Few-body dynamics of baryons and tetraquarks

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Based on recent work with J. Vijande, A. Valcarce,
older work with J.P.Ader, P. Taxil, S. Zouzou, J.L. Ballot, S. Fleck,
C. Gignoux, B. Silvestre-Brac, Fl. Stancu, Cafer Ay
& Hyam Rubinstein.

Introduction

- Delicate subject
  - You are treated as a precursor
  - And the next day, your model is dismantled

- But before improving or replacing the quark model, one should know precisely what it predicts
- Literature: often a poor treatment of the 3- or 4-body

1

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  - Sometimes consistent, wrong tetraneutron & wrong tetraquark
  - Sometimes surprising: talented experts of quarkonium less convincing for baryons and tetraquarks

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  - *Corruptio optimorum pessima*[2]

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  - Sometimes surprising: talented experts of quarkonium less convincing for baryons and tetraquarks
  - *Corruptio optimorum pessima*[2]
- Change of fashion: after a decade of $(Q\bar{Q}\ldots)$
- The flavored $(QQ\ldots)$ states come back

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3-body problem with flavor independence

- Few rigorous results
- For instance, convexity, see Bertlmann & Martin, Nussinov, ...
  - for mesons (g.s., or g.s. in a sector, or sum of *n* first levels)

$$Q\bar{Q} + q\bar{q} \le 2\, Q\bar{q}\ ,$$

  - Tentatively generalized as

$$QQq' + qqq' \le 2\, Qqq'\ ,$$

  - But there are restrictions (Lieb, Nussinov, Martin, R., Taxil, ...)
  - Recently revisited
- For instance, level order
  - Perturbed h.o. (Gromes, Stamatescu, Isgur, Karl, Stancu ...)
  - $0^+ < 1^- < 0^{+,*}$ for a $\mathrm{C^b}+$ linear potential
  - Roper resonance slightly below the negative parity states: $0^+ < 0^{+,*} \lesssim 1^- <$
  - $0^+ < 1^- < 0^{+,*}$ for a $\mathrm{C^b}+$ linear not yet proved (partial proof by Høgaasen+R. using hypersherical formalism)

Doubly-heavy baryons

- Obviously $r(QQ) \ll r(Qq)$ in (QQq) for large M/m
- The two heavy quarks are clustered in the ground state
- But the naive diquark model is misleading
- The Hamiltonian

$$H = \frac{\boldsymbol{p}_1^2}{2\,M} + \frac{\boldsymbol{p}_2^2}{2\,M} + \frac{\boldsymbol{p}_3^2}{2\,m} - \text{c.o.m} + v(r_{12}) + [v(r_{13}) + v(r_{23})] \; ,$$

- is not very well approximated by

$$H' = \left[\frac{\boldsymbol{p}_x^2}{M} + v(x)\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + 2\,v(\sqrt{3}\,y/2)\right] \; ,$$

with $\boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1$, $\boldsymbol{y} = (2\,\boldsymbol{r}_3 - \boldsymbol{r}_1 - \boldsymbol{r}_2)/\sqrt{3}$, which factorizes.

- The diquark internal energy is modified by the third quark.

Doubly-heavy baryons

- For instance, in the case of the harmonic oscillator this gives

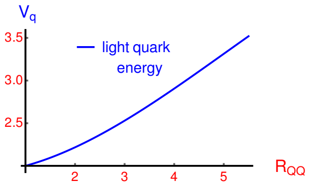
$$H' = \left[\frac{\boldsymbol{p}_x^2}{M} + x^2\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + \frac{3}{2}\,y^2\right]~,$$

instead of the exact

$$H = \left[\frac{\boldsymbol{p}_x^2}{M} + \frac{3}{2}\,x^2\right] + \left[\frac{\boldsymbol{p}_y^2}{\mu} + \frac{3}{2}\,y^2\right]~,$$

- But the *Born-Oppenheimer* treatment is very good
- Especially if done in $\boldsymbol{y}$ at fixed $\boldsymbol{x}$, instead of $\boldsymbol{r}_3$ at fixed $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$
- For instance, with a linear potential, masses $M/m = 5$,
- $E_{\text{var}} = 4.940$ $\quad E_{\text{BO}} = 4.938$ $\quad E_{Dq} = 4.749$ (arbitrary units)

Doubly-heavy baryons



Born-Oppenheimer potential for (QQq), $M/m = 5$, $V \propto \sum_{i<j} r_{ij}$
Fleck, R., PTP 82 (1989) 760

$(QQ\bar{q}\bar{q})$

- $(QQ\bar{q}\bar{q})$ becomes stable if M/m large
- As shown 37 years ago by Ader et al.

Do narrow heavy multiquark states exist?

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(Received 11 August 1981)

- And many others: Heller et al., Rosina et al., Brink et al., Lipkin, Barnea et al., Vijande et al., Oka et al., Bicudo et al., etc.
- Early papers somewhat forgotten in the recent literature!

Why $(QQ\bar{q}\bar{q})$ becomes stable?

- Very close analogy with atomic physics (R., Bad Honnef, 1992, R., Froehlich et al. 1993)

Stable multiquarks: Lessons from atomic physics
J.M. Richard (LPSC, Grenoble). 1992. 8 pp.
Published in **In *Bad Honnef 1992, Quark cluster dynamics* 84-91**
Prepared for Conference: C92-06-29.3, p.84-91 Proceedings

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Proof of Stability of the Hydrogen Molecule

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(Received 24 May 1993)

We sketch two rigorous proofs of the stability of the hydrogen molecule in quantum mechanics. The first one is based on an extrapolation of variational estimates of the ground state energy of a positronium molecule to arbitrary mass ratios. The second one is an extension of Heitler-London theory to nuclei of finite mass.

$$H = \left(\frac{1}{4\,M} + \frac{1}{4\,m}\right)\sum \boldsymbol{p}_i^2 + V + \left(\frac{1}{4\,M} - \frac{1}{4\,m}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]$$
$$= H_{\text{even}} + H_{\text{odd}}$$

- With **<u>the same threshold</u>** for H and H_{even}.
- C-symmetry breaking: $E(H) \le E(H_{\text{even}})$.
- In atomic physics H_2 more stable than Ps_2
- Quark models with flavor indep.: $QQ\bar{q}\bar{q}$ becomes stable.

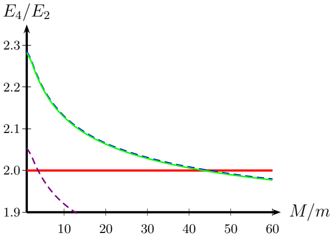
Illustration with a toy model

- Pure chromo-electric

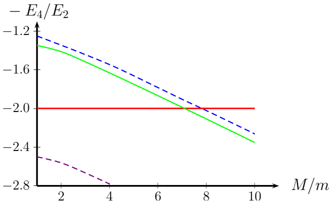
$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\,m_i} - \text{c.o.m.} - \frac{3}{16}\sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij})\ ,$$

with masses $\{m_i\} = \{M, M, m, m\}$.

- Two color wave functions (notation by Chan H-M et al. in the 70s) $T = \bar{3}3$ and $M = 6\bar{6}$
- Assume either pure T, or pure M or include color-mixing
- Stability reached and improved as $M/m \nearrow$



linear (pure T)



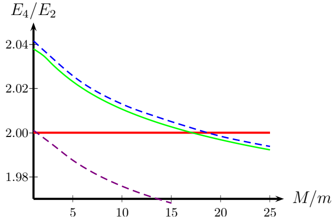
Coulomb (pure T)

Lessons from the toy model

- The critical M/m depends on the shape of the potential
- Perfect control of the 4-body dynamics $E_{\text{low}} < E < E_{\text{Variational}}$
- The diquark *approximation* fails
- Born-Oppenheimer very good, again
- Questions: spin-corrections? color mixing? 3- and 4-body forces?

Lessons from the toy model: Diquark approximation

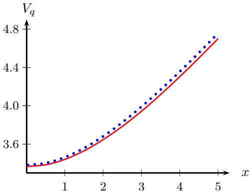
- Dramatic overestimate of the binding



$r^{0.1}$ potential (T color wave function)

Lessons from the toy model: Born-Oppenheimer approximation

- Works very well
- $V_{\text{eff}}(QQ\bar{q}\bar{q}) \simeq V_{\text{eff}}(QQq) + \mathrm{C}^{\mathrm{t}}$
- with $\mathrm{C}^{\mathrm{t}} = Qqq - Q\bar{q}$
- i.e., Eichten and Quigg's identity when one solves for QQ



B.O. potential for $QQ\bar{q}\bar{q}$ (solid red line) and shifted QQq (dotted blue line).

Lessons from the toy model: Hall-Post lower bound

- Invented in the 50's for few-nucleon systems
- Discovered independently in studies of boson systems (Fisher-Ruelle, Dyson-Lenard, Lévy-Leblond, ...)
- And for comparing mesons and baryons (Ader et al., Nussinov, ...)
- Simple form

$$H_3(m) = \sum \left[\frac{\boldsymbol{p}_1^2}{4\,m} + \frac{\boldsymbol{p}_2^2}{4\,m} + V_{12}\right] = \sum_{i<j} H_2^{(i,j)}(2\,m)$$

- Implies (g.s.)

$$E_3(m) \geq 3\,H_2(2\,m)$$

- Many refinements to remove c.m. motion and optimize the decomposition to improve the lower bound (Basdevant, Martin, R., Wu, Zouzou, Krikeb, ...)

Application to all-heavy $QQ\bar{Q}\bar{Q}$

- Hall-Post method shows rigorously that with the T color wave function, $QQ\bar{Q}\bar{Q}$ is unbound
- Equal masses m, T color wavefunction

$$H_4(m) = \frac{1}{2} h_{12}(m) + \frac{1}{2} h_{34}(m) + \frac{1}{4} \sum_{\substack{i=1,2\\ j=3,4}} h_{ij}(m)$$

  where h is the 2-body Hamiltonian, thus

$$E_4(m) \geq 2E_2(m)$$

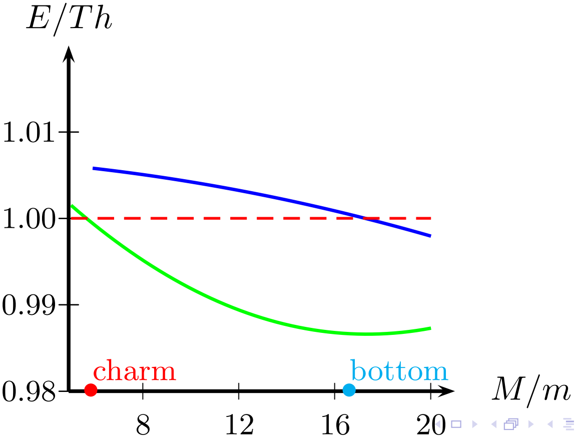
- Removing the center-of-mass properly leads to the better

$$E_4(m) \geq E_2(m) + E_2(m/2)$$

  e.g., $E_4 \geq 2.26\, E_2$ for a linear potential
- Numerical calculations show that M state is also unbound, and also the ground-state with $T - M$ mixing

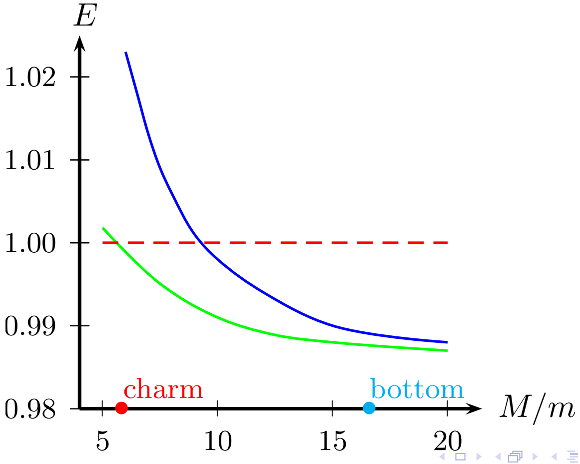
$(QQ\bar{q}\bar{q})$ spin effects

- Use an explicit model tuned to ordinary hadrons, and including an explicit short-range spin-spin term
- Chromoelectric interaction favors $(QQ\bar{q}\bar{q})$ vs. $(Q\bar{q}) + (Q\bar{q})$
- Chromomagnetic interactions also helps in some cases, e.g., 1^+



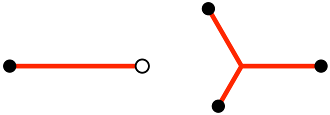
Toy model of $(QQ\bar{q}\bar{q})$ color mixing

- Chromoelectric and chromomagnetic transitions from T to M type of states
- Crucial in particular near the critical M/m ratio



Improved chromoelectric model

- Based on the string model
- Linear confinement interpreted as



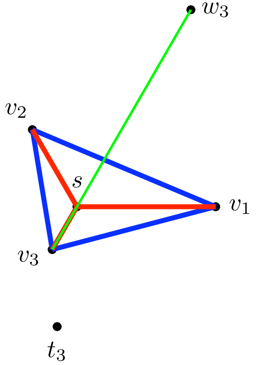
- Not very visible in baryon spectroscopy as compared to

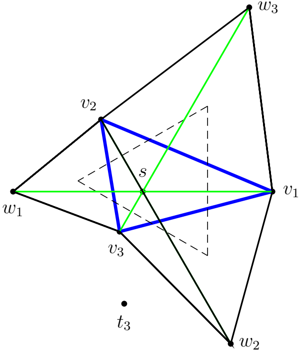
$$V_{\text{conf}} = \frac{1}{2}(r_{12} + r_{23} + r_{31})$$

of the naive additive model.

Steiner tree for baryons baryons

- This baryon potential is the solution of the famous Fermat-Torricelli problem of the minimal path linking three points, with an interesting symmetry restoration, intimately related to a theorem by Napoleon.





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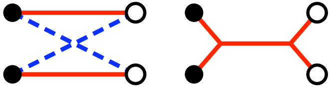
String potential for $QQ\bar{Q}\bar{Q}$

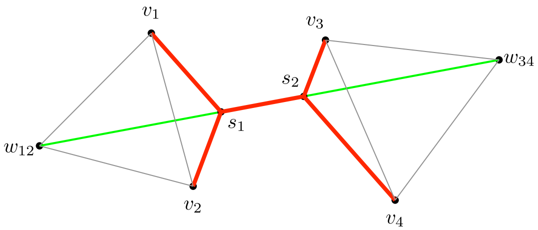
- Instead of $\propto \sum \tilde{\lambda}_i.\tilde{\lambda}_j r_{ij}$, use

$$V = \min\left\{ r_{13} + r_{24}, r_{14} + r_{23}, \min_{J,K}(r_{1J} + r_{2J} + r_{JK} + r_{K3} + r_{K4}) \right\} \ ,$$



- Not so difficult (one does not need to compute the location of the junctions (Ay, R.,Rubinstein (2009), Bicudo et al.)
- gives more attraction (R., Vijande and Valcarce, 2007), and even binding for *equal masses* not submitted to the Pauli principle, say $(QQ'\bar{Q}\bar{Q}')$ with $M(Q) = M(Q')$ but $Q \neq Q'$.
- This restriction was forgotten in some recent papers

Steiner tree: tetraquarks



In the planar case, very simple construction of the connected term of the potential (this speeds up the computation).

$$V_4 = \sigma \left\| w_{12} w_{34} \right\| ,$$

maximal distance between the two Melzak points.

Steiner tree: tetraquarks

$$V_4 = \sigma \, \| w_{12} w_{34} \| \ ,$$

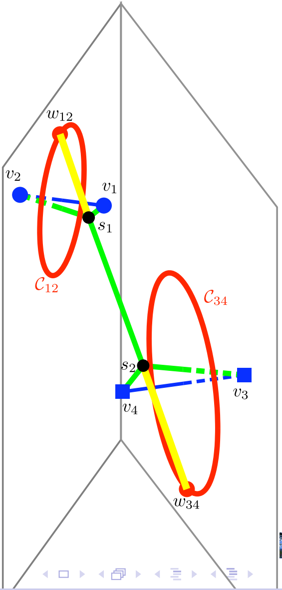
maximal distance between the two Melzak circles.

$$V_4 \le \sigma \left\{ \frac{\sqrt{3}}{2} \left[\| \boldsymbol{x} \| + \| \boldsymbol{y} \| \right] + \| \boldsymbol{z} \| \right\} \ ,$$

which is exactly solvable. The Jacobi var.

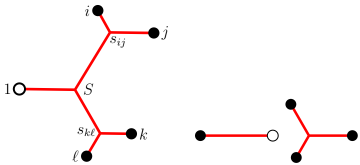
$$\boldsymbol{x} = v_1 v_2, \quad \boldsymbol{y} = v_3 v_4,$$
$$\boldsymbol{z} = (v_1 + v_2)/2 - (v_3 + v_4)/2 \ ,$$

This shows rigorously that the connected string alone binds for $M/m \gtrsim 6000$ (in practice about 200). The “flip-flop” improves significantly.



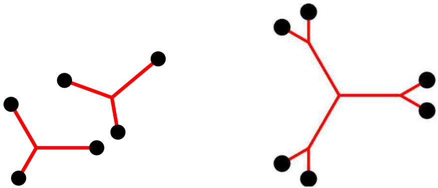
Higher configurations: pentaquark

- Same finding for pentaquark. In absence of constraints from antisymmetrization, pentaquark binding below the meson + baryon thresholds



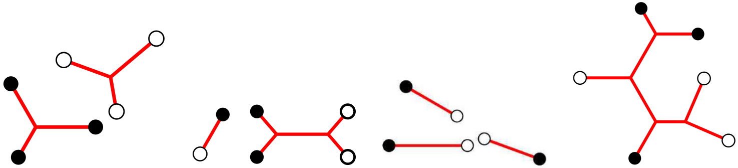
Higher configurations: dibaryon

- Same finding for pentaquark. In absence of constraints from antisymmetrization, dibaryon binding below the baryon+ baryon thresholds



Higher configurations: baryonium

- Same finding for $(3q, 3\bar{q})$. In absence of constraints from antisymmetrization, at least for some mass configurations, binding below the various thresholds (baryon-antibaryon, 3 mesons, meson + tetraquark)



Outlook

- The four-body problem is delicate, even for simple models
- Usually cannot be solved as $(a, b) \to ab$, $(c, d) \to cd$ and $ab + cd \to abcd$
- Years of methods elaborated by Faddeev, Yakubosky, Delves, Fabre de la Ripelle, Kamimura, Suzuki, Varga, etc. cannot be ignored
- Already 37 years of study of $(QQ\bar{q}\bar{q})$
- Stable if M/m large enough
- $(cc\bar{u}\bar{d})$ with 1^+ at the edge in some specific models
- Main uncertainty: extrapolation from $Q\bar{Q}'$ and $QQ'Q''$ to multiquarks.
- Multibody forces suggested in the string model.

Backup slides

Bad Honnef 1992

Stable multiquarks: Lessons from atomic physics
J.M. Richard (LPSC, Grenoble). 1992. 8 pp.
Published in **In *Bad Honnef 1992, Quark cluster dynamics* 84-91**
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STABLE MULTIQUARKS:
LESSONS FROM ATOMIC PHYSICS

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Abstract. In Atomic Physics, some configurations like $H_2^+(ppe^-)$ are particularly stable, while others, like (pe^-e^+), break into smaller subsystems. The mechanisms by which collective binding does or does not occur can tentatively be extended to hadron spectroscopy in quark models: this suggests which flavour configurations are the most likely to form stable multiquark hadrons.

More on symmetry breaking

- Role of symmetry breaking
- $\min(p^2 + x^2 + \lambda\, x) < \min(p^2 + x^2)$ in elementary QM
- $\min(H_{\text{even}} + H_{\text{odd}}) < \min H_{\text{even}}$
- $\min(M^+ M^+ m^- m^-) < \min(\mu^+ \mu^+ \mu^- \mu^-)\ , \quad 2\,\mu^{-1} = M^{-1} + m^{-1}$
- $(M^+ M^+ m^- m^-)$ and $(\mu^+ \mu^+ \mu^- \mu^-)$ have the same threshold

$$\frac{\boldsymbol{p}_1^2}{2M} + \frac{\boldsymbol{p}_2^2}{2M} + \frac{\boldsymbol{p}_3^2}{2m} + \frac{\boldsymbol{p}_4^2}{2m} + V = \left[\sum \frac{\boldsymbol{p}_i^2}{2\mu} + V\right] + \left(\frac{1}{4M} - \frac{1}{4m}\right)\left[\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2\right]$$

$$\Rightarrow \qquad \min(H_{C-\text{even}} + H_{C-\text{odd}}) < \min H_{C-\text{even}}$$

and this explains why H_2 is more stable than Ps_2.

Breaking particle identity?

- same reasoning?

$$\frac{\boldsymbol{p}_1^2}{2M}+\frac{\boldsymbol{p}_2^2}{2m}+\frac{\boldsymbol{p}_3^2}{2M}+\frac{\boldsymbol{p}_4^2}{2m}+V=$$
$$\left[\sum\frac{\boldsymbol{p}_i^2}{2\mu}+V\right]+\left(\frac{1}{4M}-\frac{1}{4m}\right)\left[\boldsymbol{p}_1^2+\boldsymbol{p}_3^2-\boldsymbol{p}_2^2-\boldsymbol{p}_4^2\right]$$

- Thus $(M^+m^+M^-m^-)$ more stable than $(\mu^+\mu^+\mu^-\mu^-)$????
- No!
- Since symmetry breaking benefits more to $(M^+M^-)+(m^+m^-)$!!!
- But some kind of metastability below the other threshold, $(M^+m^-)+\text{c.c.}$
- In short (un)favorable symmetry breaking can (spoil) generate stability.

More on the equal-mass case

- asymmetry in the kinetic energy $\Rightarrow$ (M, M, m, m) better than (μ, μ, μ, μ)
- Similar for the potential energy

$$H = \sum \boldsymbol{p}_i/(2m) + \sum g_{ij} v(r_{ij}) \,, \quad \sum g_{ij} = 2 \,.$$

If g_{ij} are equal: highest energy, and, roughly speaking, the broader the distribution of g_{ij}, the lower the energy.

- Now, if you compare Ps_2 and quark models: Ps_2 favored

$(abcd)$	$v(r)$	g_{ij}	$\bar{g}$	Δg
Thr (1,3)+(2,4)	$-1/r, r$	$\{0, 0, 1, 0, 1, 0\}$	$1/3$	0.22
Ps_2	$-1/r$	$\{-1, -1, 1, 1, 1, 1\}$	$1/3$	0.89
$[(qq)_{\bar{3}}(\bar{q}\bar{q})_3]$	$-1/r, r$	$\{1/2, 1/2, 1/4, 1/4, 1/4, 1/4\}$	$1/3$	0.01
$[(qq)_6(\bar{q}\bar{q})_{\bar{6}}]$	$-1/r, r$	$\{-1/4, -1/4, 5/8, 5/8, 5/8, 5/8\}$	$1/3$	0.17

All-heavy tetraquarks: Second proof

- Hall-Post inequalities developed in the 50s in nuclear physics, and discovered in the framework of studies on the stability of matter, or meson/baryon inequalities (Ader et al., Basdevant et al., Nussinov et al., …). Here (scale set to $M = 1/2$)

$$H_T = \sum \boldsymbol{p}_i^2 + \frac{1}{2}(V_{12} + V_{34}) + \frac{1}{4}(V_{13} + \cdots) ,$$

  where $V_{ij} = v(r_{ij})$ is the *quarkonium* potential.

- $$H_T = \frac{1}{2}(h_{12} + h_{34}) + \frac{1}{4}(h_{13} + h_{14} + h_{23} + h_{24}) ,$$

  where $h_{ij} = \boldsymbol{p}_i^2 + \boldsymbol{p}_j^2 + V_{ij}$ is the quarkonium Hamiltonian.

- $$\min(H_T) \geq 2 \min(h_{13}) = 2\, E_{\min}(Q\bar{Q}) .$$

- Improved by removing c.o.m. in H_4 and in each h.

$$\min(H_T) \geq 2.3\, E_{\min}(Q\bar{Q}) .$$

Hidden-charm pentaquarks

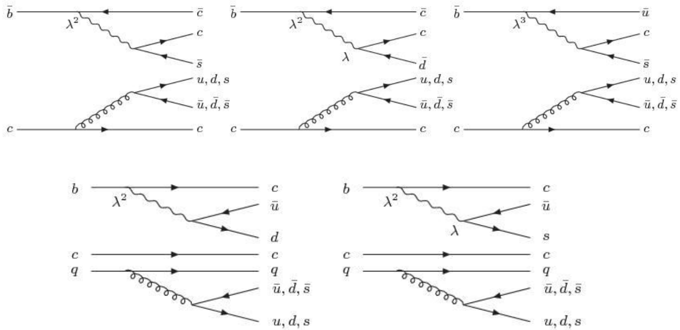
- Two recent contributions:
- Bound states below the threshold
  - Valcarce, Vijande, R., Phys. Lett. B774 (2017) 710-714 [arXiv:1710.08239]
  - $(\bar{c}cqqq)$ with $I = 1/2$ and $J = 5/2$ below the lowest S-wave threshold $\bar{D}^*\Sigma_c^*$ (but above $N\eta_c$ in D-wave)
  - For $I = 3/2$ and $J = 1/2, 3/2$ binding below S- and D-wave thresholds
  - Both chromo-electric and -magnetic parts necessary for binding
- Resonances in the quark model
  - Hiyama et al. (work in progress): real scaling, borrowed from electron-atom and electron-molecule scattering to separate, among the energies above the threshold, actual resonances from fictitious states produced by the variational method. Looks promising.
  - Similar to Luscher criteria for lattice, stability plateau in QCDSR
  - See Hiyama contribution at “Critical Stability”, Dresden, Oct. 2017

Summary for all-heavy

- $(cc\bar{c}\bar{c})$ and $(bb\bar{b}\bar{b})$ not bound in additive model nor in string-inspired variant
- Pity, would be suitable for J/ψ or Υ triggers.
- $(bb\bar{c}\bar{c})$ a little more favorable, mass ratio Q/q perhaps not large enough
- $(bc\bar{b}\bar{c})$ metastable, i.e., below its highest threshold, so a type of $(B_c\text{-}\bar{B}_c)$ molecule that can annihilate or rearrange itself into $(b\bar{b}) + (c\bar{c})$

Production of T_{cc} from B_c or Ξ_{bc}

Figs from the Roma group



Chromomagnetic binding

- In the 70s, the hyperfine splitting between hadrons ($J/\psi - \eta_c$, $\Delta - N$, etc.) explained à la Breit–Fermi, by a potential

$$V_{SS} = -A \sum_{i<j} \frac{\delta^{(3)}(\boldsymbol{r}_{ij})}{m_i\, m_j}\, \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\ ,$$

  a prototype being the magnetic part of one-gluon-exchange.
- Attractive coherences in the spin-color part: $\langle \sum \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j \rangle$ sometimes larger for multiquarks than for the threshold.
- In particular $\langle \ldots \rangle$ twice larger (and attractive) in the best ($uuddss$) as compared to $\Lambda + \Lambda$.
- But $\langle \delta^{(3)}(\boldsymbol{r}_{ij}) \rangle$ much weaker for multiquarks than for ordinary hadrons, and needs to be computed. Hence uncertainties.
- Astonishing success with > 20 experiments on H and still lattice computations of H 40 years later!

Chromomagnetic binding-2

- Other configurations found, such as the heavy P (Gignoux et al., Lipkin, 1987) ($\bar{Q}qqqq$)
- Any correction repulsive: binding is not secured
- In particular SU(3)$_F$ breaking
- Short-range factor $\langle\delta(\boldsymbol{r}_{ij})\rangle$ borrowed from baryons
- The model was improved by Høgassen et al., and later by Stancu, Zhu et al., ...

$$H = -\sum_{i<j} C_{ij}\, \lambda_i^{(c)}.\lambda_j^{(c)}\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j\ ,$$

  C_{ij} tuned to qq, cq, cs, ... in ordinary hadrons
- Astonishing picture of the $X(3872)$
- Further studies, e.g., PKU
- *b*-sector, and/or all-heavy systems more problematic.
- as it requires interplay of chr.-elec. and magn. effects

More Latin sentences

- Nec pluribus impar
- Ne sutor ultra crepidam