Hadronic light-by-light contributions to $(g-2)_\mu$

Christoph Lehner (BNL)

June 1, 2018 – Simons Center for Geometry and Physics

Collaborators (RBC/UKQCD)

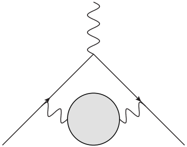
Tom Blum (Connecticut)
Norman Christ (Columbia)
Masashi Hayakawa (Nagoya)
Taku Izubuchi (BNL/RBRC)
Luchang Jin (RBRC, Connecticut)
Christoph Lehner (BNL)
Chulwoo Jung (BNL)

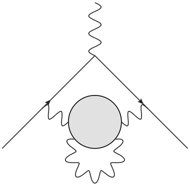
There is a tension of 3.7σ for the muon $a_\mu = (g_\mu - 2)/2$:

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 27.4 \underbrace{(2.7)}_{\text{HVP LO}} \underbrace{(2.6)}_{\text{HLbL}} \underbrace{(0.1)}_{\text{other}} \underbrace{(6.3)}_{\text{EXP}} \times 10^{-10}$$

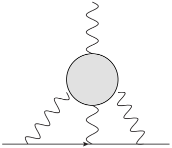
HVP here is by RBC/UKQCD 2018 (compatible with equally precise KNT 2018), HLbL is the "Glasgow consensus"

HVP LO





HLbL

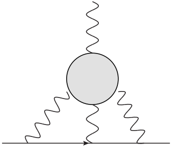


$\Leftarrow$ This talk

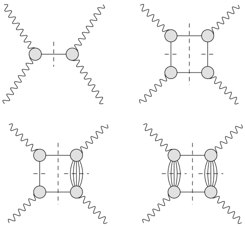
Experimental updates aim to reduce experimental errors by factor of 4 (Fermilab update in spring 2019)

Two new avenues for a model-independent value for the HLbL

Dispersive analysis +
Experimental/lattice input

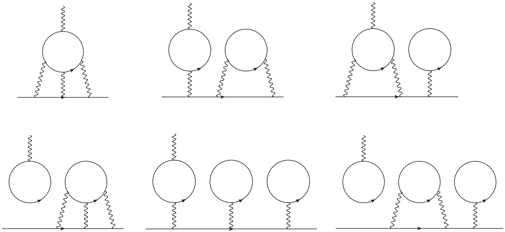


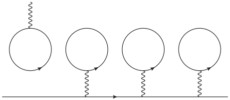
Direct lattice calculation



. . .

Truncation of cuts and states





7 quark-level topologies

Dispersive analysis

JHEP 1509 (2015) 074: Colangelo, Hoferichter, Procura, Stoffer

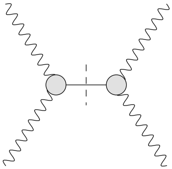
- Start with four-point function

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3) = -i\int d^4x\, d^4y\, d^4z\, e^{-i(q_1\cdot x+q_2\cdot y+q_3\cdot z)}\langle 0|T\{j^{\mu}_{\mathrm{em}}(x)j^{\nu}_{\mathrm{em}}(y)j^{\lambda}_{\mathrm{em}}(z)j^{\sigma}_{\mathrm{em}}(0)\}|0\rangle.$$

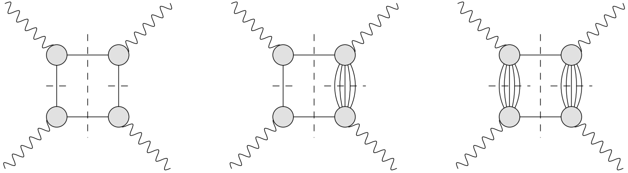
- A-priori 138 basic Lorentz structures (compare to 2 for HVP)
- Gauge invariance imposes 95 linear relations
- Special care needs to be taken (Tarrach) such that the resulting scalar functions are free of kinematic singularities that would complicate a dispersive discussion; a redundant basis satisfying this following Bardeen, Tung, and Tarrach with 54 elements can be chosen
- Crossing symmetry imposes additional constraints such that only **7** distinct structures remain

Organizing principle: systematic cuts and state truncation

- Estimate of truncation of this procedure is crucial and still being developed; ideas to use lattice for this are being explored (RBC 2018)
- Dominant contributions from pion-pole (needs $\pi \to \gamma^*\gamma^*$ form factors)



- next leading contribution from two-pion states (box topologies)



Recent results

- PRD94(2016)074507 (Mainz): Pion-pole contribution
  $a_\mu^{\pi-pole} = 6.50(83) \times 10^{-10}$ using a model parametrization of the $\pi \to \gamma^*\gamma^*$ form factor constrained by lattice data

$$\mathcal{F}^{\mathrm{LMD+V}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{\widetilde{h}_0\, q_1^2 q_2^2 (q_1^2+q_2^2) + \widetilde{h}_1 (q_1^2+q_2^2)^2 + \widetilde{h}_2\, q_1^2 q_2^2 + \widetilde{h}_5\, M_{V_1}^2 M_{V_2}^2\, (q_1^2+q_2^2) + \alpha\, M_{V_1}^4 M_{V_2}^4}{(M_{V_1}^2-q_1^2)(M_{V_2}^2-q_1^2)(M_{V_1}^2-q_2^2)(M_{V_2}^2-q_2^2)}$$

- JHEP1704(2017)161 (Colangelo et al.): Pion-box plus S-wave rescattering
  $a_\mu^{\pi-box} + a_\mu^{\pi\pi,\pi-pole\ LHC,J=0} = -2.4(1) \times 10^{-10}$

- arXiv:1805.01471 (Hoferichter et al.): Pion-pole contribution
  $a_\mu^{\pi-pole} = 6.26(30) \times 10^{-10}$ reconstructing $\pi \to \gamma^*\gamma^*$ form factor from $e^+e^- \to 3\pi, e^+e^-\pi^0$ and $\pi^0 \to \gamma\gamma$ width

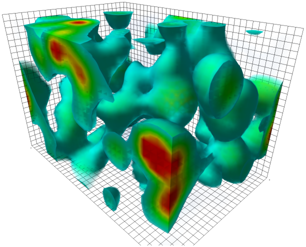
Combining these results one finds: $a_\mu^{\pi-pole} + a_\mu^{\pi-box} + a_\mu^{\pi\pi} = 3.9(3) \times 10^{-10}$

Compare to Glasgow consensus of $a_\mu^{\mathrm{HLbL}} = 10.5(2.6) \times 10^{-10}$ which also models contributions of heavier states and includes a matching with an high-energy quark picture. Control of truncation error very important.

Direct lattice calculation

Hadronic contributions from lattice QCD

- Simulate QFT in terms of fundamental quarks and gluons (QCD) on a supercomputer with discretized four-dimensional space-time lattice
- Hadrons are emergent phenomena of statistical average over background gluon configurations to which quarks are coupled
- In this framework draw diagrams only with respect to quarks, photons, and leptons; gluons and their effects are generated by the statistical average.



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

Computing resources

The RIKEN-BNL-Columbia (RBC) $g-2$ project has used on the order of 10^9 core hours (100k years on a single core) on the Mira supercomputer at Argonne, USQCD clusters at JLab and BNL, the BNL CSI KNL cluster, and the Oakforest and Hokusai supercomputers in Japan.

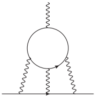
We have processed on the order of 5 petabytes of QCD data related to this project.

Top 10 positions of the 49th TOP500 in June 2017[11]

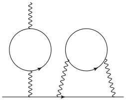
Rank	Rmax Rpeak (PFLOPS)	Name	Model	Processor	Interconnect	Vendor	Site country, year	
1	93.015 125.436	*Sunway TaihuLight*	Sunway MPP	SW26010	Sunway[12]	NRCPC	National Supercomputing Center in Wuxi China, 2016[12]	Li
2	33.863 54.902	*Tianhe-2*	TH-IVB-FEP	Xeon E5–2692, Xeon Phi 31S1P	TH Express-2	NUDT	National Supercomputing Center in Guangzhou China, 2013	Li
3	19.590 25.326	*Piz Daint*	Cray XC50	Xeon E5-2690v3, Tesla P100	Aries	Cray	Swiss National Supercomputing Centre Switzerland, 2016	Li
4	17.590 27.113	*Titan*	Cray XK7	Opteron 6274, Tesla K20X	Gemini	Cray	Oak Ridge National Laboratory United States, 2012	Li ba
5	17.173 20.133	*Sequoia*	Blue Gene/Q	A2	Custom	IBM	Lawrence Livermore National Laboratory United States, 2013	Li C
6	14.015 27.881	*Cori*	Cray XC40	Xeon Phi 7250	Aries	Cray	National Energy Research Scientific Computing Center United States, 2016	Li
7	13.555 24.914	*Oakforest-PACS*	Fujitsu	Xeon Phi 7250	Intel Omni-Path	Fujitsu	Kashiwa, Joint Center for Advanced High Performance Computing Japan, 2016	Li
8	10.510 11.280	*K computer*	Fujitsu	SPARC64 VIIIfx	Tofu	Fujitsu	Riken, Advanced Institute for Computational Science (AICS) Japan, 2011	Li
9	8.587 10.066	*Mira*	Blue Gene/Q	A2	Custom	IBM	Argonne National Laboratory United States, 2012	Li

7 quark-level topologies of direct lattice calculation

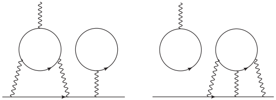
Hierarchy imposed by QED charges of dominant up- and down-quark contribution



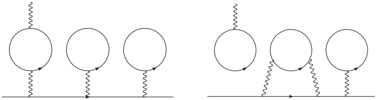
$Q_u^4 + Q_d^4 = 17/81$



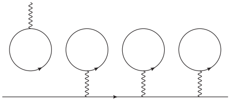
$(Q_u^2 + Q_d^2)^2 = 25/81$



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$

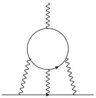


$(Q_u + Q_d)^4 = 1/81$

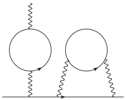
Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

7 quark-level topologies of direct lattice calculation

Hierarchy imposed by QED charges of dominant up- and down-quark contribution

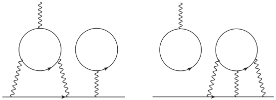


$Q_u^4 + Q_d^4 = 17/81$

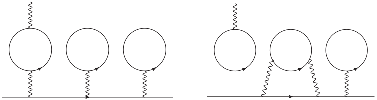


$(Q_u^2 + Q_d^2)^2 = 25/81$

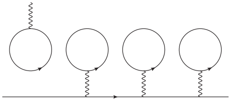
Dominant diagrams in top row: connected and leading disconnected diagram



$(Q_u^3 + Q_d^3)(Q_u + Q_d) = 9/81$



$(Q_u^2 + Q_d^2)(Q_u + Q_d)^2 = 5/81$



$(Q_u + Q_d)^4 = 1/81$

Further insight for magnitude of individual topologies can be gained by studying long-distance behavior of QCD correlation functions (Bijnens, RBC, ...)

Finite-volume and infinite-volume formulations

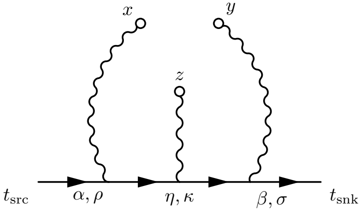
- a_μ^{HLbL} in finite-volume QCD and QED:
  - PRD93(2016)014503 (RBC/UKQCD): Connected diagram with $m_\pi = 171$ MeV; $a_\mu^{\mathrm{HLbL}} = 13.21(68) \times 10^{-10}$
  - PRL118(2017)022005 (RBC/UKQCD): Connected and leading disconnected diagram with $m_\pi = 139$ MeV; $a_\mu^{\mathrm{HLbL}} = 5.35(1.35) \times 10^{-10}$ (potentially large finite-volume systematics)

  Strategy: extrapolate away $1/L^n$ ($n \geq 2$) errors

- a_μ^{HLbL} in finite-volume QCD and infinite-volume QED:
  - Method proposed and successfully tested against the lepton-loop analytic result: arXiv:1510.08384 (Mainz), arXiv:1609.08454 (Mainz)
  - Similar method plus subtraction scheme to reduce systematic errors; successfully tested against lepton-loop analytic result: PRD96(2017)034515 (RBC/UKQCD)

  Strategy: FV errors exponentially suppressed but still may be significant, effect on noise?

Finite-volume QED (PRD93(2016)014503 (RBC/UKQCD))



- The finite-volume $\mathrm{QED_L}$ prescription uses the photon propagator

$$G_{\mathrm{L}}^{\mu\nu}(x) = \frac{\delta^{\mu\nu}}{V} {\sum_k}' \frac{1}{\hat{k}^2} e^{ikx} \,, \tag{1}$$

  where $\hat{k}^2 = \sum_\mu 4\sin^2(k_\mu/2)$ and $V = \prod_\mu L_\mu$ with lattice dimensions L_μ. The sum is over all momenta with components $k_\mu = 2\pi n_\mu / L_\mu$ with $n_\mu \in [0, \ldots, L_\mu - 1]$ and the restriction that $k_0^2 + k_1^2 + k_2^2 \neq 0$.
- For fixed x and y can get result for all z in $\mathcal{O}(V \log V)$ time using convolutions starting at t_{src} and t_{snk}; has statistical advantage for leading disconnected diagram (M^2 trick)

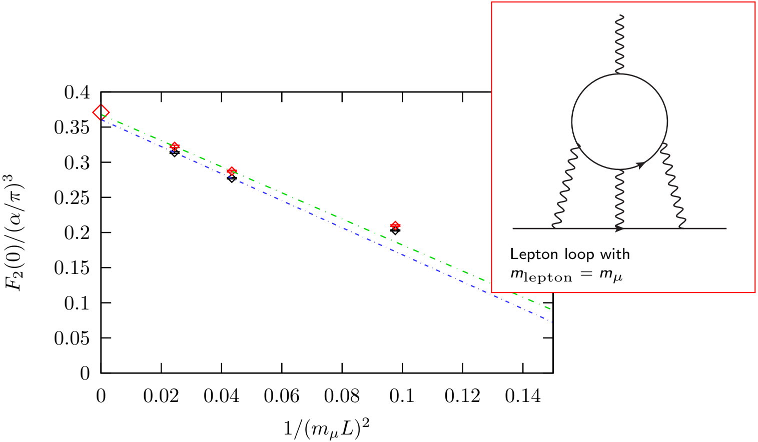
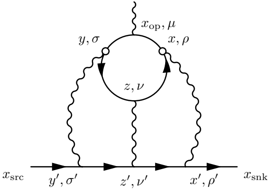
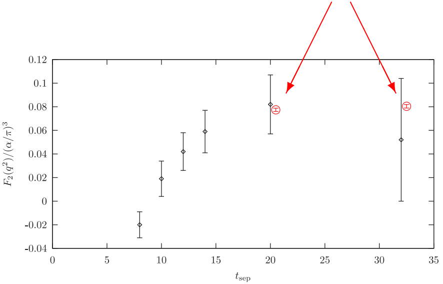


Figure 11. Results for $F_2(0)$ from QED connected light-by-light scattering. These results have been extrapolated to the $a^2 \to 0$ limit using two methods. The upper points use the quadratic fit to all three lattice spacings shown in Fig. 10 while the lower point uses a linear fit to the two left most points in that figure. Here we extrapolate to infinite volume using the linear fits shown to the two, left-most of the three points in each case.

PRD93(2015)014503 (RBC/UKQCD):

New sampling strategy with 10x reduced noise for same cost (red versus black):





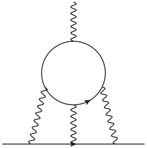
Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

PRL118(2016)022005 (RBC/UKQCD):

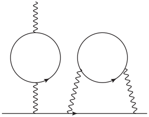
- Calculation at physical pion mass with finite-volume QED prescription (QED_{L}) at single lattice cutoff of $a^{-1} = 1.73$ GeV and lattice size $L = 5.5$ fm.

- Connected diagram:



$$a_\mu^{\mathrm{cHLbL}} = 11.6(0.96) \times 10^{-10}$$

- Leading disconnected diagram:

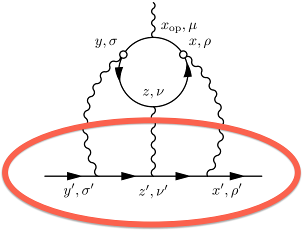


$$a_\mu^{\mathrm{dHLbL}} = -6.25(0.80) \times 10^{-10}$$

- Large cancellation expected from pion-pole-dominance considerations is realized: $a_\mu^{\mathrm{HLbL}} = a_\mu^{\mathrm{cHLbL}} + a_\mu^{\mathrm{dHLbL}} = 5.35(1.35) \times 10^{-10}$

Potentially large systematics due to finite-volume QED!

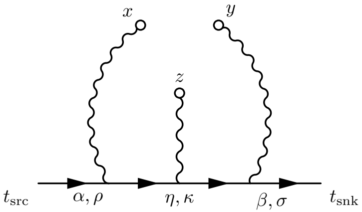
Infinite-volume QED prescription (QED_∞)



Remove power-law like finite-volume errors by computing the muon-photon part of the diagram in infinite volume (C.L. talk at lattice 2015 and Green et al. 2015, PRL115(2015)222003; Asmussen et al. 2016, PoS,LATTICE2016 164)

Now completed PRD96(2017)034515 (RBC/UKQCD) with improved weighting function.

Details:



We define

$$i^3 \mathcal{G}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \mathfrak{G}_{\sigma,\kappa,\rho}(y,z,x) + \text{other 4 permutations}\,.$$

and add the Hermitian conjugate with permuted indices (does not alter F_2 but makes this kernel infrared finite)

$$\mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) \quad = \quad \frac{1}{2}\mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \frac{1}{2}[\mathfrak{G}_{\kappa,\sigma,\rho}(z,y,x)]^\dagger$$

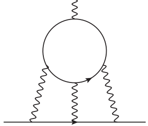
For $m_{\mathrm{line}} = 1$ this yields the kernel

$$
\begin{aligned}
\mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) &= \frac{\gamma_0+1}{2} i\gamma_\sigma \left(-\,\not{\partial}_y + \gamma_0 + 1\right) i\gamma_\kappa \left(\not{\partial}_x + \gamma_0 + 1\right) i\gamma_\rho \frac{\gamma_0+1}{2} \\
&\quad \times \frac{1}{4\pi^2} \int d^4\eta \frac{1}{(\eta-z)^2} f(\eta-y) f(x-\eta).
\end{aligned}
$$

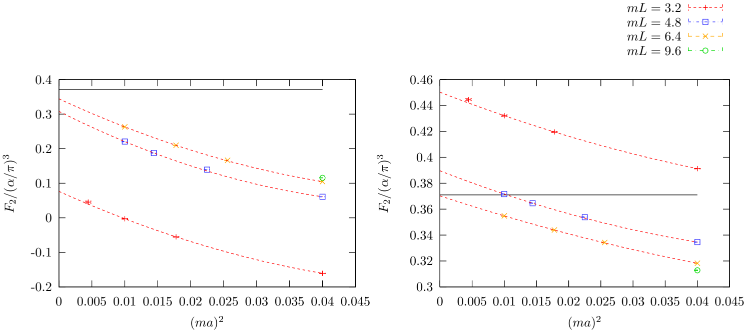
Due to current conservation, we can also devise a subtraction scheme that we found suppresses significantly finite-volume and discretization errors (demonstrated in the lepton loop case)

$$
\mathfrak{G}^{(2)}_{\rho,\sigma,\kappa}(x,y,z) = \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,z) - \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(x,y,y) + \mathfrak{G}^{(1)}_{\rho,\sigma,\kappa}(y,y,y)
$$

Test of lepton-loop for infinite-volume method

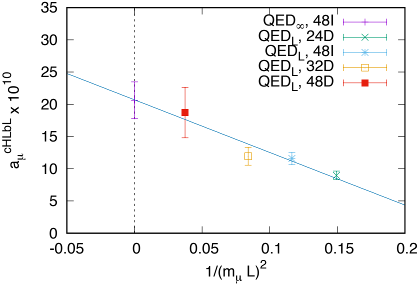


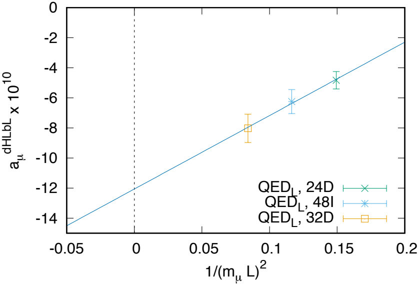
Lepton loop with
$m_{\text{lepton}} = m_\mu$

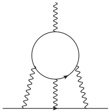


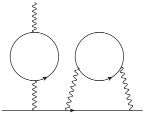
Without subtraction (left), with subtraction (right)

Preliminary QCD results for infinite-volume extrapolation (RBC/UKQCD 2018)



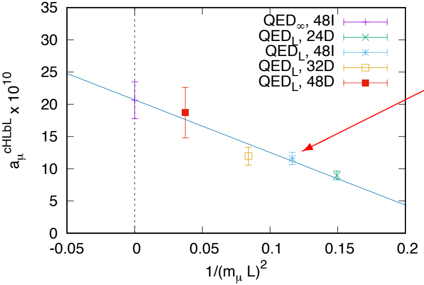


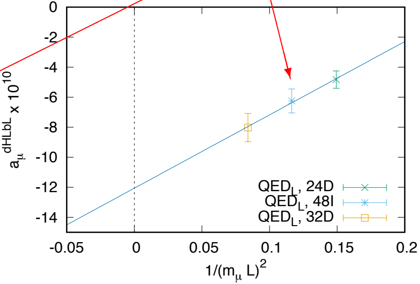


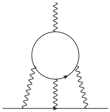


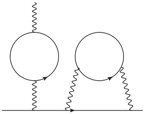
Preliminary QCD results for infinite-volume extrapolation (RBC/UKQCD 2018)

Data used for finite-volume result in PRL118(2016)022005









Roadmap to complete first-principles light-by-light calculation with all errors controlled (RBC/UKQCD 2018)

- Calculation of connected plus leading disconnected diagram at physical pion mass completed
- Infinite-volume extrapolation done (to be published)
- Discretization errors are now controlled for (four different lattice spacings over two different actions, to be published)
- Calculation of sub-leading disconnected diagrams, starting with 3-1 topology started within next month or so
- Crosscheck of dispersive versus lattice (see, e.g., arXiv:1712.00421) desirable

Summary

Summary

- Hadronic light-by-light contribution precision needs to be improved for Fermilab E989 target precision
- A model-independent first-principles calculation is needed: dispersive methods or lattice QCD
- Dispersive one and two-pion intermediate states essentially done
- Truncation error of dispersive method challenging to estimate; lattice methods for this estimate under development
- Pure lattice calculation at physical pion mass of connected and leading disconnected contribution completed, publication of infinite-volume and continuum limit imminent (RBC/UKQCD 2018)
- 5 sub-leading disconnected contributions need to be controlled as well, will be started in next month or so