Five quarks coupled with open channels
— Compact exotic or extended molecule? —

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SCGP Workshop on Exotic Hadrons at Stony Brook
May 28 - June 1, 2018

1. Prologue; Θ^+
2. qqqcc for Pc, hard to exist as a compact 5q's
3. KN interaction for molecule

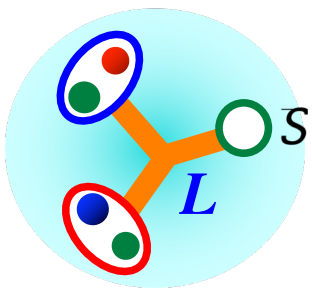
Prologue, could have been the main: Θ^+ $uudd\bar{s}$

E. Hiyama, M. Kamimura, A. Hosaka, H. Toki, M. Yahiro
Phys.Lett. B633 (2006) 237-244 , e-Print: hep-ph/0507105 | PDF

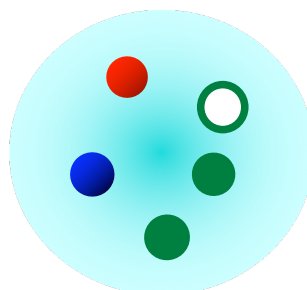
Two slides from "Pentaquark05", J-Lab Oct 20-22, 2005

Hadronic (color-singlet) or colored correlations?

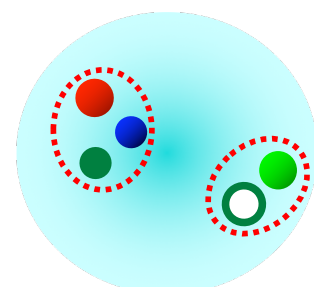
SU(3) **qqq** or **qqbar** are enough to make color singlets



Diquark correl.



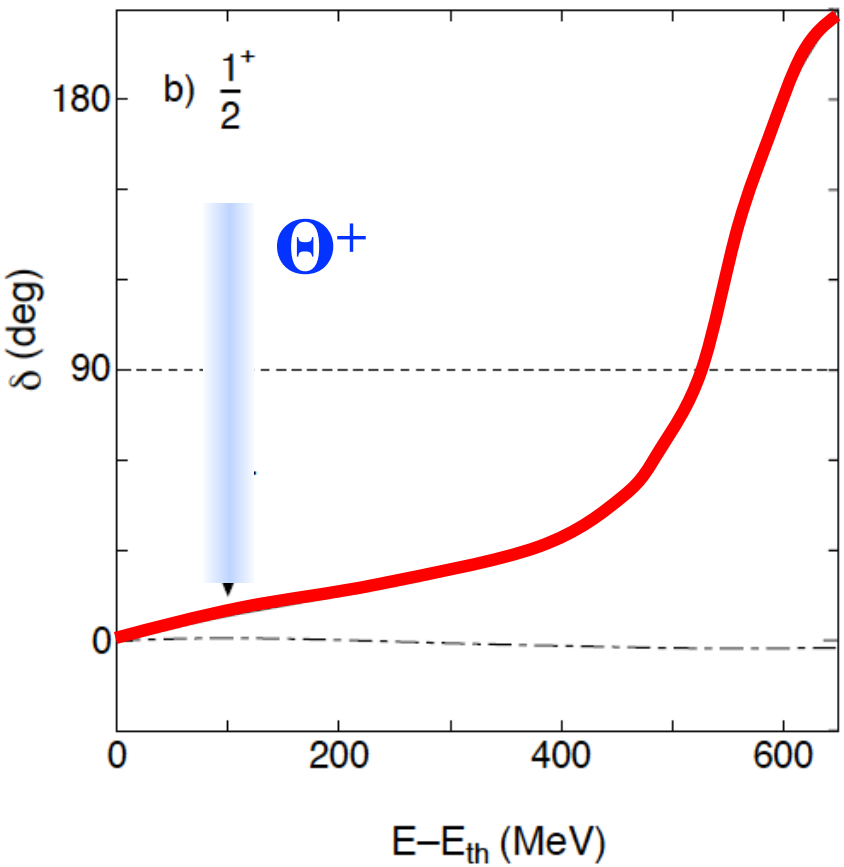
Five quarks random



Hadronic correl.

Difficult to decay ⟷ ***Easy to decay***

K^+N Phase shift



$E_{res} \sim 530$ MeV
$\Gamma_{res} \sim 110$ MeV

$(0s)^4 1p$

- Strong $q\bar{s}$ correlation favors KN Rather than $[ud][ud]\bar{s}$

=>

- Formation of the JW type conf. is a dynamical problem

Quark model estimate of hidden-charm pentaquark resonances

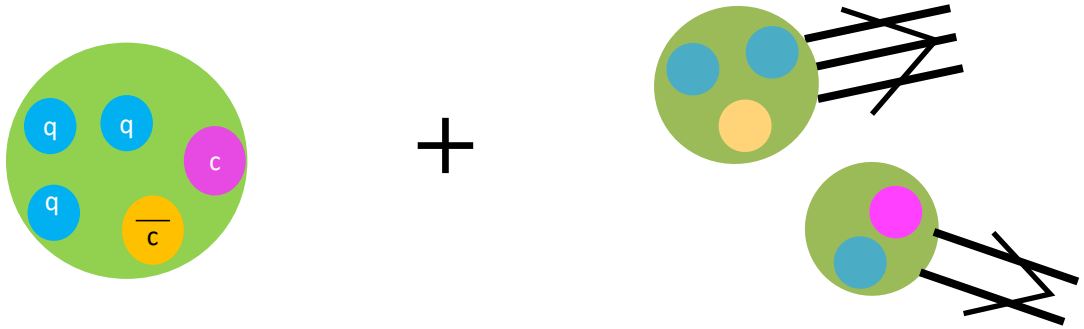
Emiko Hiyama , Atsushi Hosaka, Makoto Oka, Jean-March Richard
arXiv:1803.11369 [nucl-th] | PDF

For Pc(4380, 4450), PRL 115, 072001 (2015)

Full **five-body calculation** with the **confined** + **scattering**

Closed-multiquark + Fallapart

Feshbach resonance



Hamiltonian

$$H = \sum_i (m_i + \frac{\boldsymbol{p}_i^2}{2m_i}) - T_G - \frac{3}{16} \sum_{i<j} \lambda_i . \lambda_j \, V_{ij}(r_{ij})$$

$$V_{ij}(r) = -\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi\kappa'}{3m_i m_j} \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \boldsymbol{\sigma}_i . \boldsymbol{\sigma}_j$$

$$r_0(m_i, m_j) = A(\frac{2m_i m_j}{m_i + m_j})^{-B}$$

Parameters B. Silvestre-Brac and C. Semay, Z. Phys. C61 (1994) 271

	p	$m_{u,d}$(GeV)	m_s(GeV)	Λ(GeV)
AP1	3/2	0.277	0.553	1.851
AL1	1	0.315	0.577	1.836

B	$A(\mathrm{GeV}^{B-1})$	κ	κ'	$\lambda(\mathrm{GeV}^{5/3})$
0.3263	1.5296	0.5871	1.8025	0.3898
0.2204	1.6553	0.5069	1.8609	0.1653

Threshold masses

hadron	J^P	cal.	exp.
η_c	0^-	2984	2983
J/ψ	1^-	3103	3096
D	0^-	1882	1869
D^*	1^-	2033	2007
N	$1/2^+$	937	938
Λ_c	$1/2^+$	2290	2286
Σ_c	$1/2^+$	2472	2455
Σ_c^*	$3/2^+$	2545	2520

5q Wave function q(1)q(2)q(3) c(4)$\bar{c}$(5)

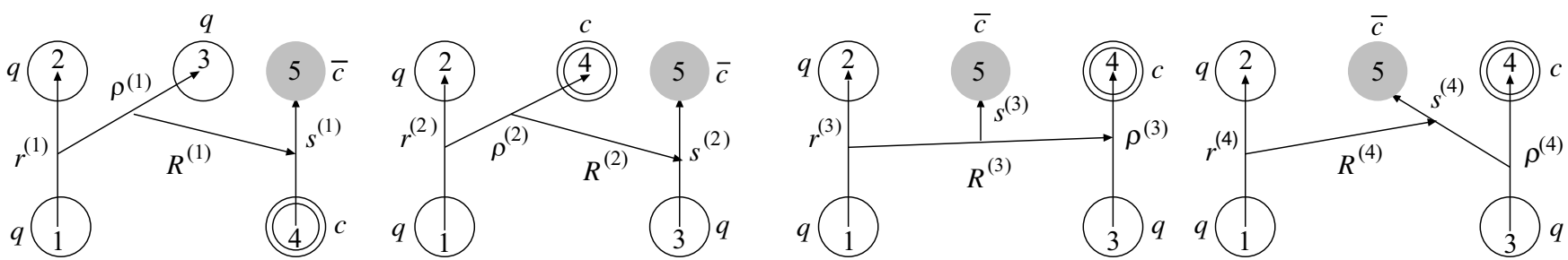
$$\Psi_{JM} = \sum \mathcal{A}_{123}\, \xi_1^{(C)} \eta_T^{(C)} \Big[\chi^{(C)}_{S(s\bar{s}\sigma)} \times \Big[\big[[\phi_{nl}^{(C)}(\boldsymbol{r}^{(C)})\ \varphi_{\nu\lambda}^{(C)}(\boldsymbol{\rho}^{(C)})]_\Lambda \psi_{NI}^{(C)}(\boldsymbol{R}^{(C)}) \big]_{I'} \tilde{\psi}_{N'K}^{(C)}(\boldsymbol{s}^{(C)}) \Big]_L \Big]_{J^P M}$$

C = 1, …, 4

$\mathcal{A}$ntisymmetrizer For 1, 2, 3 (qqq)

Color Isospin Spin Orbital

Four Jacobi configurations

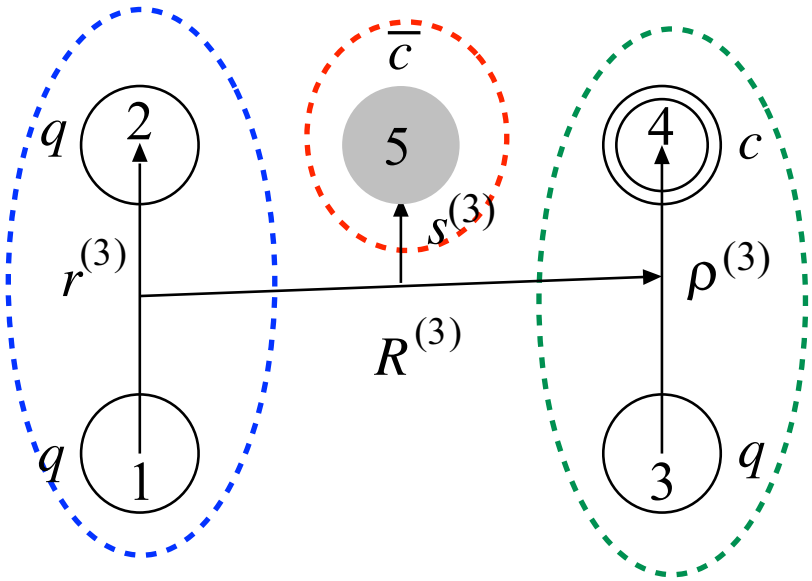


C = 1 C = 2 C = 3 C = 4

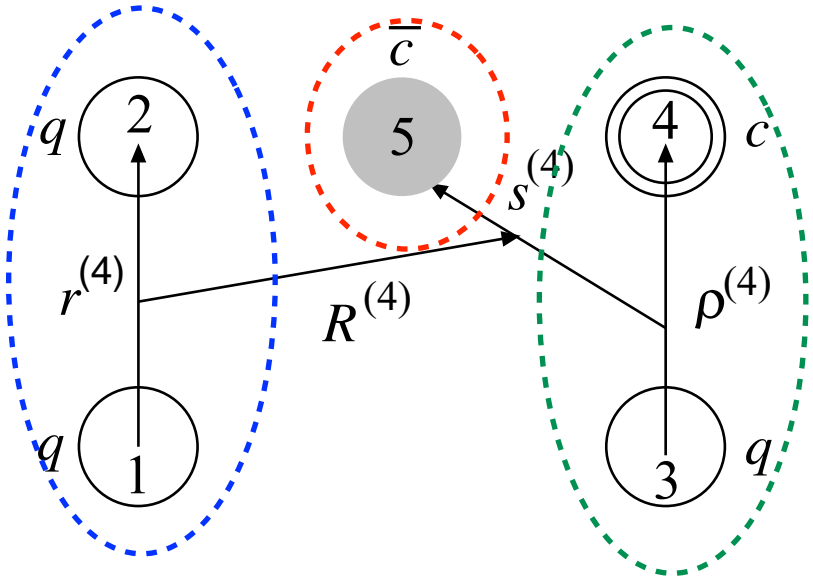
Open/scattering Confined

Confining configurations

C = 3



C = 4

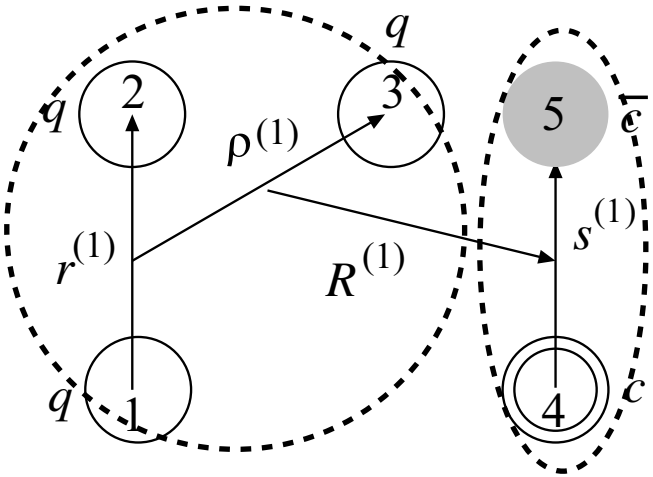


Color: $\xi_1^{(3)} = [(12)_{\bar{3}}(34)_{\bar{3}}]_3 5]_1 \longrightarrow (\bar{3} \times \bar{3}) \times \bar{3} \longrightarrow 1$

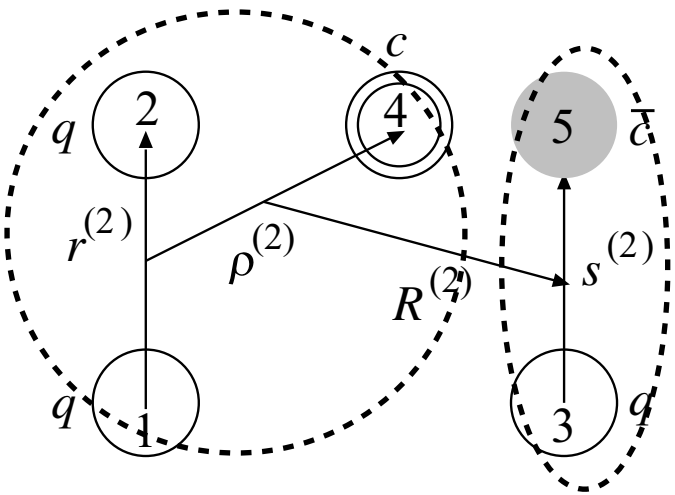
Spin Isospin: $\chi^{(3)}_{S(s\bar{s}\sigma)}(123,4,5) = [[(12)_s(34)_{\bar{s}}]_\sigma 5]_S$

Open/Scattering channels

C = 3



C = 4



Color: $\xi_1^{(1)} = [(123)_1(45)_1]_1$

$$\rightarrow (3 \times 3 \times 3) \times (3 \times \bar{3}) \rightarrow 1 \times 1$$

Spin

Isospin: $\chi^{(1)}_{S(s\bar{s}\sigma)}(123,4,5) = [[(12)_s 3]_\sigma (45)_{\bar{s}}]_S$

Orbital $\Psi_{JM} =$

$$\sum \mathcal{A}_{123}\, \xi_1^{(C)} \eta_T^{(C)} \Big[\chi^{(C)}_{S(s\bar{s}\sigma)} \times \Big[\underbrace{\big[[\phi^{(C)}_{nl}(\boldsymbol{r}^{(C)})\ \varphi^{(C)}_{\nu\lambda}(\boldsymbol{\rho}^{(C)})]_\Lambda \psi^{(C)}_{NI}(\boldsymbol{R}^{(C)}) \big]_{I'} \tilde{\psi}^{(C)}_{N'K}(\boldsymbol{s}^{(C)}) \Big]_L \Big]_{J^P M}$$

C = 1,.., 4
n l m, …

$$\phi_{\boldsymbol{nlm}}(\boldsymbol{r}) = N_{nl}\, r^l\, e^{-(r/r_n)^2}\, Y_{lm}(\hat{\boldsymbol{r}})$$

$$r_n = r_1\, a^{n-1} \qquad (n = 1 \ldots n_{\max})$$

and similar for other parts

$n_{\max} \sim 5$ for confining and ~ 10 for scattering

Coefficients are obtained by diagonalizing H

By about 40,000 basis functions

Strategy:

Study s-wave dominant negative parity states

1st: Solutions only with confining channels included
c = 3, 4 ~ Corresponds to standard QM calc.

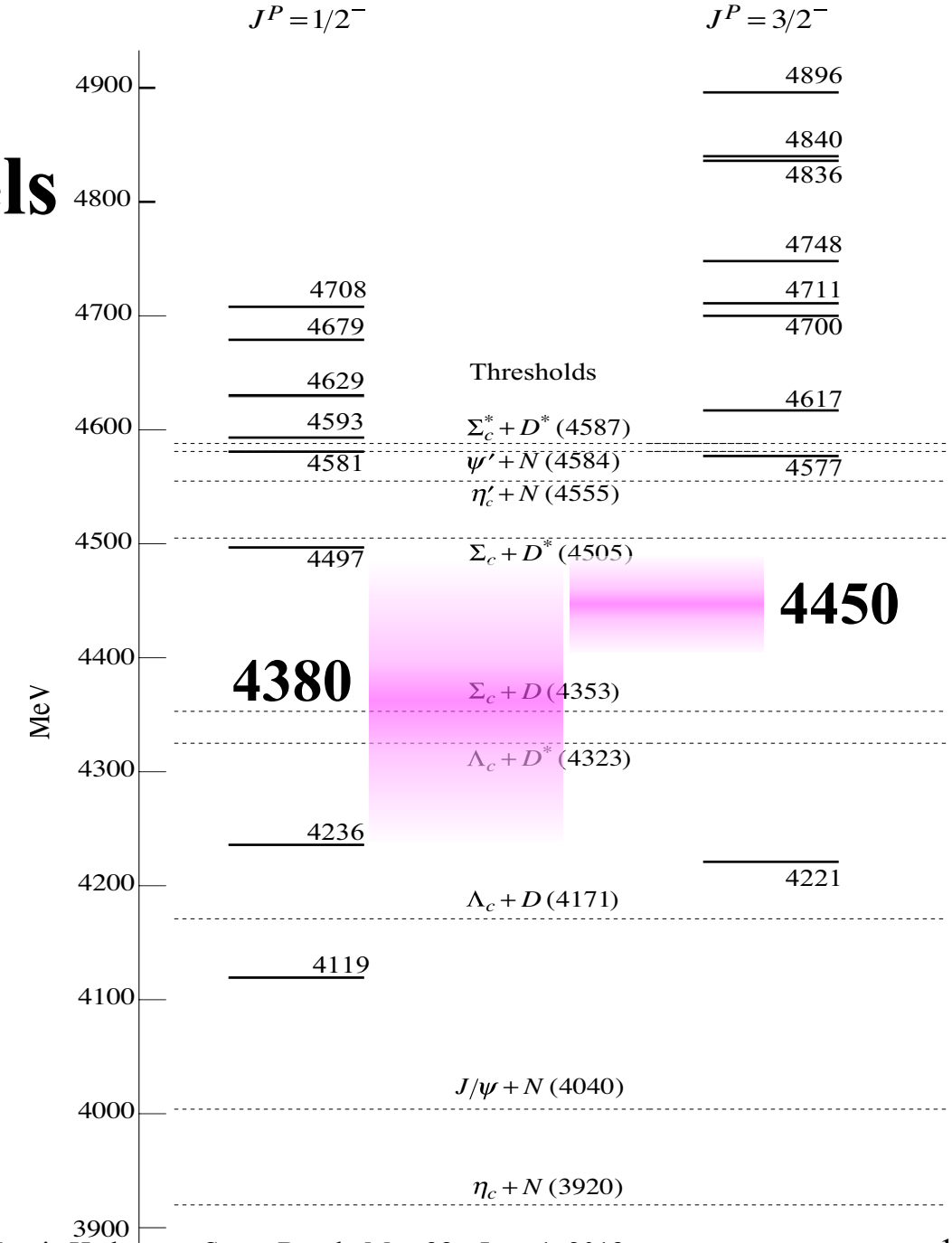
2nd: Couple the scattering channels c = 1, 2
All possible open channels:
$\eta_c N$, $J/\psi N$, $\Lambda_c D$, $\Lambda_c D^*$, $\Sigma_c D$, $\Sigma_c D^*$, $\eta_c' N$, $\psi' N$, $\Sigma_c^* D^*$

Result with confining channels

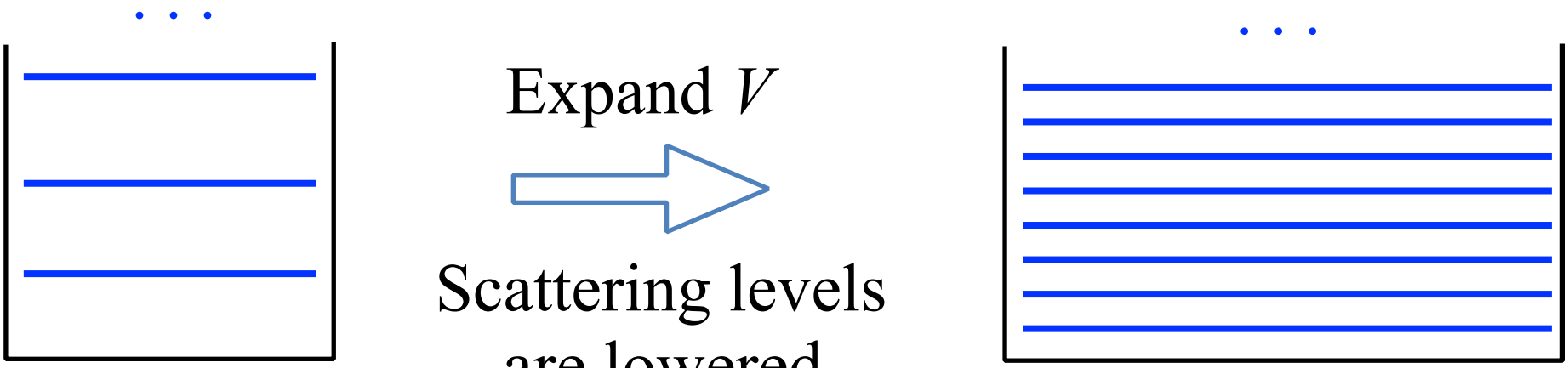
Many bound states are found.

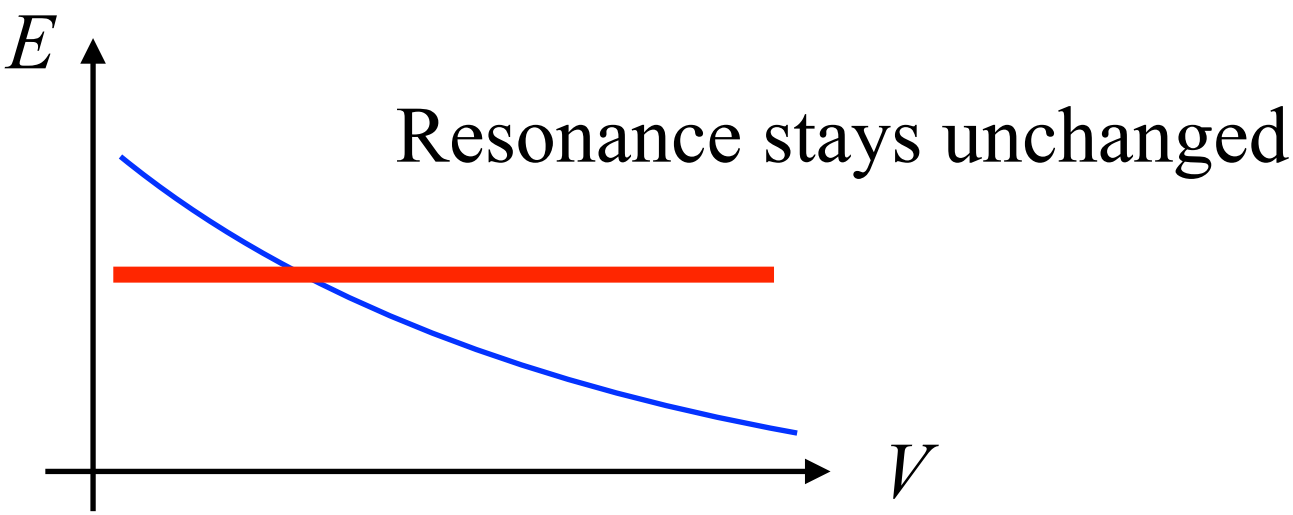
BUT

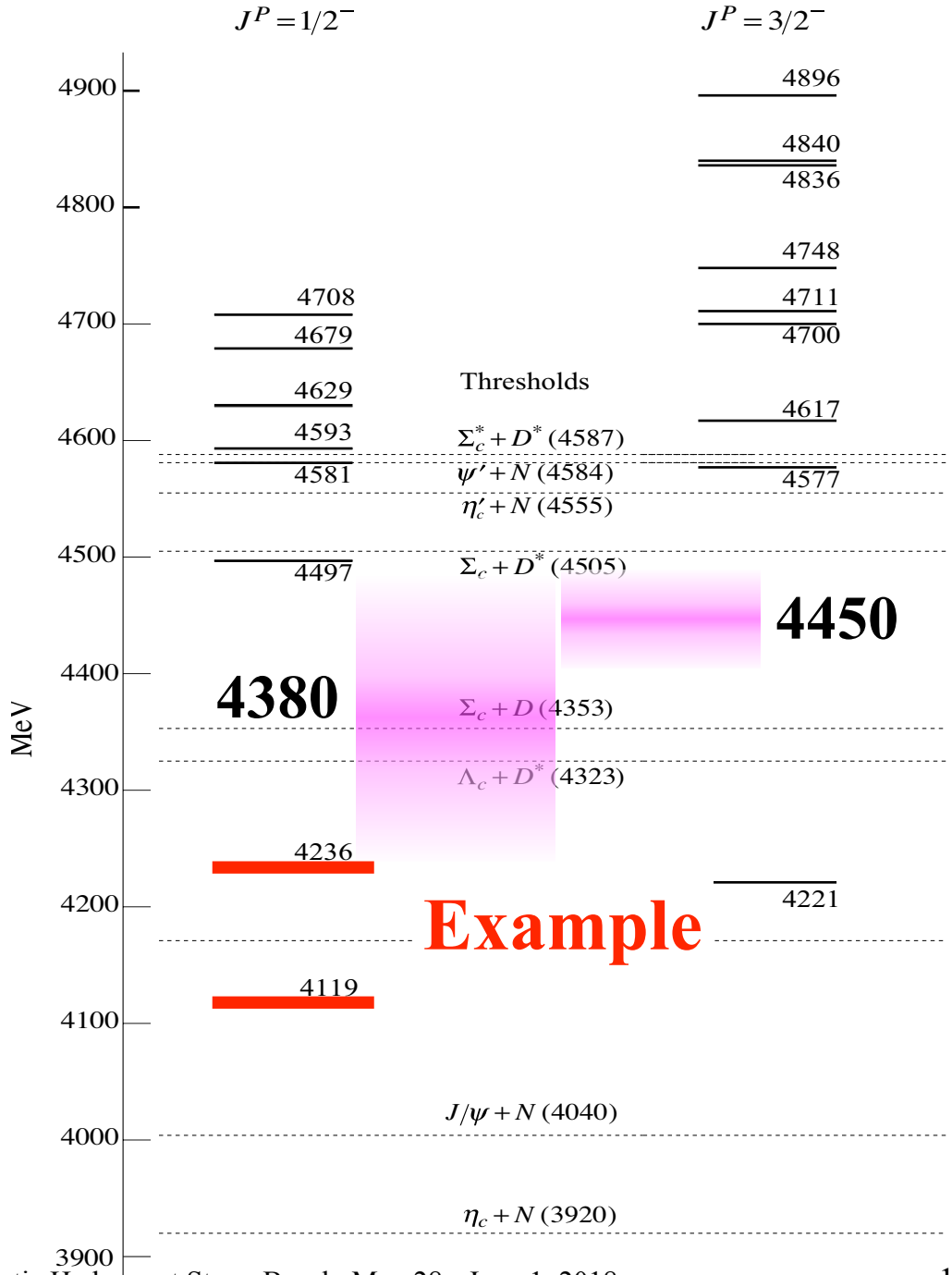
Do they survive ?



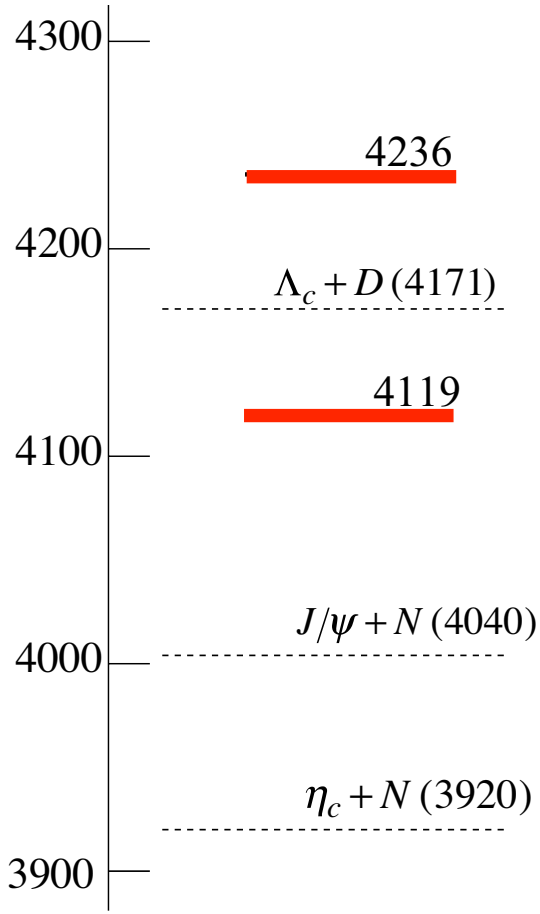
Scaling method





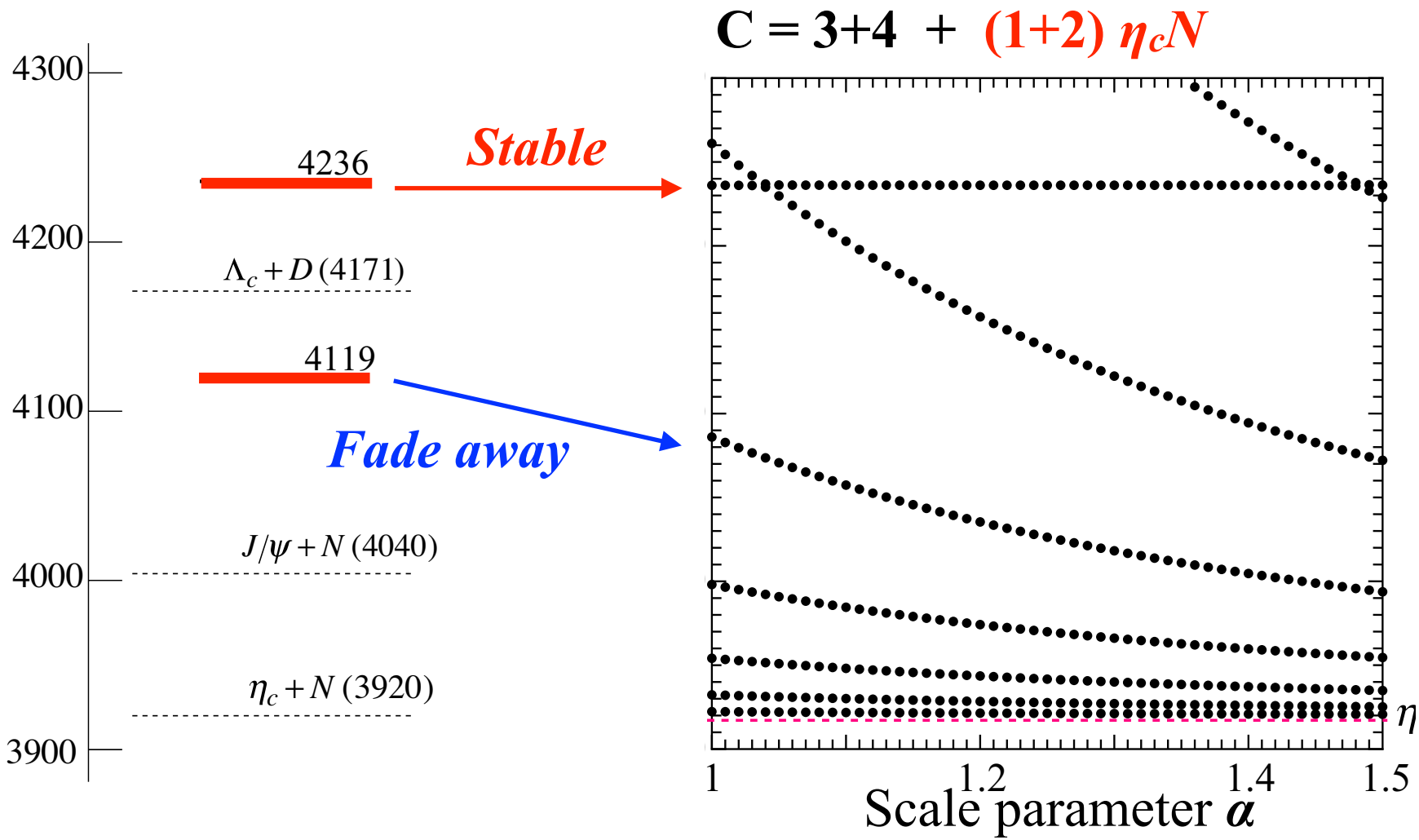

$J^P = 1/2^-$
$J^P = 3/2^-$
MeV
4900
4800
4700
4600
4500
4400
4300
4200
4100
4000
3900
4896
4840
4836
4748
4711
4700
4617
4577
4221
4708
4679
4629
4593
4581
4497
4236
4119
Thresholds
$\Sigma_c^* + D^*$ (4587)
$\psi' + N$ (4584)
$\eta_c' + N$ (4555)
$\Sigma_c + D^*$ (4505)
$\Sigma_c + D$ (4353)
$\Lambda_c + D^*$ (4323)
$J/\psi + N$ (4040)
$\eta_c + N$ (3920)
4380
4450
Example

Confined states and their fate



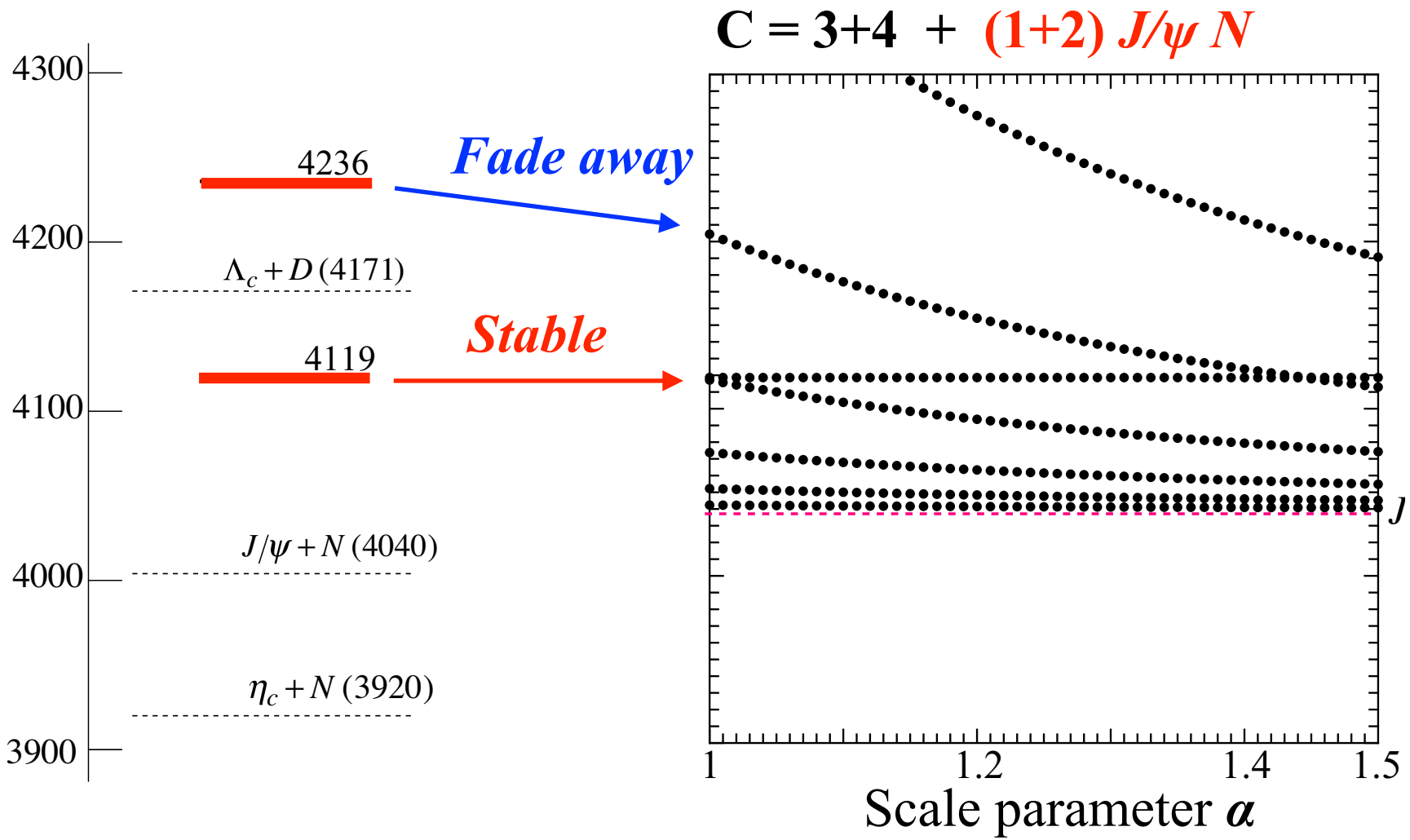
4119 couples strongly with $\boldsymbol{\eta_c N} \rightarrow$ Dominates it

Confined states and their fate



4119 couples strongly with $\boldsymbol{\eta_c N} \rightarrow$ Dominates it

Confined states and their fate



4236 couples strongly with ***J/ψ N*** → Dominates it

All confined states

$J = 1/2^-$	
4119	$\eta_c + N$
4236	$J/\psi + N$, $\Lambda_c + D$
4497	$\eta_c + N$,$\Lambda_c + D^*$, $\Sigma_c + D$
4581	$J/\psi + N$
4593	$\Lambda_c + D$
4629	$\psi' + N$
4679	$\Sigma_c + D^*$
4708	-

$J = 3/2^-$	
4221	$J/\psi + N$
4577	$\Lambda_c + D^*, \Sigma_c * D^*, \Sigma_c^* + D^*$,$J/\psi + N$
4617	$\Sigma_c^* + D$,$\Sigma_c^* + D^*$
4700	$\Sigma_c^* + D$,$\Sigma_c^* + D^*$
4711	$\Sigma_c + D^*$
4748	$\Sigma_c^* + D$
4836	$\Sigma_c^* + D^*$ or $\Sigma_c^* + D$
4840	$\Sigma_c^* + D^*$ or $\Sigma_c^* + D$
4896	-

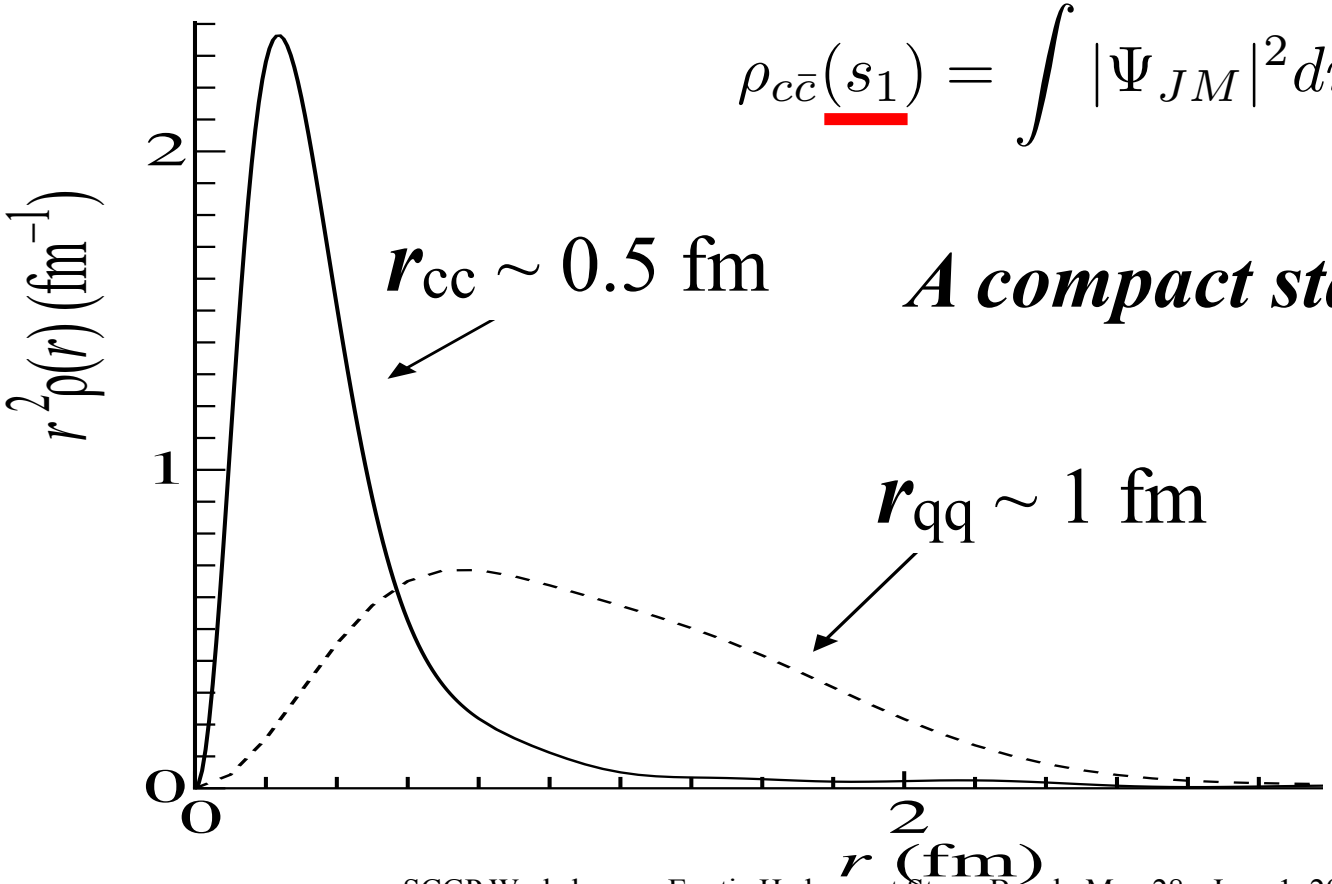
All states fade away after coupling to scattering states
Except for **4708** and **4896**; No strong coupling to any
They show at **4690** and **4920 as resonances**

Structure of 1/2⁻ (4690)

qq and $c\bar{c}$ distributions

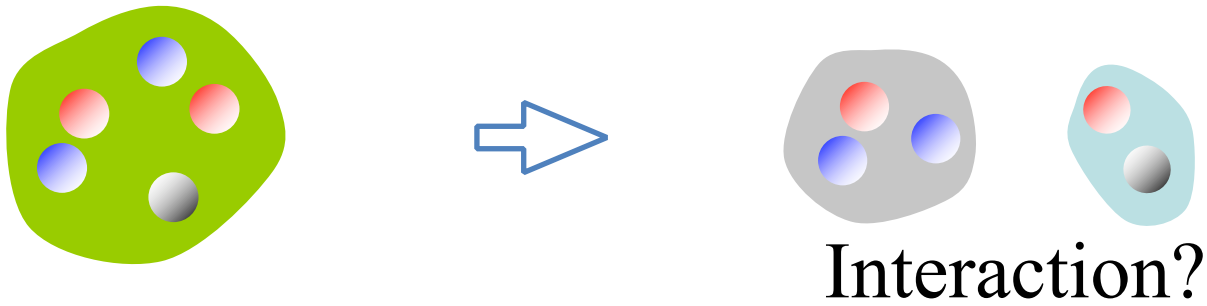
$$\rho_{qq}(r_1) = \int |\Psi_{JM}|^2 d\boldsymbol{s}_1 d\boldsymbol{R}_1 d\boldsymbol{\rho}_1 d\hat{\boldsymbol{r}}_1$$

$$\rho_{c\bar{c}}(s_1) = \int |\Psi_{JM}|^2 d\boldsymbol{r}_1 d\boldsymbol{R}_1 d\boldsymbol{\rho}_1 d\hat{\boldsymbol{s}}_1$$



A compact state

- $qqqc\bar{c}$ system does not support states in the Pc region in the quark model
- Color singlet hadrons are formed to fall apart in the absence of inter-hadron interactions



- Coupling to scattering states is important

Interaction between the color singlet hadrons

The key issue for molecular states
It took 70 years after Yukawa for NN from QCD

Necessity of a good and practical framework

An example
of **K($\bar{K}$)N interaction in the Skyrme model**

$s\bar{q}$ qqq ~ qqq requires lattice calc. huge sources

Skyrme **Lagrangian** supplemented by the WZ term

$$L = \frac{F_\pi^2}{16}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e^2}\mathrm{tr}\left[\left(\partial_\mu U\right)U^\dagger, \left(\partial_\nu U\right)U^\dagger\right]^2 + L_{SB} + L_{WZ}$$

- Respects chiral symmetry and large *Nc*
- Nucleons as ***solitons***
- Strong spin-isospin force forms the ***hedgehog***
- ***Nucleons as rotating hedgehogs***

Kaons around the nucleon

Around the hedgehog ~ Body fixed frame

Callan-**K**lebanov, Nucl. Phys. B262 (1985)

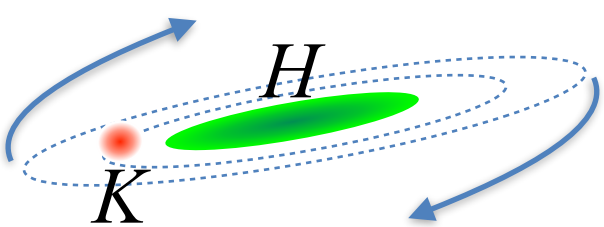
Around the rotating hedgehog ~ Lab frame

Ezoe-**H**osaka, Phys Rev D94, 034022 (2016); D96, 054002 (2017)

$$U_{CK} = A\,(t)\sqrt{U_\pi}U_K\sqrt{U_\pi}A^\dagger\,(t) \qquad U_\pi = \exp(i\vec{\tau}\cdot\hat{r}F(r))$$

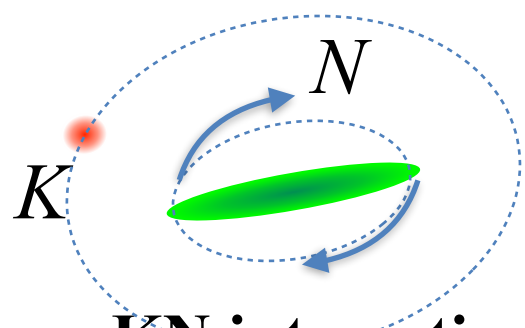
$$U_{EH} = A\,(t)\sqrt{U_\pi}A^\dagger\,(t)U_K A\,(t)\sqrt{U_\pi}A^\dagger\,(t)$$

Body-fixed frame



KH interaction

Lab frame



KN interaction

Kaon around the rotating hedgehog

$$L(U \to A\sqrt{U_\pi}A^\dagger U_K A\sqrt{U_\pi}A^\dagger)$$

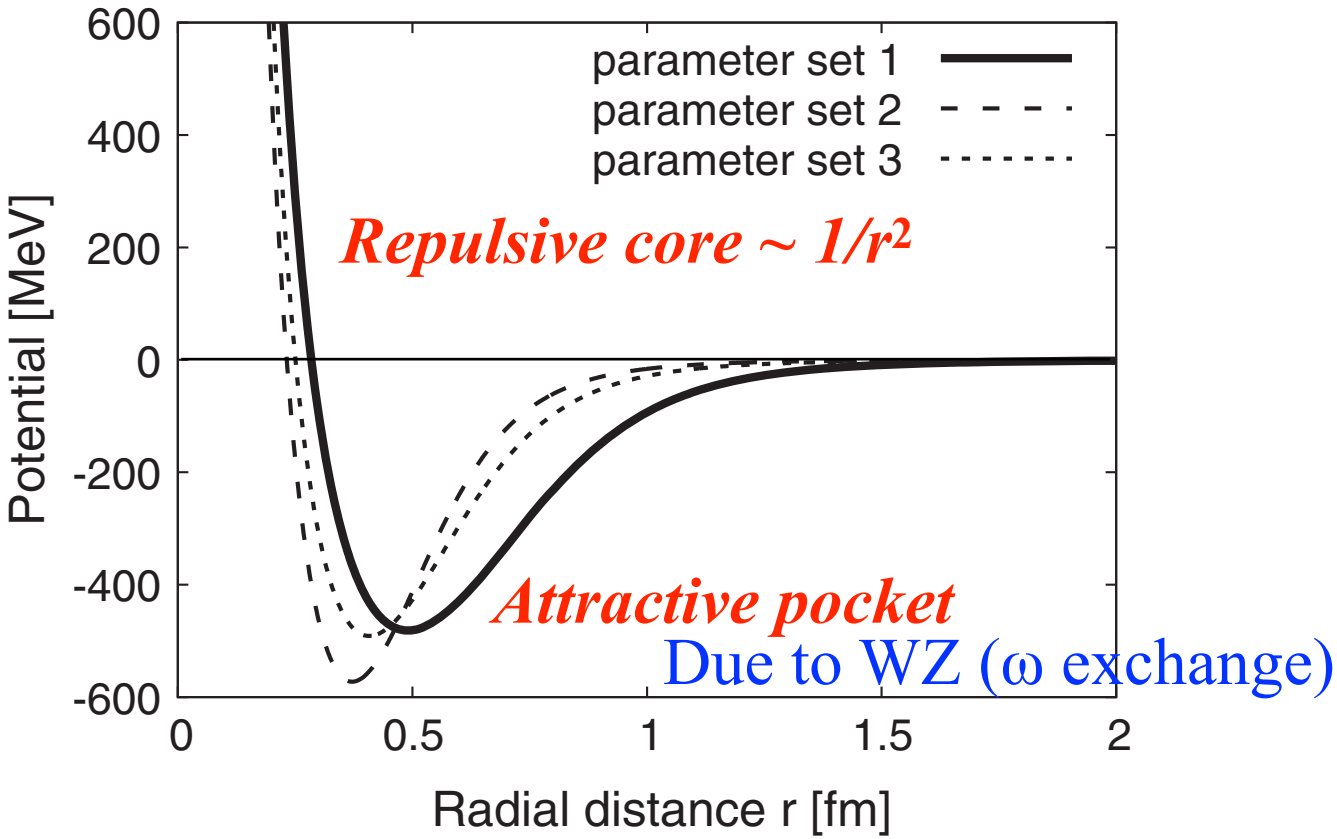
$$L = L_{SU(2)} + L_{KN}$$

$$L_{SU(2)} = \frac{1}{16}{F_\pi}^2 \mathrm{tr}\left[\partial_\mu \tilde{U}^\dagger \partial^\mu \tilde{U}\right] + \frac{1}{32e^2}\mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2$$

$$\begin{aligned} L_{KN} \quad = \quad & (D_\mu K)^\dagger D^\mu K - K^\dagger a_\mu^\dagger a^\mu K - m_K^2 K^\dagger K \\ & + \frac{1}{(eF_\pi)^2}\left\{-K^\dagger K \mathrm{tr}\left[\partial_\mu \tilde{U}\tilde{U}^\dagger, \partial_\nu \tilde{U}\tilde{U}^\dagger\right]^2 - 2\left(D_\mu K\right)^\dagger D_\nu K \mathrm{tr}\left(a^\mu a^\nu\right)\right. \\ & \left. - \frac{1}{2}\left(D_\mu K\right)^\dagger D^\mu K \mathrm{tr}\left(\partial_\nu \tilde{U}^\dagger \partial^\nu \tilde{U}\right) + 6\left(D_\nu K\right)^\dagger \left[a^\nu, a^\mu\right] D_\mu K\right\} \\ & + \frac{3i}{F_\pi^2} B^\mu \left[\left(D_\mu K\right)^\dagger K - K^\dagger \left(D_\mu K\right)\right] \end{aligned}$$

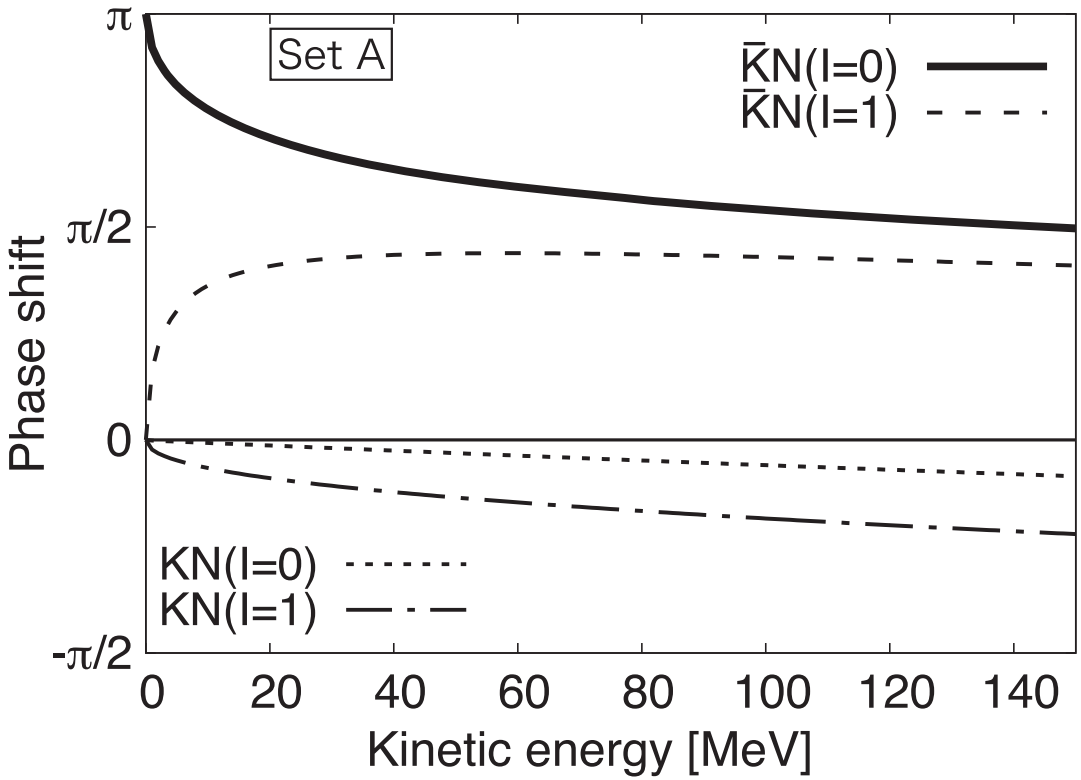
S-wave $\bar{K}N$ interaction

Lennard-Jones like potential



Produces a loosely bound state for Λ(1405)

Phase shifts for $\bar{K}N$ and KN



$\bar{K}N$
strong attraction

KN
Weak repulsion

$$a_0 = -0.03 \text{ fm}$$
$$a_1 = -0.46 \text{ fm}$$

a_0 = 0.00 fm
a_1 = -0.33 fm
(J. S. Hyslop *et al.*, Phys. Rev. D, **46**, 961 (1992).)

Summary

- $qqqc\bar{c}$ system does not support states in the Pc region in the quark model
- Color singlet hadrons are formed to fall apart in the absence of inter-hadron interactions

As an example of interaction for color singlet hadrons

- K and $\bar{K}$ N interaction may have a structure of the Lennard-Jones type shape → weakly bound Λ(1405)

- Coupling to scattering states is crucial for resonances.
- We need good description for interaction for hadronic molecule to answer/predict **when and how near-threshold molecules are developed**

Exotic triple-charm deuteron-like hexaquarks
e-Print: arXiv:1804.02961

$\Xi_{cc}\Lambda_c/\Xi_{cc}\Sigma_c/\Xi_{cc}\Sigma_c^*$ coupled-channel $I(J^P) = 1/2(0^+, 1^+)$

$\Xi_{cc}\Xi_c/\Xi_{cc}\Xi_c'/\Xi_{cc}\Xi_c^*$ $I(J^P) = 1(0^+, 1^+)$

Prediction of triple-charm molecular pentaquarks; arXiv:1711.09579
Phys.Rev. D96 (2017) no.11, 114030

$\Xi_{cc}D$ state with $I(J^P) = 0(1/2^-)$ and a $\Xi_{cc}D^*$ state with $I(J^P) = 0(3/2^-)$

Ωc-like molecular states from meson-baryon interaction; arXiv:1711.07650
Phys.Rev. D97 (2018) no.3, 036016

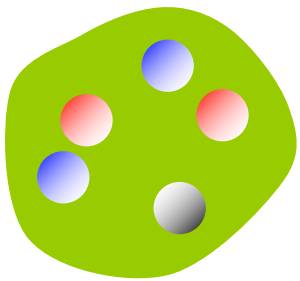
$\Xi_c^*\bar{K}/\Omega_c\eta/\Omega_c^*\eta/\Xi_c\bar{K}^*/\Xi_c'\bar{K}^*/\Omega_c\omega$ with $I(J^P) = 0(3/2^-)$
mainly composed of the $\Xi_c^*\bar{K}$

Heavy molecules and one-σ/ω-exchange model $\Lambda_c D(\bar{D})$, and $\Lambda_c\Lambda_c(\bar{\Lambda}_c)$
Phys.Rev. D96 (2017) no.11, 116012; arXiv:1707.08306

By including one-π-exchange forces providing additional attraction when coupled channels are included,
we expect many molecular states in heavy-quark sectors.

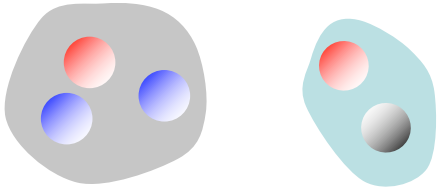
$uudd\bar{s}$

Gaussian expansion method



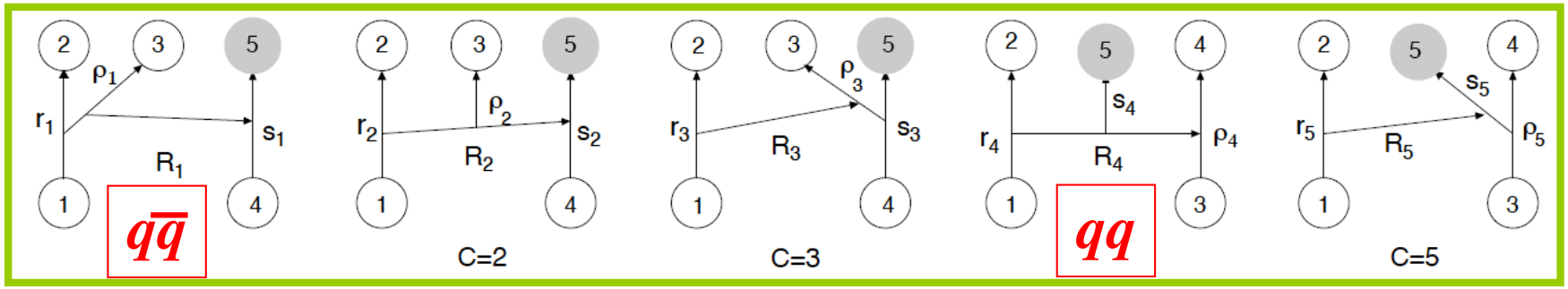
Θ^+-confined

+



NK-scattering

Compute phase shifts



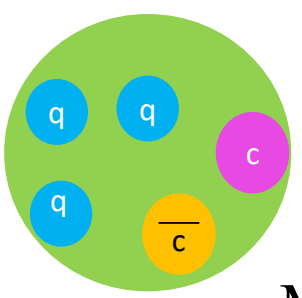
Quark model estimate of hidden-charm pentaquark resonances

Emiko Hiyama (Kyushu U. & Nishina Ctr., RIKEN & JAERI, Tokai & RCNP),
Atsushi Hosaka (RCNP & JAERI, Tokai),
Makoto Oka (Tokyo Inst. Tech. & JAERI, Tokai),
Jean-March Richard (Lyon, IPN). Mar 30, 2018. 9 pp.
e-Print: **arXiv:1803.11369 [nucl-th] | PDF**

JAEA

A quark model, which reproduces the ground-state mesons and baryons, i.e., the threshold energies, is applied to the $qqqc\bar{c}$ configurations, where q is a light quark and c the charmed quark. In the calculation, several open channels are explicitly included such as $J/\psi + N$, $\eta_c + N$, $\Lambda_c + D$, etc. To distinguish genuine resonances and estimate their width, we employ Gaussian Expansion Method supplemented by the real scaling method (stabilization). No resonance is found at the energies of the $P_c(4380)$ and $P_c(4450)$ pentaquarks. On the other hand, there is a sharp resonant state at 4690 MeV with $J = 1/2^-$ state and another one at 4920 MeV with $J = 3/2^-$ state, which have a compact structure.

For Pc, PRL 115, 072001 (2015)



State	Mass (MeV)	Width (MeV)	Fit fraction (%)	Significance
$P_c(4380)^+$	4380±8±29	205±18±86	8.4±0.7±4.2	9σ
$P_c(4450)^+$	4449.8±1.7±2.5	39± 5±19	4.1±0.5±1.1	12σ

MB molecule, Baryocharmonium, Tightly bound, Cusp,…