1)^2 - s_3].$$

Larger values of Δ_s^{max} means more significant effects from the ATS mechanism!

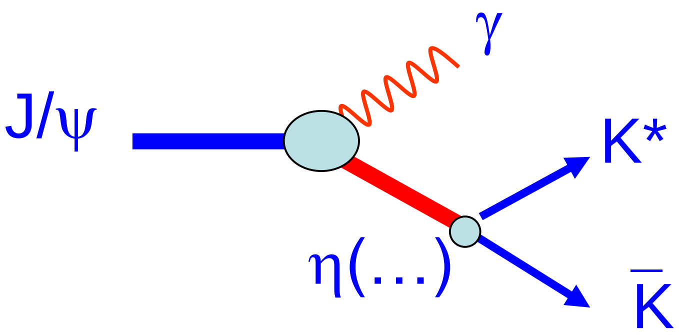
- **Problems arising from two-state solutions:**

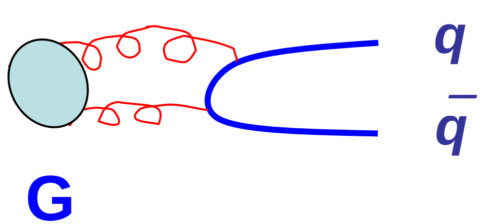
$\eta(1405)$ and $\eta(1475)$ both can decay into $K\bar{K}\pi$ as suggested by the Mark III analysis. However, BES-II analysis suggests that if an energy-dependent width is applied, it is not necessary to have two states in $J/\psi \to K\bar{K}\pi$.

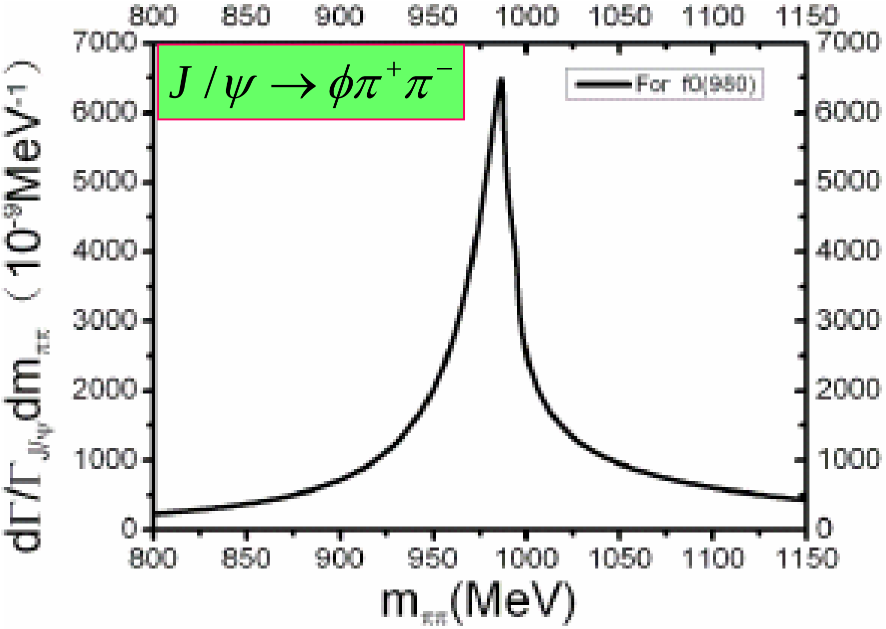
If so, it lacks evidence for $\eta(1405)$ and $\eta(1475)$ to appear in the same decay channel.

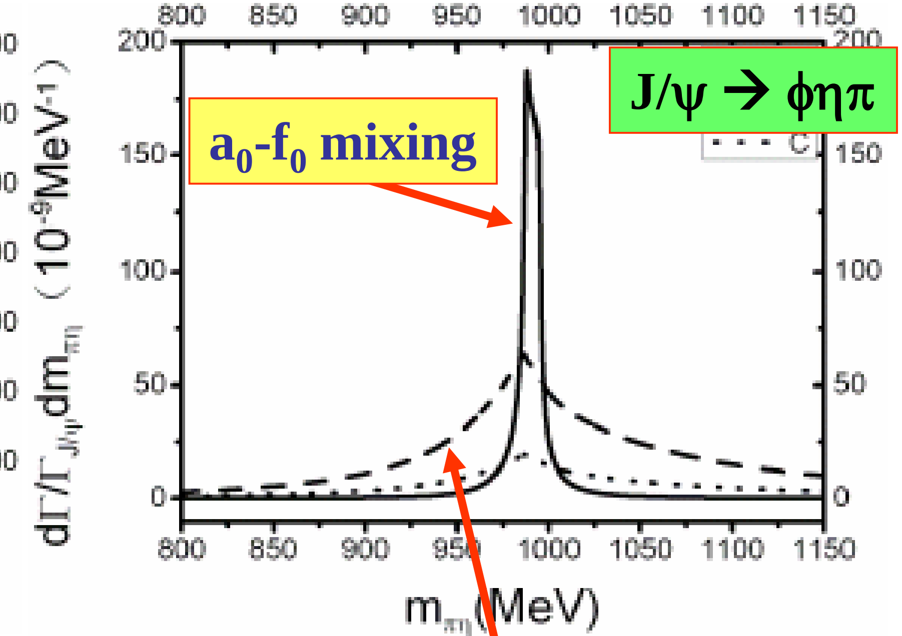
Actual observation:

The high-statistics experiments (CLEO-c, BESII and BESIII) have never observed two states ($\eta(1405)$ and $\eta(1475)$) to appear together in any exclusive channel.

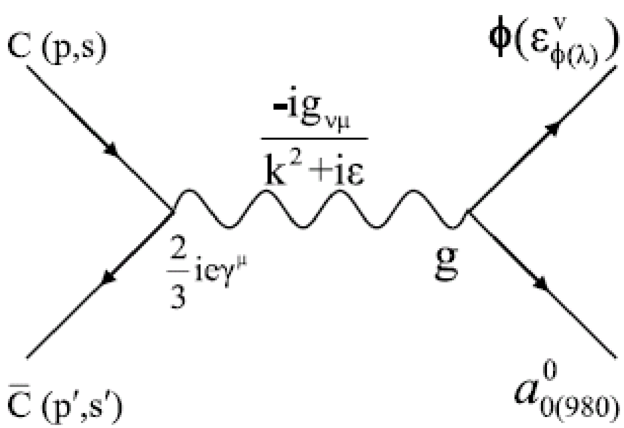


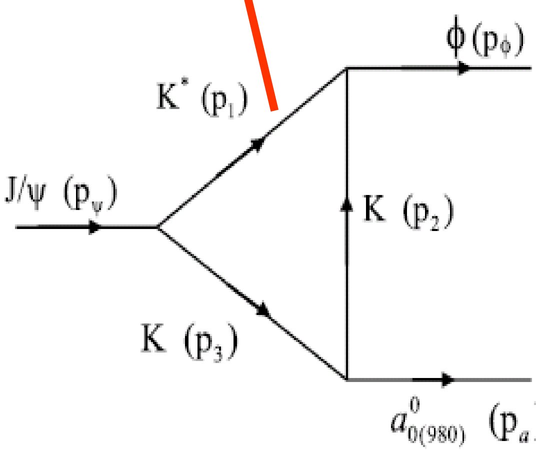






BG contributions

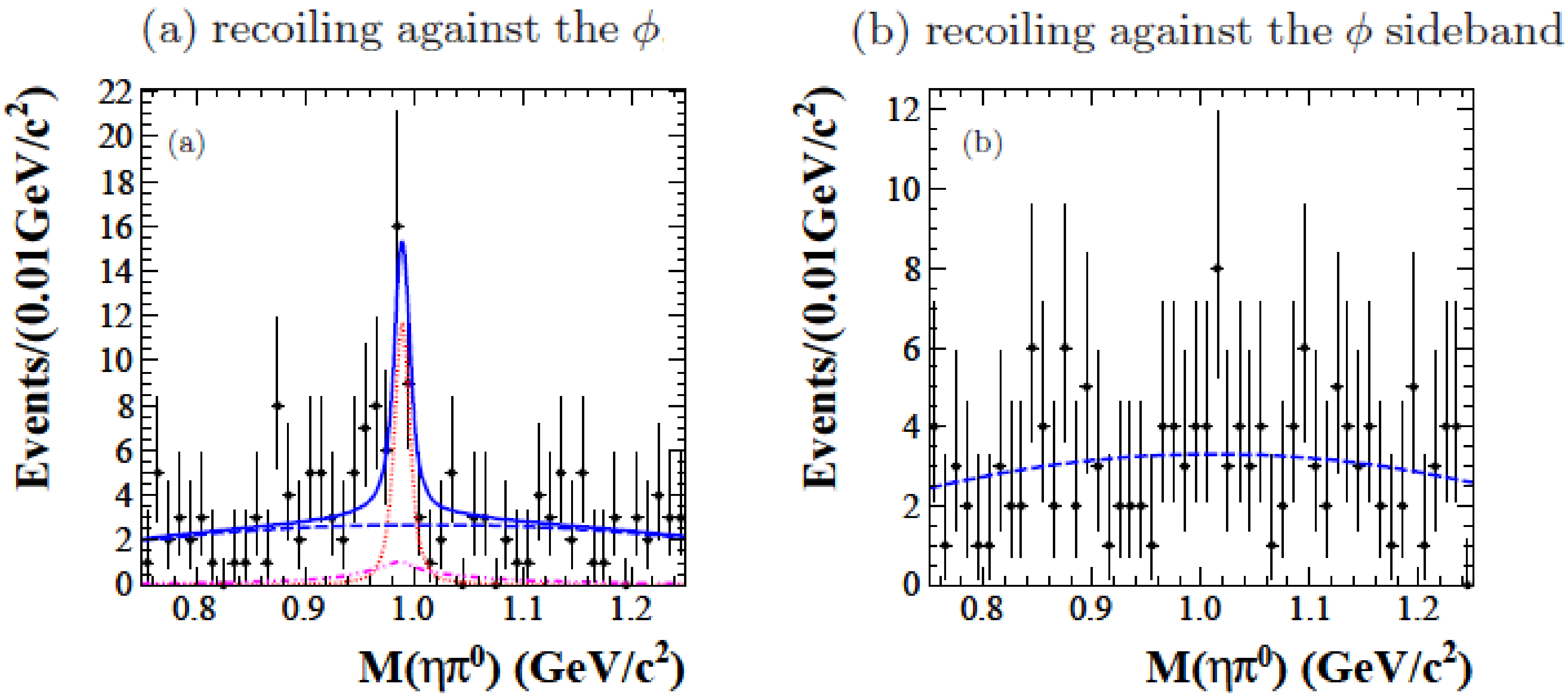




BR: a_0-f_0 mixing ~ (2-20)*10^{-6} , γ* ~ 2.6*10^{-7} , K*K ~ (3.8 -12)*10^{-6}

J.J. Wu, Q.Z, and B.S. Zou, PRD75, 114012 (2007)

• **Measurement of a0-f0 mixing intensity in $J/\psi \to \phi\, f0(980) \to \phi\, a0(980) \to \phi\eta\pi^0$**



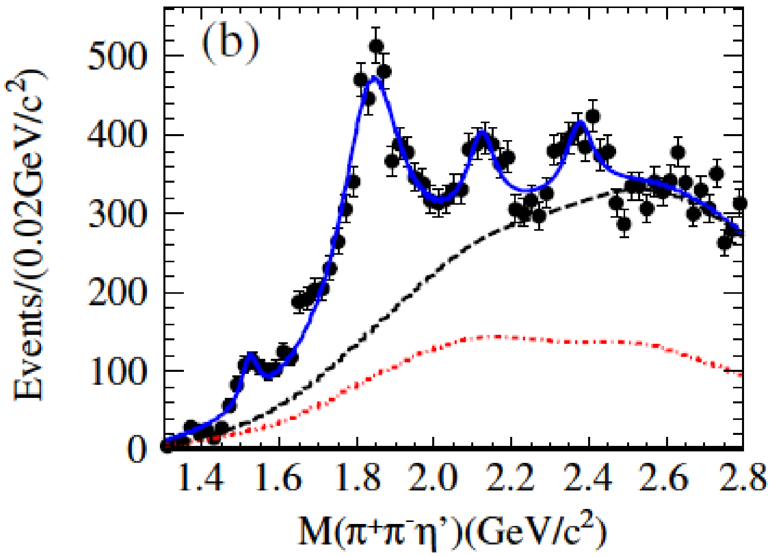
a0(980) is extremely narrow: $\Gamma \cong$ 10 MeV.

PDG: $\Gamma \cong$ 50~100 MeV.

-- Narrow width is due to the charged and neutral $K\bar{K}$ thresholds.

BES-III, PRD(2011)

$J/\psi \to \gamma\pi^+\pi^-\eta'$

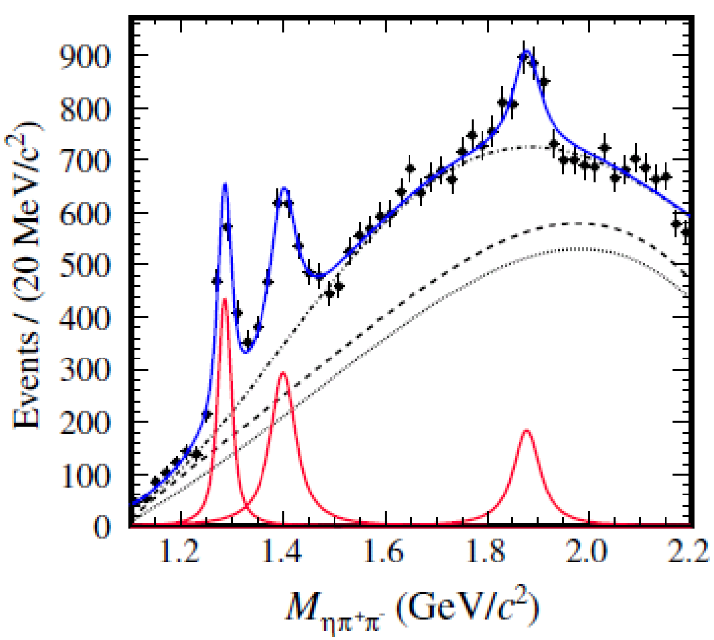


PRL **106**, 072002 (2011)

Resonance	M(MeV/c^2)	Γ(MeV/c^2)	N_{event}
$f_1(1510)$	1522.7 ± 5.0	48 ± 11	230 ± 37
$X(1835)$	1836.5 ± 3.0	190.1 ± 9.0	4265 ± 131
$X(2120)$	2122.4 ± 6.7	83 ± 16	647 ± 103
$X(2370)$	2376.3 ± 8.7	83 ± 17	565 ± 105

Incoherent fit of the resonance and non-resonance background!

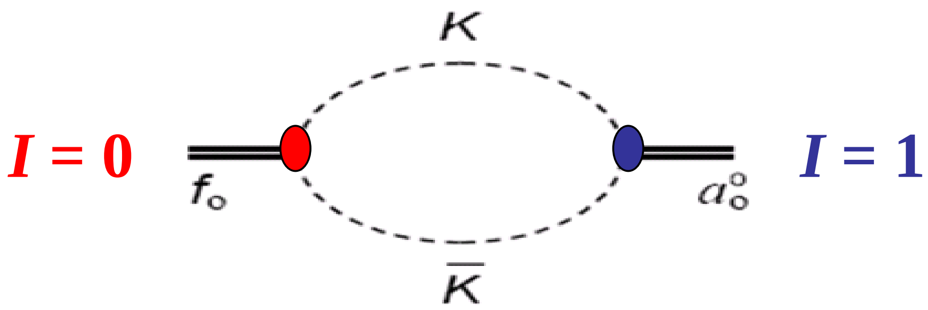
$J/\psi \to \omega\eta\pi^+\pi^-$



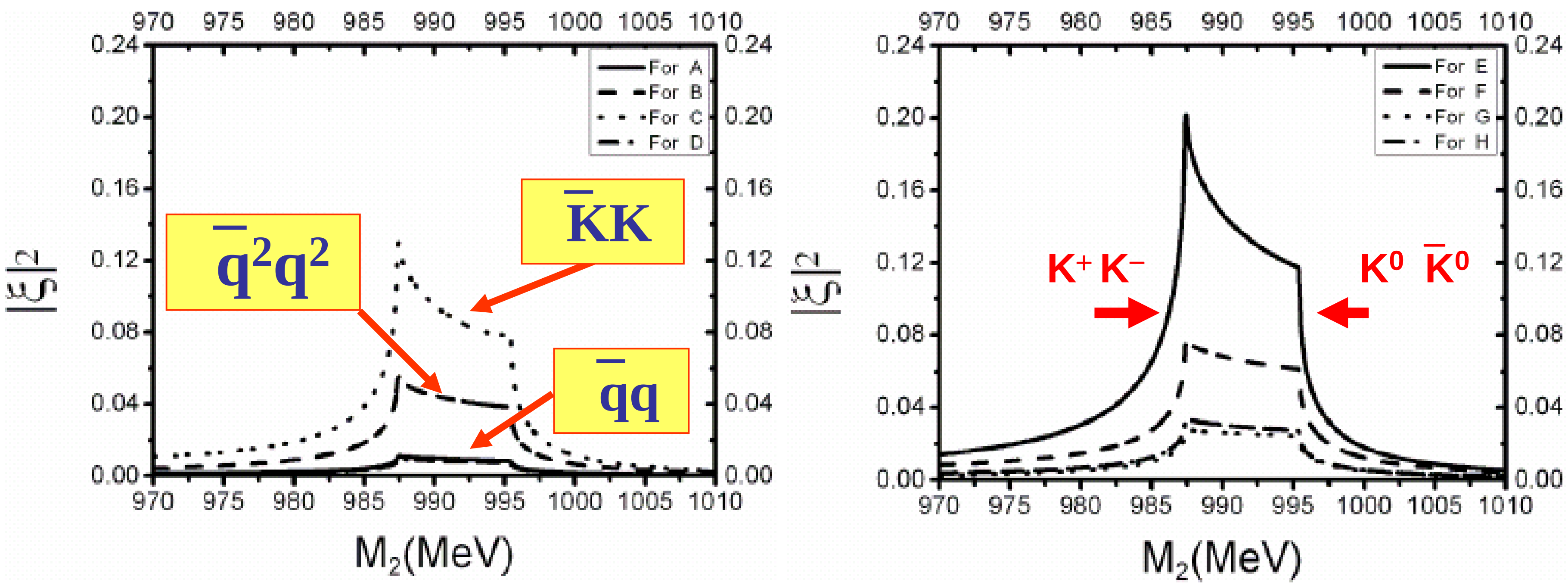
PRL **107**, 182001 (2011)

Resonance	Mass (MeV/c^2)	Width (MeV/c^2)	$\mathcal{B}(10^{-4})$
$f_1(1285)$	$1285.1 \pm 1.0^{+1.6}_{-0.3}$	$22.0 \pm 3.1^{+2.0}_{-1.5}$	$1.25 \pm 0.10^{+0.19}_{-0.20}$
$\eta(1405)$	$1399.8 \pm 2.2^{+2.8}_{-0.1}$	$52.8 \pm 7.6^{+0.1}_{-7.6}$	$1.89 \pm 0.21^{+0.21}_{-0.23}$
$X(1870)$	$1877.3 \pm 6.3^{+3.4}_{-7.4}$	$57 \pm 12^{+19}_{-4}$	$1.50 \pm 0.26^{+0.72}_{-0.36}$

• $a_0(980)$-$f_0(980)$ mixing mechanism



$g(a_0K^+K^-)\ g(f_0K^+K^-)$
$= -g(a_0K^0\ \bar{K}^0)\ g(f_0K^0\ \bar{K}^0)$

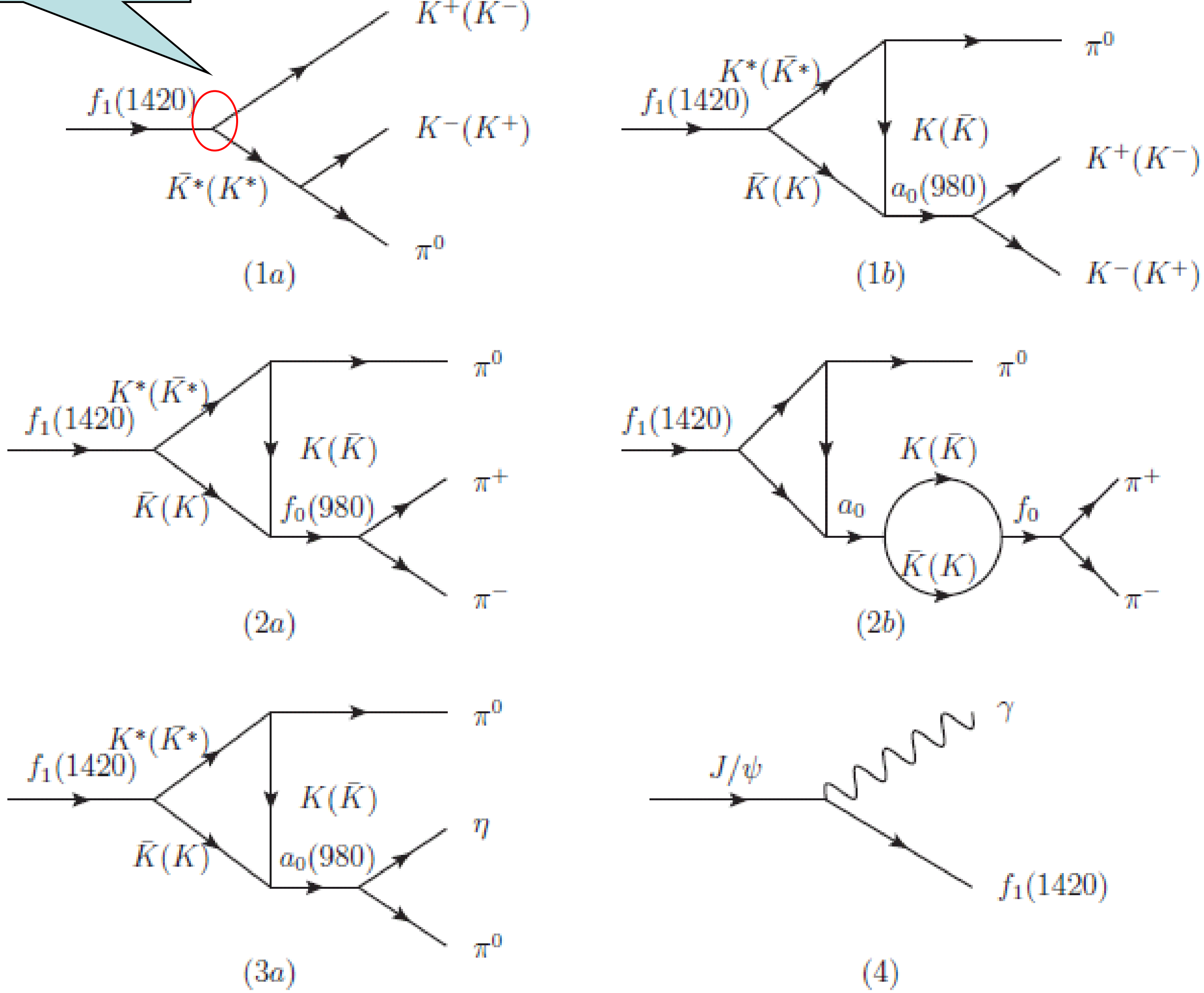


J.J. Wu, Q.Z, and B.S. Zou, PRD75, 114012 (2007)

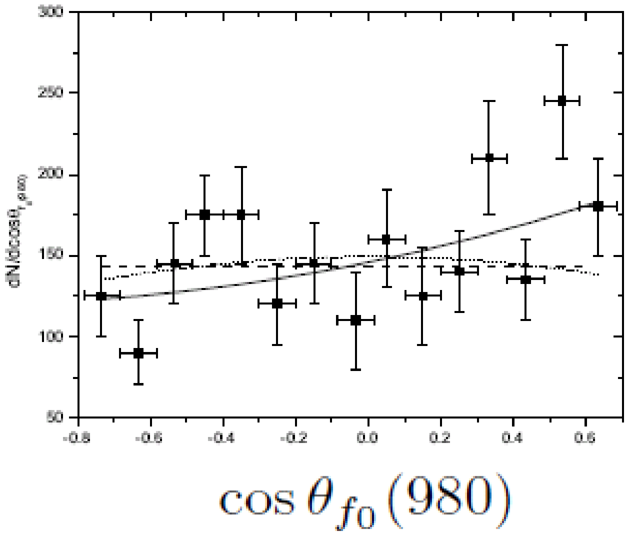
Observables and predictions?

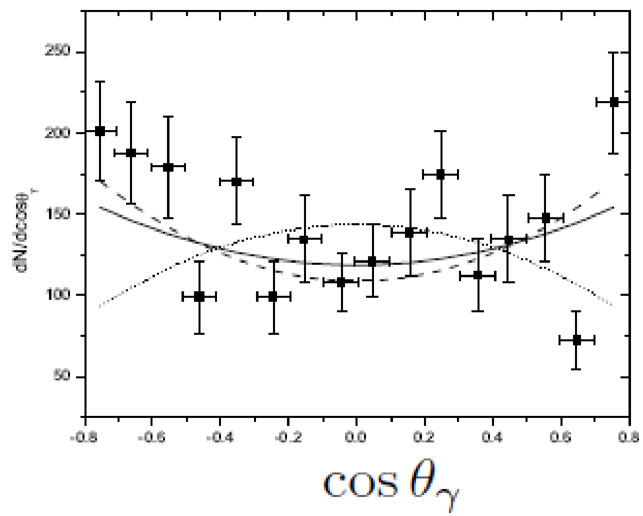
- **There must be $f_1(1420)$ contributing in $J/\psi \rightarrow \gamma\,\pi\pi\pi$**

Relative S wave



Partial wave analysis of J/ψ → γ η(1405)/f_1(1420) → γ πππ

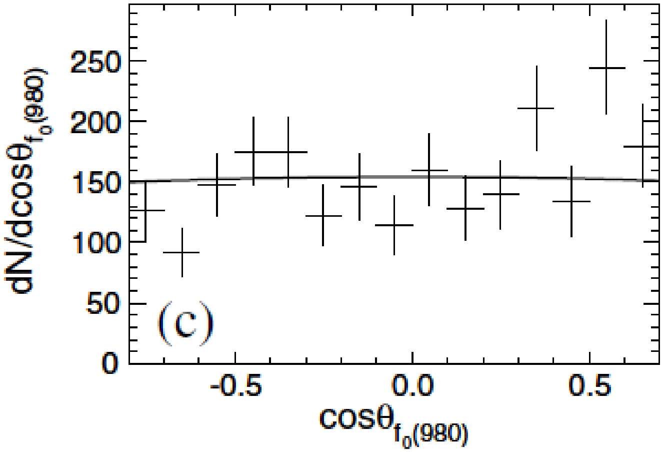


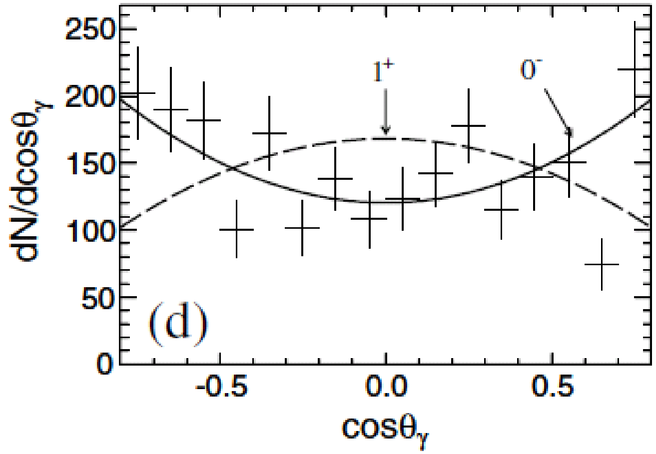


Dashed: eta(1440)
Dotted: f1(1420)
Solid: eta(1440) + f1

$$\chi^2/d.o.f = 38.3/14; \quad b_\gamma = 118.5 \pm 8.8,\ c = 0.538 \pm 0.312$$

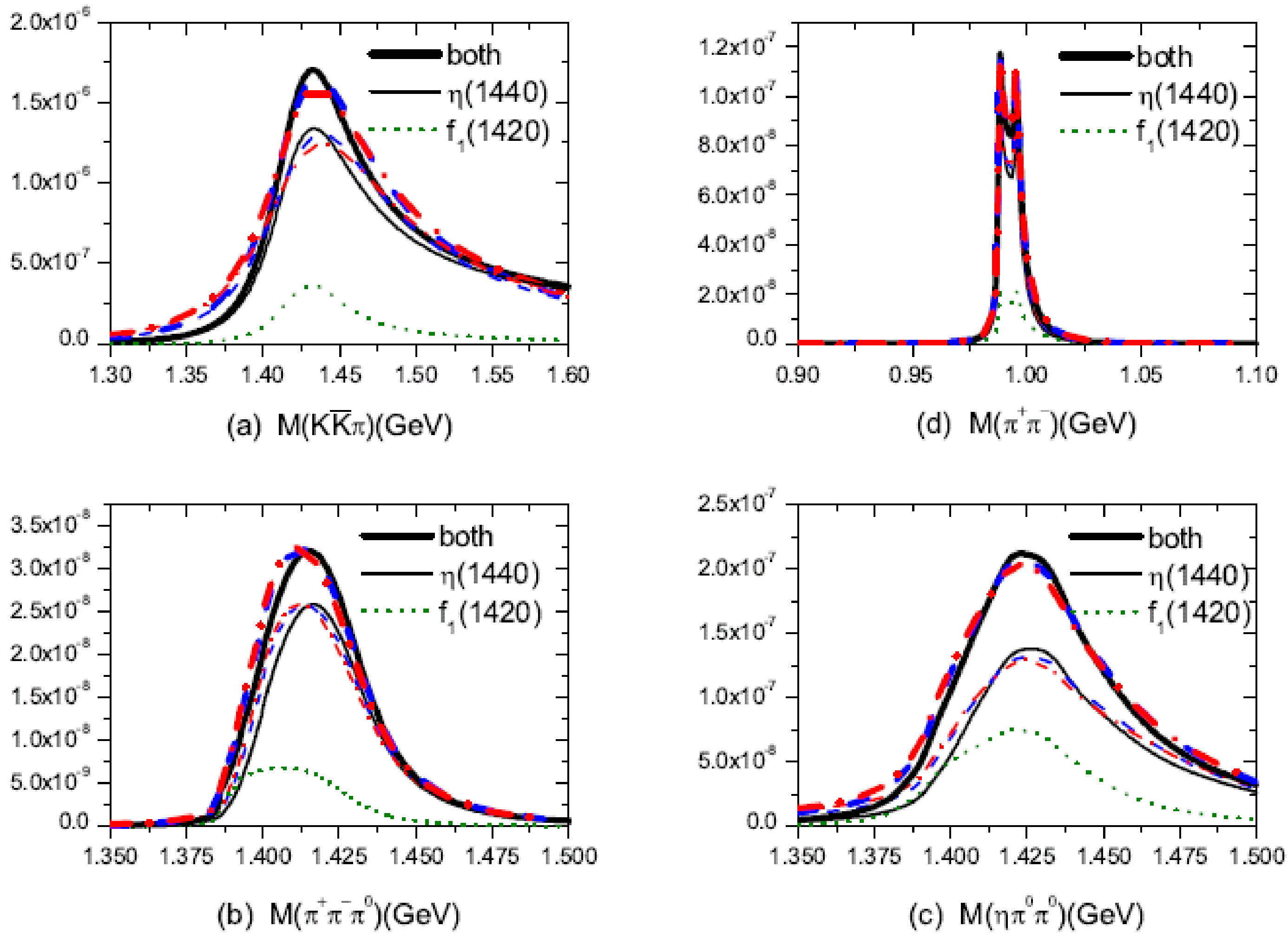
$$\chi^2/d.o.f = 19.8/12; \quad b_{f_0} = 145.7 \pm 10.7,\ c_1 = 0.314 \pm 0.128,\ c_2 = 0.141 \pm 0.317$$



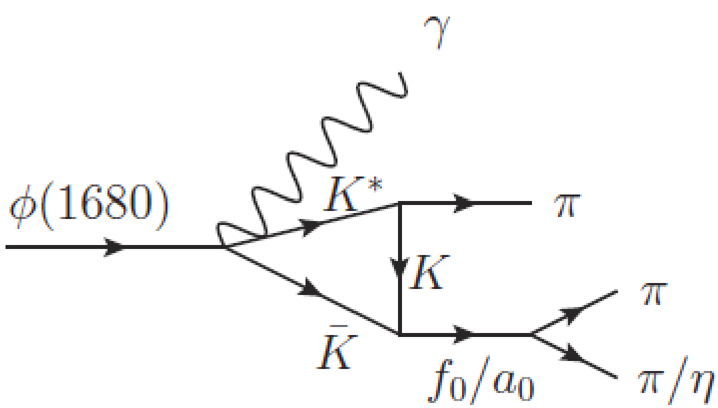


BESIII results:

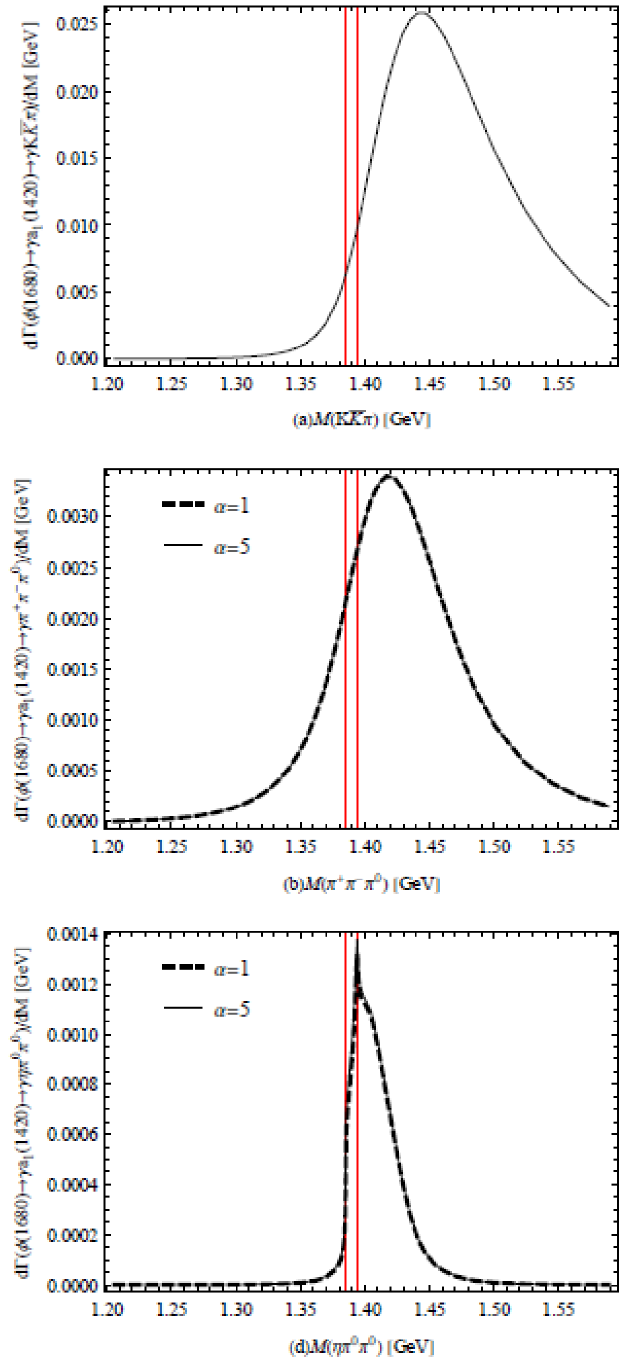
immediate states	$\chi^2/d.o.f$ for $\cos\theta_\gamma$	$\chi^2/d.o.f$ for $\cos\theta_{f_0}$
$\eta(1440)$	40.2/15	26.8/14
$f_1(1420)$	59.0/15	26.4/13
$\eta(1440)$ and $f_1(1420)$	38.3/14	19.8/12



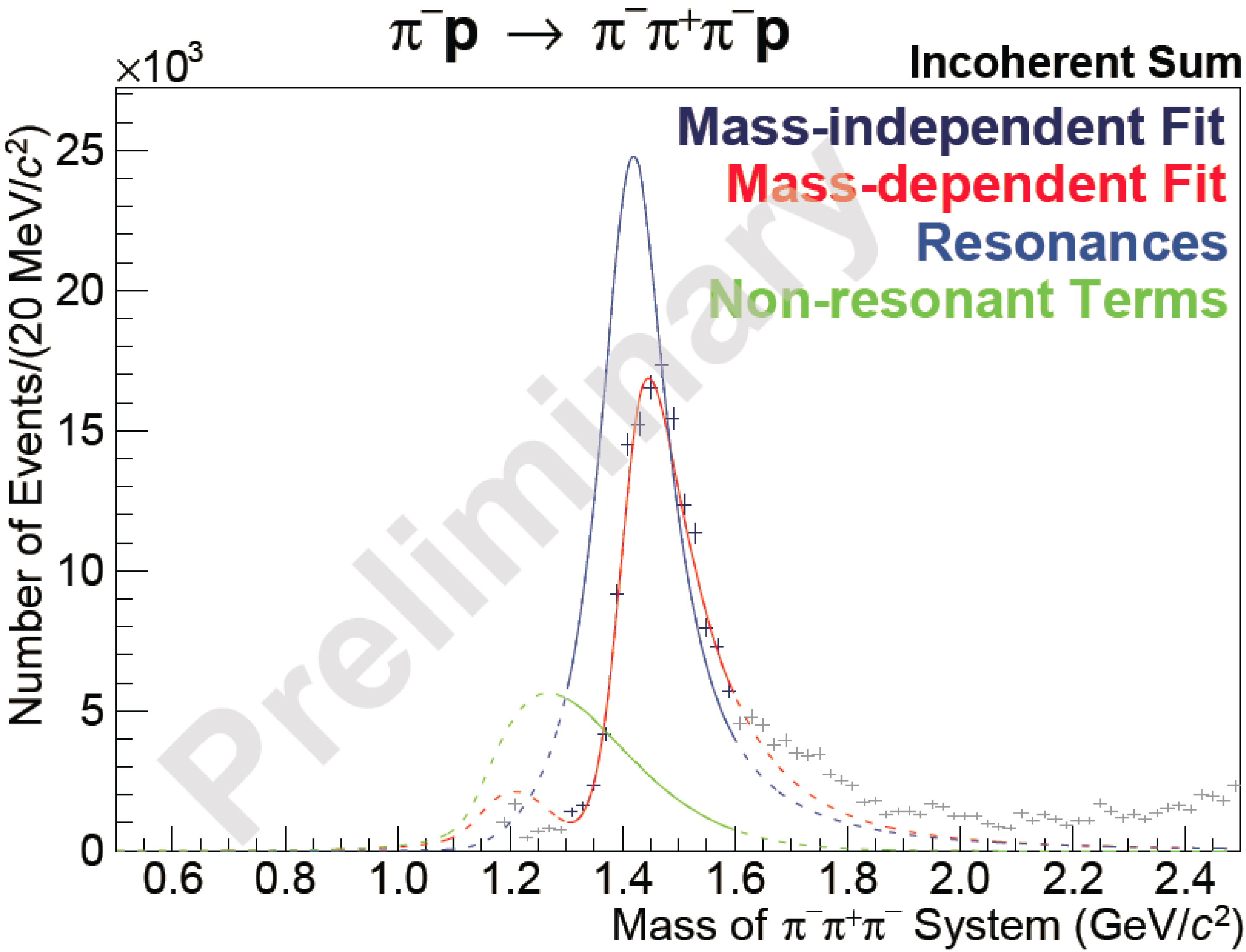
- **Implication of existence of $a_1(1420)$**

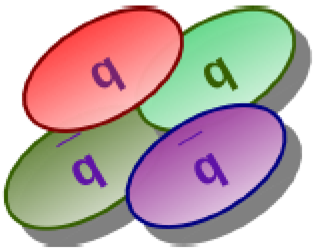


Due to the "triangle singularity", the same "state" produces different resonance-like lineshapes in different channels!



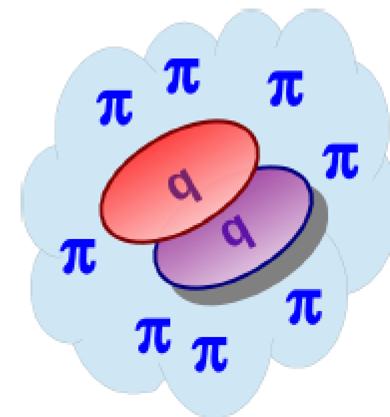
Observation of a new state $a_1(1420)$ at COMPASS

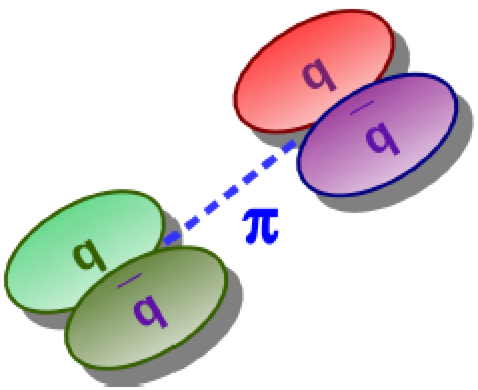




Tetraquark
Compact state of four quarks

Hadrocharmonium
Heavy Quarkonium Core
Surrounded by pion cloud





Hadronic Molecule
Formed from interactions of two hadrons
Classic Example for Baryons: Deuteron

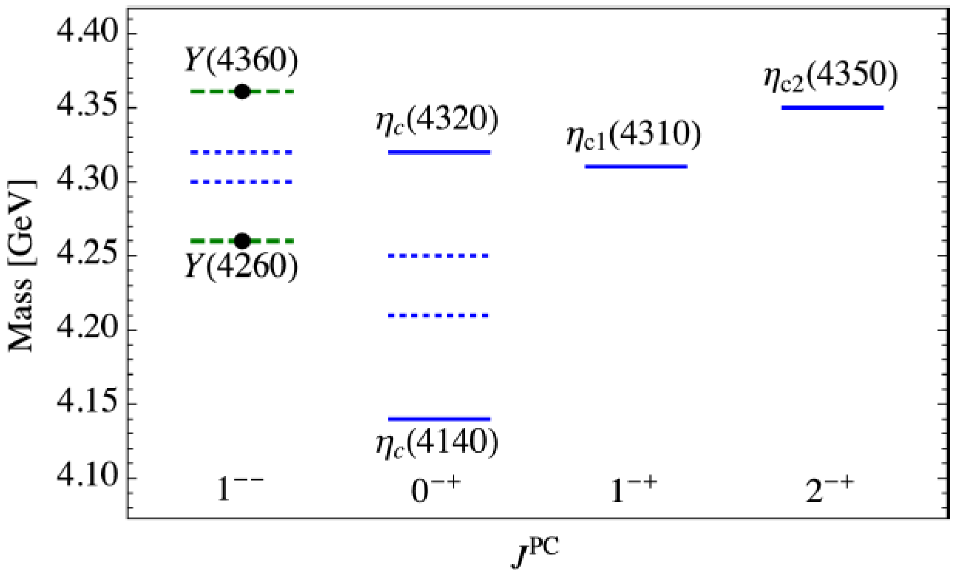
In the heavy quark spin symmetry (HQSS) limit these models have different predictions for the spectrum.

Slide from talk by M. Cleven

- **Hadro-quarkonium states (Voloshin)**

$$\begin{pmatrix} Y(4260) \\ Y(4360) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_3 \end{pmatrix} \qquad \theta \sim 40^0 \qquad \begin{cases} \psi_1 \sim h_c \times (0^{-+})_{q\bar{q}} \\ \psi_3 \sim \psi' \times (0^{++})_{q\bar{q}} \end{cases}$$

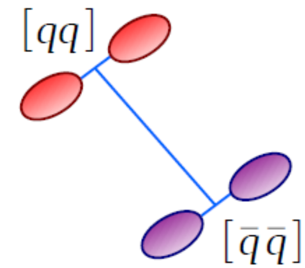
Heavy spin doublets: (h_c, χ_{cJ}), (ψ, η_c)



- Possible decay channel: $\eta_c\pi\pi$, $\chi_{cJ}\pi\pi$
- Exotic quantum number: $J^{PC}=1^{-+}$
- Two η_c states

M. Cleven, F.-K. Guo, C. Hanhart, Q. Wang, Q.Z., PRD92, 014005 (2015); arXiv:1505.01771 [hep-ph]

- **Tetraquark states (Maiani et al.)**

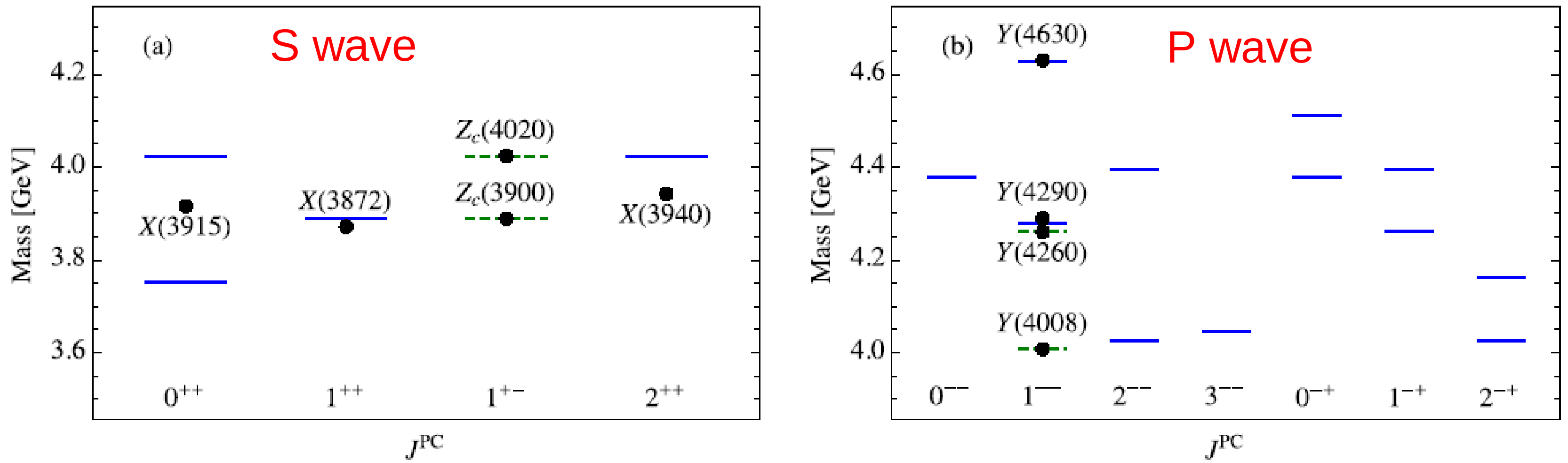


The mass of a tetraquark is given by

$$M = M_{00} + B_c \frac{\boldsymbol{L}^2}{2} - 2a\boldsymbol{L}\cdot\boldsymbol{S} + 2\kappa_{cq}\left[(\boldsymbol{s}_q \cdot \boldsymbol{s}_c + (\boldsymbol{s}_{\bar{q}} \cdot \boldsymbol{s}_{\bar{c}}))\right]$$

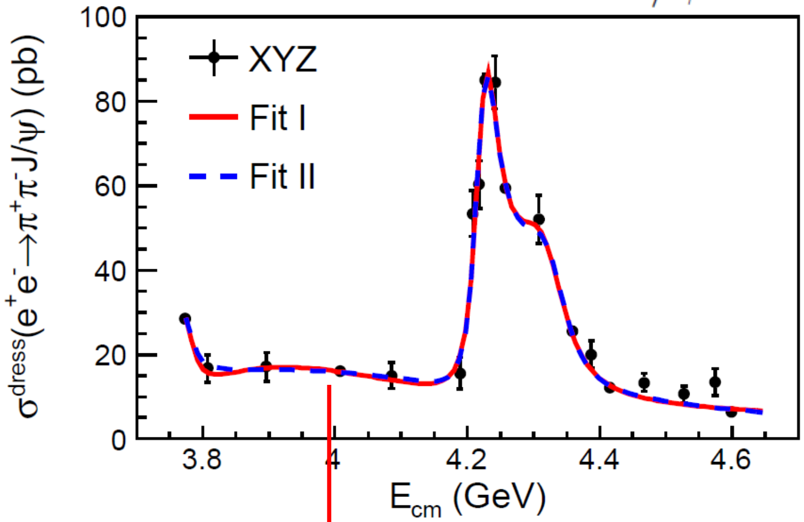
For a state with given ***J***, the mass can be estimated:

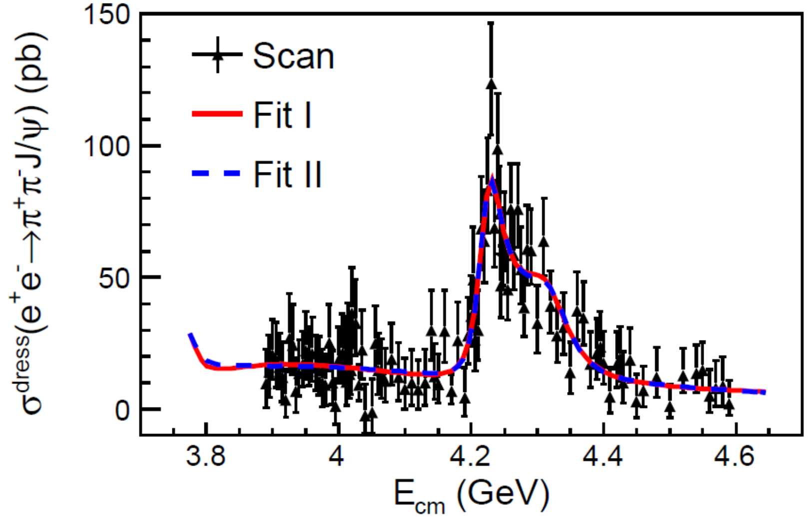
$$M = M_{00} + B_c \frac{L(L+1)}{2} + a[L(L+1) + S(S+1) - J(J+1)] + \kappa_{cq}\left[s(s+1) + \bar{s}(\bar{s}+1) - 3\right]$$



Extremely rich spectrum is predicted!

$e^+e^- \to \pi^+\pi^- J/\psi$



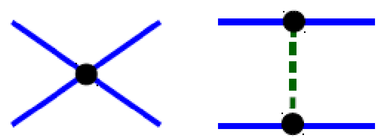


No Y(4008) observed!

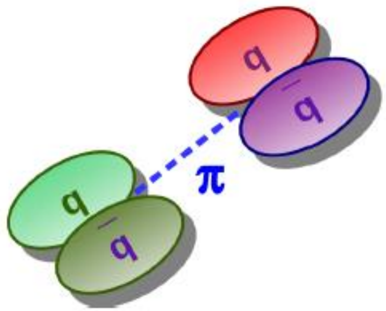
BESIII, arXiv:1611.01317 [hep-ex]

• Hadronic molecules (Cleven et al.)

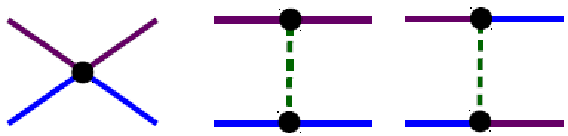
- (D,D*) + (D,D*)



$J^{PC} = 0^{++}, 1^{++}, 1^{+-}, 2^{++}$



- (D_1,D_2) + (D,D*)



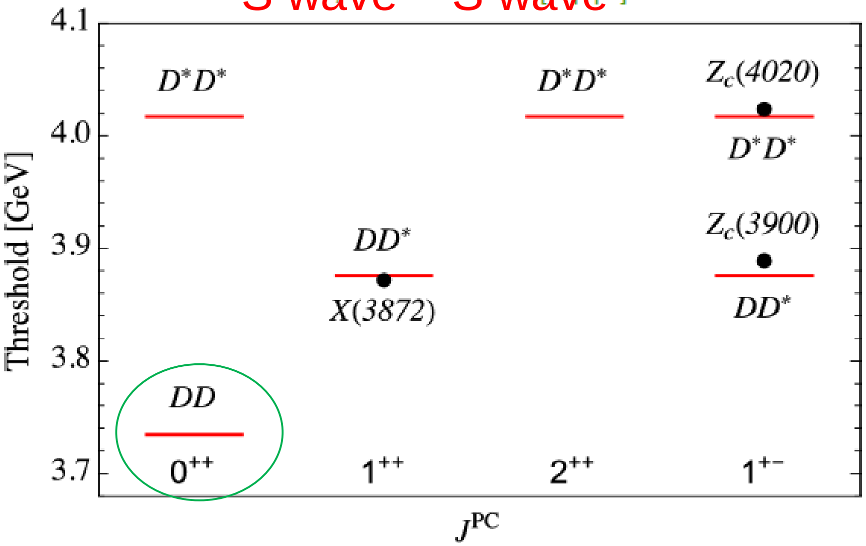
$J^{PC} = 1^{--}, 1^{-+}$

- Long-range pion exchange;
- Isoscalar and isovector may not bind simultaneously;

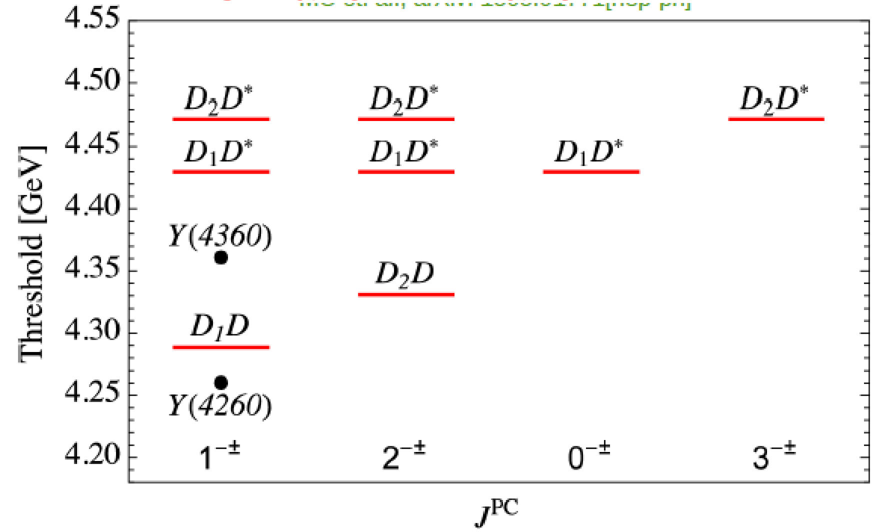
$$\langle I, I_3 | \vec{\tau}_{(1)} \cdot \vec{\tau}_{(2)} | I, I_3 \rangle = 2\left[I(I+1) - 3/2\right] = \begin{cases} -3 & I=0 \\ 1 & I=1 \end{cases}$$

M. Cleven, F.- K. Guo, C. Hanhart, Q. Wang and Q. Zhao, **PRD 92**, 014005 (2015);
Q. Wang, **PRD 89**, 114013 (2014)

S wave – S wave



S wave – P wave



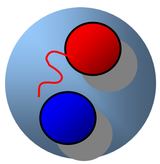
- **States appear at S-wave thresholds;**
- **The J=3 state has significantly higher mass than for tetraquarks;**
- **Only one $J^{PC}=0^{-+}$ state is predicted;**
- **Scalar state of $\bar{D}D$ may not exist;**
- **Exotic partners of $J^{PC}= 1^{--}$;**
- **……**

Quantum Chromo-Dynamics:

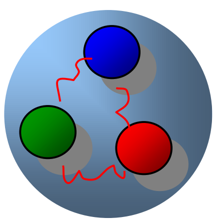
a highly successful theory for Strong Interactions _

Conventional hadrons

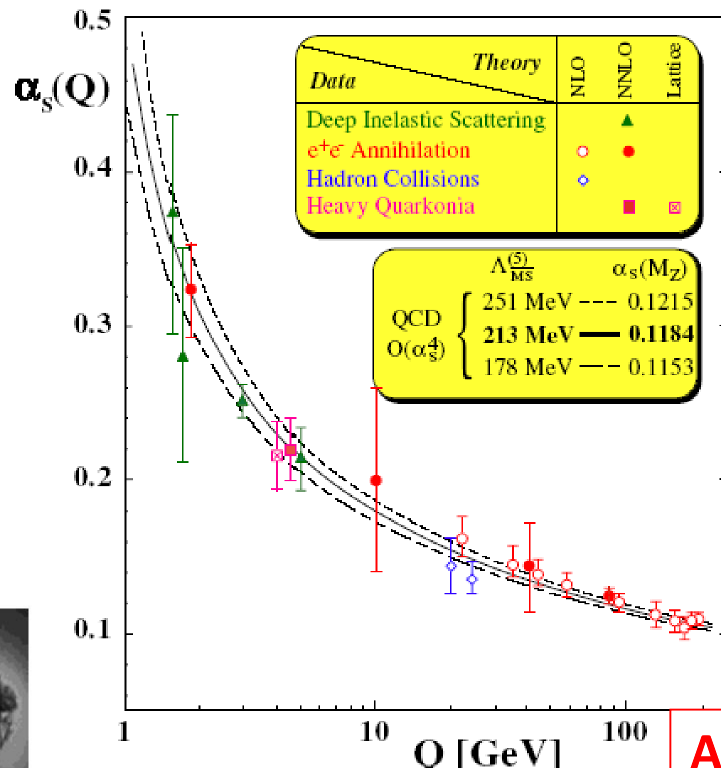
Meson

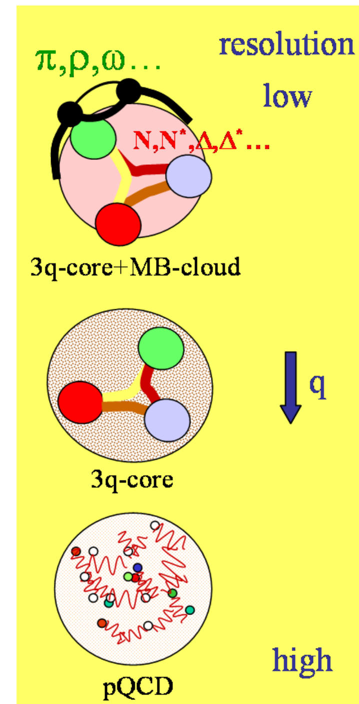


Baryon



Confinement

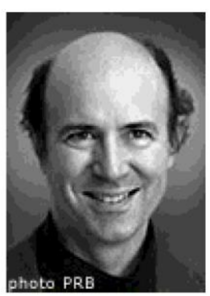




Asymp. freedom

David J. Gross

H. David Politzer



Frank Wilczek

Outline

1. Hadrons beyond the conventional quark model and three types of exotics signals
2. Do not forget the nearby S-wave thresholds, and the presence of the “triangle singularity”
3. Story of the pseudoscalar glueball puzzle
4. Observables sensitive to the underlying dynamics
5. Brief summary

1. Why study QCD exotics?
2. What are the key issues that we have known and what we may have missed?
3. What are the criteria for exotic hadrons?
4. How to put together pieces of the “jigsaw puzzle” for QCD exotics?
5. Prospects