States with Four Heavy Quarks

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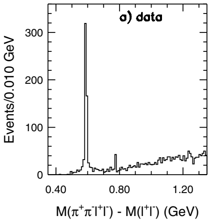
Outline

1. Tetraquarks
2. Observing stable $QQ\bar{Q}\bar{Q}$ states at the LHC *
3. Lattice QCD with all heavy quarks $bb\bar{b}\bar{b}$ *
4. Approaches for heavy-light tetraquarks

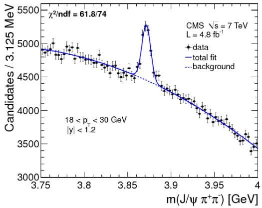
* Zhen Liu and E. E., [arXiv:1709.09605]
* Ciaran Hughes, E. E. and Christine Davies, PRD97 (2018), 054505

XYZ states.

- BELLE observed $X(3872)$ in $B^{\pm} \to K^{\pm}\pi^{+}\pi^{-} J/\psi$.
  PRL 91 (26), 2003
- BABAR

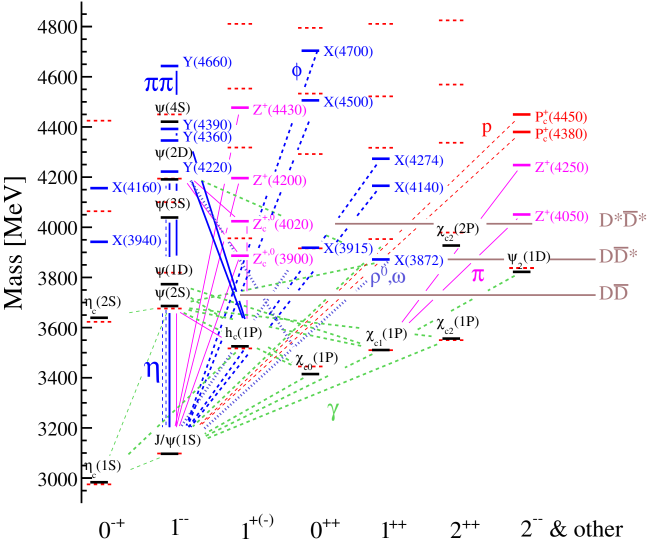


- Direct production observed CDF, DZero
- CMS, ATLAS, LHCb



XYZ States Today

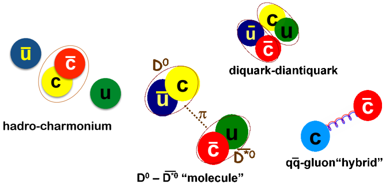
$c\bar{c}$



S. L. Olsen, T. Skwarnicki, D. Zieminska, Reviews of Modern Physics **90**, 1 (2018)

QCD dynamics - XYZ states

- For heavy quark-antiquark ($Q\bar{Q}$) systems the QCD effects of gluon excitations and light quark pairs become manifest above $(Q\bar{q} + q\bar{Q})$ threshold.
- Theoretical tools
  - Heavy Quark Symmetry (HQS)
  - Lattice QCD
- Model approaches:
  - tetraquark states with various dynamic models
  - molecules and cusp effects
  - hybrid states - excited gluonic degrees of freedom



Tetraquarks

$Q\bar{Q}q\bar{q}$ $QQ\bar{q}\bar{q}$ $Q\bar{Q}Q\bar{Q}$
$Qq\bar{q}\bar{q}$ $Q\bar{Q}Q\bar{q}$

- All the presumed tetraquark states observed so far have strong decays.
- Only stable ordinary mesons: π, K, D, D_s, D_s^*, B, B_s, B_s^*, B_c, B_c^*
- Are there any stable tetraquarks?

YES

Levels of stability for tetraquarks

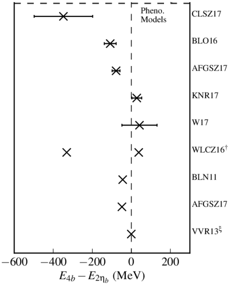
A) Unstable
  - Resonance with OZI allowed strong decays.
  - Typically large width
  - Analog in $Q\bar{Q}$ systems are states above two heavy light meson threshold

B) Metastable
  - Narrow states with strong decays (but none OZI allowed).
  - Analog in $Q\bar{Q}$ systems: states below heavy-light pair threshold

C) Stable
  - No strong decays.
  - Analog in $Q\bar{Q}$ systems is B_c

$QQ\bar{Q}\bar{Q}$

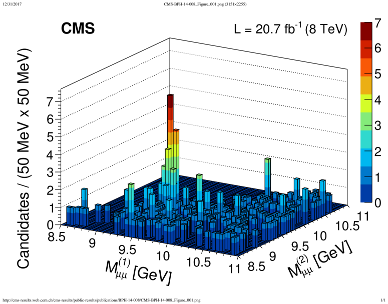
- Any stable tetraquarks for $Q = b$?
- Many phenomenological models for $bb\bar{b}\bar{b}$ tetraquarks.



Upsilon Pair Production

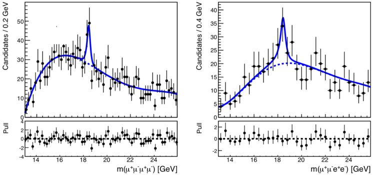
- CMS 2Υ production
  - $\sigma_{fid} = 68.8 \pm 12.7(stat) \pm 7.4(syst) \pm 2.8(BR)pb$
  - Fraction of DPS $\approx 30\%$
- Encouraging for observation of possible $bb\bar{b}\bar{b}$ tetraquarks as well as bbq baryons and $bb\bar{q}\bar{q}'$ tetraquarks.

[arXiv:1610.07095]



- Thesis talk Suleyman Durgut (CMS) APS meeting April 16, 2018. Note: The results are taken from my thesis work and they are not approved but CMS yet. The analysis is still in progress.

Combined Result



- Do a simultaneous fit to both channels, with fixed signal shapes but floating mass value.
- Best mass : 18.4 ± 0.1 (stat.) ± 0.2 (syst.) GeV
- Local Significance: 4.86σ ($p\_value = 5.8\times10^{-7}$)

- In order to calculate global significance, Look-Elsewhere-Effect must be taken into account. Lots of toy MC generations are required, not an efficient method.
- Global significance is calculated using Gross-Vitells method which is used in Higgs discovery.

Eur.Phys.J.C70:525-530,2010

- The returned global significance was 3.6σ.

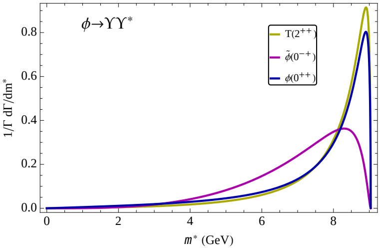
12

- LHCb does not see any signal for such a $bb\bar{b}\bar{b}$ in the range 16 − 26 GeV in the final state $\Upsilon + \mu^+\mu^-$. (Marco Pappagallo's talk)

Observing a $\phi = bb\bar{b}\bar{b}$ tetraquark ground state at the LHC

Zhen Liu & EE, arXiv:1709.09605

- Production ϕ with $J^{PC} = O^{++}$ via gluon fusion
- Peaked forward in rapidity.
  - Assume VMD
  - $\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}\Lambda\phi\Upsilon^\mu\Upsilon_\mu + \ldots$
  - $\sigma(pp \to \phi \to 4\ell) \sim$
    - ★ $3\left(\frac{\Lambda}{0.2\ \mathrm{GeV}}\right)^2$ fb for 8 TeV
    - ★ $5\left(\frac{\Lambda}{0.2\ \mathrm{GeV}}\right)^2$ fb for 13 TeV



- Invariant mass distribution for $m(\phi) = 18.5$ GeV: $\phi \to \Upsilon\Upsilon^* \to l^+l^-l^+l^-$

Observing a $\phi = bb\bar{b}\bar{b}$ tetraquark ground state at the LHC

Compare cross section $H \to ZZ^* \to l^+l^-l^+l^-$.

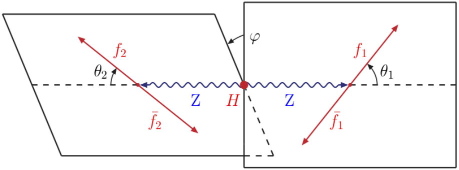
$$\frac{d\Gamma_H}{d\cos\theta_1 d\cos\theta_2} \sim \sin^2\theta_1 \sin^2\theta_2 + \frac{1}{2\gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2}\left[(1+\cos^2\theta_1)(1+\cos^2\theta_2) + 4\,\eta_1\eta_2\,\cos\theta_1\cos\theta_2\right] \quad (21)$$

and

$$\frac{d\Gamma_H}{d\varphi} \sim 1 - \eta_1\eta_2\frac{1}{2}\left(\frac{3\pi}{4}\right)^2 \frac{\gamma_1\gamma_2(1+\beta_1\beta_2)}{\gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2+2}\cos\varphi + \frac{1}{2}\frac{1}{\gamma_1^2\gamma_2^2(1+\beta_1\beta_2)^2+2}\cos 2\varphi \quad (22)$$

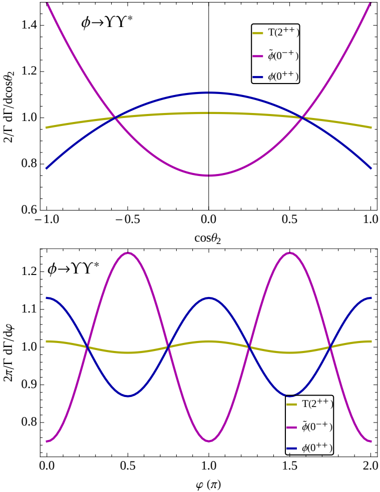
S.Choi, D. Miller, M. Muhlleitner, &P. Zerwas, [arXiv:hep-ph/0210066]

ϕ decays to $4l$ has only the vector contribution ($\eta_i = 0$).



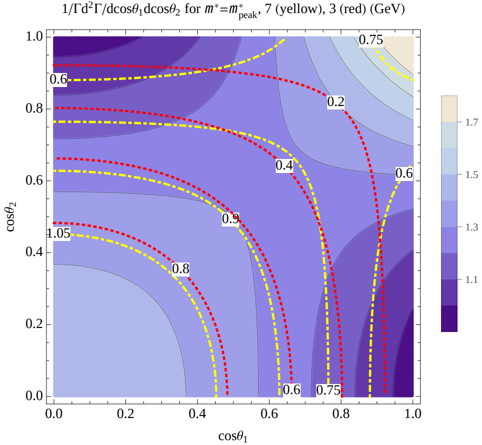
Observing a $\phi = bb\bar{b}\bar{b}$ tetraquark ground state at the LHC

- Expected angular distributions: $\phi \to \Upsilon\Upsilon^{(*)} \to l^+l^-l^+l^-$
  - Can determine J^{PC} from the angular distributions



Observing a $\phi = bb\bar{b}\bar{b}$ tetraquark ground state at the LHC

Double differential angular distributions of the tetraquark state $\phi(0^{++})$ for different values of the off-shell Υ^* dilepton invariant masses.



Does a Stable $bb\bar{b}\bar{b}$ Tetraquark State Exist within QCD?

The Lattice Approach Provides the Answer

Ciaran Hughes, E. E. and Christine Davies, PRD97 (2018), 054505

Lattice NRQCD

For the evolution of heavy quarks use the highly improved lattice action:

- $G(\mathbf{x}, t+1) = e^{-aH} G(\mathbf{x}, t)$

$$e^{-aH} = \left(1 - \frac{a\delta H|_{t+1}}{2}\right)\left(1 - \frac{aH_0|_{t+1}}{2n}\right)^n U_t^\dagger(x) \times \left(1 - \frac{aH_0|_t}{2n}\right)^n \left(1 - \frac{a\delta H|_t}{2}\right)$$

- $aH_0 = -\frac{\Delta^{(2)}}{2am_b}$
- $n = 4$ used to extend range of validity
- $a\delta H = a\delta H_{v^4} + a\delta H_{v^6}$
- Only spin dependent terms in $O(v^6)$ are retained.

$$a\delta H_{v4} = -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) - c_3 \frac{1}{8(am_b)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) - c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}$$

$$a\delta H_{v6} = -c_7 \frac{1}{8(am_b)^3}\left\{\boldsymbol{\Delta}^{(2)}, \sigma \cdot \tilde{\mathbf{B}}\right\} - c_8 \frac{3}{64(am_b)^4}\left\{\boldsymbol{\Delta}^{(2)}, \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right)\right\} - c_9 \frac{i}{8(am_b)^3} \sigma \cdot \tilde{\mathbf{E}} \times \tilde{\mathbf{E}}$$

Lattice ensembles studied

Set	β	a (fm)	am_l	am_s	am_c	$N_s \times N_T$	n_{cfg}
1 coarse	6.00	0.1219(9)	0.0102	0.0509	0.635	24×64	1052
2 coarse	6.00	0.1189(9)	0.00184	0.0507	0.628	48×64	1000
3 fine	6.30	0.0884(6)	0.0074	0.037	0.440	32×96	1008
4 superfine	6.72	0.0592(3)	0.0048	0.024	0.286	48×144	400

- β is the gauge coupling. a (fm) is the lattice spacing.
- Full QCD. am_q are the sea quark masses. All sets have $m_\pi = 300$ MeV except set 2 which has physical m_π.
- $N_s \times N_T$ gives the spatial and temporal extent of the lattices in lattice units
- n_{cfg} is the number of configurations used for each ensemble. 16 time sources on each configuration was used to increase statistics.

NRQCD parameters

Set	am_b	u_{0L}	c_1, c_6	c_2	c_4	c_5
1	2.73	0.8346	1.31	1.02	1.19	1.16
2	2.66	0.8350	1.31	1.02	1.19	1.16
3	1.95	0.8525	1.21	1.29	1.18	1.12
4	1.22	0.8709	1.15	1.00	1.12	1.10

- am_b is the bare mass; u_{0L} is the tadpole parameter.
- The c_i coefficients for the NRQCD action are tree-level except c_1, c_6, c_2, c_4 and c_5 above.

Correlators

Euclidean two-point correlators

$$C_{i,j}(t, \mathbf{P}tot = \mathbf{0}) = \int d^3x \langle \mathcal{O}_i(t, \mathbf{x}) \mathcal{O}_j(0, \mathbf{0})^\dagger \rangle$$

- Projected to zero spatial momentum.
- i, j label potential different operators at the source and sink with the same J^{PC}.

Single particle contributions to the correlator determined by inserting a complete set of single-particle states

$$C_{i,j}(t, \mathbf{P}tot = 0) = \sum_n Z_n^i Z_n^{j,*} e^{-E_n t}$$

- $Z_n^i = \langle 0 | \mathcal{O}_i^{J,m_i} | n \rangle$
- $E_n | n \rangle = H | n \rangle$

Effective Mass Plots

Effective mass plots help visualize the data

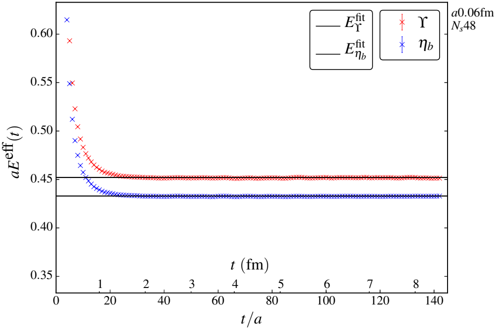
$$
\begin{aligned}
aE\text{eff} &= \log\left(\frac{C_{i,j}(t)}{C_{i,j}(t+1)}\right) \\
&= aE + \frac{Z_1^i Z_1^{j,*}}{Z_0^i Z_0^{j,*}} e^{-(E_1-E_0)t}(1 - e^{-(E_1-E_0)}) + \ldots \\
&\xrightarrow{t\to\infty} aE.
\end{aligned}
$$

where

$$
C(t, \mathbf{P}tot = \mathbf{0}) = \int d^3x \langle \mathcal{O}(t, \mathbf{x}) \mathcal{O}(0, \mathbf{0})^\dagger \rangle
$$

$$
\begin{aligned}
J^{PC} &= 0^{-+} : \quad \mathcal{O}(t, \mathbf{x}) : \quad \bar{b}\gamma_5 b(t, \mathbf{x}) \qquad (\eta_b) \\
J^{PC} &= 1^{--} : \quad \mathcal{O}(t, \mathbf{x}) : \quad \bar{b}\gamma_\mu b(t, \mathbf{x}) \qquad (\Upsilon_b)
\end{aligned}
$$

Low-Lying Energy Eigenstates of the S-wave $b\bar{b}$ System



- Superfine data (Set 4). Time extent > 8 fm.
- Error bars too small to show on plot
- The excited states are well separated $\rightarrow$ rapid convergence to the ground state.
- Noise does not increase with time

Heavy Quark Local Operators for Tetraquarks

We can construct meson interpolating operators as

$$\mathcal{O}_M^{1(8)}(t,\mathbf{x}) = \mathcal{G}_{efg}^{1(8)}\bar{b}_f\Gamma_M b_g(t,\mathbf{x}) \tag{1}$$

where $\Gamma_M = i\gamma^5, \gamma^k$ projects onto the quantum numbers of the η_b and Υ respectively, and $\mathcal{G}_{efg}^{1(8)}$ is the colour projection onto the singlet (octet). In addition, it is also possible to construct a (anti-) diquark operator as

$$\mathcal{O}_D^{\bar{3}(6)}(t,\mathbf{x}) = \mathcal{G}_{efg}^{\bar{3}(6)}\bar{b}_f^{\hat{C}}\Gamma_D b_g(t,\mathbf{x}) \tag{2}$$

$$\mathcal{O}_A^{3(\bar{6})}(t,\mathbf{x}) = \mathcal{G}_{efg}^{3(\bar{6})}\bar{b}_f\Gamma_A b_g^{\hat{C}}(t,\mathbf{x}) \tag{3}$$

where $(b^{\hat{C}})_\alpha = C_{\alpha\beta}\bar{b}_\beta$ is the charge-conjugated field with $C = -i\gamma^0\gamma^2$.As the two quarks have the same flavour, the Pauli-exclusion principle applies and the wavefunction has to be completely anti-symmetric. With our choice to focus on S-wave combinations of particles, the spatial wave-function must be symmetric. As the colour (triplet) sextet has a (anti-) symmetric colour wavefunction, this forces the spin-wavefunction to be in a (triplet) singlet with ($\Gamma = \gamma^k$) $\Gamma = i\gamma^5$.

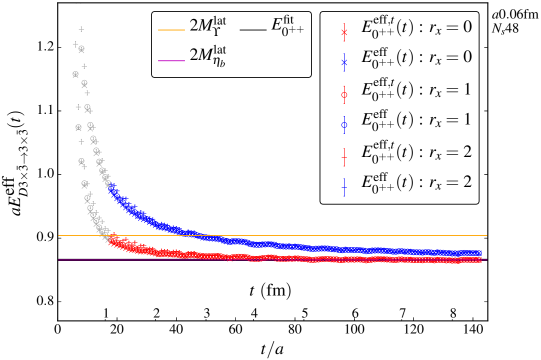
Tetraquark Color Projections

Table: The colour representations of the different quark combinations. Note that, as described in the text, once the colour representation of the (anti-) diquark is chosen, the Pauli-exclusion principle enforces certain spin combinations in S-wave. Also given are the $SU(3)$ colour contractions needed for the $\bar{b}\bar{b}bb$ operators.

	b	$\bar{b}$	$\bar{b}b$	$\bar{b}\bar{b}$	bb
Colour Irrep	3_c	$\bar{3}_c$	$1_c, 8_c$	$3_c, \bar{6}_c$	$\bar{3}_c, 6_c$
$\mathcal{G}^1_{efg}\mathcal{G}^1_{ef'g'}$	$\delta_{fg}\delta_{f'g'}$				
$\mathcal{G}^8_{efg}\mathcal{G}^8_{ef'g'}$	$2\delta_{fg'}\delta_{f'g} - 2\delta_{fg}\delta_{f'g'}/3$				
$\mathcal{G}^3_{efg}\mathcal{G}^3_{ef'g'}$	$(\delta_{ff'}\delta_{gg'} - \delta_{fg'}\delta_{gf'})/2$				
$\mathcal{G}^6_{efg}\mathcal{G}^6_{ef'g'}$	$(\delta_{ff'}\delta_{gg'} + \delta_{fg'}\delta_{gf'})/2$				

Sample Tetraquark Correlator

$$\bar{\mathbf{3}} \otimes \mathbf{3} \rightarrow \bar{\mathbf{3}} \otimes \mathbf{3}$$



Comments

- Two meson contribution to correlators:

$$I = \sum_{X^2} \int \frac{d^3 P_{tot}}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3} \frac{1}{2E(X^2)} |X^2_{(\mathbf{P}_{tot},\mathbf{k})}\rangle\langle X^2_{(\mathbf{P}_{tot},\mathbf{k})}|$$

  where $|X^2_{(\mathbf{P}_{tot},\mathbf{k})}\rangle = |M_1(\mathbf{k})M_2(\mathbf{P}_{tot} - \mathbf{k})\rangle$. The internal relative momentum, $\vec{\mathbf{k}}$, contributes an additional three-integral.

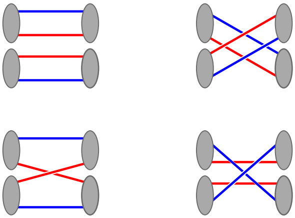
- The effective mass plot has a slow convergence to the ground state (blue). The lowest excited energy $|\mathbf{k}|^2/2\mu_r$ 20 MeV or 0.0092 (lattice units).
- The form appropriate for a dominate two meson contribution (red)

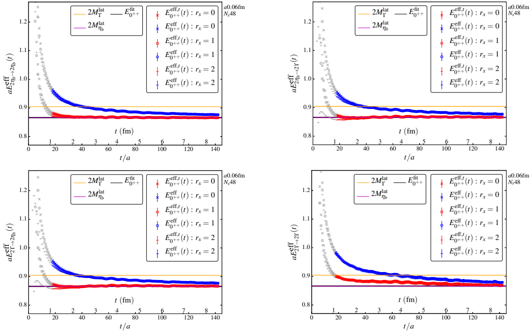
$$aE^{\text{eff},t}_{J^{PC}} = \log\left(\frac{t^{\frac{3}{2}} C^{J^{PC}}_{i,j}(t)}{(t+1)^{\frac{3}{2}} C^{J^{PC}}_{i,j}(t+1)}\right).$$

- There is no difference between local and non-local operators in the correlators
- Gray points not used in fits.

Tetraquark Correlators

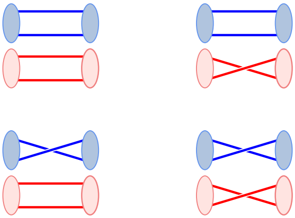
$J^{PC} = 0^{++}$ Basis $[\mathbf{1} \bigotimes \mathbf{1},\ \mathbf{8} \bigotimes \mathbf{8}]$



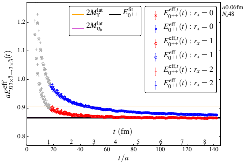


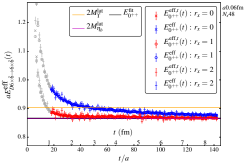
Tetraquark Correlators

Basis $[\bar{\mathbf{3}} \otimes \bar{\mathbf{3}}, \mathbf{6} \otimes \bar{\mathbf{6}}]$

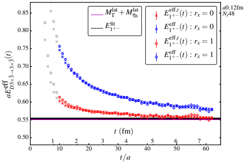


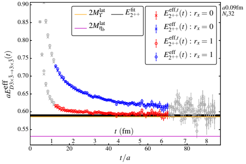
$J^{PC} = 0^{++}$



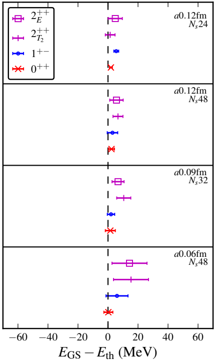


$J^{PC} = 1^{+-}, 2^{++}$





Summary for $bb\bar{b}\bar{b}$ Tetraquarks



No stable $bb\bar{b}\bar{b}$ tetraquarks found: 0^{++}, 1^{+-} or 2^{++}

Other stable tetraquarks

- Appropriate action for each mass range:

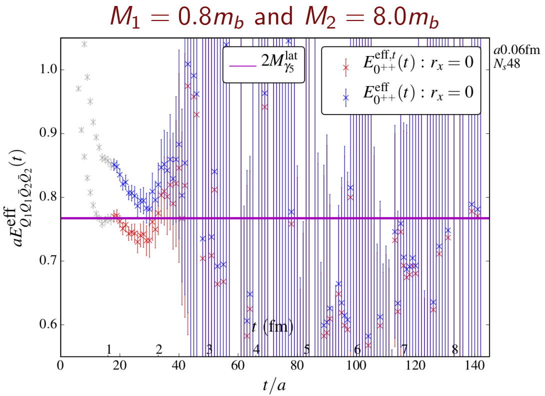
$m_u, m_d\ \ m_s$	m_c	m_b
LQCD	fermilab	NRQCD
$ss\bar{s}\bar{s}$	$cc\bar{c}\bar{c}$	$bb\bar{b}\bar{b}$

- Signal to noise for unequal masses:

$$S/N \sim |< \mathcal{O}(t)\mathcal{O}^\dagger(0) > |/\sqrt{< \mathcal{O}(t)\mathcal{O}^\dagger(t)\mathcal{O}^\dagger(0)\mathcal{O}(0) >}$$

$$\begin{aligned}
\mathcal{O} = bb\bar{c}\bar{c} \quad &: \quad S/N \to \exp\{-[m(bb\bar{c}\bar{c}) - m(b\bar{b}) - m(c\bar{c})]t\} \to \exp(-1.17t(fm)) \\
\mathcal{O} = bb\bar{u}\bar{d} \quad &: \quad S/N \to \exp\{-[m(bb\bar{u}\bar{d}) - m(b\bar{b}) - 2m(\pi^0)]t\} \to \exp(-0.22t(fm)) \\
\mathcal{O} = c\bar{c}u\bar{u} \quad &: \quad S/N \to \exp\{-[m(cc\bar{u}\bar{u}) - m(\eta_c) - 2m(\pi^+)]t\} \to \exp(-0.32t(fm)) \\
\mathcal{O} = bud \quad &: \quad S/N \to \exp\{-[m(bud) - 0.5m(b\bar{b}) - m(\pi^0)\}]t \to \exp(-0.25t(fm))
\end{aligned}$$

Example for Unequal Masses $Q_1 Q_1 \bar{Q}_2 \bar{Q}_2$.



How to Deal with the Noise

Three basic approaches:

- Use many operators and diagonalize to extract low states reliably at shorter times.
- Try to limit the noise itself. Use heavy pions and extrapolate.
  Multi-boson block factorization - break the lattice into a set a blocks and match physics on the interfaces. [M. C, Leonardo Giusti and S. Schaefer, PRD 93 (2016) 094507 [1601.04587], PRD 95 (2017) 034503 [1609.02419]]
- Use some hybrid method. For example, compute the "potentials including light quarks" for static heavy quarks. Then solve SE for the heavy quarks; or HAL approach.

Summary

- If stable $QQ\bar{Q}\bar{Q}$ tetraquark states with $J^{PC} = 0^{++}$ exist, they can be observed at the LHC by their decay into a pair of the 1^3S_1 quarkonium states $[(Q\bar{Q}) + (Q\bar{Q})^{(*)}]$ in the four lepton final state.
- Lattice QCD finds no stable $bb\bar{b}\bar{b}$ tetraquarks exist with $J^{PC} = 0^{++}, 1^{+-}$ or 2^{++}.
- Lattice studies for $Q_1 Q_2 \bar{q}_1 \bar{q}_2$ heavy-light tetraquarks are difficult because of signal/noise issues.

BACKUP

- Effect of Four Quark Contact Terms.

  Four quark contact terms enter in order $\alpha_s^2 v^3$.
  We estimate these effects using the method of :
  R. J. Dowdall et.al. Phys.Rev. D89 (2014) no.3, 031502, (E) Phys.Rev. D92 (2015) 039904.

$$\mathcal{L}_{4q} = z\psi^\dagger \eta \psi + z^* \chi^T \eta^\dagger \chi^*, \quad z = -\frac{d_1^{\frac{1}{2}} \alpha_s}{a m_b},$$

  We find that the shift was consistent with expectations from perturbation theory. ($d_1 \alpha_s^2 = -0.026$). The shift of the $bb\bar{b}\bar{b}$ states was equal to twice the shift in th η_b mass within errors. No additional binding was introduced.