Simons Center `Exotic Hadrons and Flavor Physics', Stony Brook, 1st June 2018

g-2 Hadronic VP: off and on the lattice

Thomas Teubner

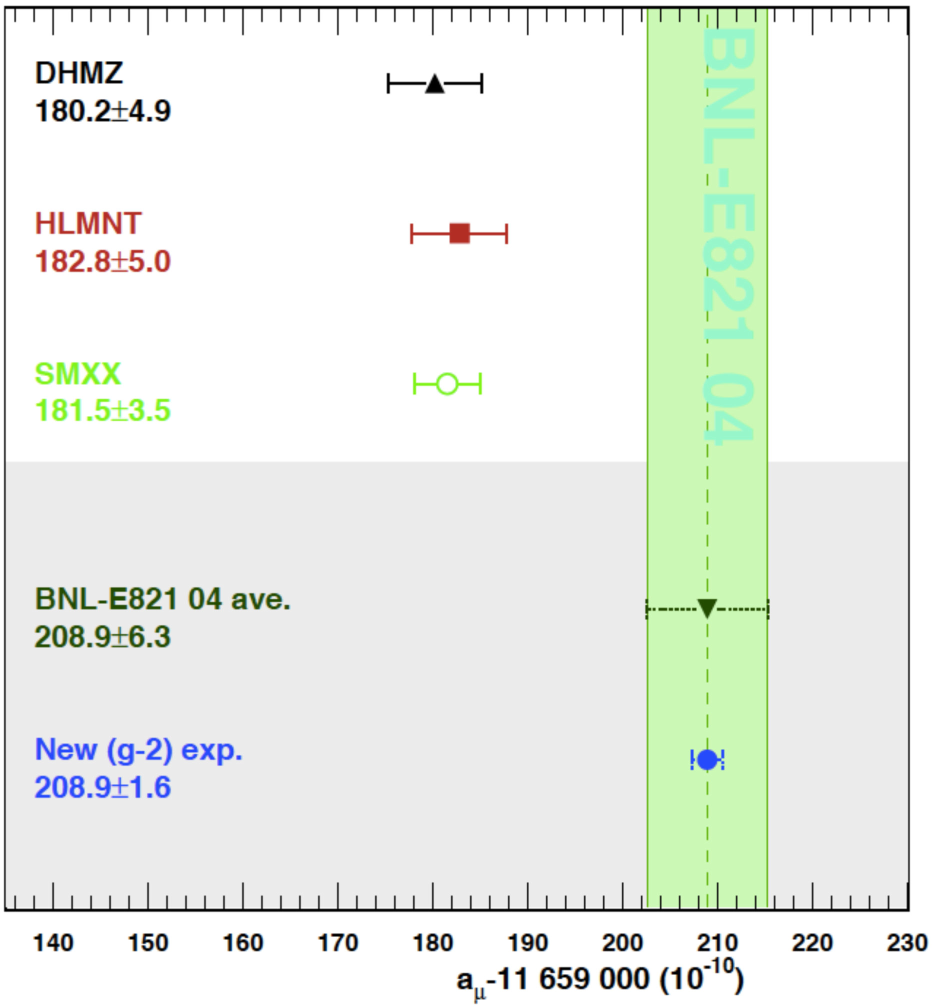
- Introduction
- a_μ^{HVP} : Data driven approach; overview and new updates
- HVP from the lattice
- Outlook

a_μ: Status and future projection → charge for SM TH

$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{hadronic}} + a_\mu^{\mathrm{NP?}}$$

From: arXiv:1311.2198
`The Muon (g-2) Theory Value: Present and Future'

- if mean values stay and with no a_μ^{SM} improvement: 5σ discrepancy
- if also EXP+TH can improve a_μ^{SM} \`as expected' (consolidation of L-by-L on level of Glasgow consensus, about factor 2 for HVP): NP at 7-8σ
- or, if mean values get closer, very strong exclusion limits on many NP models (extra dims, new dark sector, xxxSSSM)...



PRD (in press)

The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis

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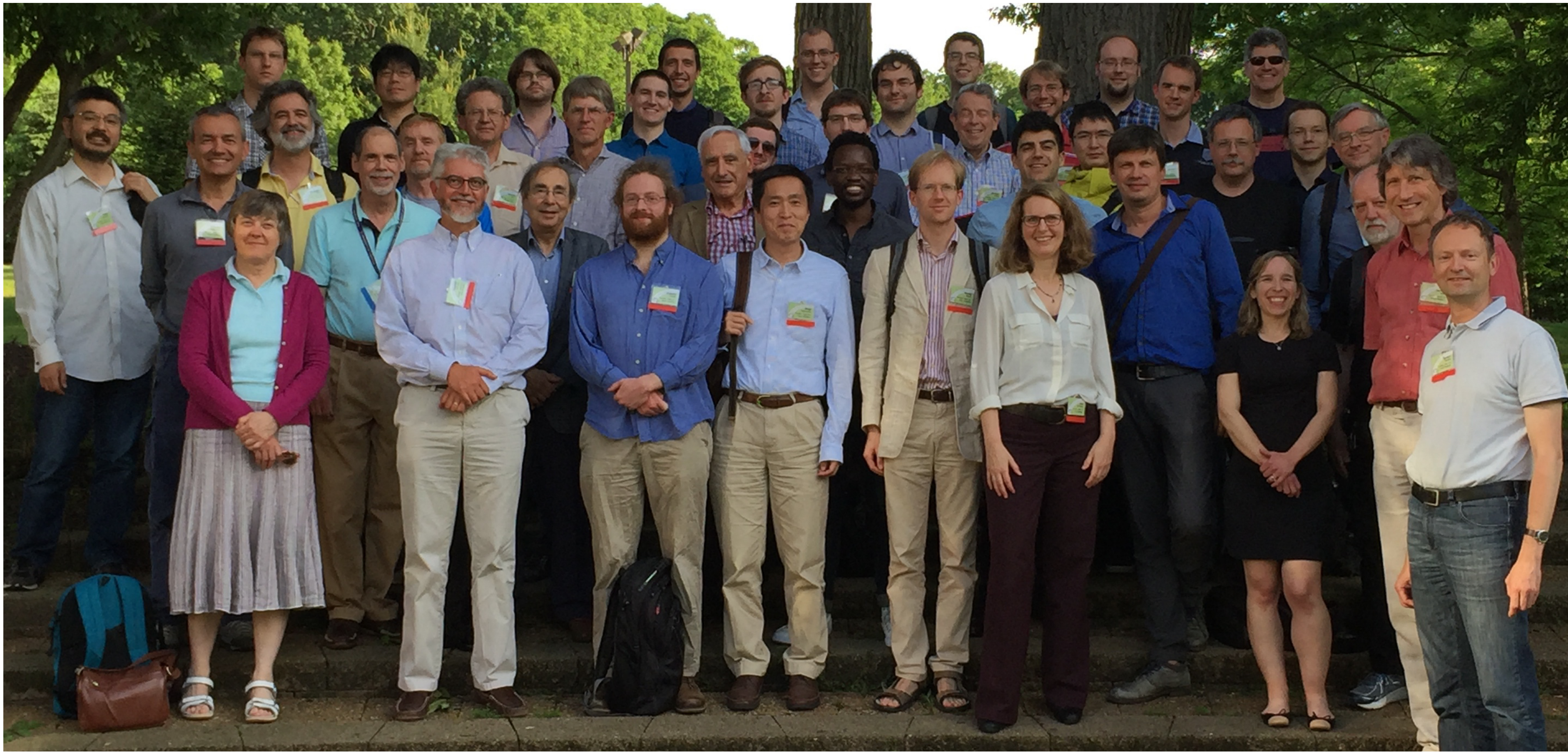
Abstract

This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon, $a_\mu^{\rm had,\,VP}$ and the hadronic contributions to the effective QED coupling at the mass of the Z boson, $\Delta\alpha_{\rm had}(M_Z^2)$, from the combination of $e^+e^- \to$ hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available $e^+e^- \to$ hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic R-ratio and its covariance matrix in the energy range $m_\pi \leq \sqrt{s} \leq 11.2$ GeV. Using these combined data and pQCD above that range results in estimates of the hadronic vacuum polarisation contributions to $g-2$ of the muon of $a_\mu^{\rm had,\,LO\,VP} = (693.27 \pm 2.46) \times 10^{-10}$ and $a_\mu^{\rm had,\,NLO\,VP} = (-9.82 \pm 0.04) \times 10^{-10}$. The new estimate for the Standard Model prediction is found to be $a_\mu^{\rm SM} = (11\,659\,182.05 \pm 3.56) \times 10^{-10}$, which is 3.7σ below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$, resulting in $\alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$. Detailed comparisons with results from similar related works are given.

“Muon g-2 theory initiative” formed in June 2017



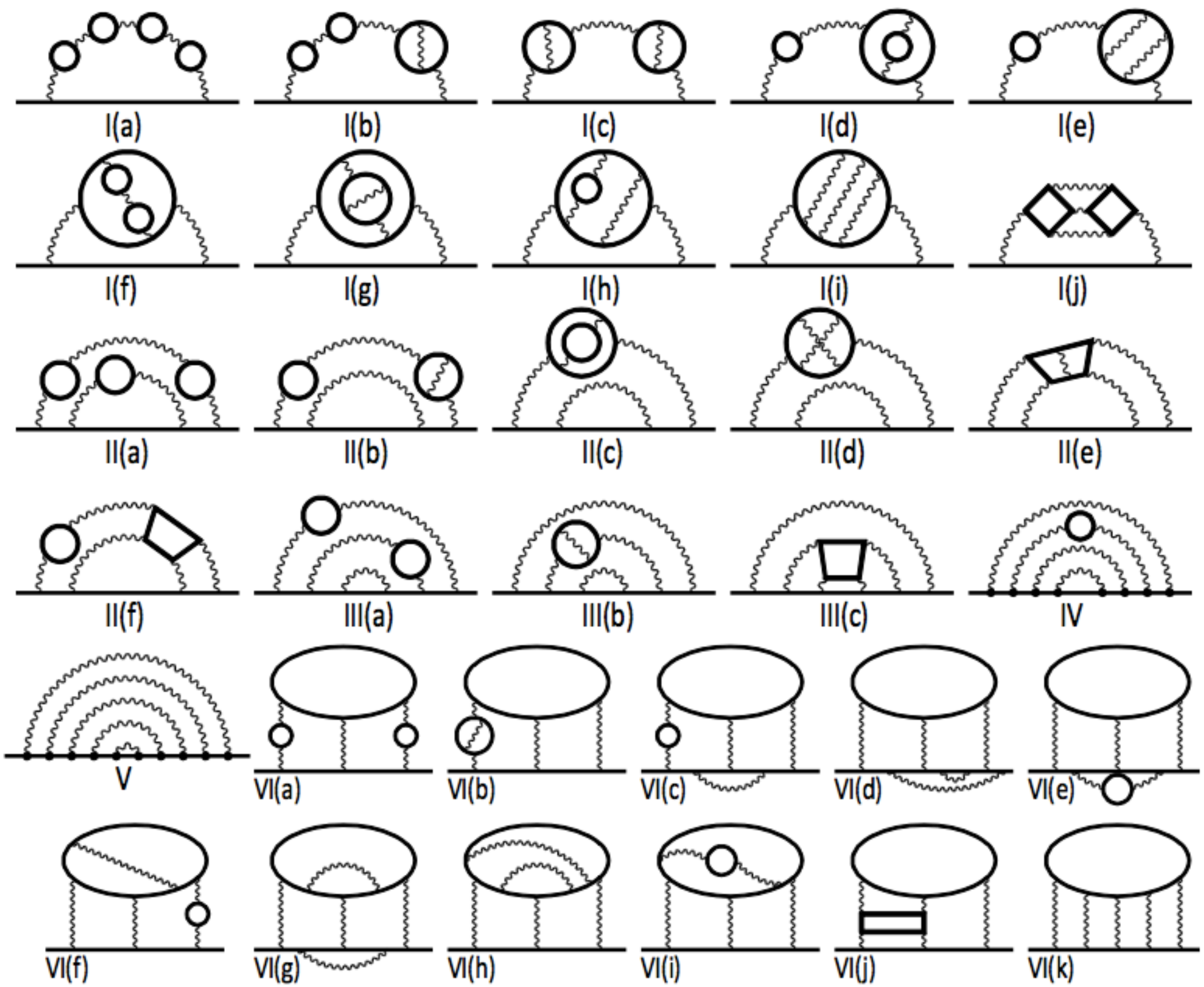
“map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental results”

a_μ^{QED} Kinoshita et al.: g-2 at 1, 2, 3, 4 & 5-loop order

T. Aoyama, M. Hayakawa,
T. Kinoshita, M. Nio (PRLs, 2012)

A triumph for perturbative QFT and computing!

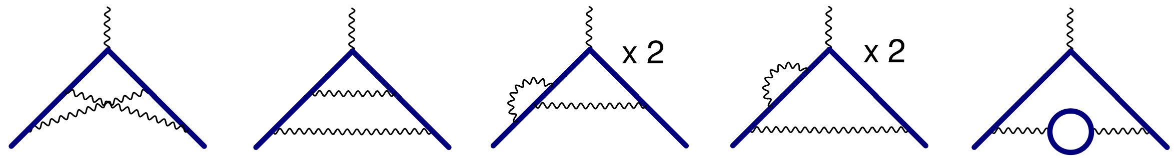
10th
12672
diagrams



- code-generating code, including
- renormalisation
- multi-dim. numerical integrations

a_μ^{QED}

- Schwinger 1948: 1-loop $a = (g-2)/2 = \alpha/(2\pi) = 116\,140\,970 \times 10^{-11}$
- 2-loop graphs:



- 72 3-loop and 891 4-loop diagrams …
- Kinoshita et al. 2012: 5-loop completed numerically (12672 diagrams):

  $a_\mu^{QED} = 116\,584\,718.951\ (0.009)\ (0.019)\ (0.007)\ (0.077) \times 10^{-11}$

  errors from: lepton masses, 4-loop, 5-loop, α from ^{87}Rb

- QED extremely accurate, and the series is stable: $$a_\mu^{\mathrm{QED}} = C_\mu^{2n} \sum_n \left(\frac{\alpha}{\pi}\right)^n$$

$$C_\mu^{2,4,6,8,10} = 0.5,\ 0.765857425(17),\ 24.05050996(32),\ 130.8796(63),\ 753.29(1.04)$$

- Could a_μ^{QED} still be wrong?
  Some classes of graphs known analytically (Laporta; Aguilar, Greynat, deRafael),

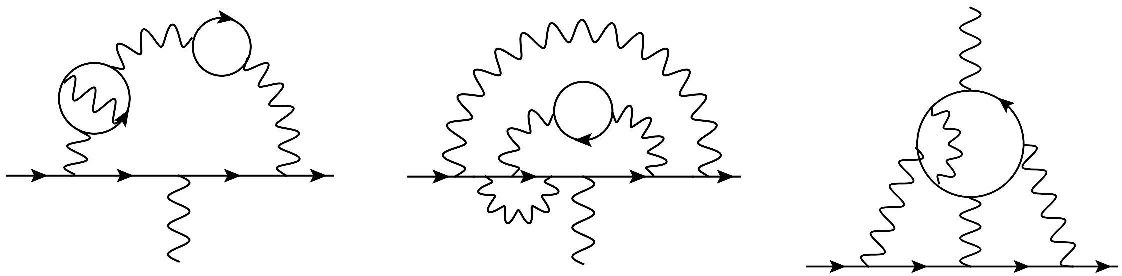
a_μ^{QED}

- … but 4-loop and 5-loop rely heavily on numerical integrations

- Recently several independent checks of 4-loop and 5-loop diagrams:

  Baikov, Maier, Marquard [NPB 877 (2013) 647], Kurz, Liu, Marquard, Smirnov AV+VA, Steinhauser [NPB 879 (2014) 1, PRD 92 (2015) 073019, 93 (2016) 053017]:

- all 4-loop graphs with internal lepton loops now calculated independently, e.g.



(from Steinhauser et al., PRD 93 (2016) 053017)

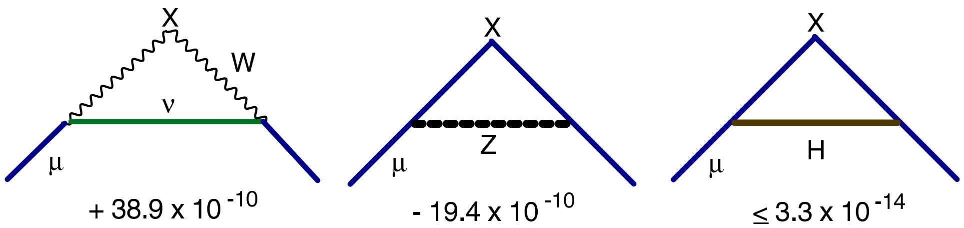
- 4-loop universal (massless) term calculated semi-analytically to 1100 digits (!) by Laporta, arXiv:1704.06996, also new numerical results by Volkov, 1705.05800

- all agree with Kinoshita et al.'s results, so QED is on safe ground ✓

  (and further consolidated with recent update by Kinoshita et al., PRD97(2018)036001)

a_μ Electro-Weak

- Electro-Weak 1-loop diagrams:



$a_\mu^{EW(1)} = 195\times10^{-11}$

- known to 2-loop (1650 diagrams, the first full EW 2-loop calculation):
  Czarnecki, Krause, Marciano, Vainshtein; Knecht, Peris, Perrottet, de Rafael

- agreement, a_μ^{EW} relatively small, 2-loop relevant: $a_\mu^{EW(1+2\ loop)} = (154\pm2)\times10^{-11}$

- Higgs mass now known, update by Gnendiger, Stoeckinger, S-Kim,
  PRD 88 (2013) 053005

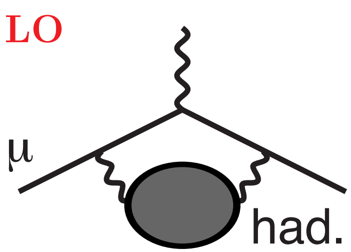
$a_\mu^{EW(1+2\ loop)} = (153.6\pm1.0)\times10^{-11}$ ✓

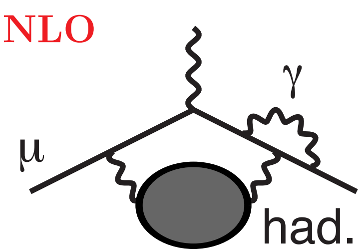
compared with $a_\mu^{QED} = 116\ 584\ 718.951\ (80) \times10^{-11}$

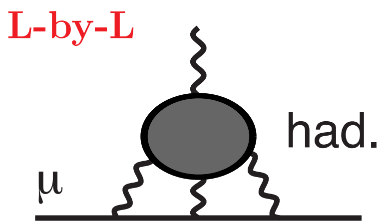
a_μ^{hadronic}

- Hadronic: non-perturbative, the limiting factor of the SM prediction ✗ → ✓

$$a_\mu^{\text{had}} = a_\mu^{\text{had,VP LO}} + a_\mu^{\text{had,VP NLO}} + a_\mu^{\text{had,Light-by-Light}}$$

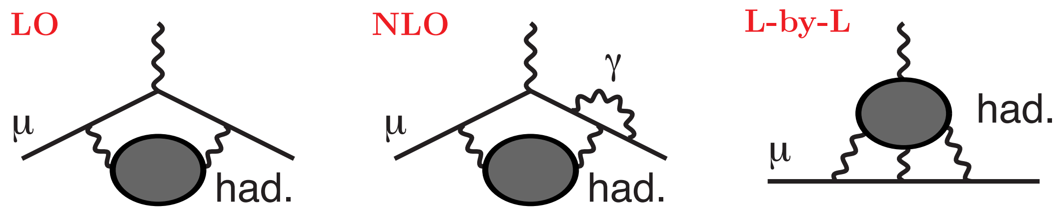






$a_\mu^{\text{had, VP}}$: Hadronic Vacuum Polarisation

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$

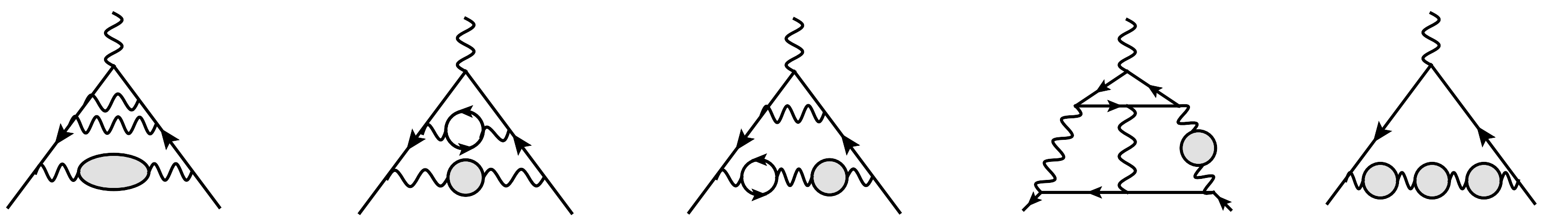


HVP: - most precise prediction by using e^+e^- hadronic cross section (+ tau) data and well known dispersion integrals

- done at LO and NLO (see graphs)

- and recently at NNLO [Steinhauser et al., PLB 734 (2014) 144, also F. Jegerlehner]

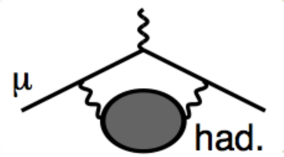
$a_\mu^{\text{HVP, NNLO}} = +\,1.24 \times 10^{-10}$ not so small, from e.g.:



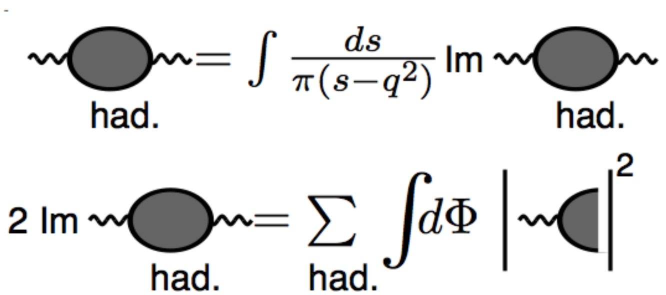
- Alternative: lattice QCD, but need QED and iso-spin breaking corrections
Lots of activity by several groups, errors coming down, see later.

Hadronic Vacuum Polarisation, essentials:

Use of data compilation for HVP:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$\boxed{a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)}$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

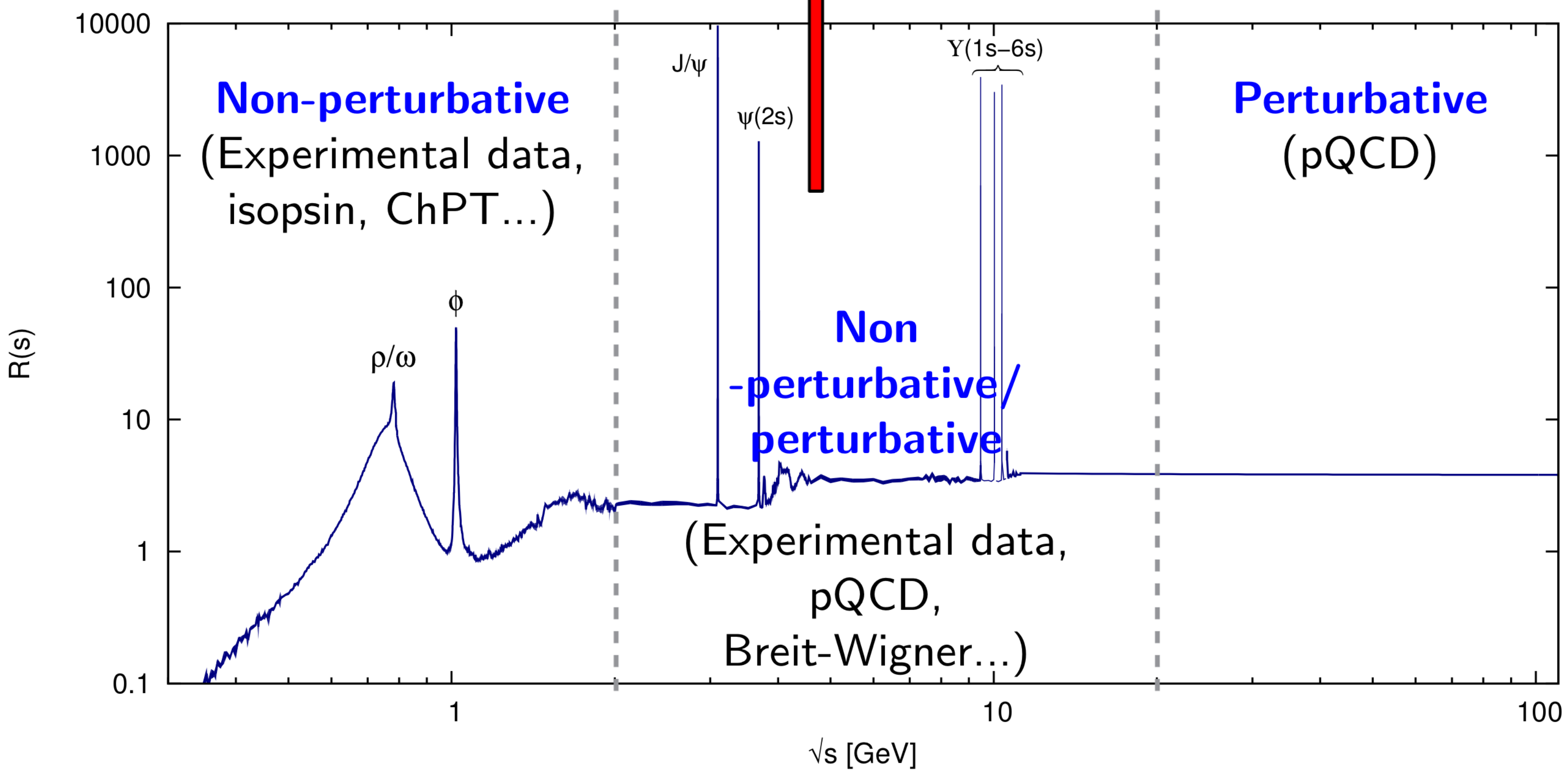
$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

How to get the most precise σ^0_{had}? e^+e^- data:

- Low energies: sum ~35 exclusive channels, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KK$\pi\pi$, $\eta\pi$, ..., use iso-spin relations for missing channels
- Above ~1.8 GeV: can start to use pQCD (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)
- Challenge of data combination (locally in √s): many experiments, different energy bins, stat+sys errors from different sources, correlations; must avoid inconsistencies/bias
- traditional `direct scan' (tunable $e^+e^-$ beams) vs. `Radiative Return' [+ τ spectral functions]
- σ^0_{had} means `bare' σ, but WITH FSR: RadCorrs [HLMNT '11: $\delta a_\mu^{\mathrm{had,\ RadCor\ VP+FSR}} = 2 \times 10^{-10}$!]

Hadronic cross section input

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

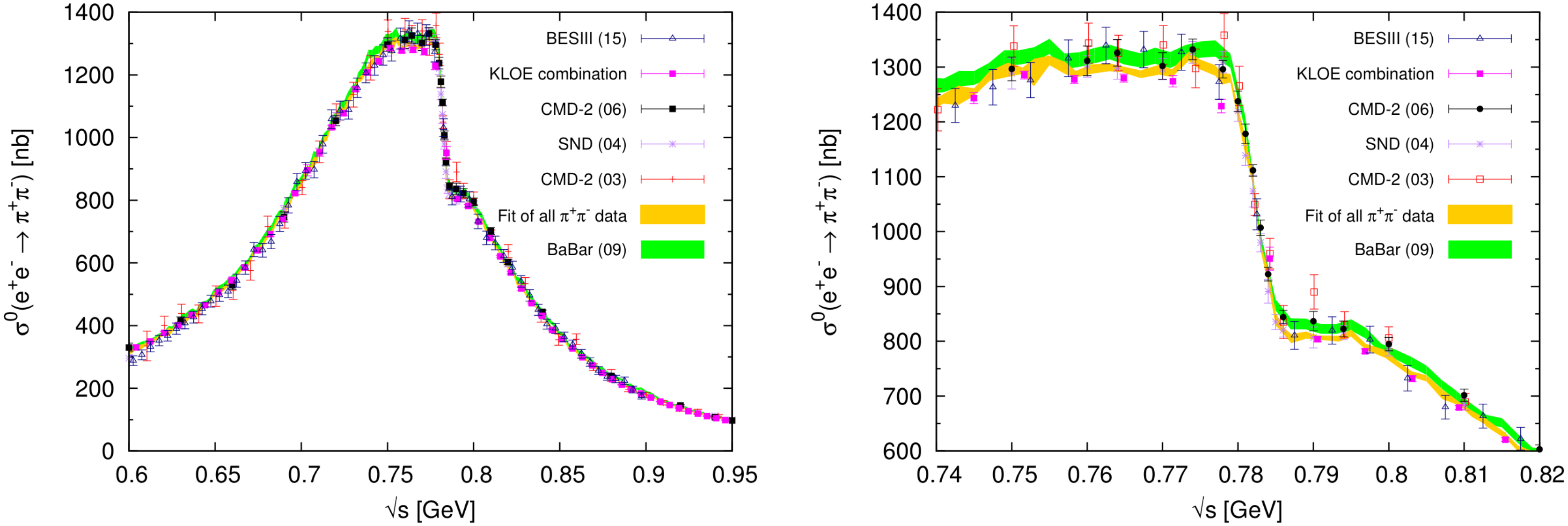


Must build full hadronic cross section/R-ratio...

$\pi^+\pi^-$ channel [KNT18: arXiv:1802.02995]

$\Rightarrow$ $\pi^+\pi^-$ accounts for over 70% of $a_\mu^{\mathrm{had,\,LO\,VP}}$

$\rightarrow$ Combines 30 measurements totalling nearly 1000 data points



$\Rightarrow$ Correlated & experimentally corrected $\sigma^0_{\pi\pi(\gamma)}$ data now entirely dominant

$$a_\mu^{\pi^+\pi^-}[0.305 \le \sqrt{s} \le 1.937\ \text{GeV}] = 502.97 \pm 1.14_{\text{stat}} \pm 1.59_{\text{sys}} \pm 0.06_{\text{vp}} \pm 0.14_{\text{fsr}}$$

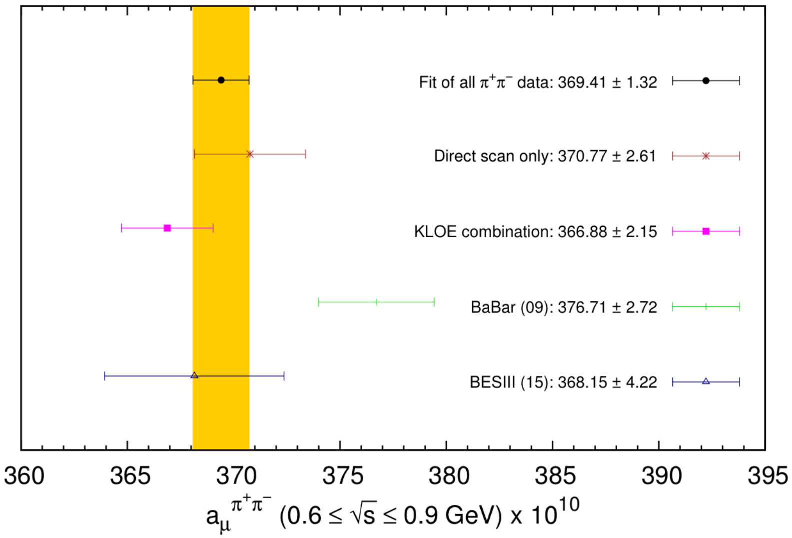
$$= 502.97 \pm 1.97_{\text{tot}} \quad \text{HLMNT11: } 505.77 \pm 3.09$$

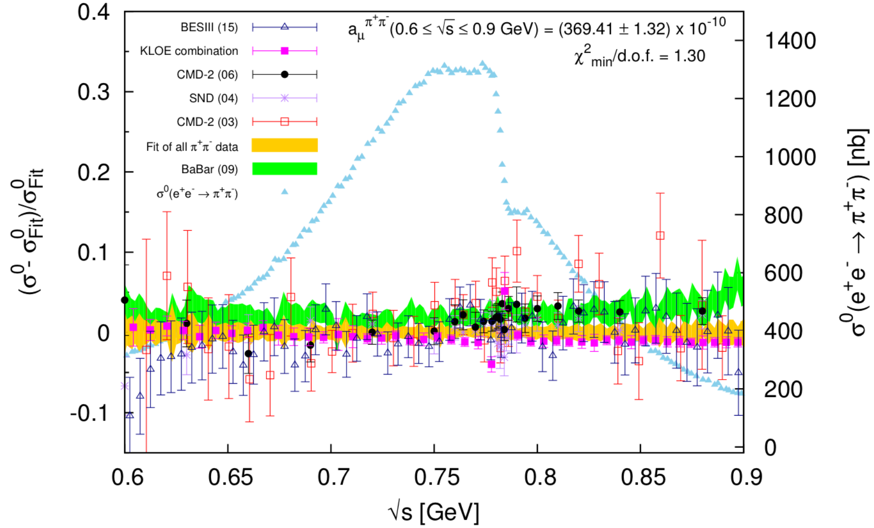
$\Rightarrow$ 15% local $\chi^2_{\text{min}}/\text{d.o.f.}$ error inflation due to tensions in clustered data

$\pi^+\pi^-$ channel [KNT18: arXiv:1802.02995]

$\Rightarrow$ Tension exists between BaBar data and all other data in the dominant ρ region.

$\rightarrow$ Agreement between other radiative return measurements and direct scan data largely compensates this.



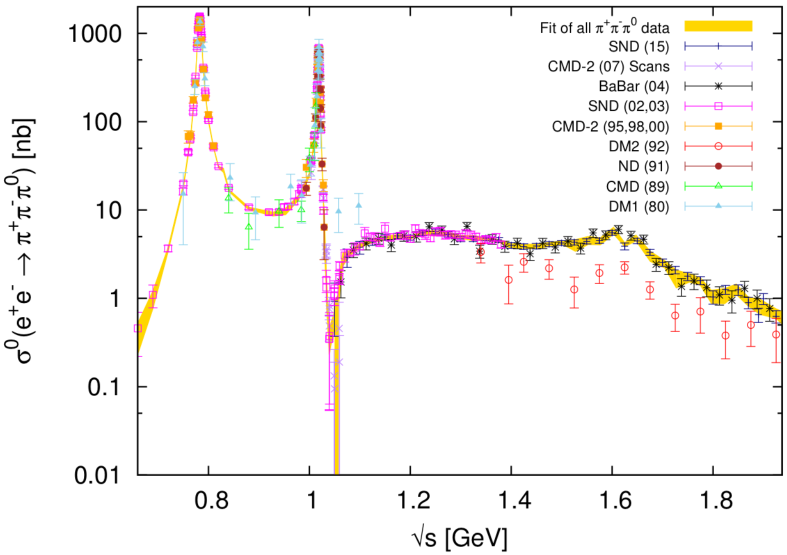


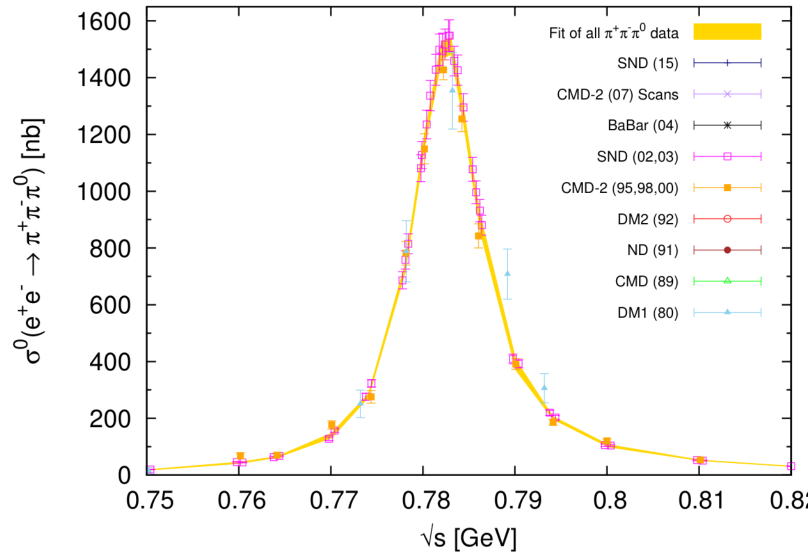
BaBar data alone $\Rightarrow a_\mu^{\pi^+\pi^-}(\text{BaBar data only}) = 513.2 \pm 3.8.$

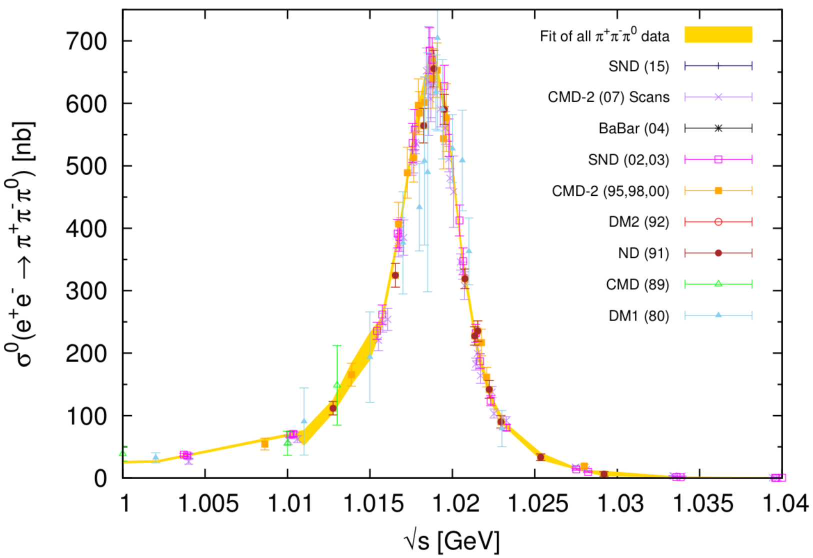
Simple weighted average of all data $\Rightarrow a_\mu^{\pi^+\pi^-}(\text{Weighted average}) = 509.1 \pm 2.9.$
(i.e. - no correlations in determination of mean value)

BaBar data dominate when no correlations are taken into account for the mean value
Highlights importance of fully incorporating all available correlated uncertainties

$\pi^+\pi^-\pi^0$ channel [KNT18: arXiv:1802.02995]







Improvement for 3π also

New data:

SND: [J. Exp. Theor. Phys. 121 (2015), 27.]

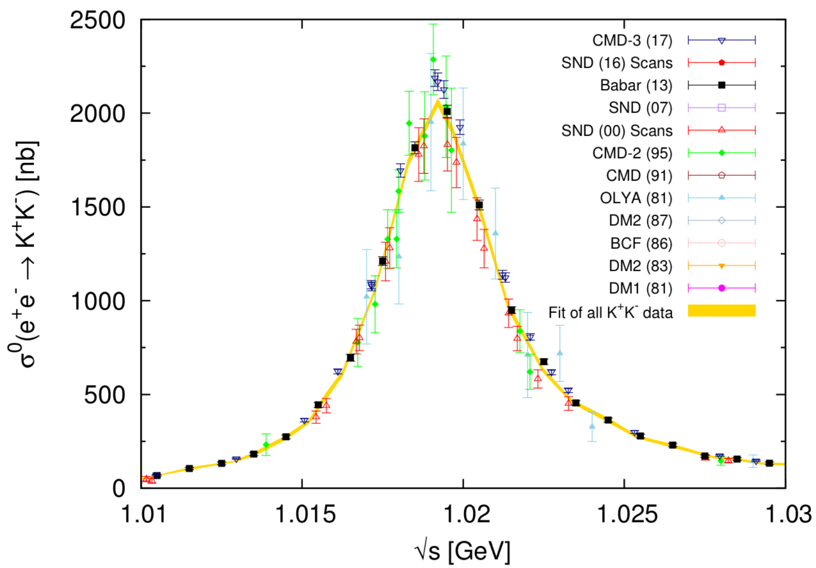
$$a_\mu^{\pi^+\pi^-\pi^0} = 47.79 \pm 0.22_{\rm stat} \pm 0.71_{\rm sys} \pm 0.13_{\rm vp} \pm 0.48_{\rm fsr}$$

$$= 47.79 \pm 0.89_{\rm tot}$$

HLMNT11: $47.51 \pm 0.99_{\rm tot}$

$K\bar{K}$ channels [KNT18: arXiv:1802.02995]

K^+K^-



New data:

BaBar: [Phys. Rev. D 88 (2013), 032013.]
SND: [Phys. Rev. D 94 (2016), 112006.]
CMD-3: [arXiv:1710.02989.]

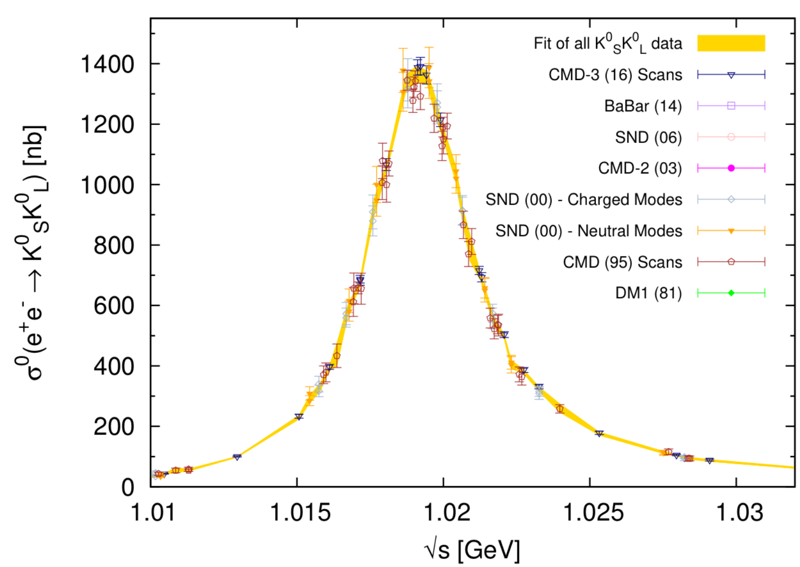
Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.] omitted as waiting reanalysis.

$$a_\mu^{K^+K^-} = 23.03 \pm 0.22_{\text{tot}}$$

HLMNT11: $22.15 \pm 0.46_{\text{tot}}$

Large increase in mean value

$K_S^0 K_L^0$



New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.]
CMD-3: [Phys. Lett. B 760 (2016) 314.]

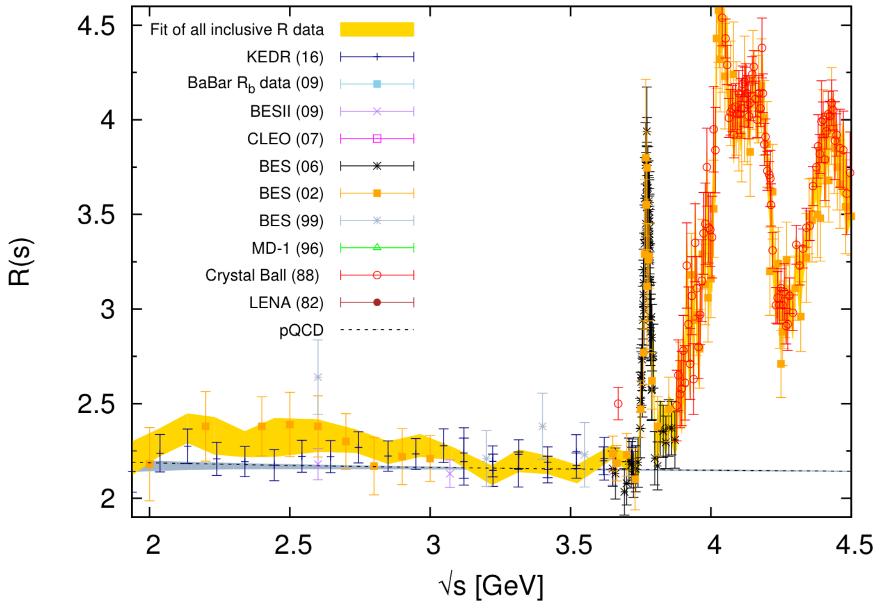
$$a_\mu^{K_S^0K_L^0} = 13.04 \pm 0.19_{\text{tot}}$$

HLMNT11: $13.33 \pm 0.16_{\text{tot}}$

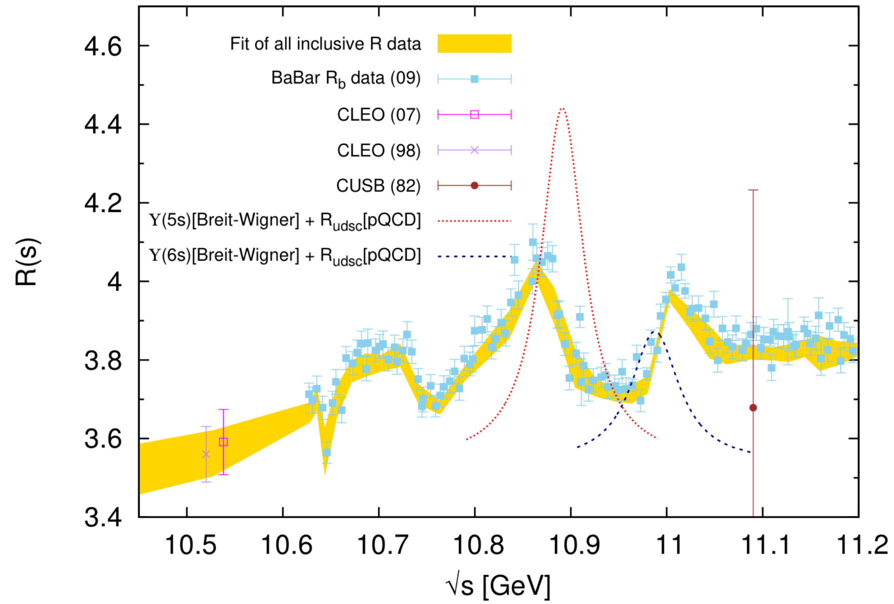
Large changes due to new precise measurements on ϕ

Inclusive

⇒ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

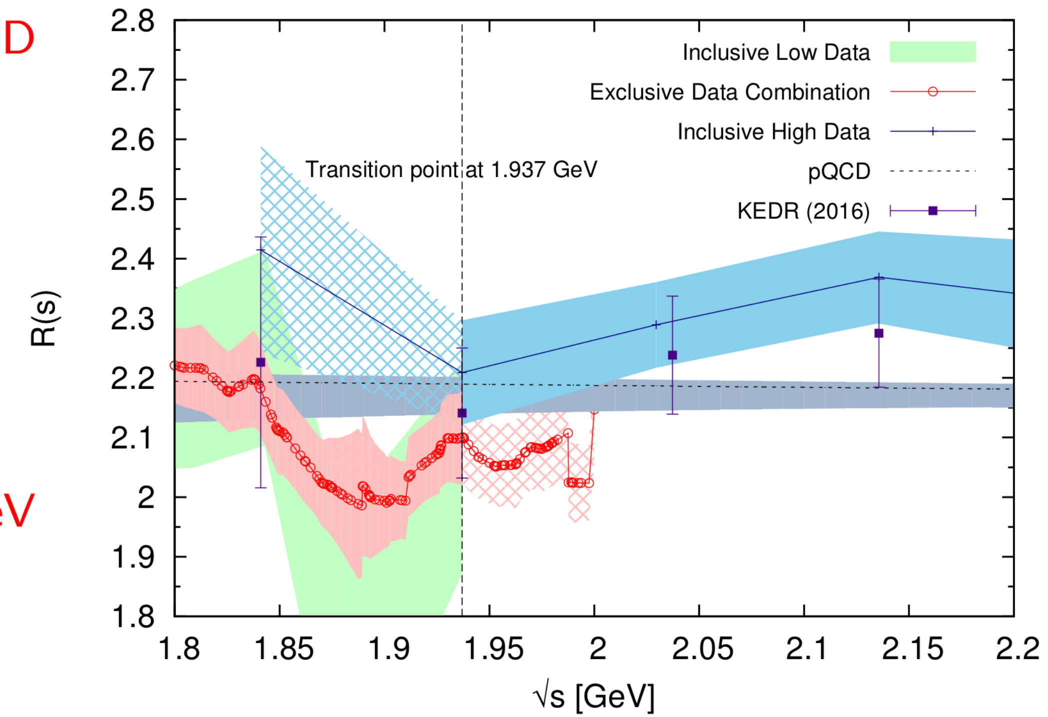
$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\mathrm{Inclusive}} = 43.67 \pm 0.17_{\mathrm{stat}} \pm 0.48_{\mathrm{sys}} \pm 0.01_{\mathrm{vp}} \pm 0.44_{\mathrm{fsr}} = 43.67 \pm 0.67_{\mathrm{tot}}$$

Exclusive/inclusive transition point

$\Rightarrow$ New KEDR data allow reconsideration of exclusive/inclusive transition point

- $\rightarrow$ KNT18 aim to avoid use of pQCD and keep a data-driven analysis
- $\rightarrow$ Disagreement between sum of exclusive states and inclusive data/pQCD
- $\rightarrow$ New $\pi^+\pi^-\pi^0\pi^0$ data result in reduction of the cross section
- $\rightarrow$ Previous transition point at 2 GeV no longer the preferred choice
- $\rightarrow$ More natural choice for this transition point at 1.937 GeV



Input	$a_\mu^{\mathrm{had,\,LO\,VP}}[1.841 \leq \sqrt{s} \leq 2.00\ \mathrm{GeV}] \times 10^{10}$
Exclusive sum	6.06 ± 0.17
Inclusive data	6.67 ± 0.26
pQCD	6.38 ± 0.11
Exclusive (< 1.937 GeV) + inclusive (> 1.937 GeV)	6.23 ± 0.13

KNT18 $a_\mu^{\text{had, VP}}$ update [KNT18: arXiv:1802.02995]

HLMNT(11): 694.91 ± 4.27

$\downarrow$

This work: $a_\mu^{\text{had, LO VP}} = 693.27 \pm 1.19_{\text{stat}} \pm 2.01_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}}$

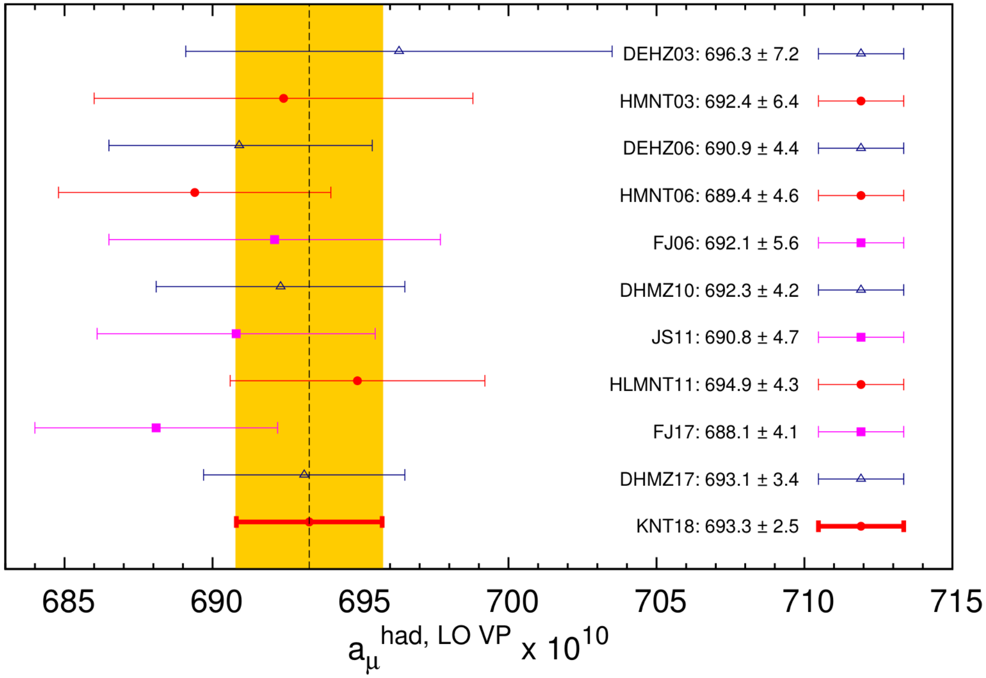
$= 693.27 \pm 2.34_{\text{exp}} \pm 0.74_{\text{rad}}$

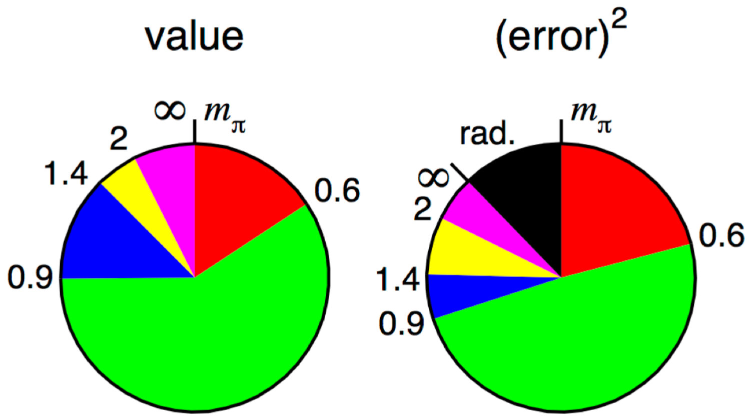
$= 693.27 \pm 2.46_{\text{tot}}$

$a_\mu^{\text{had, NLO VP}} = -9.82 \pm 0.04_{\text{tot}}$

$\Rightarrow$ Accuracy better then 0.4%

(uncertainties include all available correlations)





$\Rightarrow 2\pi$ dominance

Comparison with other similar works

Channel	This work (KNT18)	DHMZ17	Difference
$\pi^+\pi^-$	503.74 ± 1.96	507.14 ± 2.58	-3.40
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	22.81 ± 0.41	0.19
$K^0_S K^0_L$	13.04 ± 0.19	12.82 ± 0.24	0.22
$1.8 \leq \sqrt{s} \leq 3.7$ GeV	34.54 ± 0.56 (data)	33.45 ± 0.65 (pQCD)	1.09
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

$\Rightarrow$ Total estimates from two analyses in very good agreement

$\Rightarrow$ Masks much larger differences in the estimates from individual channels

$\Rightarrow$ Unexpected tension for 2π considering the data input likely to be similar

- $\rightarrow$ Points to marked differences in way data are combined
- $\rightarrow$ From 2π discussion: $a_\mu^{\pi^+\pi^-}$ (Weighted average) $= 509.1 \pm 2.9$

$\Rightarrow$ Compensated by lower estimates in other channels

- $\rightarrow$ For example, the choice to use pQCD instead of data above 1.8 GeV

$\Rightarrow$ FJ17: $a_{\mu,\,\mathrm{FJ17}}^{\mathrm{had,\,LO\,VP}} = 688.07 \pm 41.4$

- $\rightarrow$ Much lower mean value, but in agreement within errors

Comparison tables

Channel	KNT18	DHMZ17	Difference
Data based channels ($\sqrt{s} \leq 1.8$ GeV)			
$\pi^+\pi^-$	503.74 ± 1.96	506.70 ± 2.58	-2.96
$\pi^+\pi^-\pi^0$	47.70 ± 0.89	46.20 ± 1.45	1.50
$\pi^+\pi^-\pi^+\pi^-$	13.99 ± 0.19	13.68 ± 0.31	0.31
$\pi^+\pi^-\pi^0\pi^0$	18.15 ± 0.74	18.03 ± 0.54	0.12
K^+K^-	23.00 ± 0.22	23.06 ± 0.41	-0.06
$K_S^0K_L^0$	13.04 ± 0.19	12.82 ± 0.24	0.22
Total	693.3 ± 2.5	693.1 ± 3.4	0.2

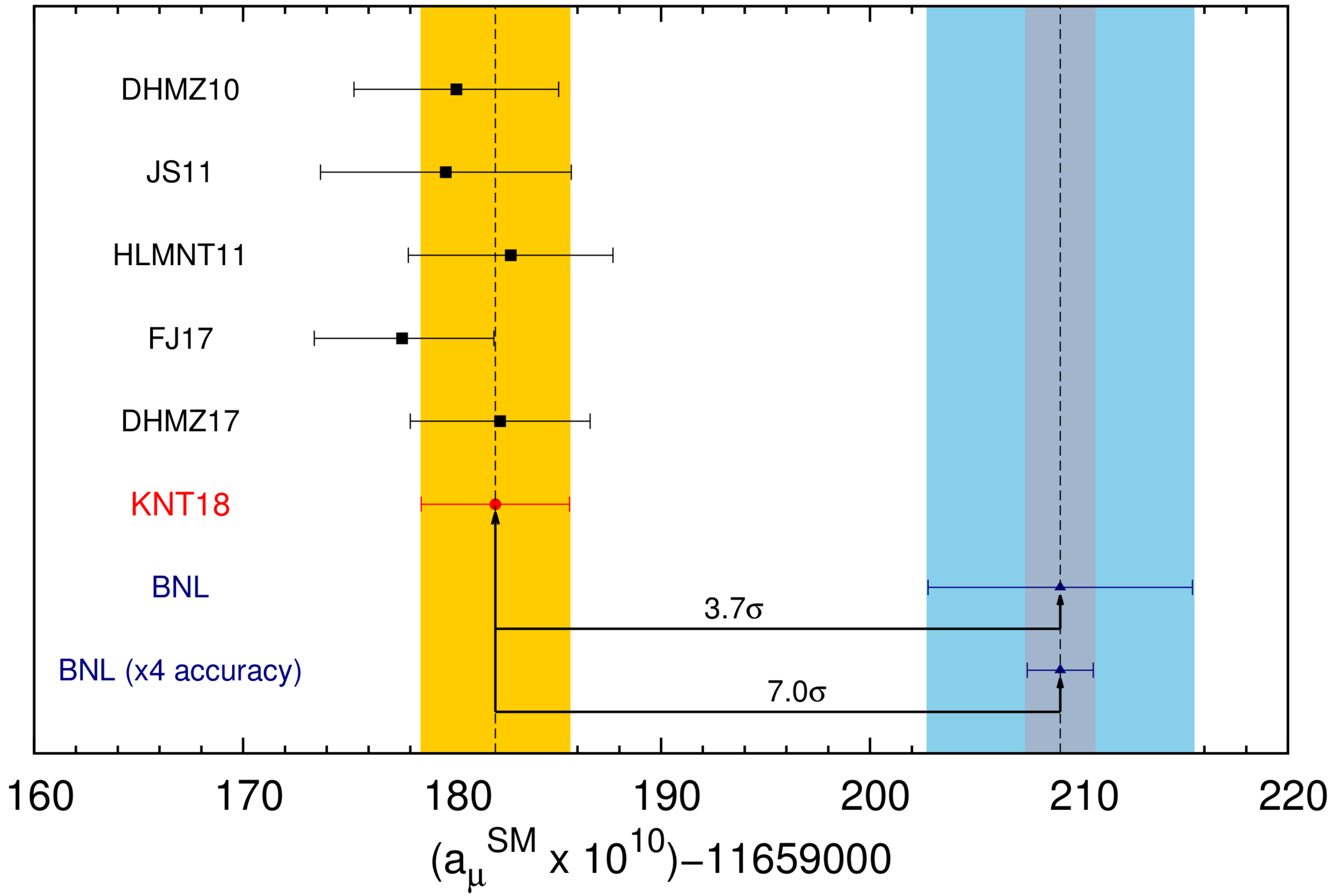
Channel	KNT18	FJ17	Difference
Data based channels ($0.318 \leq \sqrt{s} \leq 2$ GeV)			
$\pi^+\pi^-$	501.68 ± 1.71	502.16 ± 2.44	-0.48
$\pi^+\pi^-\pi^0$	47.83 ± 0.89	44.32 ± 1.48	3.51
$\pi^+\pi^-\pi^+\pi^-$	15.17 ± 0.21	14.80 ± 0.36	0.37
$\pi^+\pi^-\pi^0\pi^0$	19.80 ± 0.79	19.69 ± 2.32	0.11
K^+K^-	23.05 ± 0.22	21.99 ± 0.61	1.06
$K_S^0K_L^0$	13.05 ± 0.19	13.10 ± 0.41	-0.05
Total	693.27 ± 2.46	688.07 ± 4.14	5.20

Channel	KNT18	Benayoun et. al	Difference
Data based channels ($\sqrt{s} \leq 1.05$ GeV)			
$\pi^+\pi^-$	495.86 ± 1.94	489.83 ± 1.22	6.03
$\pi^+\pi^-\pi^0$	44.49 ± 0.80	42.94 ± 0.52	1.55
K^+K^-	18.12 ± 0.18	17.18 ± 0.25	0.94
$K_S^0K_L^0$	11.97 ± 0.17	11.87 ± 0.25	0.10

KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995]

	2011		2017	
QED	11658471.81 (0.02)	$\longrightarrow$	11658471.90 (0.01)	[arXiv:1712.06060]
EW	15.40 (0.20)	$\longrightarrow$	15.36 (0.10)	[Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	$\longrightarrow$	9.80 **(2.60)**	[EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20)	[Phys. Lett. B 735 (2014) 90]
	HLMNT11		**KNT18**	
LO HVP	694.91 (4.27)	$\longrightarrow$	693.27 **(2.46)**	this work
NLO HVP	-9.84 (0.07)	$\longrightarrow$	-9.82 (0.04)	this work
NNLO HVP			1.24 (0.01)	[Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	$\longrightarrow$	11659182.05 (3.56)	this work
Experiment			11659209.10 (6.33)	world avg
Exp - Theory	26.1 (8.0)	$\longrightarrow$	27.1 (7.3)	this work
Δa_μ	3.3σ	$\longrightarrow$	3.7σ	this work

KNT18 a_μ^{SM} update [KNT18: arXiv:1802.02995]



HVP from the lattice

A non-expert's re-cap of the lattice talks at the TGm2 HVP meeting at KEK in February.

- Complementary to data-driven (`pheno') DR.
- Need high statistics, and control highly non-trivial systematics:
  - need simulations at physical pion mass,
  - control continuum limit and Finite Volume effects,
  - need to include full QED and Strong Isospin Breaking effects (i.e. full QED+QCD including disconnected diagrams).

- There has been a lot of activity on the lattice, for HVP and HLbL:
  - Budapest-Marseille-Wuppertal (staggered q's, also moments)
  - RBC / UKQCD collaboration (Time-Momentum-Representation, DW fermions, window method to comb. `pheno' with lattice)
  - Mainz (CLS) group (O(a) improved Wilson fermions, TMR)
  - HPQCD & MILC collaborations (HISQ quarks, Pade fits)

HVP from LQCD: introduction

Consider in Euclidean spacetime (Blum '02)

$$\Pi_{\mu\nu}(Q) =$$

$$= \int d^4x\, e^{iQ\cdot x}\langle J_\mu(x)J_\nu(0)\rangle$$

$$= \left(Q_\mu Q_\nu - \delta_{\mu\nu}Q^2\right)\Pi(Q^2)$$

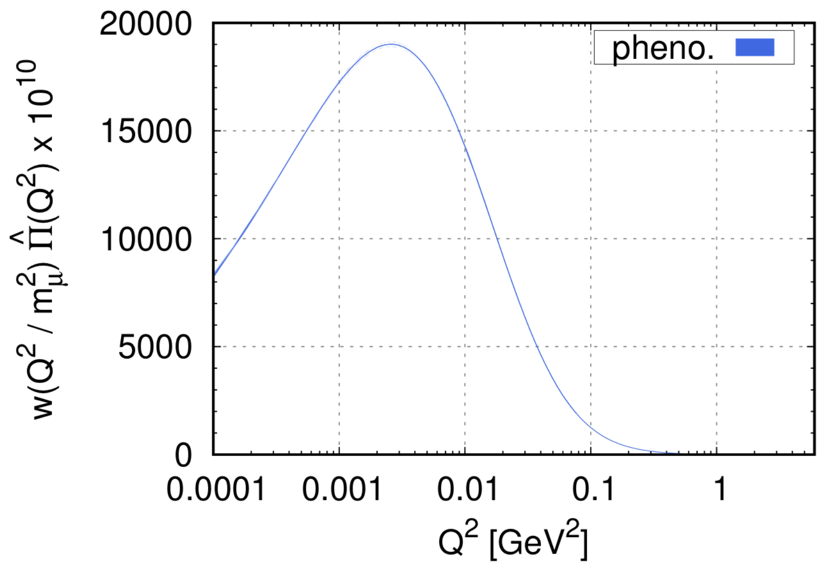
w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

Then (Lautrup et al '69, Blum '02)

$$a_\ell^{\text{LO-HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{m_\ell^2}\, w(Q^2/m_\ell^2)\hat{\Pi}(Q^2)$$

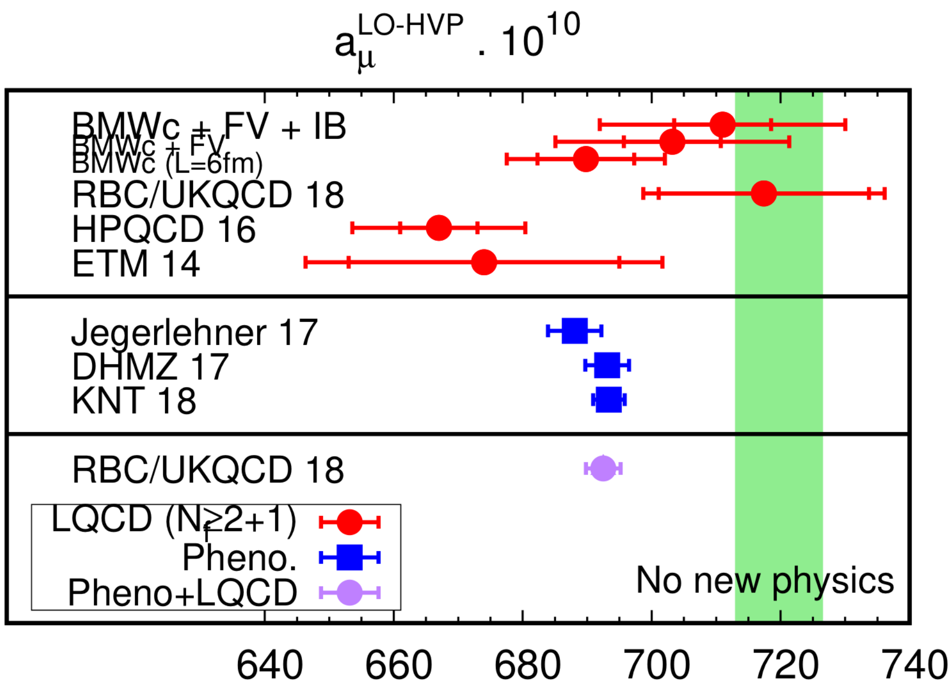
w/ $\hat{\Pi}(Q^2) \equiv \left[\Pi(Q^2) - \Pi(0)\right]$

Integrand peaked for $Q \sim (m_\ell/2)$



(HVP from Jegerlehner, "alphaQEDc17" (2017))

Comparison



- "No New Physics" scenario: $= (720 \pm 7) \times 10^{-10}$
- BMWc '17 consistent w/ "No new physics" scenario & pheno.
- Total uncertainty of 2.7% is $\sim 6\times$ pheno. error
- BMWc '17 is larger than other $N_f = 2+1+1$ results
  $\rightarrow$ difference w/ HPQCD '16 is $\sim 1.9\sigma$

From Christoph Lehner's talk at the TGm2 meeting at KEK

`Results from the RBC / UKQCD collaborations'

- They use the window method to combine pheno+lattice
- See their recent paper for more information:
  arXiv:1801.07224

Calculation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment

T. Blum,[1] P.A. Boyle,[2] V. Gülpers,[3] T. Izubuchi,[4,5] L. Jin,[1,5]
C. Jung,[4] A. Jüttner,[3] C. Lehner,[4,*] A. Portelli,[2] and J.T. Tsang[2]
(RBC and UKQCD Collaborations)
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(Dated: January 22, 2018)

Regions of precision (R-ratio data here is from **Fred Jegerlehner** 2017)

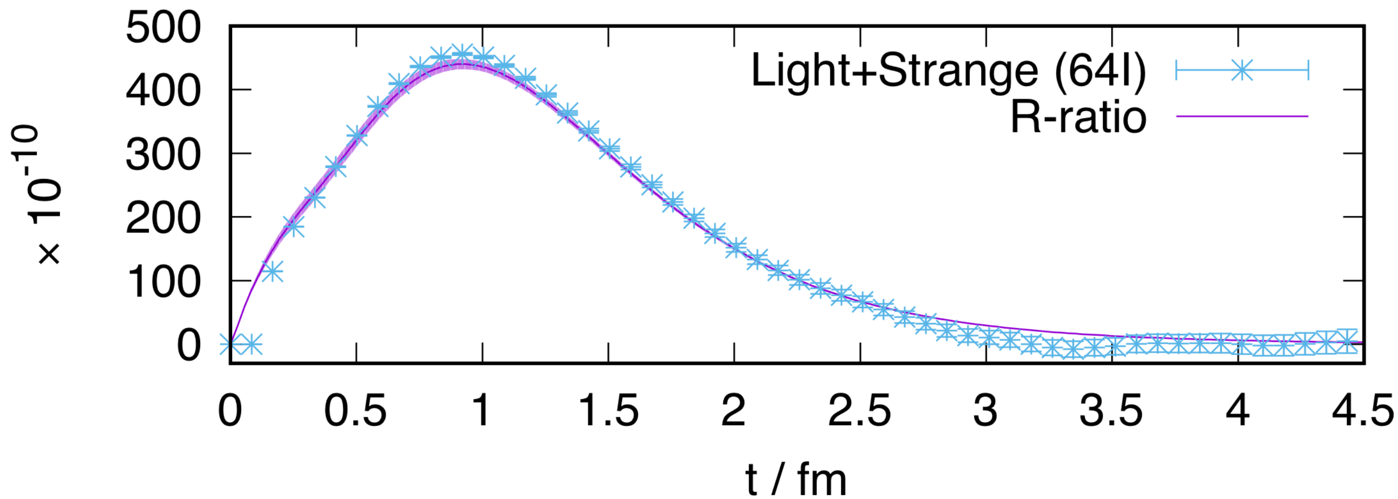


FIG. 4. Comparison of $w_t C(t)$ obtained using R-ratio data [1] and lattice data on our 64I ensemble.

The precision of lattice data deteriorates exponentially as we go to large t, however, is precise at intermediate distances. The R-ratio is very precise at long distances.

Note: in this plot a direct comparison of R-ratio and lattice data is not appropriate. Continuum limit, infinite-volume corrections, charm contributions, and IB corrections are missing from lattice data shown here.

Window method

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

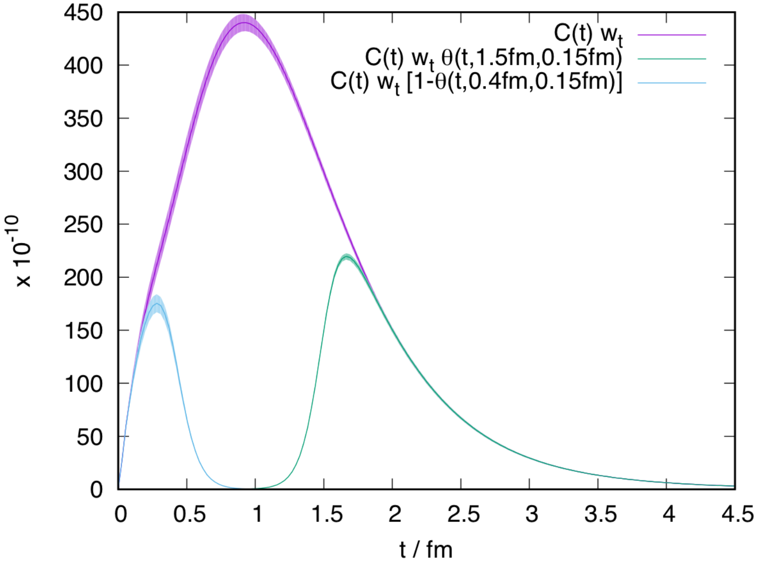
$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

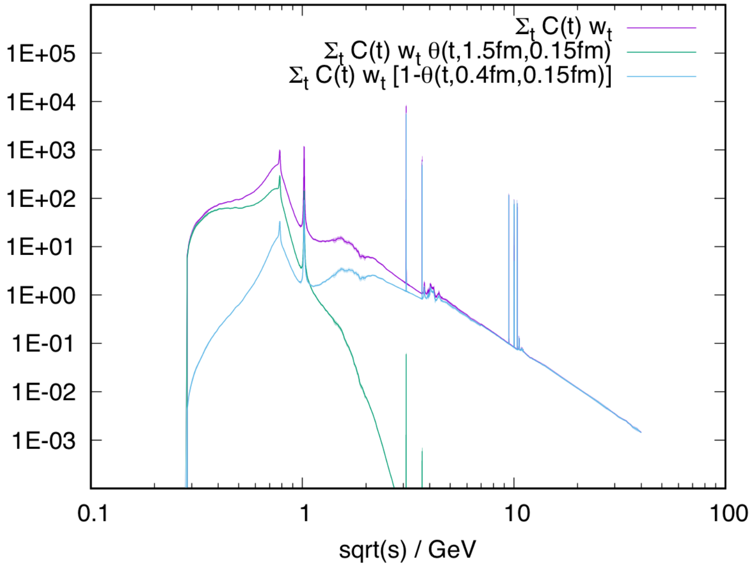
with

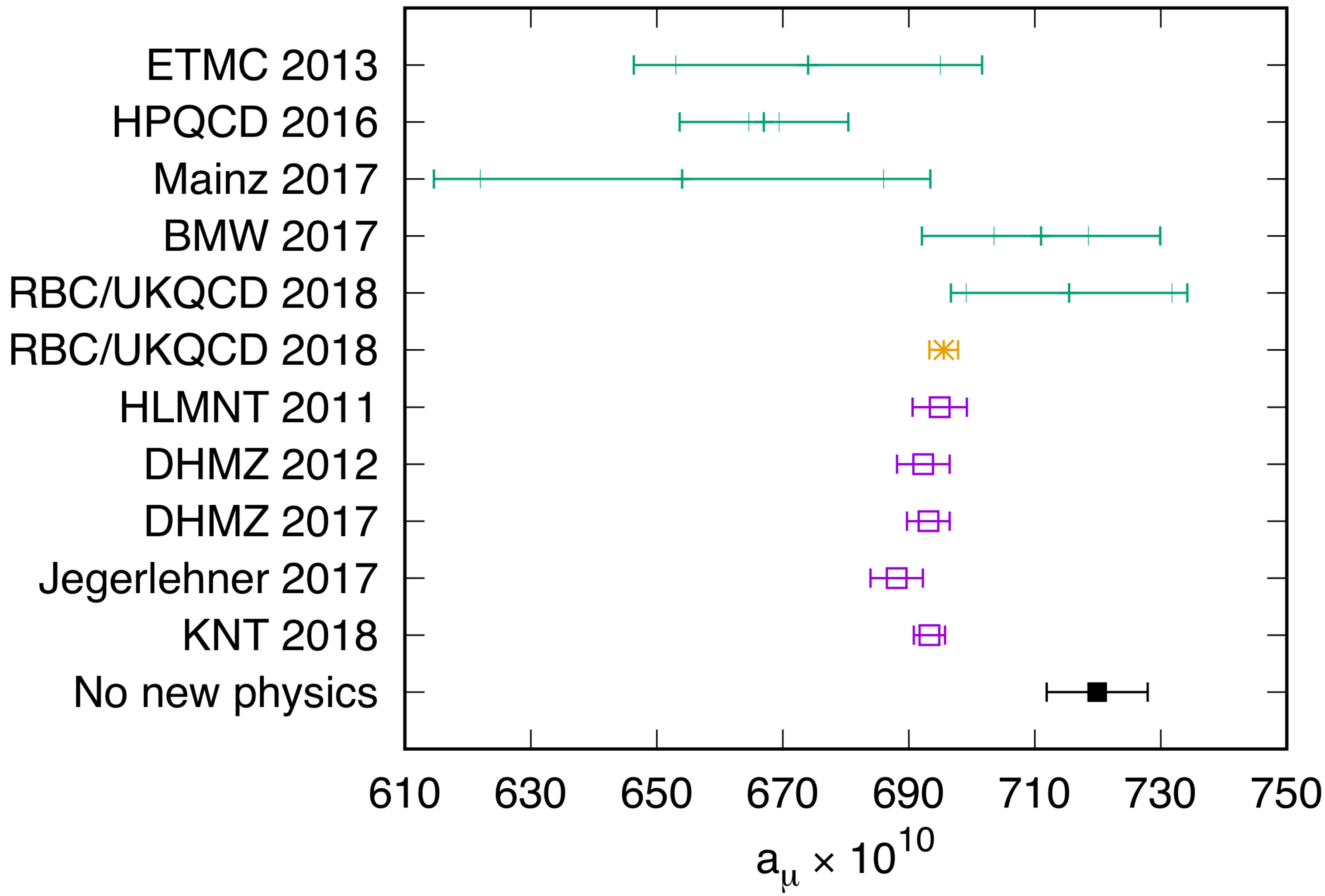
$$\begin{aligned}
a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]\,/2\,.
\end{aligned}$$

In this version of our calculation, we use $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})$ to compute a_μ^{SD} and a_μ^{LD}.

How does this translate to the time-like region?







We need to improve the precision of our pure lattice result so that it can distinguish the "no new physics" results from the cluster of precise R-ratio results.

HVP predictions: Outlook

- More data expected;
  - in the 2π channel from BaBar, CMD-3, SND,
  - in subleading channels, 3π, 4π, KK
  - in the inclusive region from BESIII and KEDR,
  - BELLE II will be able to contribute with ISR measurements.

- If new data produce no new tensions/puzzles, further improvement should be significant within few years
  (but ultimately hit a limit with experimental systematics)

- Lattice expected to become a competitive alternative and check/challenge direct data-driven analyses;
- combined methods may provide the best HVP predictions.

- Still room for global combined fits with with more TH input?
- Long term: a direct measurement in the space-like: MuonE

SM prediction: Summary

- All sectors of the Standard Model prediction of g-2 have been scrutinised a lot in recent years.

- The basic picture has not changed, but recent data, many from Radiative Return, significantly improve the prediction for a_μ^{HVP}.

- A discrepancy ~ 3 -> 4 σ is consolidated.

- With further hadronic data in the pipeline, and very promising progress on the lattice, the HVP contribution are expected to get even more accurate in time for the new experimental measurements.

- Now the error on the HLbL contributions needs to get under better control. This is happening already; over to Christoph.

Extras

Where we are

- Calculation of all relevant contributions to $a_\mu^{\text{LO-HVP}}$ directly at physical m_{ud}

$$a_\mu^{\text{LO-HVP}} = 711.0(7.5)(17.5) \times 10^{-10} \quad [2.7\%]$$

- Also have slope and curvature of $\hat{\Pi}(Q^2)$ at $Q^2 = 0$ (PRD96 '17)
- Fully controlled continuum limit and matching to perturbation theory
- Only model/pheno. assumptions for FV, QED and $m_u \neq m_d$ corrections, but dominate error
- Consistent with "no new physics" and dispersive methods, but error $\sim (6 \div 7)\times$ larger; some tension with HPQCD 16 on $a_{\mu,\,ud}^{\text{LO-HVP}}$
- Total error is 2.7%, dominated by poorly controlled FV effects
- Need $\sim 0.2\%$ to match upcoming experiments !
- With same methods, compute (see also ETM '16)

$$a_e^{\text{LO-HVP}} = 188.5(2.6)(5.5) \times 10^{-14}[3.2\%] \quad \leftrightarrow \quad 184.6(1.2) \times 10^{-14}[0.7\%] \qquad \text{(Jegerlehner '15)}$$

$$a_\tau^{\text{LO-HVP}} = 341.3(0.8)(3.2) \times 10^{-8}[1.0\%] \quad \leftrightarrow \quad 338(4) \times 10^{-8}[1.2\%] \qquad \text{(Eidelman et al. '07)}$$

$\hat{\Pi}(Q^2)$ vs Q^2: LQCD vs phenomenology (preliminary)

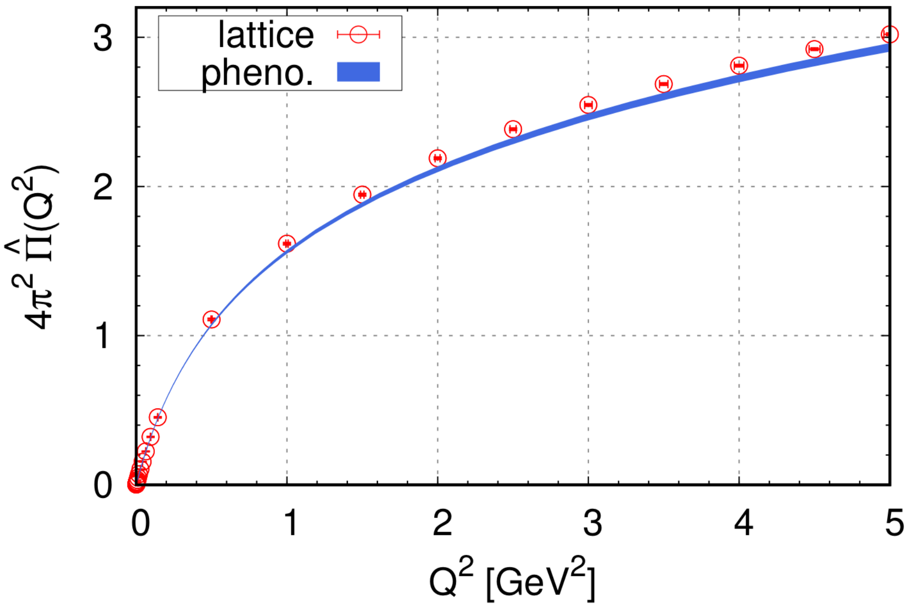
LQCD

$$\hat{\Pi}_{\mathrm{lat}}(Q^2) = \lim_{a\to 0, L\to\infty} \sum_{t=0}^{T/2} \left[t^2 - \frac{4}{Q^2}\sin\frac{Qt}{2} \right] \mathrm{Re}\, C_L(t)$$

Phenomenology

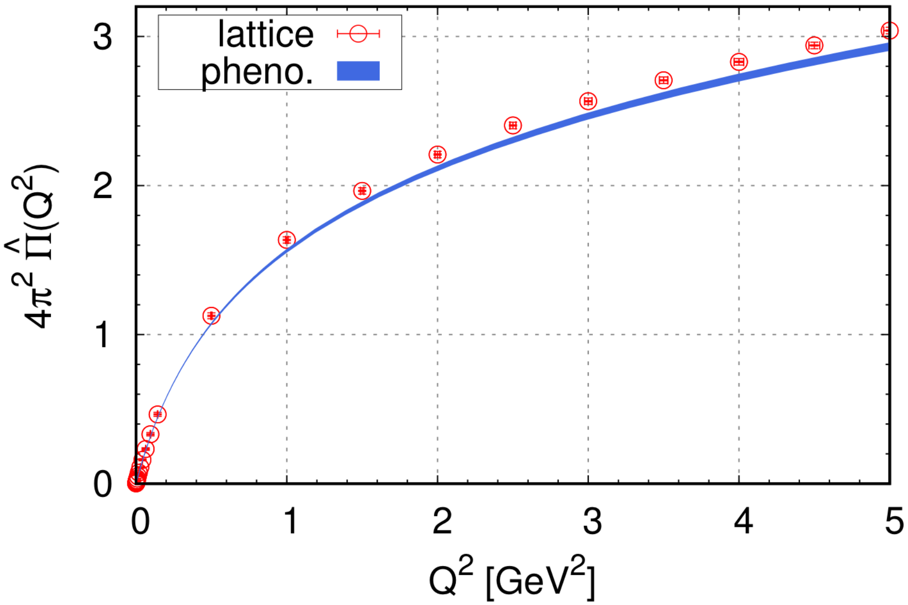
$$\hat{\Pi}_{\mathrm{pheno}}(Q^2) = \frac{Q^2}{12\pi^2}\int_0^\infty ds\, \frac{R_{\mathrm{had}}(s)}{s(s+Q^2)}$$

LQCD w/ stat., $a \to 0$, $\delta_a Q^2$, t_C & Q_{max} errors



(BMWc vs Jegerlehner '17)

LQCD w/ additional FV correction (still missing IB corrections)



(BMWc vs Jegerlehner '17)

What next?

- Increase statistics
- Understand and control FV effects much better
- Compute QED and $m_d \neq m_u$ correction to relevant observables
- Need high precision scale setting
- Detailed comparison to phenomenology to understand where we agree and why if we don't
- Combine LQCD and phenomenology to improve overall uncertainty (RBC/UKQCD '18), only if the two agree statistically with comparable errors, locally

Features of the recent RBC / UKQCD work;
  slides from Christoph's talk at the TGm2 meeting at KEK:

Starting from the vector current

$$J_\mu(x) = i \sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$$

we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

We perform the calculation as a perturbation around an isospin-symmetric lattice QCD computation with two degenerate light quarks with mass $m_{\rm light}$ and a heavy quark with mass $m_{\rm heavy}$ tuned to produce a pion mass of 135.0 MeV and a kaon mass of 495.7 MeV.

The correlator is expanded in the fine-structure constant α as well as $\Delta m_{\rm up,\ down} = m_{\rm up,\ down} - m_{\rm light}$, and $\Delta m_{\rm strange} = m_{\rm strange} - m_{\rm heavy}$. We write

$$C(t) = C^{(0)}(t) + \alpha C^{(1)}_{\rm QED}(t) + \sum_f \Delta m_f C^{(1)}_{\Delta \rm m_f}(t) + \mathcal{O}(\alpha^2, \alpha\Delta m, \Delta m^2)\,.$$

The correlators of this expansion are computed in lattice QCD with dynamical up, down, and strange quarks. We compute the missing contributions to a_μ from charm sea quarks in perturbative QCD (RHAD) by integrating the time-like region above 2 GeV and find them to be smaller than 0.3×10^{-10}.

We tune the bare up, down, and strange quark masses $m_{\rm up}$, $m_{\rm down}$, and $m_{\rm strange}$ such that the π^0, π^+, K^0, and K^+ meson masses computed in our calculation agree with the respective experimental measurements. The lattice spacing is determined by setting the Ω^- mass to its experimental value.

We perform the lattice calculations for the light quark contributions using RBC/UKQCD's 48I and 64I lattice configurations with lattice cutoffs $a^{-1} = 1.730(4)$ GeV and $a^{-1} = 2.359(7)$ GeV and a larger set of ensembles with up to $a^{-1} = 2.774(10)$ GeV for the charm contribution.

From the parameter tuning procedure on the 48I we find $\Delta m_{\rm up} = -0.00050(1)$, $\Delta m_{\rm down} = 0.00050(1)$, and $\Delta m_{\rm strange} = -0.0002(2)$.

The shift of the Ω^- mass due to the QED correction is significantly smaller than the lattice spacing uncertainty and its effect on $C(t)$ is therefore not included separately.

Conclusions

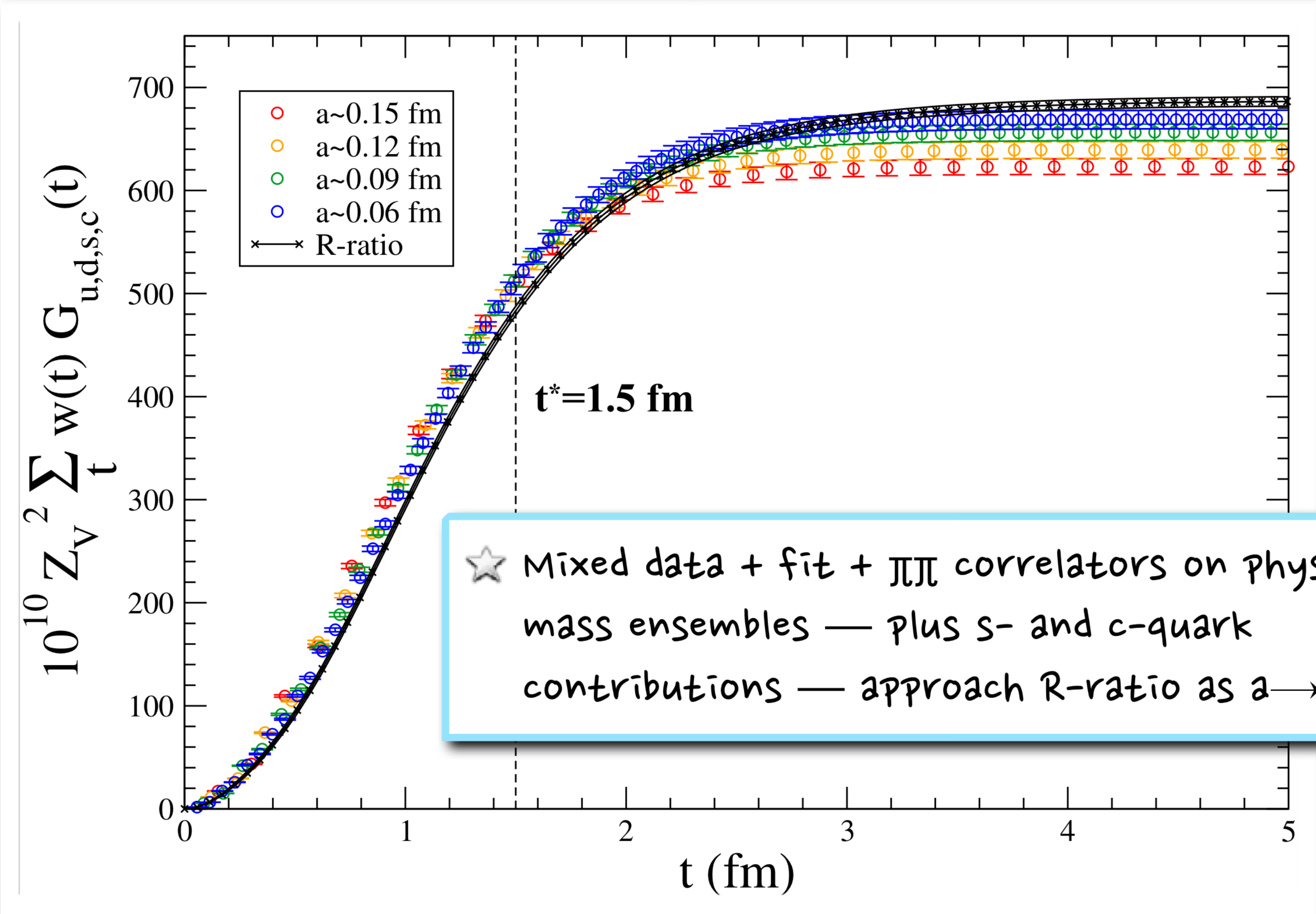
We now have a lattice calculation that is sufficiently precise that we can start to compare to the R-ratio data and we can provide a joint analysis.

The next target is to reduce the uncertainty of the pure lattice number to the order of 5×10^{-10} such that it can resolve the "no new physics" scenario from the cluster of precise R-ratio results. This requires in particular improvements in the treatment of long distances and finite-volume corrections.

To address these effects we are now combining the bounding method with an exclusive study of finite-volume energy states.

The lattice may also be able with a wise selection of observables to help resolve tensions of individual data sets.

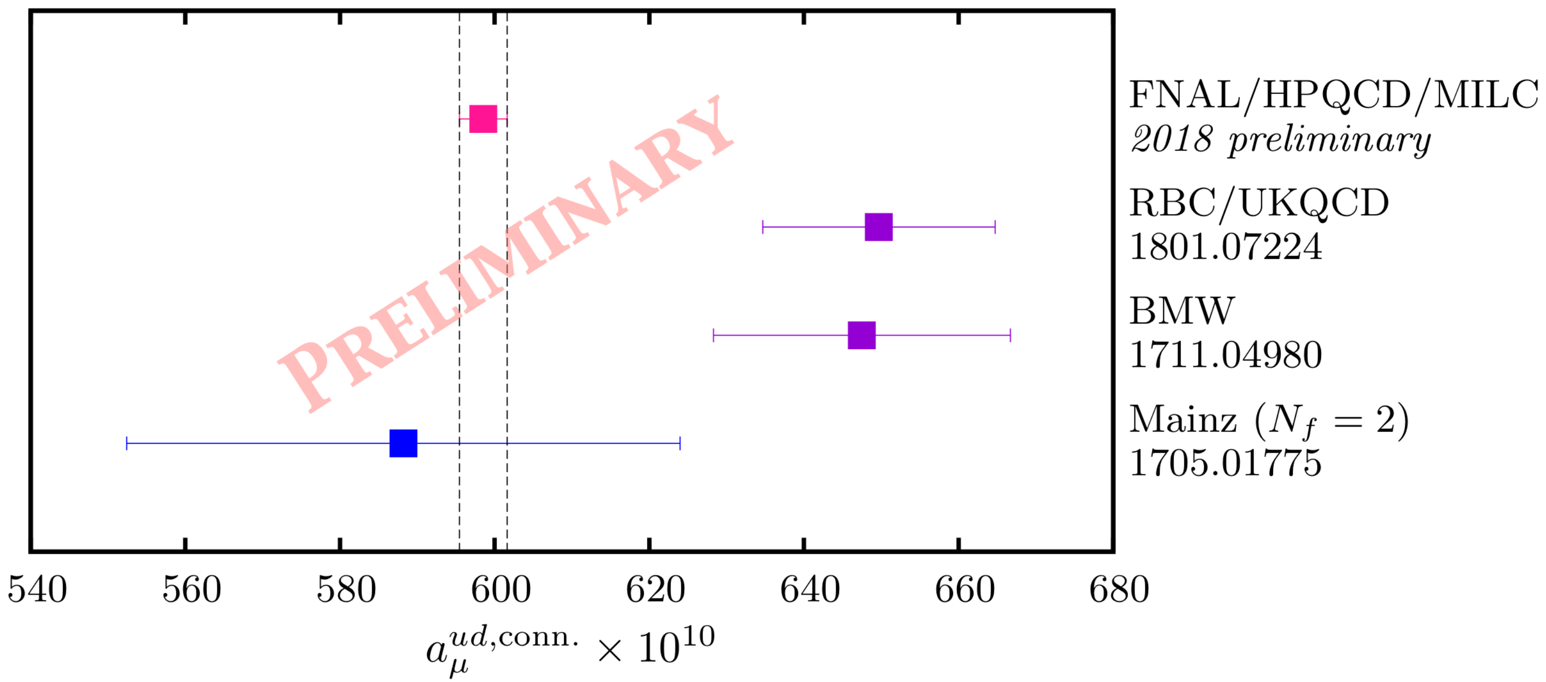
Comparison with *R-ratio data*



☆ Mixed data + fit + ππ correlators on physical-mass ensembles — plus s- and c-quark contributions — approach R-ratio as a⟶0

Comparison with other work

- Results shown are for isospin-limit quantity without QED & isospin-breaking corrections



Significant difference with BMW & RBC/UKQCD requires further scrutiny within collaborations & comparisons of more intermediate quantities

Outlook

Forthcoming:

- light contribution at the physical pion mass,
- chiral and continuum extrapolation,
- careful study of scale setting errors,
- more complete spectroscopic studies in the vector channel,

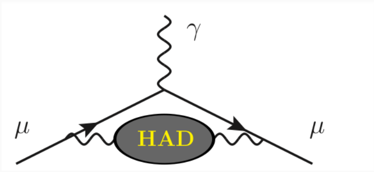
  → MITP workshop "Scattering Amplitudes and Resonance Properties for Lattice QCD", Mainz Institute for Theoretical physics, August 27-31, 2018
- inclusion of isospin breaking effects → A. Risch (PhD) ,
- use of covariant coordinate space method [Meyer, 2017] .

Summary

- $N_\mathrm{f} = 2 + 1$ CLS ensembles now include physical pion mass,
- full $\mathrm{O}(a)$ improvement now implemented,
- use of spectroscopic information allows control of long-distance behaviour,
- finite-volume effects appear to be under control,
- better than 1% statistical accuracy within reach,
- disconnected signal greatly improved using hierarchical probing.

We Focus:

- We investigate Moments of Hadronic Vacuum Polarization (HVP):



$$\hat{\Pi}(Q^2) = \sum_n Q^{2n}\Pi_n \ ,$$

$$\Pi_n = \frac{1}{n!}\frac{d^n\hat{\Pi}(Q^2)}{(dQ^2)^n}\bigg|_{Q^2\to 0} = \sum_x \frac{(-\hat{x}_\nu^2)^{n+1}}{(2n+2)!}\langle j_\mu(x) j_\mu(0)\rangle \equiv \Pi_n[\mu\nu] \ ,$$

where $j_\mu(x) = \sum_f Q_f(\bar{\psi}\gamma_\mu\psi)(x)$.

- In the asymmetric box $(T \sim 1.5L)$, the $\mu\nu$ dependent moments $(\Pi_n[\mu\nu])$ are classfied into three irreducible reps:

$$\Pi_{n,ss} = \frac{1}{6}\sum_{i\neq j}\Pi_n[ij] \ , \Pi_{n,ts} = \frac{1}{3}\sum_j \Pi_n[4j] \ , \Pi_{n,st} = \frac{1}{3}\sum_i \Pi_n[i4] \ . \qquad (1)$$

To be evaluated is the average: $\Pi_n = (\Pi_{n,ss} + \Pi_{n,ts} + \Pi_{n,st})/3$.

- Π_n are composed of the connected $(l = ud, s, c)$ and disconnected parts:

$$\Pi_n = \left(\frac{5}{9}\Pi_n^l + \frac{1}{9}\Pi_n^s + \frac{4}{9}\Pi_n^c\right)_{conn} + \frac{1}{9}\Pi_n^{disc} \ . \qquad (2)$$

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

$\Rightarrow$ When combining data...

- $\rightarrow$ ...how to best combine large amounts of data from different experiments
- $\rightarrow$ ...the correct implementation of correlated uncertainties (statistical and systematic)
- $\rightarrow$ ...finding a solution that is free from bias

d'Agostini bias [Nucl.Instrum.Meth. A346 (1994) 306-311]

$$x_1 = 0.9 \pm p x_1 \qquad C = \begin{pmatrix} p^2 x_1^2 & p^2 x_1 x_2 \\ p^2 x_2 x_1 & p^2 x_1^2 \end{pmatrix}$$
$$x_2 = 1.1 \pm p x_2$$

(Normalisation uncertainties defined by data)

$\Rightarrow \bar{x} \simeq 0.98$ (systematic bias)

Effect worsened with full, iterative data combination

Data combination consideration

Question:

What are the main points of concern when combining experimental data to evaluate $a_\mu^{\text{had, VP}}$?

$\Rightarrow$ When combining data...

- $\rightarrow$ ...how to best combine large amounts of data from different experiments
- $\rightarrow$ ...the correct implementation of correlated uncertainties (statistical and systematic)
- $\rightarrow$ ...finding a solution that is free from bias

Fixed matrix method [R. D. Ball et al. [NNPDF Collaboration], JHEP 1005 (2010) 075.]

$$x_1 = 0.9 \pm px_1 \qquad C = \begin{pmatrix} p^2\bar{x}^2 & p^2\bar{x}^2 \\ p^2\bar{x}^2 & p^2\bar{x}^2 \end{pmatrix}$$
$$x_2 = 1.1 \pm px_2$$

(Normalisation uncertainties defined by estimator)

$\Rightarrow \bar{x} = 1.00$ (systematic bias)

Redefinition repeated at each stage of iterative data combination

Data combination: setup

$\Rightarrow$ Re-bin data into *clusters*

$\rightarrow$ Scan cluster sizes for optimum solution (error, χ^2, check by sight...)

$\Rightarrow$ Correlated data beginning to dominate full data compilation...

$\rightarrow$ Non-trivial, energy dependent influence on both mean value and error estimate

KNT18 prescription

- Construct full covariance matrices for each channel & entire compilation
  $\Rightarrow$ Framework available for inclusion of any and all inter-experimental correlations

- If experiment does not provide matrices...
  $\rightarrow$ Statistics occupy diagonal elements only
  $\rightarrow$ Systematics are 100% correlated

- If experiment does provide matrices...
  $\rightarrow$ Use correlations to full capacity

Systematic bias and use of the data/covariance matrix

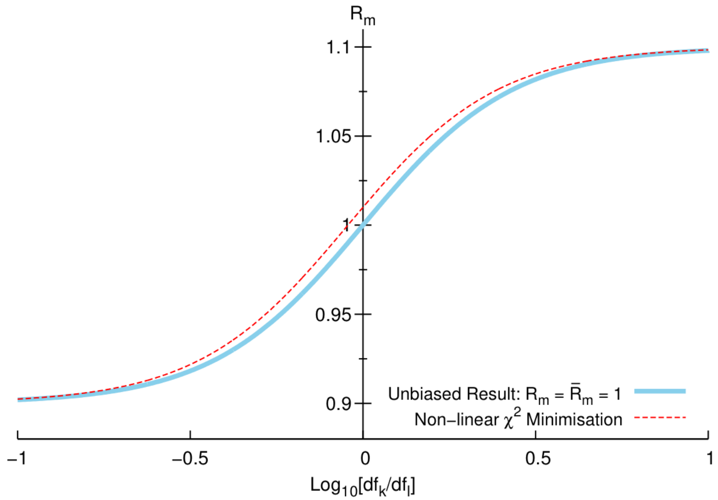
⇒ Data is re-binned using an adaptive clustering algorithm

⇒ Iterative fit of covariance matrix as defined by data → D'Agostini bias

[Nucl.Instrum.Meth. A346 (1994) 306-311]

HLMNT11

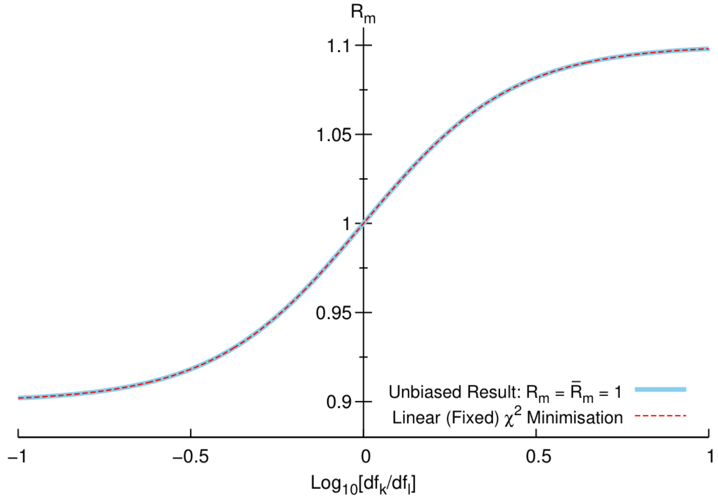
⇒ Non-linear χ^2 minimisation fitting nuisance parameters
→ Penalty trick bias



$\Longrightarrow$

KNT18

⇒ Fix the covariance matrix in an iterative χ^2 minimisation
→ Free from bias



Allows for increased fit flexibility and full use of energy dependent, correlated uncertainties

Linear χ^2 minimisation [KNT18: arXiv:1802.02995]

⇒ Clusters are defined to have linear cross section

→ Fix covariance matrix with linear interpolants at each iteration (extrapolate at boundary)

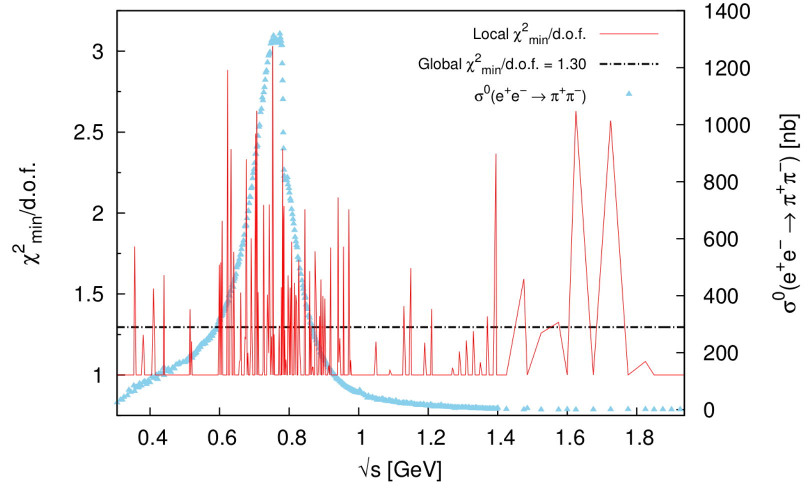
$$\chi^2 = \sum_{i=1}^{N_{\rm tot}} \sum_{j=1}^{N_{\rm tot}} \left(R_i^{(m)} - \mathcal{R}_m^i\right) \mathbf{C}^{-1}\left(i^{(m)}, j^{(n)}\right) \left(R_j^{(n)} - \mathcal{R}_n^j\right)$$

⇒ Through correlations and linearisation, result is the minimised solution of all available uncertainty information

→ ... and solution is shown to be free of systematic bias

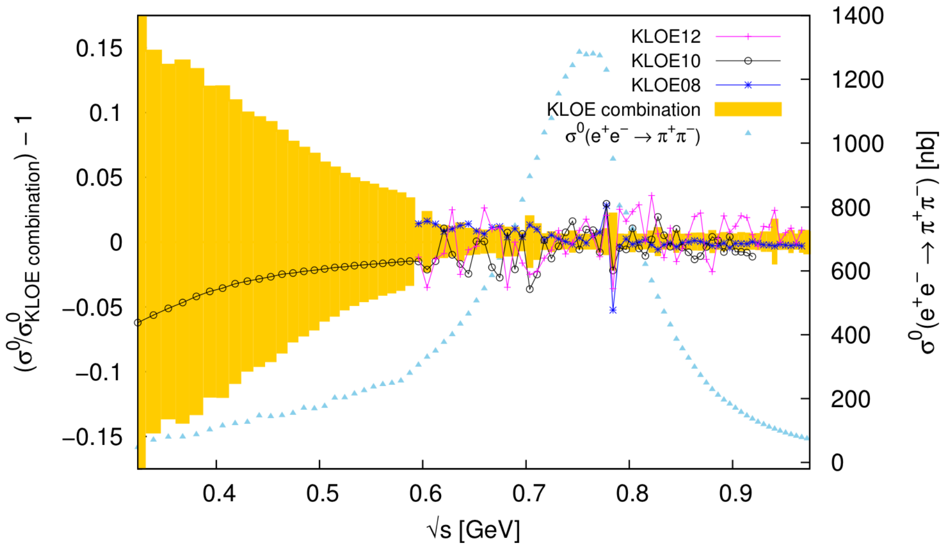
⇒ The flexibly of the fit to vary due to the energy dependent, correlated uncertainties benefits the combination

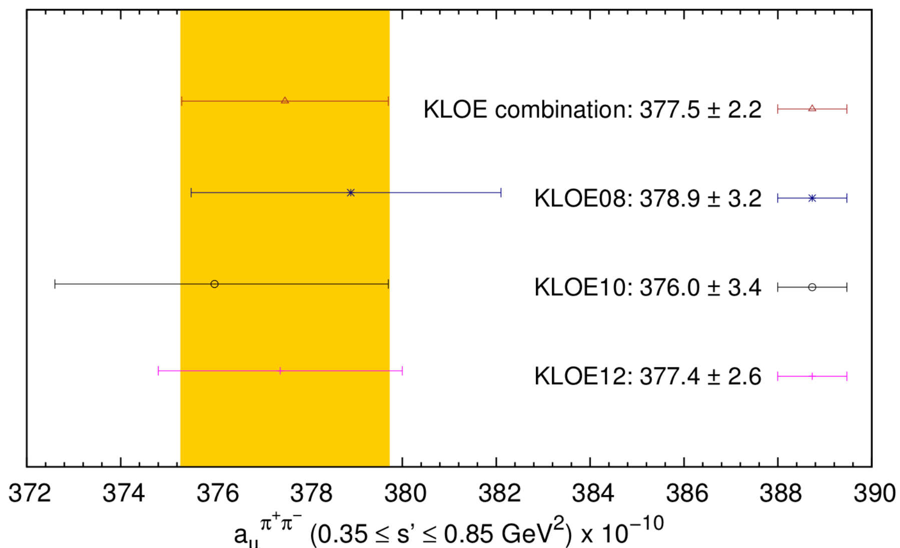
→ ...and any data tensions are reflected in a local and global $\chi^2_{\rm min}/{\rm d.o.f.}$ error inflation



The resulting KLOE $\pi^+\pi^-\gamma(\gamma)$ combination [arXiv:1711.0308]

$\Rightarrow$ Combination of KLOE08, KLOE10 and KLOE12 gives 85 distinct bins between $0.1 \leq s \leq 0.95$ GeV^2





$\rightarrow$ Covariance matrix now correctly constructed

$\Rightarrow$ a positive semi-definite matrix

$\rightarrow$ Non-trivial influence of correlated uncertainties on resulting mean value

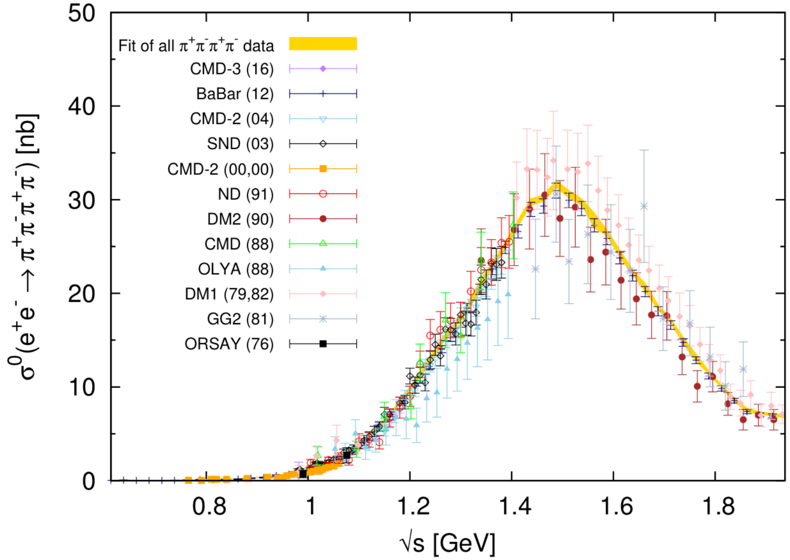
$$a_\mu^{\pi^+\pi^-}(0.1 \leq s' \leq 0.95 \text{ GeV}^2) = (489.9 \pm 2.0_{\text{stat}} \pm 4.3_{\text{sys}}) \times 10^{-10}$$

$\rightarrow$ All previous combinations issues now eliminated...

...and consistency between measurements and combination

4π channels [KNT18: arXiv:1802.02995]

$\pi^+\pi^-\pi^+\pi^-$



New data:

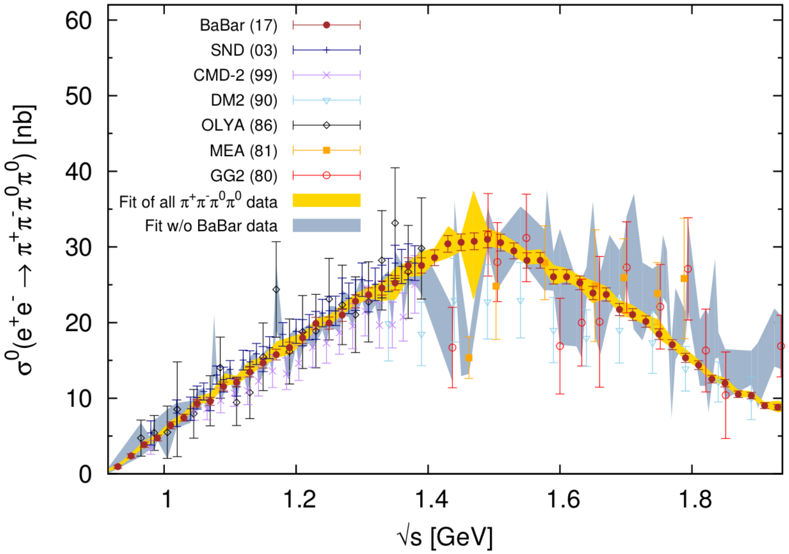
BaBar: [Phys. Rev. D 85 (2012) 112009.]
CMD-3: [Phys. Lett. B 768 (2017) 345.]

$$a_\mu^{\pi^+\pi^-\pi^+\pi^-} = 14.87 \pm 0.20_{\rm tot}$$

HLMNT11: $14.65 \pm 0.47_{\rm tot}$

Large improvement here

$\pi^+\pi^-\pi^0\pi^0$



New data:

BaBar: [Phys. Rev. D 96 (2017), 092009.]

$$a_\mu^{\pi^+\pi^-\pi^0\pi^0} = 19.39 \pm 0.78_{\rm tot}$$

HLMNT11: $20.37 \pm 1.26_{\rm tot}$

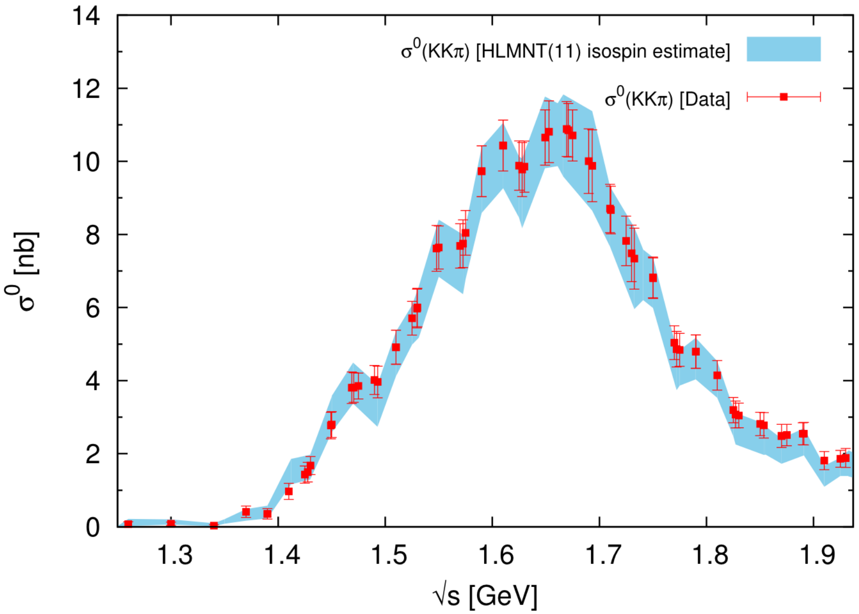
Requires better new data

$KK\pi$, $KK\pi\pi$ and isospin

$\Rightarrow$ New data for $KK\pi$ and $KK\pi\pi$ removes reliance on isopsin (only $K_S^0 \cong K_L^0$)

$\underline{KK\pi}$

$K_S^0 K_L^0 \pi^0$ [Phys.Rev. D95 (2017), 052001, arXiv:1711.07143]

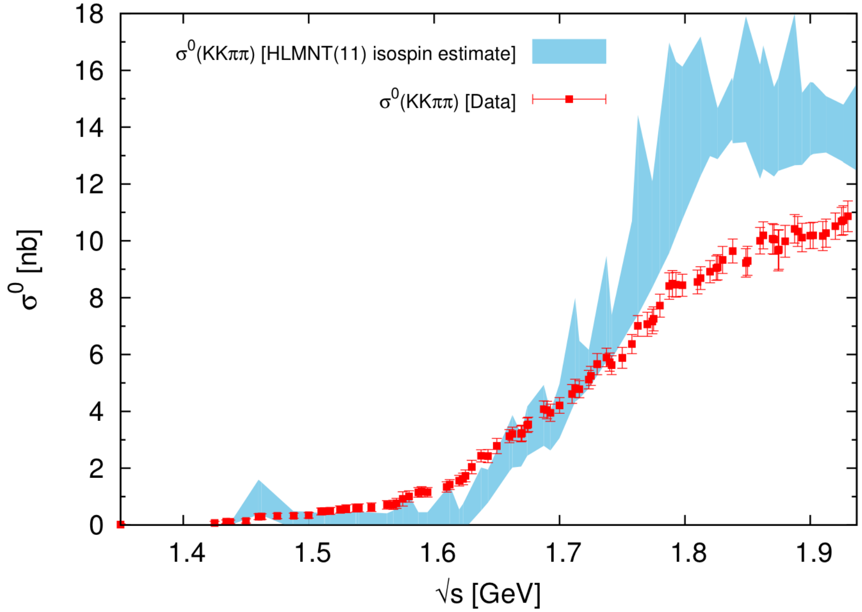


HLMNT11: 2.65 ± 0.14

KNT18: 2.71 ± 0.12

$\underline{KK\pi\pi}$

$K_S^0 K_L^0 \pi^+ \pi^-$ [Phys.Rev. D80 (2014), 092002]
$K_S^0 K_S^0 \pi^+ \pi^-$ [Phys.Rev. D80 (2014), 092002],
$K_S^0 K_L^0 \pi^0 \pi^0$ [Phys.Rev. D95 (2017), 052001]
$K_S^0 K^\pm \pi^\mp \pi^0$ [Phys.Rev. D95 (2017), 092005]

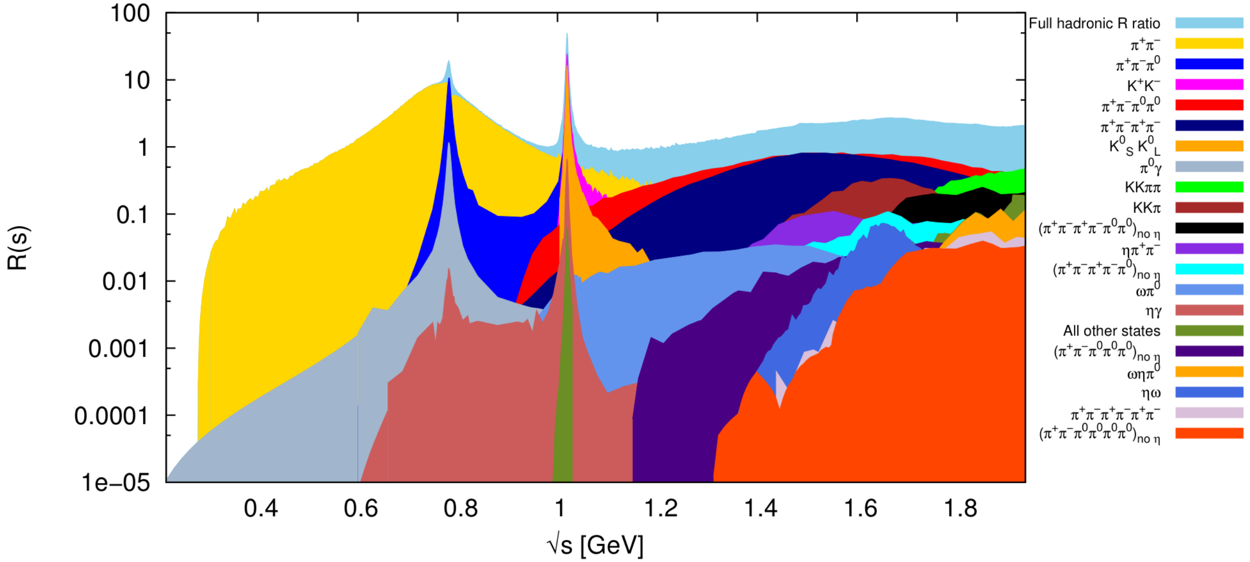


HLMNT11: 2.51 ± 0.35

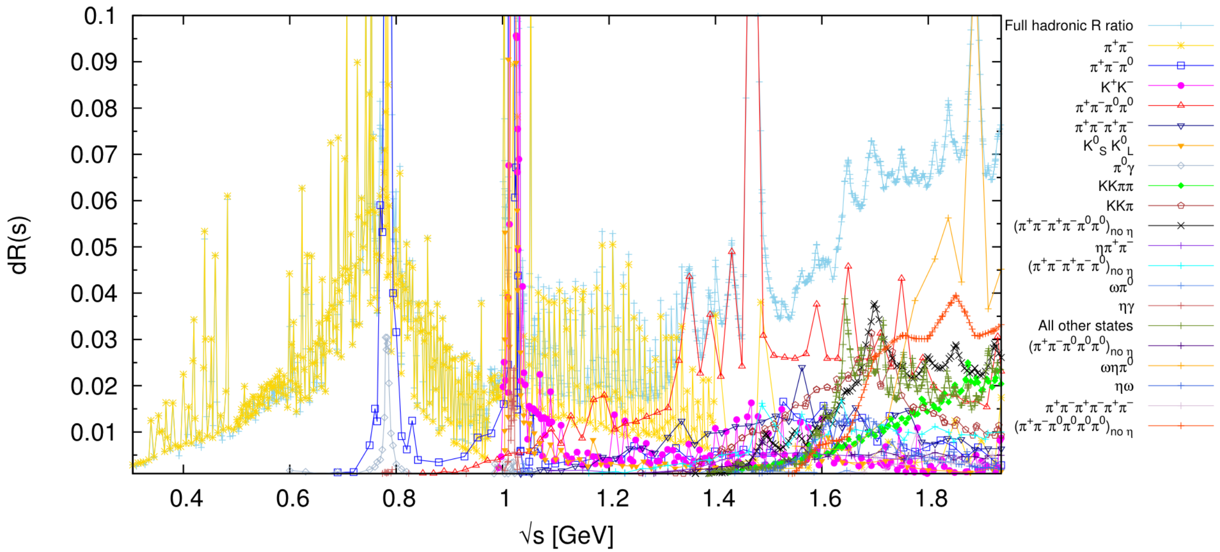
KNT18: 1.93 ± 0.08

$\Rightarrow$ **But**, still reliant on isospin estimates for $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-4\pi^0$, $KK3\pi$...

Contributions below 2GeV [KNT18: arXiv:1802.02995]



→ Dominance of 2π below 0.9 GeV evident for both cross section and uncertainty

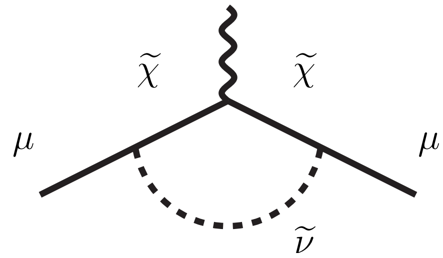


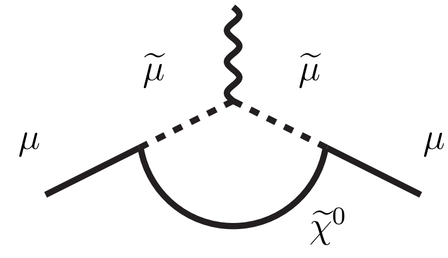
→ Large improvement to cross section and uncertainty from new 4π data

a_μ: New Physics?

- Many BSM studies use g-2 as constraint or even motivation

- SUSY could easily explain g-2
  - Main 1-loop contributions:





  - Simplest case:

$$a_\mu^{\mathrm{SUSY}} \simeq sgn(\mu)\, 130 \times 10^{-11}\, \tan\beta \left(\frac{100\,\mathrm{GeV}}{\Lambda_{\mathrm{SUSY}}}\right)^2$$

  - Needs $\mu>0$, `light' SUSY-scale Λ and/or large $\tan\beta$ to explain 281×10^{-11}

  - This is already excluded by LHC searches in the simplest SUSY scenarios (like CMSSM); causes large χ^2 in simultaneous SUSY-fits with LHC data and g-2

  - However:
    * SUSY does not have to be minimal (w.r.t. Higgs),
    * could have large mass splittings (with lighter sleptons),
    * be hadrophobic/leptophilic,
    * or not be there at all, but don't write it off yet…

New Physics? just a few of many recent studies

- Don't have to have full MSSM (like coded in GM2Calc [by Athron, ..., Stockinger et al., EPJC 76 (2016) 62], which includes all latest two-loop contributions), and

- extended Higgs sector could do, see, e.g. Stockinger et al., JHEP 1701 (2017) 007, `The muon magnetic moment in the 2HDM: complete two-loop result'

➔ lesson: 2-loop contributions can be highly relevant in both cases; one-loop analyses can be misleading

- 1 TeV Leptoquark Bauer + Neubert, PRL 116 (2016) 141802

one new scalar could explain several anomalies seen by BaBar, Belle and LHC in the flavour sector (e.g. violation of lepton universality in B -> Kll, enhanced B -> Dτν) and solve g-2, while satisfying all bounds from LEP and LHC