Stability of Exotic Heavy Mesons

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Outline

- $Q\bar{Q}q\bar{q}$ vs $QQ\bar{q}\bar{q}$
- 'Double Coulomb' limit, $M_Q, m_q \gg \Lambda_{QCD}/\alpha_s$
  - Two color components, 'Coulomb-like' potential
  - $M \gg m$ limit
  - Mixing of color components and critical value of m/M
  - Symmetries in the problem and numerical results
- Application to $QQ\bar{q}\bar{q}$ with light (massless) $\bar{q}$.
- Conclusions

Known (?) heavy exotic:

Exotic: not fitting the Peterman - Gell-Mann - Zweig template, i.e. Mesons = $(q\tilde{q})$, Baryons = (qqq).

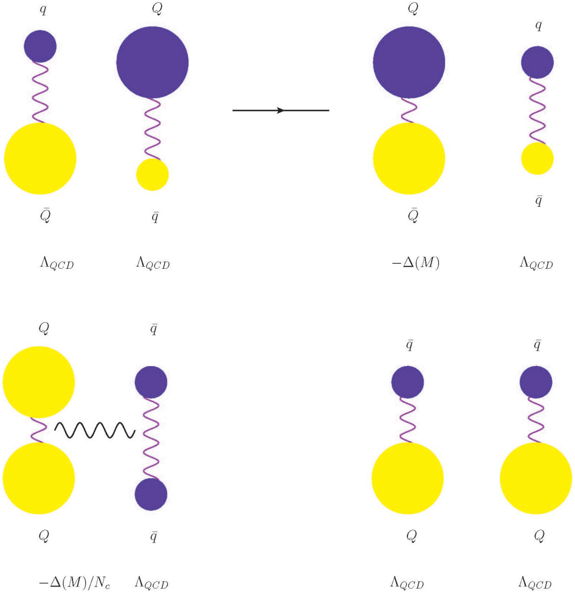
- Charmonium-like
  - $X(3872)$ $(D^0 D^{*0})$, $\to J/\psi\rho$ *and* $J/\psi\omega$, isospin badly broken,
  - $Z_c^{\pm,0}(3900)$ (DD^*), $\to J/\psi\pi$,
  - $Z_c^{\pm}(4020)$, $D^*\bar{D}^*$, $\to h_c\pi^{\pm}$,
  - $Z_1^{\pm}(4050)$, $Z_2^{\pm}(4250)$ $\to \chi_{c1}\pi^{\pm}$,
  - $Z^{\pm}(4430)$, $\to \psi(2S)\pi^{\pm}$
  - Pentaquark(s):
    $P_c(4380)$, $P_c(4450)$, $\to J/\psi p$
- Bottomonium-like
  - $Z_b^{\pm,0}(10610)$, (BB^*), $\to \Upsilon(nS)\pi$ $(n = 1, 2, 3)$, $h_b(kP)\pi$ $(k = 1, 2)$,
  - $Z_b^{\pm,0}(10650)$, $(B^*\bar{B}^*)$, $\to \Upsilon(nS)\pi$ $(n = 1, 2, 3)$, $h_b(kP)\pi$ $(k = 1, 2)$

Heavy and charged $\Rightarrow$ four quarks: $Q\bar{Q}u\bar{d}$.

Stable double-heavy exotic mesons (Tetrons). $QQ\bar{q}\bar{q}$

- LHCb (July 2017) found double-charm (non-exotic) hyperon $\Xi_{cc}^{++} \sim ccu$ with mass 3621 MeV.
- cc pair makes a tightly bound diquark in the $\bar{3}$ color state. The binding force is 1/2 of that in $c\bar{c}$.
- cc effectively behaves as a heavy antiquark, so that $(cc)u$ is like a 'meson'.
- Then an analogue of a '(anti)baryon' is $(cc)\bar{q}\bar{q}$ (e.g. $(cc)\bar{u}\bar{d}$).
- The mass difference $(cc)\bar{u}\bar{d} - ccu$ should be somewhere between $cud - c\bar{u} = \Lambda_c - D \approx 400$ MeV and $bud - b\bar{u} = \Lambda_b - B \approx 340$ MeV.
- Even if 340 MeV, the mass of a cc tetron ~ 3960 MeV - ustable with respect to decay into DD^*.
- bb in a $\bar{3}$ state stronger bound than $cc \Rightarrow$ the $bb\bar{u}\bar{d}$ tetron is expected stable w.r.t. decay into BB. (Decays only weakly). Mass predictions: Karliner & Rosner — 10389 ± 12 MeV, Eichten & Quigg — 10468 MeV. The mixed $bc\bar{u}\bar{d}$ tetron is likely just above the threshold - unstable.
- Caveat: unlike in baryon QQq, the QQ in tetron $QQ\bar{q}\bar{q}$ does not have to stay in $\bar{3}$. Generally a mixture of $\bar{3}$ and 6.

Difference between $Q\bar{Q}q\bar{q}$ and $QQ\bar{q}\bar{q}$



$Q\bar{Q}$, $Q\bar{q}$ mesons in QCD reminder

- Start with very heavy quarks. $\alpha_s M \gg \Lambda_{QCD} \Rightarrow$ lower states of $Q\bar{Q}$ are Coulomb-like. One gluon exchange between Q and $\bar{Q}$:

$$V(r) = T^a_{(Q)} T^a_{(\bar{Q})} \frac{\alpha_s}{r} = \begin{cases} -\frac{N_c^2-1}{2N_c} \frac{\alpha_s}{r} & \text{singlet} \\ \frac{1}{2N_c} \frac{\alpha_s}{r} & \text{adjoint} \end{cases}$$

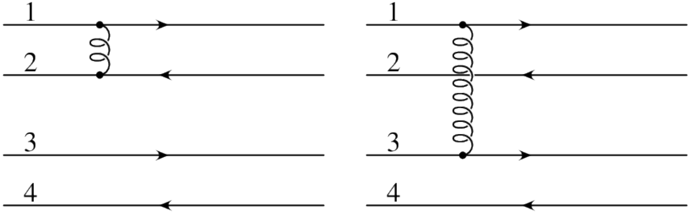
  Bound at distances $\sim 1/(\alpha_s M) \ll \Lambda_{QCD}$ — justifies one gluon exchange. Goes back to a 1974 paper by Applequist and Politzer about $c\bar{c}$...
- Subsequent improvements:
  - Systematic — loop corrections Up to 3 loops, Chetyrkin, A.V. Smirnov, V.A. Smirnov, M. Steinhauser circa 2010
  - Nonperturbative corrections (gluon condensate) H. Leutwyler, M.V. circa 1980
  - Heuristic — potential models Cornell, Martin, ...
- Bottomline — $Q\bar{q}$ color singlets are bound, and the constituents are somehow confined.

Untangling interactions in a (very) heavy tetron

- Consider very heavy quarks. (Both $M = M_Q$ and $m = m_q$ justify Colomb-like approximation.) Pair-wise interaction at short distances with the potential

$$V_{ij} = T^a_{(i)} T^a_{(j)} d_{ij}$$

with T^a color generators and $d_{ij} = \alpha_s/|r_i - r_j|$.



- Overall color singlet - two possibilities: $Q_1 \bar{q}_2$ and $Q_3 \bar{q}_4$ - singlets, or $Q_1 \bar{q}_4$ and $Q_3 \bar{q}_2$ - singlets.

$$\Psi = \left(\bar{q}_{(2)\,\alpha} Q^{\alpha}_{(1)}\right) \left(\bar{q}_{(4)\,\beta} Q^{\beta}_{(3)}\right) / N_c \,, \quad \Phi = \left(\bar{q}_{(4)\,\alpha} Q^{\alpha}_{(1)}\right) \left(\bar{q}_{(2)\,\beta} Q^{\beta}_{(3)}\right) / N_c$$

- These two are not orthogonal and are mixed by the interaction.

$$\langle \Phi | \Psi \rangle = \langle \Psi | \Phi \rangle = 1/N_c$$

- Use the sum and the difference instead.

$$u = \frac{1}{\sqrt{2(1+1/N_c)}}(\Psi + \Phi), \quad w = \frac{1}{\sqrt{2(1-1/N_c)}}(\Psi - \Phi)$$

- Two components: $u \sim \{QQ\}\{\bar{q}\bar{q}\}$ ($\{\ldots\}$ - color symmetric), $w \sim [QQ][\bar{q}\bar{q}]$ ($[\ldots]$ - color antisymmetric)
- Potential for $Q(\vec{r}_1)Q(\vec{r}_3)\bar{q}(\vec{r}_2)\bar{q}(\vec{r}_4)$:

$$V\begin{pmatrix} u \\ w \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} \frac{N_c^2-1}{N_c}\,r - \frac{N_c-1}{N_c}\,t & \sqrt{N_c^2-1}\,s \\ \sqrt{N_c^2-1}\,s & \frac{N_c^2-1}{N_c}\,r + \frac{N_c+1}{N_c}\,t \end{pmatrix}\begin{pmatrix} u \\ w \end{pmatrix}$$

N_c - number of colors, $r = d_{12} + d_{34} + d_{14} + d_{23}$,
$s = d_{12} + d_{34} - d_{14} - d_{23}$, $t = 2d_{13} + 2d_{24} - d_{12} - d_{14} - d_{23} - d_{34}$

- Consider large N_c ('t Hooft limit): $N_c \to \infty$, $N_c\alpha_s \sim O(1)$
- The t term describes the interaction within the di-(anti)quark (small r_{13} and/or r_{24}). The s term describes the $u \leftrightarrow w$ mixing.
- At large N_c the coefficient of s is N_c larger than of t.

- Consider $M \gg m$. Main effect the attraction between QQ in the color antisymetric w state:

$$V_{13} = -\frac{N_c + 1}{2N_c}\, d_{13}$$

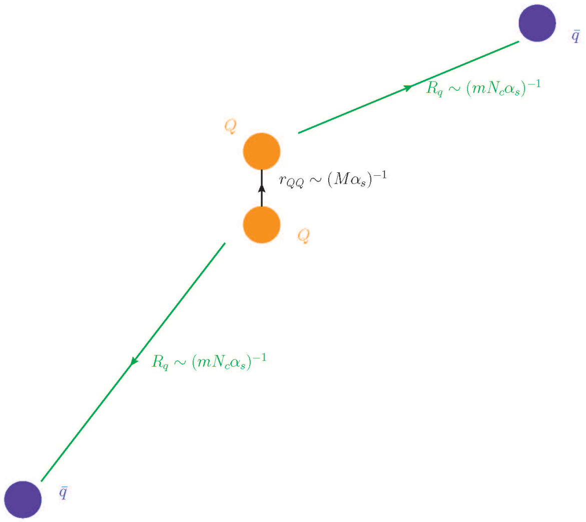
- Coulomb-like bound state, $r_{QQ} \sim (M\alpha_s)^{-1}$, $E_{QQ} \sim M\alpha_s^2$.
- The $\bar{q}$ are at larger distance $R_q \gg r_{QQ}$ from the QQ diquark. Set $\vec{r}_3 = \vec{r}_1$, and the leading in N_c diagonal term in the potential is

$$V_{qQ} = -\frac{N_c}{2}\,(d_{12} + d_{14})$$

  Independent Coulomb-like interaction of each $\bar{q}$ with the QQ diquark. $\Rightarrow$

$$R_q \sim (mN_c\alpha_s)^{-1}, \quad E_q \sim mN_c^2\alpha_s^2$$

- The tetron is stable as long as mixing between w and u can be neglected.



$\bar{q}$
$R_q \sim (mN_c\alpha_s)^{-1}$
Q
$r_{QQ} \sim (M\alpha_s)^{-1}$
Q
$R_q \sim (mN_c\alpha_s)^{-1}$
$\bar{q}$

- The mixing s term:

$$\sqrt{N_c^2-1}\,(d_{12}+d_{34}-d_{14}-d_{23}) \sim N_c\alpha_s r_{QQ}/R_r^2$$

- The mixing s can be considered small as long as the energy shift due to s is smaller than E_{QQ}:

$$(N_c\,\alpha_s\,r_{QQ}/R_q^2)^2/E_{QQ}^2 \sim N_c^6\,(m/M)^4 \ll 1$$

- In other words, existence of a stable tetron is guaranteed as long as the mass ratio $f = m/M$ is small enough, $f < f_c$ with

$$f_c \sim N_c^{-3/2}$$

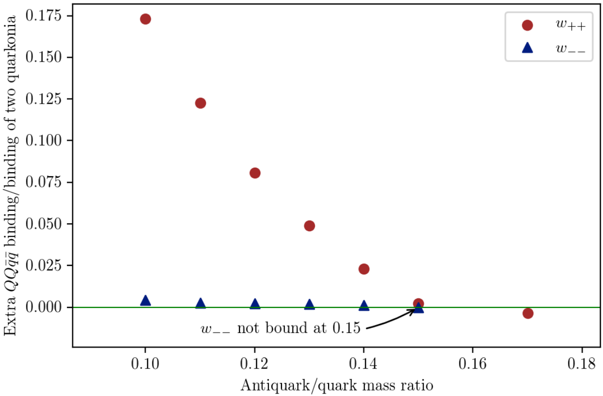
.

Numerical computations

- The potential V has a $Z_2 \times Z_2$ symmetry under independent permutations of coordinates of the quarks and the antiquarks $\vec{r}_1 \leftrightarrow \vec{r}_3$, $\vec{r}_2 \leftrightarrow \vec{r}_4$. The solutions to the Schrödinger eqn. are of four types: w_{++}, w_{--}, w_{+-}, and w_{-+}, where the first (second) sign is the coordinate symmetry for quarks (antiquarks).
- The symmetry of the u component in each Z_2 is opposite to that of the w (as insured by the antisymmetry of the mixing s term).
- Variational numerical computation: Both u and w are represented as a sum of Gaussians, e.g.

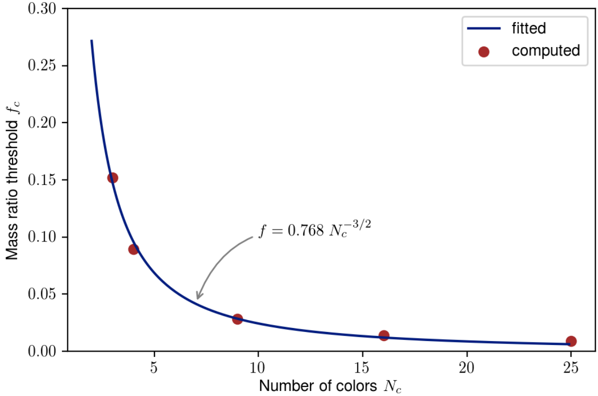
$$\psi_u = \sum_{k=1}^{N_B} c_k \exp\left(-\sum_{i,j} a_{ijk} r_{ij}^2\right)$$

  with a_{ijk} and c_k being the variational coefficients, and N_B — the number of trial functions (typically $N_B = 200$ was used).
- Challenge for variational approach — slow convergence near threshold (critical value f_c).



Extra binding energy of a tetron (in units of the total binding for two independent $Q\bar{q}$ mesons) as a function of the antiquark/quark mass ratio f. The number of colors is $N_c = 3$. The state with the symmetry w_{++} (circles) is bound more strongly than w_{--} (triangles). Even the state w_{++} is no longer bound when the mass ratio is higher than about $f_c \simeq 0.152$.

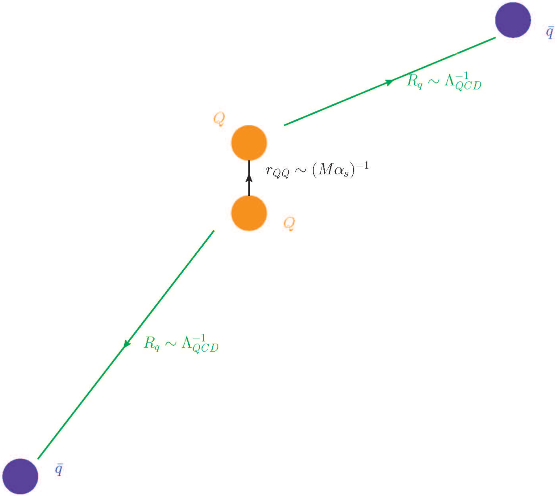
No double bottomonium $bb\bar{b}\bar{b}$...



Mass threshold f_c for tetrons as a function of the number of colors. The curve is the fit of the numerically computed values of f_c to the formula $f_c = A\,N_c^{-3/2}$.

$QQ\bar{q}\bar{q}$ with realistically light $\bar{q}$

- Q is still very heavy, but $\bar{q}$'s are light $m \ll \Lambda_{QCD}$.
- Cannot rely on the Coulomb-like (or any potential) picture for $\bar{q}$'s.
- Use parametric estimates in the large N_c limit.
- $R_q \sim \Lambda_{QCD}^{-1}$, $E_q \sim \Lambda_{QCD}$



- The mixing (s) is not described by a potential, but still is of order one in the large N_c limit and vanishes at $r_{QQ} \to 0$.
- Thus can estimate: $\langle u|H|w\rangle \sim r_{QQ}/R_q^2 \sim r_{QQ}\Lambda_{QCD}^2$.
- The perturbation parameter for the mixing is then

$$\xi \sim \frac{|\langle u|H|w\rangle|^2}{E_{QQ}^2} \sim \frac{\Lambda_{QCD}^4}{M^4\alpha_s^6} \sim N_c^6\left(\Lambda_{QCD}/M\right)^4$$

- The parameter for u - w mixing is
- The mixing is small (and the QQ binding dominant) at $\xi \ll 1$. $\Rightarrow$ Stability condition: $M/\Lambda_{QCD} > N_c^{3/2}$
- At $N_c = 3$ the condition is (probably) satisfied only for $M = m_b$. $\Rightarrow$ Only $bb\bar{q}\bar{q}$ stable tetrons. (Agrees with phenomenological estimates.)

Fermi-Dirac selection rules

- The coordinate symmetry of the w.f. is a part of the statistics consideration. Lowest energy w_{++} — antisymmetric in color, symmetric in coordinate $\Rightarrow$ symmetric in spin of QQ: $J^P = 1^+$ for the heavy diquark
- In $bb\bar{u}\bar{d}$ if isospin $I = 0$ (antisymmetric) the spin of $(\bar{u}\bar{d})$ is 0: $J^P = 0^+$. (The same as in a purely phenomenological analysis.)
- The (lightest) double-b stable tetron is $bb\bar{u}\bar{d}$: $J^P = 1^+$ and $I = 0$. Could be the only existing stable, (unless $bb\bar{s}\bar{u}$ and $bb\bar{s}\bar{d}$ are also stable — Eichten & Quigg).
- Other symmetries: w_{--} very weak binding, w_{+-} and w_{-+} — no binding at all.

Conclusions

- The approach, where a di-quark (di-anti-quark) is considered only in color-antisymmetric state (as e.g. in hyperons), generally is not applicable to tetrons due to the mixing through the interaction with the color symmetric states.
- The mixing apparently sets the conditions for existence of stable tetrons.
- In the limit where all constituents are heavy, the ratio of two mass scales should be small enough: $m/M < 0.768 N_c^{-3/2}$ ($m/M < 0.152$ at $N_c = 3$). No double-bottomonium $bb\bar{b}\bar{b}$.
- With two light constituents a stable double-heavy tetron exists only if $M > N_c^{3/2}\,\Lambda_{QCD}$. Possibly a $bb\bar{u}\bar{d}$ tetron. (Neither $cc\bar{q}\bar{q}$ nor $bc\bar{q}\bar{q}$.)
- the much discussed diquark - anti-diquark model of charmonium- and bottomonium-like exotic states is based on color-antisymmetric quark - quark clustering, e.g $(cq)(\bar{c}\bar{q})$. Such clustering is suppressed in the large N_c limit, and generally, formation of such structures is not supported by the present analysis.