

# Status and outlook of Lattice calculations for $R(D^{(*)})$ and $R_{K^{(*)}}$

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# Introduction: Lattice QCD

$$\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times \underbrace{(\text{matrix elements})}_{\text{lattice QCD}}$$

Parametrize MEs in terms of decay constants, form factors, bag parameters, ...

**Lattice QCD**: Numerical evaluation of QCD path integral (rely only on first principles) using Monte Carlo methods.

Phenomenology (in particular **flavour physics**) needs **precise** lattice QCD calculations →

- \* Control and reliably estimate systematic errors.
- \* Not everything can be calculated **precisely** using lattice techniques, only some processes:

with estable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron

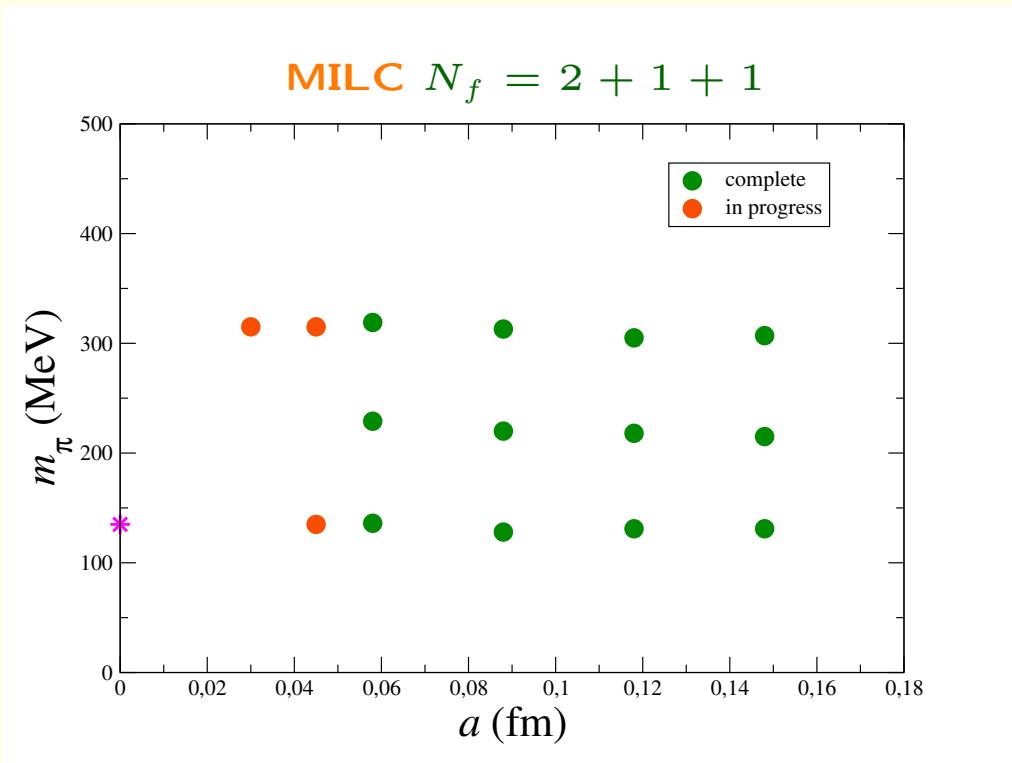
# Introduction: Lattice QCD

Development of new methods is allowing to increase the scope of LQCD calculations:

- \* Baryons
- \* Nonleptonic decays ( $K \rightarrow \pi\pi\dots$ )
- \* Resonances
- \* Scattering
- \* Long-distance effects
- \* QED effects ...

# Introduction: Lattice QCD

## Combined chiral-continuum extrapolation



Many lattice collaborations doing now simulations with **physical light-quark masses**; **PACS-CS, BMW, MILC, RBC/UKQCD, ETM...**

ChPT techniques still necessary to reduce errors and/or correct/estimate systematic effects: light and heavy quark discretization, finite volume, isospin-breaking, mass mistunings, ...

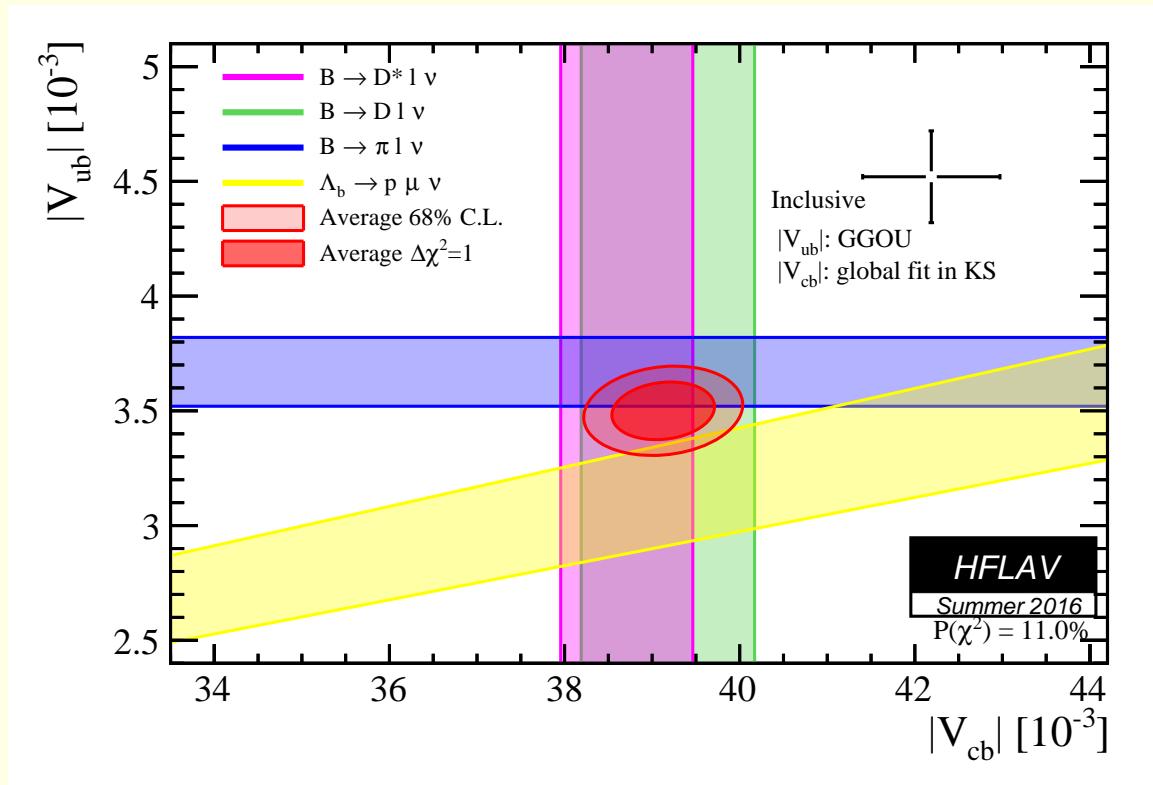
\* For  $B, D$  processes use Heavy Meson ChPT: ChPT +  $1/M$  expansion

# Next generation of gauge configurations: isospin-breaking ( $N_f = 1 + 1 + 1 + 1$ ) and QED+QCD **BMW, QCDSF, RBC, MILC ...**

# **Charged-current $b$ decays**

# Exclusive vs inclusive $|V_{ub}|$ and $|V_{cb}|$

Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements  $|V_{cb}|$  and  $|V_{ub}|$  at the  $\sim 3\sigma$  level.



$|V_{cb}|$  from exclusive  $B$  decays ( $w = v_B \cdot v_D$  velocity transfer to the leptonic pair)

$$\begin{aligned} \frac{d\Gamma(B \rightarrow D^* l \nu)}{dw} &= (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2 \\ \frac{d\Gamma(B \rightarrow D l \nu)}{dw} &= (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2 \end{aligned}$$

## LQCD inputs

- \*  $B \rightarrow \pi \ell \nu$ :  $f_+(q^2)$  ( $f_0(q^2)$ )
- \*  $\Lambda_b \rightarrow p \mu \nu / \Lambda_b \rightarrow \Lambda_c \mu \nu$ :  
Six form factors for each channel
- \*  $B \rightarrow D^* \ell \nu$ :  $\mathcal{F}(\omega = 1)$
- \*  $B \rightarrow D \ell \nu$ :  $\mathcal{G}(\omega)$  (related to  $f_+(q^2)$ )

# Exclusive determination of $|V_{cb}|$

$|V_{cb}|$  extracted from exclusive  $B$  decays ( $w = v_B \cdot v_D$  is the velocity transfer to the leptonic pair)

$$\frac{d\Gamma(B \rightarrow D^* l \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2$$
$$\frac{d\Gamma(B \rightarrow D l \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

\* How to parametrize the  $\omega$  dependence?

Model-independent  $z$ -expansion: map  $\omega$  onto a complex variable  $z$  via the conformal transformation  $z = (\sqrt{\omega + 1} - \sqrt{\omega_0 + 1}) / (\sqrt{\omega + 1} + \sqrt{\omega_0 + 1})$

\* Coefficients in the  $z$ -expansion are subject to unitarity bounds based on analyticity.

**BCL (Bourrely-Caprini-Lellouch)**, 0807.2722

**BGL (Boyd-Grinstein-Lebed)**, hep-ph/9504235, hep-ph/9508211, hep-ph/9705252

**CLN (Caprini-Lellouch-Neubert)**, hep-ph/9712417: + model-dependent NLO HQET constraints to reduce the error:

heavy-quark corrections (neglecting  $(\Lambda/m_c)^2$ ) underestimated?

# Form factors for $B \rightarrow D\ell\nu$ with $\ell = e, \mu, \tau$

Exclusive  $B \rightarrow D$  decays without disregarding lepton masses (SM)

$$\frac{d\Gamma(B \rightarrow D\ell\nu)}{dw} = (\text{known}) \cdot |V_{cb}|^2 |f_+^2(q^2)|^2 + (\text{known}) \cdot |V_{cb}|^2 |m_\ell^2 f_0^2(q^2)|^2$$

- \* Calculate the form factors  $f_0$  and  $f_+$  (also  $f_T(q^2)$ ) at low recoil (high  $q^2$ ).
- \* z-expansion: model-independent parametrization of  $q^2$  dependence.

Two LQCD calculations of  $f_+$  and  $f_0$

**FNAL/MILC** 1503.07237

MILC  $N_f = 2 + 1$  asqtad ensembles:  
14 ensembles

$u, d, s$  valence quarks: asqtad

$c, b$  valence quarks: clover (Fermilab)

**z-expansion (BGL parametrization)**  
to go to high recoil region.

**HPQCD** 1505.03925

MILC  $N_f = 2 + 1$  asqtad ensembles:  
5 ensembles

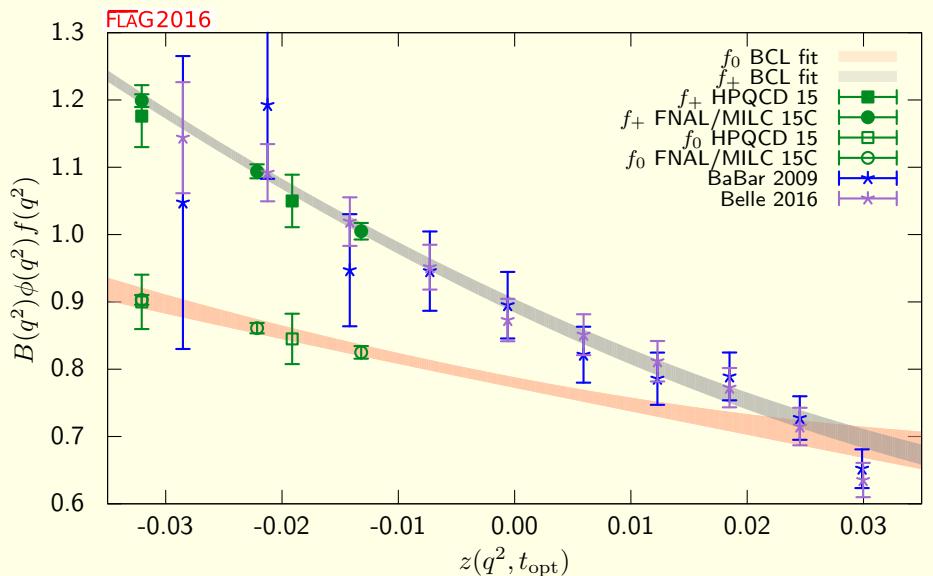
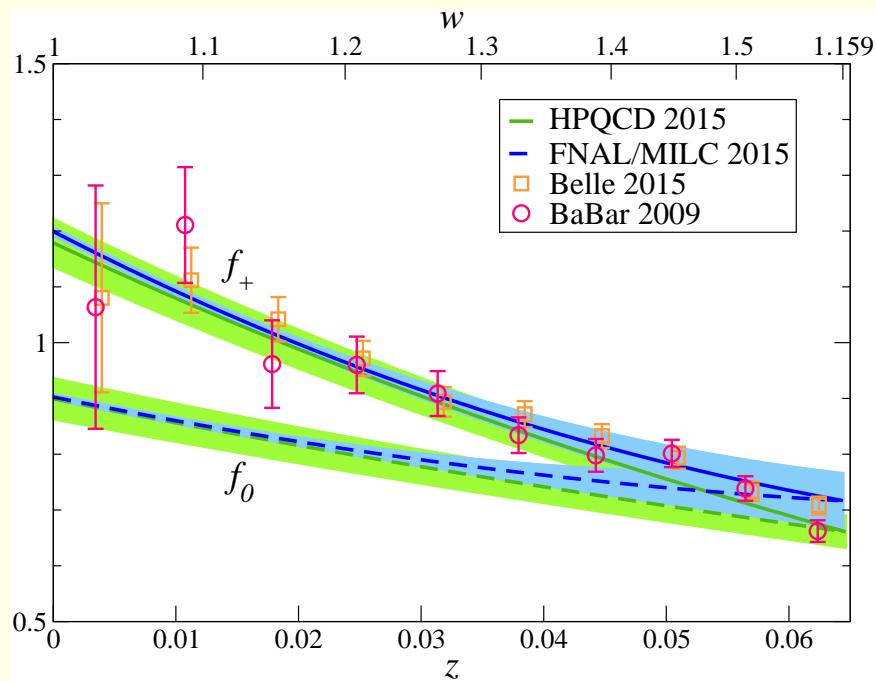
$u, d, s, c$  valence quarks: HISQ

$b$  valence quarks: NRQCD

**z-expansion (BCL parametrization)**  
to go to high recoil region.

Both quote results as coefficients of the expansion + covariance matrix.

# Form factors for $B \rightarrow D\ell\nu$ with $\ell = e, \mu, \tau$



**FLAG** global fit to lattice and experimental data with **BCL** parametrization.

$$(|V_{cb}| = 40.1(1.0) \cdot 10^{-3})$$

\* Combined **BGL** fit to experimental and lattice data [Bigi&Gambino, 1606.08030](#)

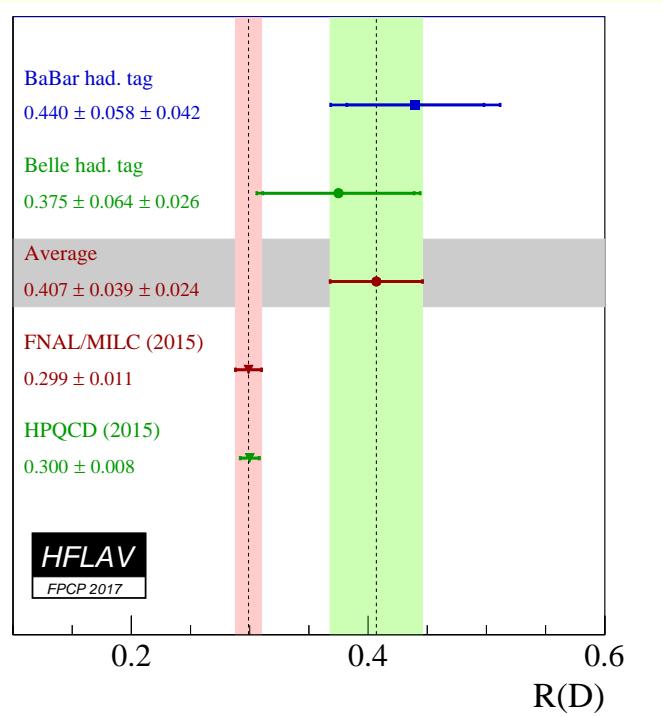
$$|V_{cb}| = 40.49(97) \cdot 10^{-3}$$

\* Lattice form factors uncertainties ( $\sim 1.2\%$ ) smaller than experiment.

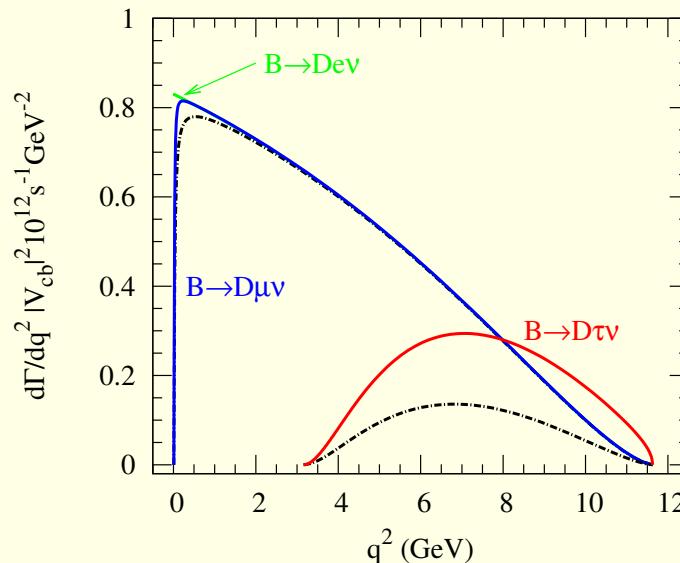
# $B \rightarrow D\ell\nu$ : Lepton Flavor Universality tests

Lattice  $f_+^{B \rightarrow D}(q^2)$  and  $f_0^{B \rightarrow D}(q^2)$  form factors can be used to calculate:

$$R(D) \equiv \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu)} \text{ FLAG2016, only lattice} = 0.300(8)$$



Very sensitive to  $f_0(q^2)$



Theory results  $\sim 2.3\sigma$  lower than experimental average

Critical to control reliability of form factor parametrization at low  $q^2$

\* Global fit experiment+lattice data with BGL parametrization:  $R(D) = 0.299(3)$ ,  $2.4\sigma$

Bigi & Gambino 1606.08030. Similar results from Bernlochner, Ligeti, Papucci, Robinson 1703.05330

# $B \rightarrow D\ell\nu$ : Lepton Flavor Universality tests

2018: Soft-photon corrections to  $R(D)$ , could amplify  $R(D^{+(0)})$  by  
 $\leq 5.5\%(3.6\%)$  Boer, Kitara, Nisandzic 1803.05881

Experimental uncertainty on  $|V_{cb}|$  from  $B \rightarrow D^{(*)}\ell\bar{\nu}$  expected to be  $\leq 1\%$  at Belle II. P.Urquijo talk at “Challenges in semileptonic B decays”, MITP, 04/2018

In progress

\* FNAL/MILC, HPQCD:  $N_f = 2 + 1 + 1$  HISQ ensembles, HISQ light quarks, Fermilab  $b, c$  (FNAL/MILC), NRQCD  $b$  and HISQ  $c$  (HPQCD).

\*\* Longer term: Going to smaller  $q^2$

E. McLean, C. Davies, B. Colquhoun, A. Lytle 1711.03487

\* LANL/SNU:  $N_f = 2 + 1 + 1$  HISQ ensembles, HISQ light quarks, Oktay-Kronfeld (OK)  $b, c$ . J. Bailey, J. Leem, W. Lee, Y. Jang 1612.09081

\* RBC/UKQCD: Domain wall  $N_f = 2 + 1$  configurations, domain wall light and  $c$ , RHQ  $b$ . J. Flynn et al. 1612.05112

# $B \rightarrow D^* \ell \nu$ at zero recoil

$$\frac{d\Gamma(B \rightarrow D^* l \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2$$

Until now, lattice data for  $\mathcal{F}(\omega)$  only available at zero recoil  $\omega = 1$  ( $q_{max}^2$ )

- \* One form factor needed  $\mathcal{F}(1) = h_{A_1}(1)$ , extracted from double ratio

$$R_{A_1}(1) = \frac{\langle D^* | A_1 | \bar{B} \rangle \langle \bar{B} | A_1 | \bar{D}^* \rangle}{\langle D^* | V_0 | D^* \rangle \langle \bar{B} | V_0 | \bar{B} \rangle} = |h_{A_1}(1)|^2$$

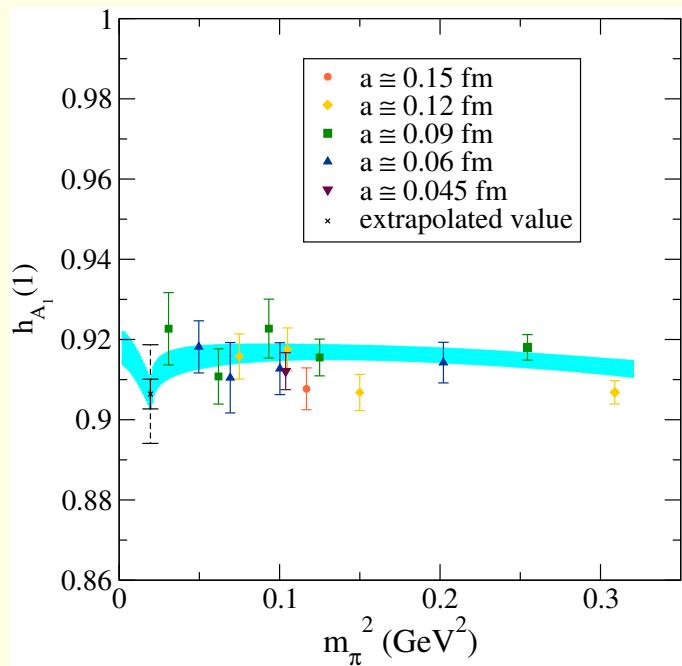
Luke's theorem: result protected from NLO power corrections.

- \* Suppression factor at zero recoil limits experimental measurements:  
Experimental data extrapolated to  $\omega = 1$  using CLN parametrization

$$\rightarrow \bar{\eta}_{EW} |V_{cb}| \mathcal{F}(\omega = 1)$$

# $B \rightarrow D^* \ell \nu$ at zero recoil

FNAL/MILC 1403.0635



MILC  $N_f = 2 + 1$  asqtad ensembles  
 $u, d, s$  valence quarks: asqtad  
 $c, b$  valence quarks: clover (Fermilab)

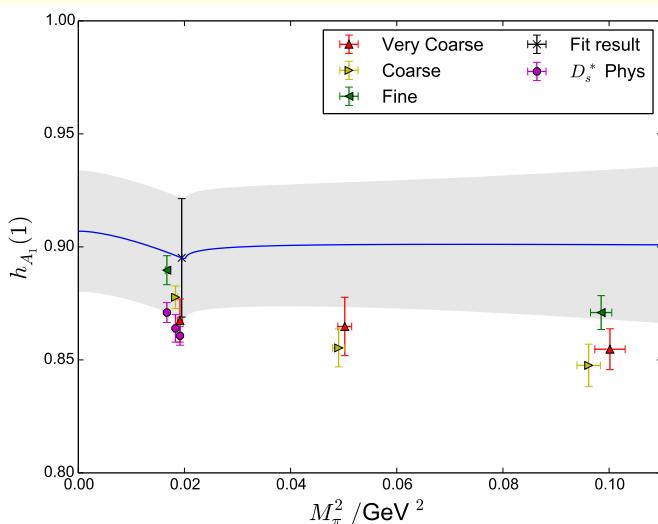
$$\mathcal{F}(1) = 0.906(4)(12)$$

Dominant error: Discretization  
 $a\Lambda^2/m_c, a^2\Lambda^2$

\*\* Cusp at the physical point due to  $D^* \rightarrow D\pi$  and  $m_{D^*} - m_D \sim m_\pi$  included using ChPT with  $D^* D\pi$  coupling as input.

Two LQCD calculations (stat. independent)

HPQCD 1711.11013



MILC  $N_f = 2 + 1 + 1$  HISQ ensembles  
 $u, d, s, c$  valence quarks: HISQ  
 $b$  valence quarks: NRQCD

$$\mathcal{F}(1) = 0.895(10)(24)$$

Dominant error: matching  $\alpha_s^2$

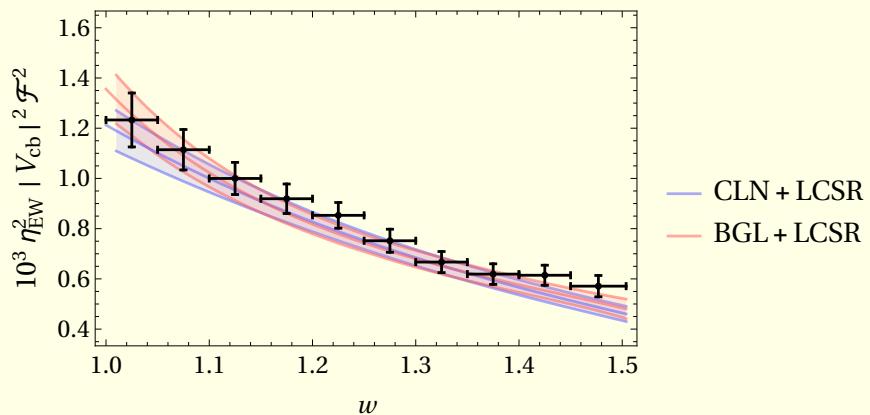
$$\mathcal{F}^{B_s \rightarrow D_s^*}(1) = 0.883(12)(28)$$

# $B \rightarrow D^* \ell \nu$ at zero recoil

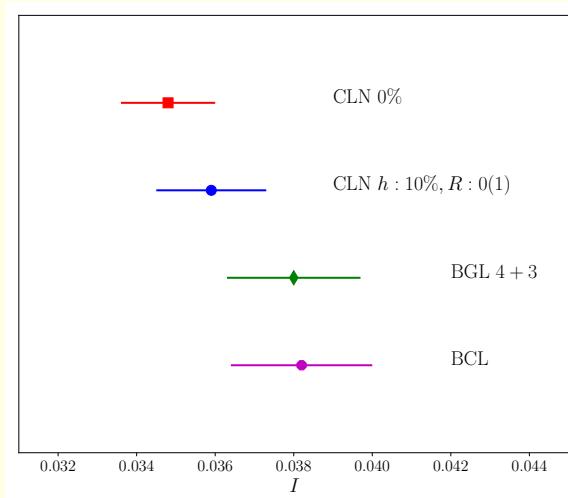
2017: For the first time, **Belle** 1702.01521 provided unfolded fully-differential decay rate and associated covariance matrix.

Intrinsic uncertainties of CLN parametrization can no be neglected.

**Bigi, Gambino & Schacht** 1703.06124



**HPQCD** 1711.11013



CLN underestimates low recoil points

$$|\bar{\eta}_{EW} V_{cb}| h_{A_1}(1)$$

**Bigi, Gambino & Schacht** 1703.06124, 1707.09509, **Grinstein & Kobach** 1703.08170

Belle+LQCD:  $|V_{cb}|_{CLN} = 38.2(1.5) \cdot 10^{-3} \rightarrow |V_{cb}|_{BGL} = 41.7(2.0) \cdot 10^{-3}$

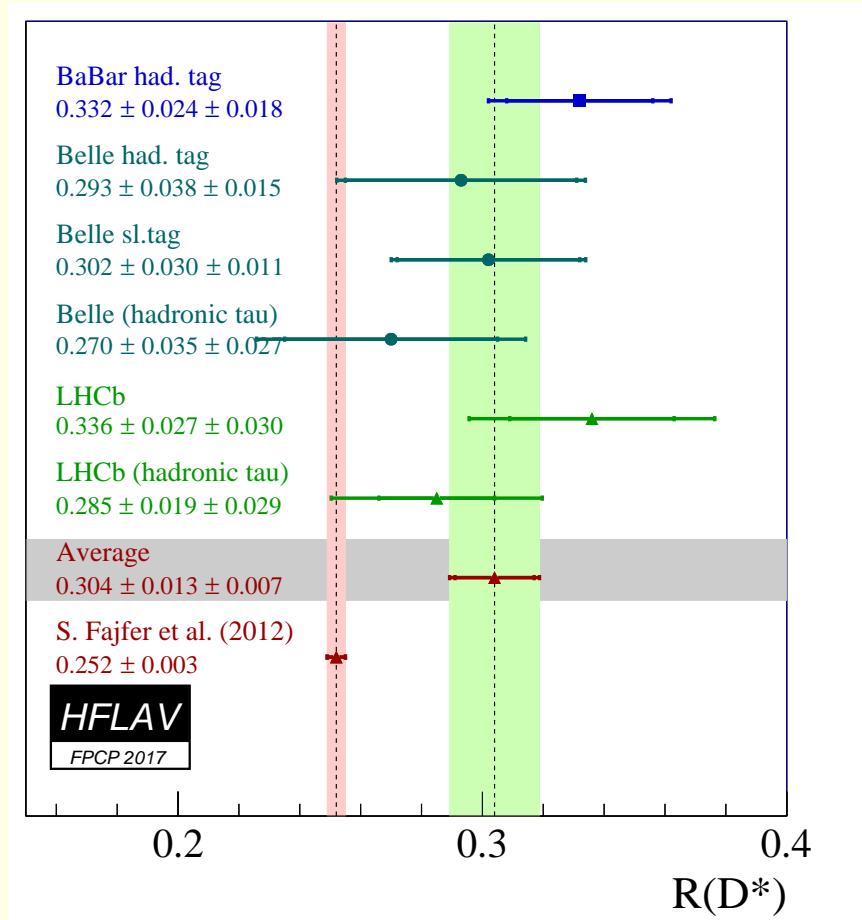
Belle+LQCD+LCSR:  $|V_{cb}|_{CLN} = 38.2(1.4) \cdot 10^{-3} \rightarrow |V_{cb}|_{BGL} = 40.4(1.6) \cdot 10^{-3}$

BGL fits agree with inclusive  $|V_{cb}|$

\* Large difference not necessarily true for other exp. data sets.

# $B \rightarrow D^* \ell \nu$ : Lepton Flavor Universality tests

No lattice inputs available to calculate  $R(D^*) \equiv \frac{\mathcal{B}(B \rightarrow D^* \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^* \ell \nu)}$



$\sim 3.4\sigma$  SM-experiment disagreement

**Needs reliable LQCD form factors at non-zero recoil**

Stabilize fits and reduce errors of  $|V_{cb}|$  and  $R(D^*)$

\* SM calculation in [Fajfer et al 1203.2654](#) based on exp.  $B \rightarrow D^*$  measurements which use CLN parametrization.

\* [Bigi, Gambino & Schacht 1707.09509](#) use BGL param. instead with [Belle](#) data

$$R(D^*) = 0.260(8) \quad (3.4 \rightarrow 2.6\sigma)$$

Systematic errors larger due to accounting for unknown NNLO corrections.

A LQCD calculation of the form factor  $P_1(\omega = 1)$  (function of  $h_{A_1, A_2, A_3}$ ) could reduce error by a factor of 2 [Bigi, Gambino & Schacht 1707.09509](#)

# $B \rightarrow D^* \ell \nu$ beyond zero recoil

In general, the form factors needed for  $\omega \neq 1$  are

$$\begin{aligned} \frac{\langle D^*(p_{D^*}, \varepsilon^\nu) | \mathcal{V}^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} &= \frac{1}{2} \varepsilon^{\nu*} \epsilon_{\rho\sigma}^{\mu\nu} v_B^\rho v_{D^*}^\sigma h_V(\omega) \\ \frac{\langle D^*(p_{D^*}, \varepsilon^\nu) | \mathcal{A}^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} &= \frac{i}{2} \varepsilon^{\nu*} \left[ g^{\mu\nu} (1 + \omega) h_{A_1}(\omega) - v_B^\nu (v_B^\mu h_{A_2}(\omega) + v_{D^*}^\mu h_{A_3}(\omega)) \right] \end{aligned}$$

$h_{A_1}(\omega)$  (and its error) will still dominate extraction of  $|V_{cb}|$  and  $R(D^*)$

**In progress:** [FNAL/MILC 1710.09817](#)

$N_f = 2 + 1$  asqtad (same ensembles, statistics and actions as for the zero recoil  
2014 [FNAL/MILC](#) calculation)

Use double ratios

$$\begin{aligned} \frac{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle \langle \bar{B}(0) | A_1 | D^*(p_\perp) \rangle}{\langle D^*(0) | V_0 | D^*(0) \rangle \langle \bar{B}(0) | V_0 | \bar{B}(0) \rangle} &= \left( \frac{1+\omega}{2} \right)^2 |h_{A_1}(\omega)|^2; & \frac{\langle D^*(p_\parallel) | V_1 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle} &= \sqrt{\frac{\omega-1}{\omega+1}} \frac{h_V(\omega)}{h_{A_1}(\omega)} \\ \frac{\langle D^*(p_\parallel) | A_4 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_4 | \bar{B}(0) \rangle} &= \frac{\sqrt{\omega^2-1} (1 - h_{A_2}(\omega) + \omega h_{A_3}(\omega))}{(1+\omega) h_{A_1}(\omega)}; & \frac{\langle D^*(p_\parallel) | A_1 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle} &= \omega - \frac{(\omega^2-1) h_{A_3}(\omega)}{(1+\omega) h_{A_1}(\omega)} \end{aligned}$$

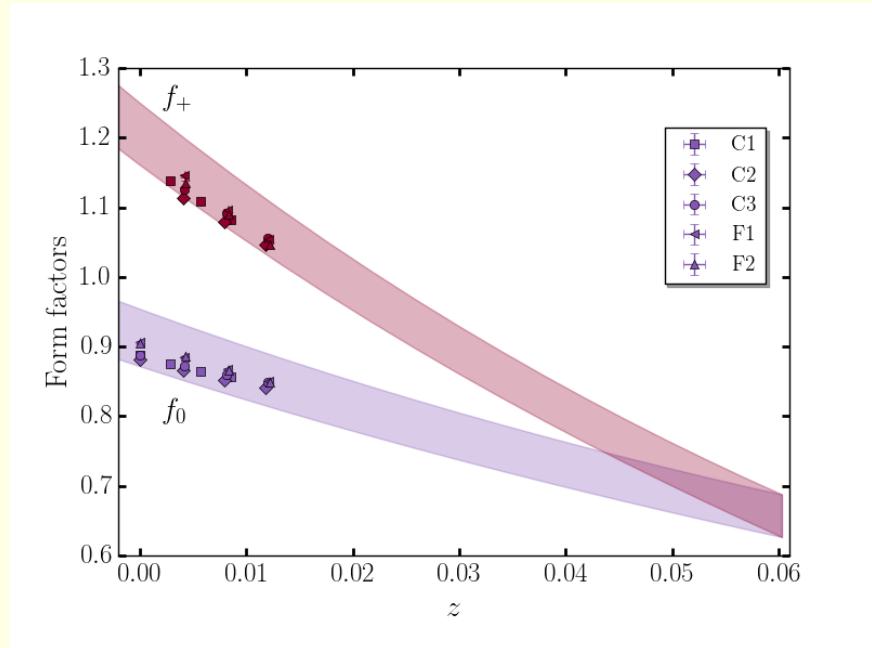
# $B \rightarrow D^* \ell \nu$ beyond zero recoil

More in progress:

- \* At zero recoil **LANL/SNU**:  $N_f = 2 + 1 + 1$  HISQ ensembles, HISQ light quarks, Oktay-Kronfeld (OK)  $b, c$ . **J. Bailey et al** 1711.01777, 1711.01786
- \*\* Can reduce  $c$  discretization error in **FNAL/MILC** calculation (dominant)  
 $1\% \rightarrow 0.4\%$
- \* **RBC/UKQCD**: Domain wall  $N_f = 2 + 1$  configurations, domain wall light and  $c$ , RHQ  $b$ .
- \* **FNAL/MILC**:  $N_f = 2 + 1 + 1$  HISQ ensembles, HISQ light quarks, Fermilab  $b, c$

# More $b \rightarrow c\ell\nu$ decays

$B_s \rightarrow D_s \ell \nu$  New HPQCD 1703.09728



$N_f = 2 + 1$  MILC configurations + HISQ  
c + NRQCD b

$$R(D_s) = 0.301(6)$$

- \* No sensitivity to spectator quark ( $R(D) = 0.300(8)$ )
- \* + Previous  $B \rightarrow D$  results → Ratios used for fragmentation fractions  $f_q/f_s$

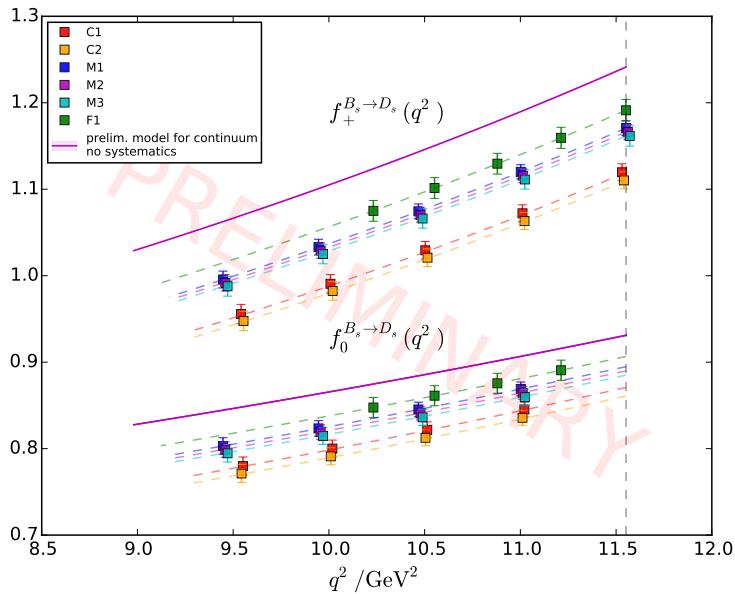
Consistent with previous results by FNAL/MILC 1703.09728 1202.6346 and ETMC 1310.5238

# More $b \rightarrow c\ell\nu$ decays

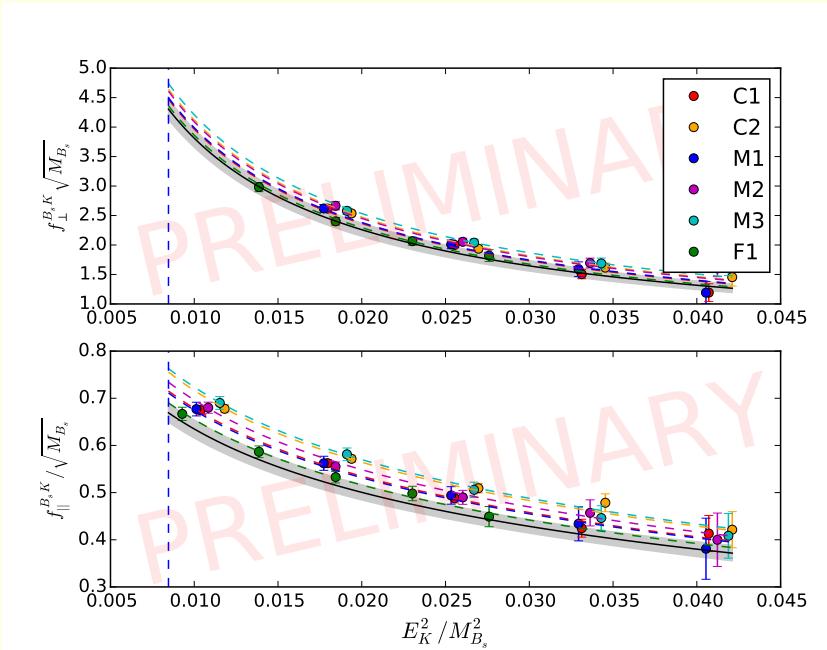
$B_s \rightarrow D_s \ell\nu$  &  $B_s \rightarrow K \ell\nu$

In progress: RBC/UKQCD

Domain wall  $N_f = 2 + 1$  configurations, domain wall light and  $c$ , RHQ  $b$ .



Will provide  $R(D_s)$



Experimental data from LHCb expected for (M. Calvi talk at Challenges in semileptonic B decays, MITP, 2018)  $\frac{\mathcal{B}(B_s^0 \rightarrow K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}$

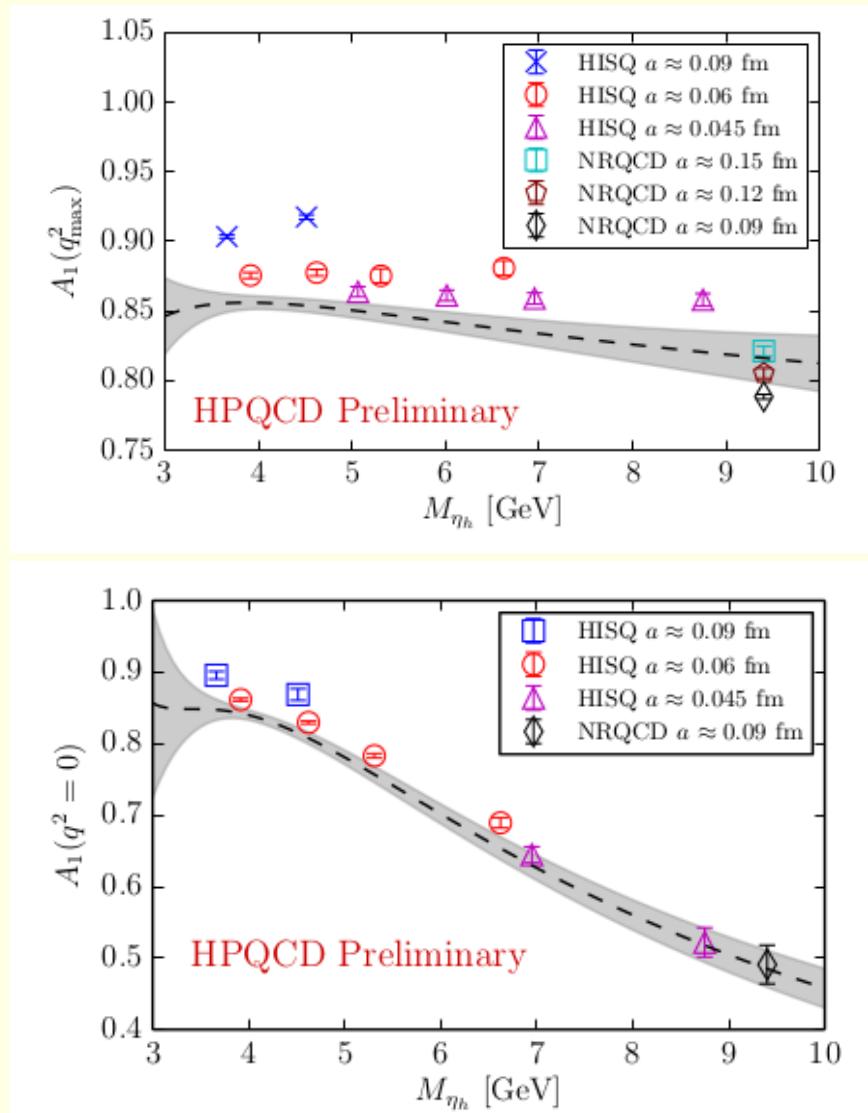
Plus LQCD form factors will provide another determination of  $|V_{ub}|/|V_{cb}|$ .

In progress: FNAL/MILC 1711.08085, ALPHA, JLQCD

# More $b \rightarrow c \ell \nu$ decays

$B_c \rightarrow J/\psi \ell \nu$  4 independent form factors

In progress/future plans: **HPQCD, A. Lytle** “Instant workshop on B-mesons anomalies”



Test lepton universality **LHCb 1711.05623**

$$R(B_c)_{\text{exp}} = \frac{\mathcal{B}(B_c \rightarrow J/\psi \tau \nu)}{\mathcal{B}(B_c \rightarrow J/\psi \mu \nu)} = 0.71 \pm 0.25$$

Help to shed light on the existing anomalies

Two approaches (on  $N_f = 2 + 1 + 1$  MILC ensembles), also for  $B \rightarrow D(D^*)$ :

- \* Relativistic (HISQ) on finer lattices with  $m_h \leq m_b$
- \* NRQCD directly at  $m_b$
- \* Can cover full  $q^2$  range accurately and compare relativistic and non relativistic approaches **A. Lytle, B. Colquhoun et al (HPQCD)**

Good agreement of both approaches

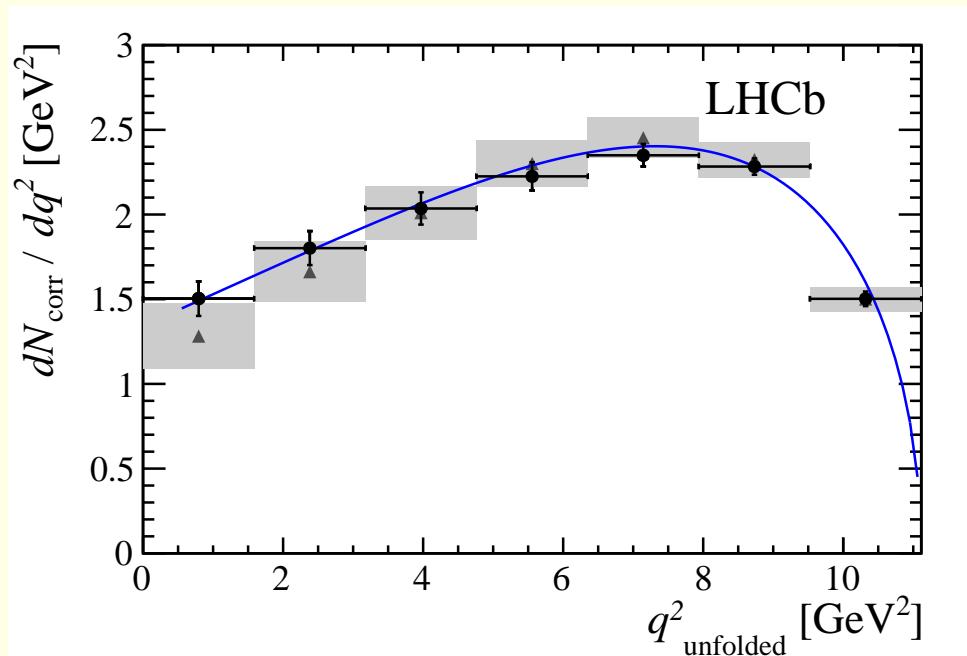
# More $b \rightarrow c\ell\nu$ decays

$\Lambda_b \rightarrow \Lambda_c \ell \nu$

W. Detmold, C. Lehner, S. Meinel 1503.01421,

A. Datta, S. Kamali, S. Meinel, A. Rashed 1702.02243

- \*  $N_f = 2 + 1$  calculation on RBC/UKQCD ensembles. Domain wall light + anisotropic clover  $c, b$ .
- \* Ten form factors in total.



Black points: decay rate measured by LHCb  
1709.01920

Gray rectangles: Lattice prediction W. Detmold,  
C. Lehner, S. Meinel 1503.01421

$$R(\Lambda_c) \equiv \frac{\Gamma(\Lambda_b \rightarrow \Lambda_c \tau^- \bar{\nu}_\tau)}{\Gamma(\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu)} = 0.3328 \pm 0.0074_{\text{stat.}} \pm 0.0070_{\text{syst.}}$$

(3.1% error)

A measurement of  $R(\Lambda_c)$  can add strong constraints to NP models  
(correlations with  $R(D^{(*)})$ ) A. Datta, S. Kamali, S. Meinel, A. Rashed 1702.02243

## More $b \rightarrow c\ell\nu$ decays

$\Lambda_b \rightarrow \Lambda_c^* \ell \nu$

S. Meinel, G. Rendon In progress.

Also in progress:  $\Lambda_b \rightarrow \Lambda_c$  results with errors a factor of 2 smaller  
expected for 2020 S. Meinel Challenges in semileptonic B decays workshop, Mainz 2018

# Neutral-current $b$ decays

# Form factors for $B \rightarrow K(\pi)\ell^+\ell^-$

Flavor-changing neutral currents  $b \rightarrow q$  transitions are potentially sensitive to NP effects  $B \rightarrow K^*\gamma$ ,  $B \rightarrow K^{(*)}\ell^+\ell^-$ ,  $B \rightarrow \pi\ell^+\ell^-$

For  $B \rightarrow P\ell\ell$ , hadronic contributions are parametrized in terms of matrix elements of current (vector, axial and tensor) operators through three form factors

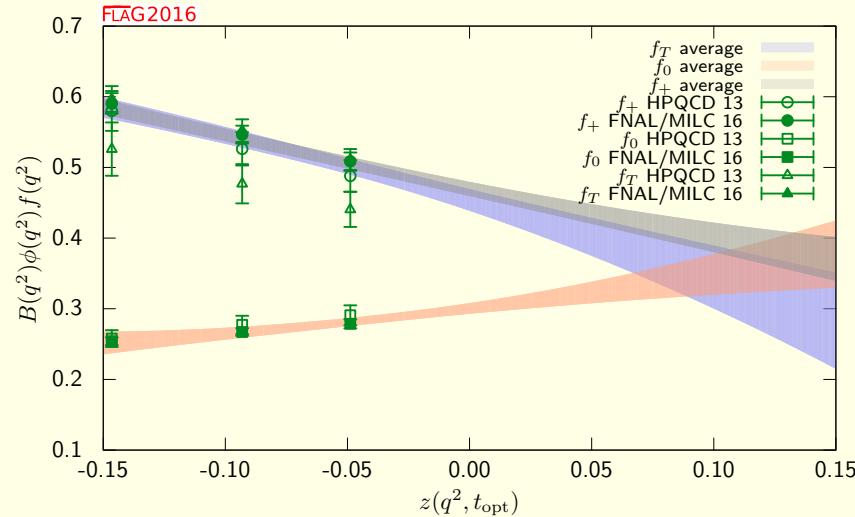
$f_+$  ,  $f_0$  (for  $m_\ell \neq 0$ ) and  $f_T$

+ non-factorizable contributions **J. Martín Camalich & G. Hiller talks**

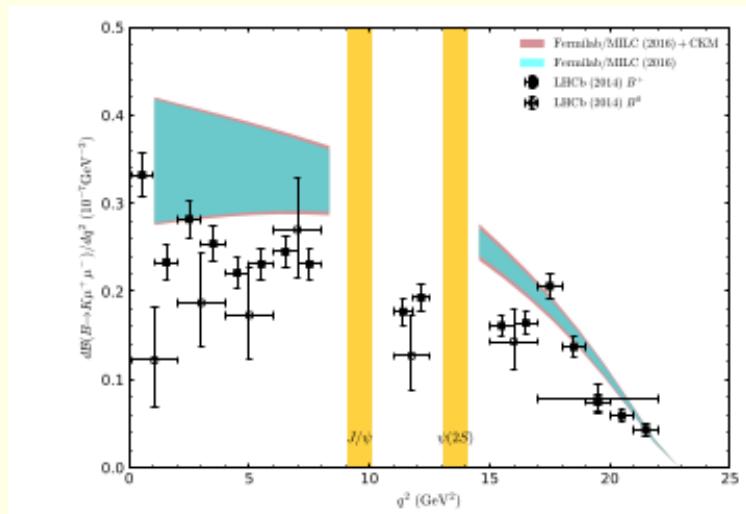
\* Determine CKM matrix elements  $V_{td,ts}$  or constrain Wilson coeff.  $C_9$  and  $C_{10}$ .

# Form factors for $B \rightarrow K\ell^+\ell^-$

$B \rightarrow K\ell^+\ell^-$ : HPQCD 1306.0434, 1306.2384, FNAL/MILC, 1509.06235

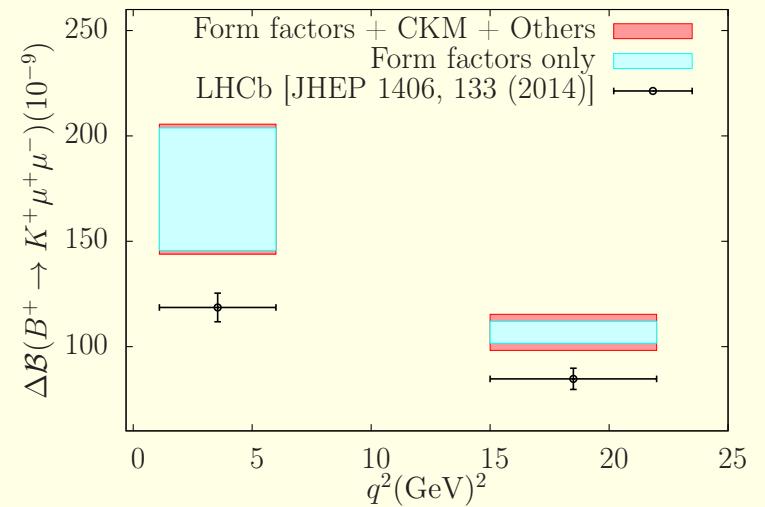


From D. Du et al 1510.02349, FNAL/MILC 1509.06235 (non-factorizable contributions under control?)



1 –  $2\sigma$  experiment-SM tensions.

Overlapping ensemble sets (asqtad MILC  
 $N_f = 2 + 1$ ) but different lattice actions:  
**HPQCD**: NRQCD  $b$  + HISQ  $u, d, s$   
**FNAL/MILC**: Fermilab  $b$  + asqtad  $u, d, s$   
Consistent results for  $f_{0,+,\text{T}}$ , and with LCSR  
**Khodjamarian et al 1006.4945**

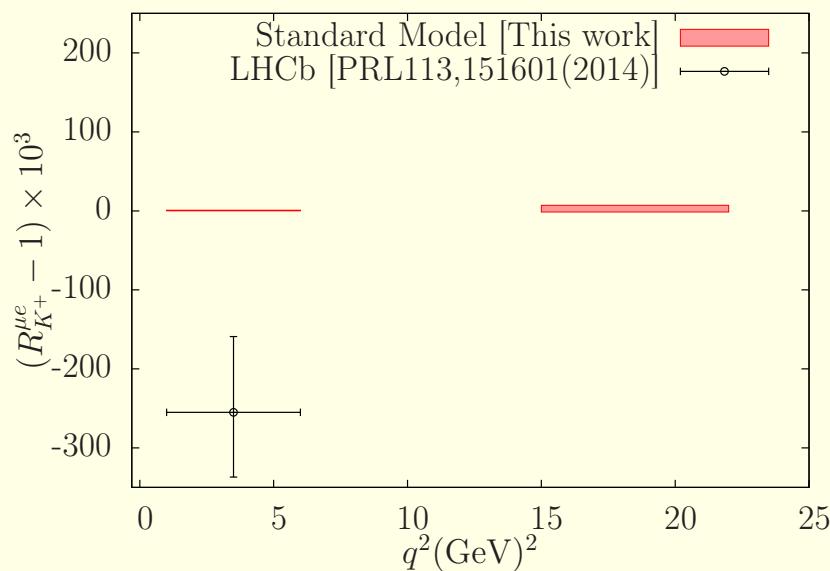


focus on large bins above and below  
charmonium resonances

# $B \rightarrow K\ell^+\ell^-$ : Lepton Flavor Universality Tests

D. Du et al. 1510.02349 with FNAL/MILC

form factors



2.6 $\sigma$  tension LHCb-SM

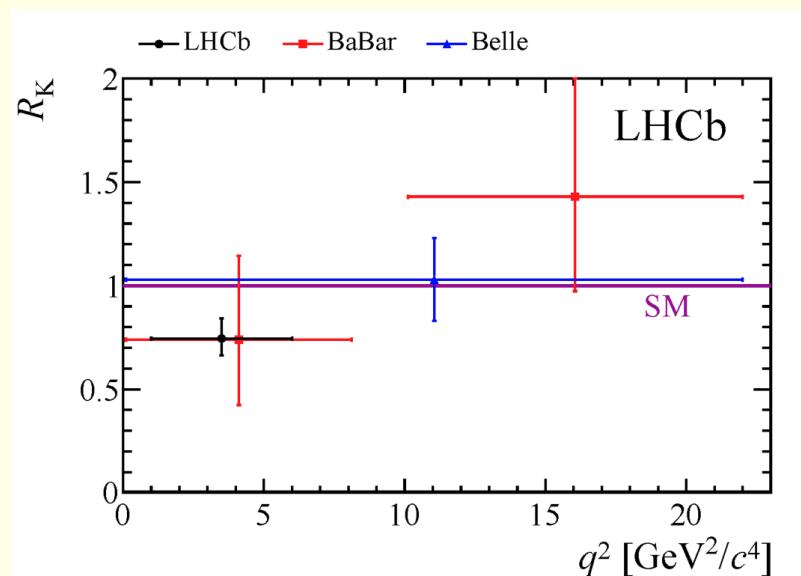
for [ $q_{min}^2 = 1 \text{ GeV}^2$ ,  $q_{max}^2 = 6 \text{ GeV}^2$ ]

$$(1 - R_K)^{\text{HPQCD}} = 0.00074 \pm 0.00035, \quad (1 - R_{K^+})^{\text{FNAL/MILC}} = 0.00050 \pm 0.00043$$

$$(1 - R_{K^+})^{\text{LHCb}} = 0.255^{+0.090}_{-0.074} \pm 0.036$$

SM predictions for these ratios pretty insensitive to form factors and non-factor contributions.

$$R_P^{\ell_1 \ell_2}(q_{min}^2, q_{max}^2) = \frac{\int_{q_{min}^2}^{q_{max}^2} dq^2 d\mathcal{B}(B \rightarrow P \ell_1^+ \ell_1^-)}{\int_{q_{min}^2}^{q_{max}^2} dq^2 d\mathcal{B}(B \rightarrow P \ell_2^+ \ell_2^-)}$$

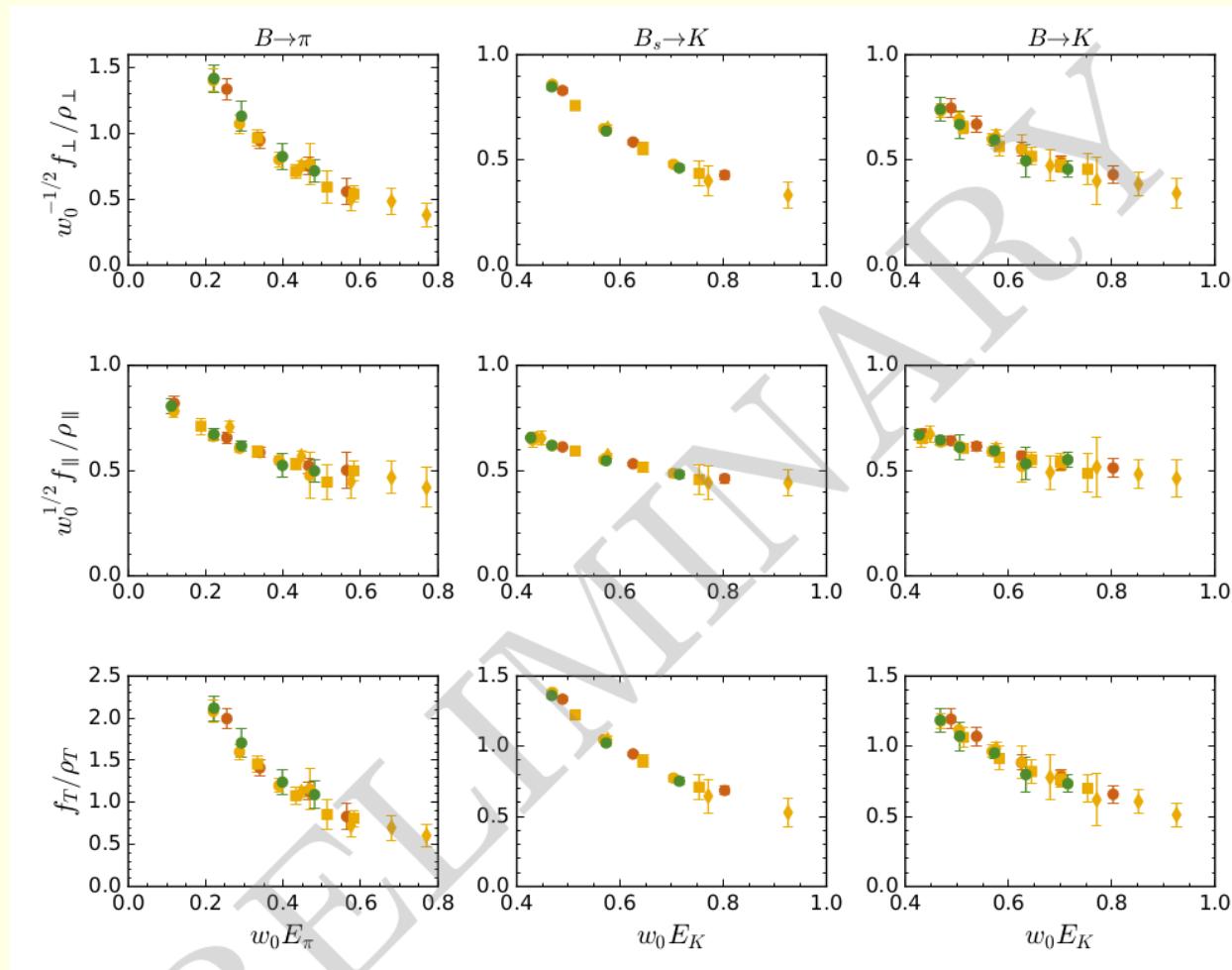


# Form factors for $B \rightarrow K\ell^+\ell^-$ : In progress

Necessary to reduce form factors uncertainties

**FNAL/MILC** on  $N_f = 2 + 1 + 1$  HISQ ensembles, HISQ light quarks, Fermilab  $b, c$ .

Form factors for  $B \rightarrow \pi(K)$  and  $B_s \rightarrow K$  [Z. Gelzer et al 1710.09442](#)



Preliminary and missing  
renorm. factors

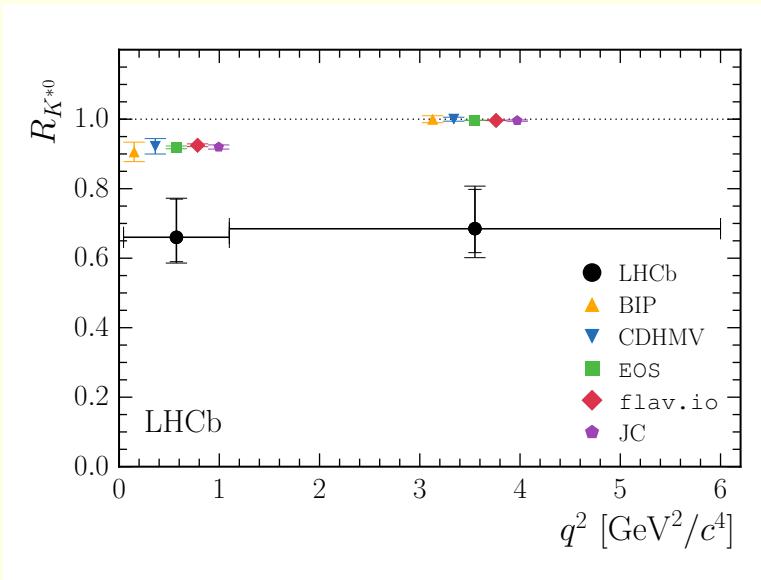
- $a \approx 0.15 \text{ fm}, m'_l/m'_s = \text{phys}$
- $a \approx 0.12 \text{ fm}, m'_l/m'_s = 0.2$
- $a \approx 0.12 \text{ fm}, m'_l/m'_s = 0.1$
- $a \approx 0.12 \text{ fm}, m'_l/m'_s = \text{phys}$
- $a \approx 0.088 \text{ fm}, m'_l/m'_s = \text{phys}$

Compared to previous **FNAL/MILC**:

Similar  $a \rightarrow$  similar statistics,  
smaller discretization (HISQ)

Physical  $m'_l$  ensembles  $\rightarrow$   
remove chiral extrapolation error

# $B \rightarrow K^* \ell^+ \ell^-$ : Lepton Flavor Universality Tests



LHCb will reach  $\sim 1.5\%$  precision for the branching fractions at both low and high  $q^2$ . [J. Albrecht et al 1709.10308](#)

\* Low  $K^*$  recoil: Lattice QCD form factors

**LHCb** measurement of  $R_{K^*}$ , [R. Ajai et](#)

[al 1705.05802](#):  $2 - 2.5\sigma$  tension with SM.

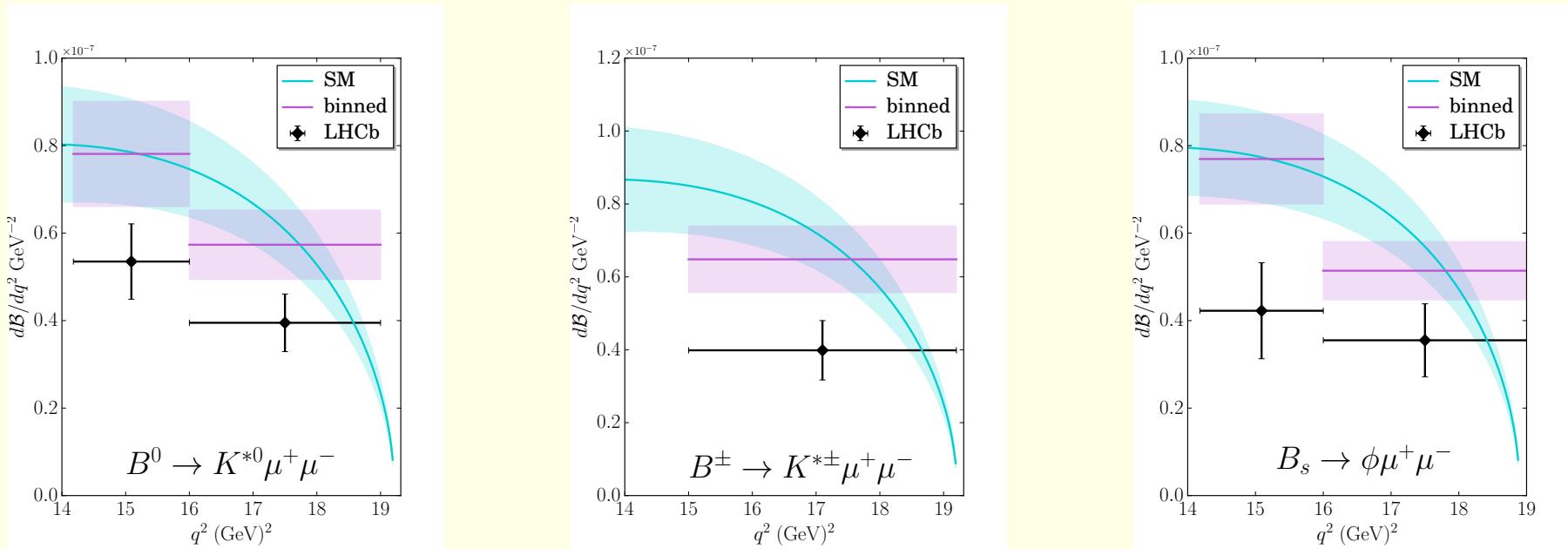
$B \rightarrow K^* \ell^+ \ell^-$  (and  $B_s \rightarrow \phi \ell^+ \ell^-$ ) provide richer set of observables but additional challenges for LQCD:  $K^*(\phi)$  unstable, more (7) form factors ...

# Form factors for $B_s \rightarrow \phi \ell^+ \ell^-$

So far,  $K^*$  and  $\phi$  treated as stable mesons on the lattice

\* First unquenched LQCD results [Horgan, Liu, Meinel and Wingate](#), 1310.3722, 1310.3887, 1501.00367

$N_f = 2 + 1$  calculation with NRQCD  $b$  + asqtad light at low recoil (high  $q^2$ )



differential branching fractions  $\sim 1 - 2\sigma$  larger than experiment

In progress: [RBC/UKQCD](#) 1612.05112  $N_f = 2 + 1$  form factors for  $B_s \rightarrow \phi$  with DW light and RHQ  $b$  quarks (include phys. mass and 3 different lattice spacings).

# Form factors for $B_{(s)} \rightarrow V\ell^+\ell^-$

Need a proper treatment of  $K^*$ : requires  $B \rightarrow K\pi$  matrix elements

- \* Finite-volume methods to include the width of an unstable final state hadron exist, but not implemented yet.

Briceño, Hansen, Walker-Loud 1406.5965, 1502.04314

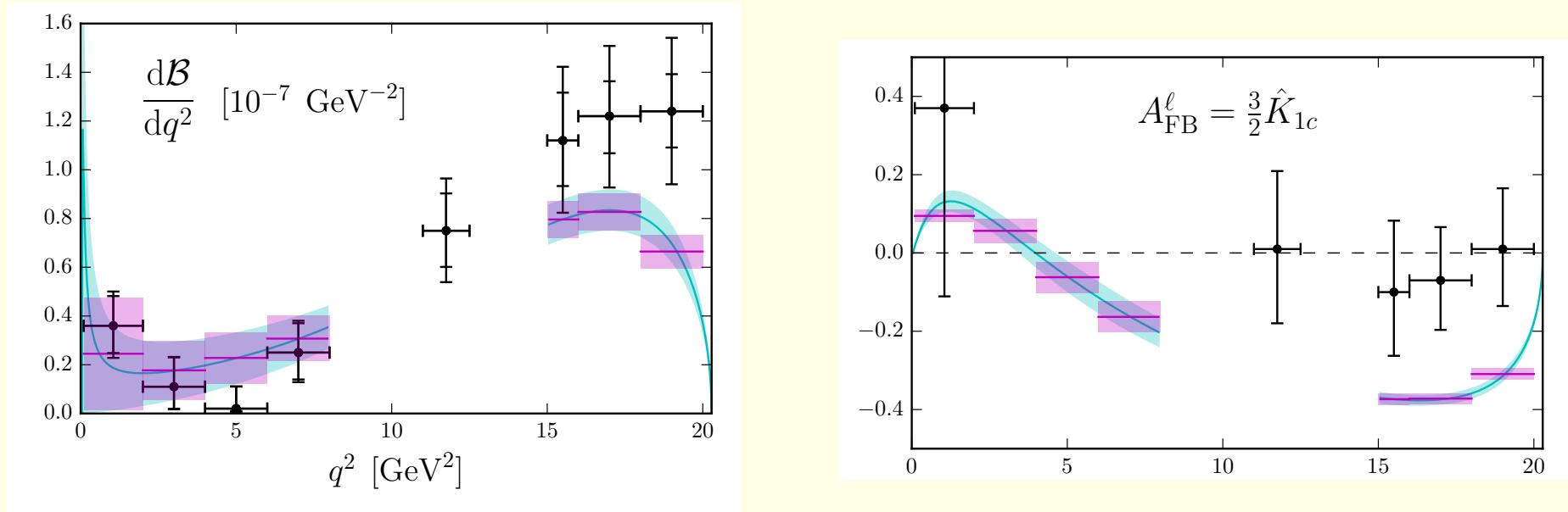
\*\* Pioneering work on  $B \rightarrow K^* \rightarrow K\pi$ : Leskovec, Meinel

\*\* Pilot study: Agadjanov et al 1605.03386

\*\* New ideas: Hansen, Meyer, Robaina 1704.08993

# More $b \rightarrow s\ell^+\ell^-$ processes: $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$

Tensions  $(1 - 3)\sigma$  also observed in the channel  $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$



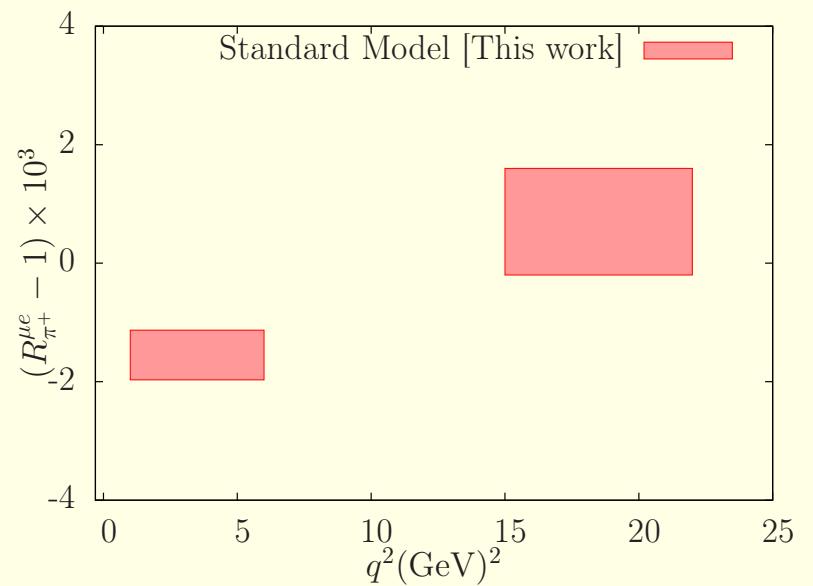
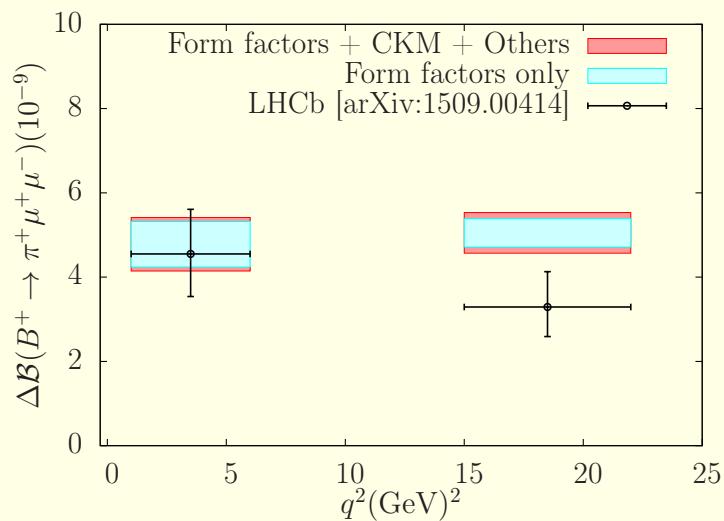
(focus on regions below and above charmonium resonances)

- \* **LHCb** 1503.07138 data vs SM LQCD calculation **Detmold & Meinel** 1602.01399 (10 form factors)
- \* Deviation in opposite direction compared to mesonic decays?

**Improvements underway** Results with errors a factor of 2 smaller expected for 2020 **S. Meinel** Challenges in semileptonic B decays workshop, Mainz 2018

# Form factors for $B \rightarrow \pi \ell^+ \ell^-$

$B \rightarrow \pi \ell^+ \ell^-$ : [FNAL/MILC](#), 1507.01618, [D. Du et al.](#) 1510.02349. Take  $f_+$  and  $f_0$  from combined fit of lattice + experimental data for  $B \rightarrow \pi \ell \nu$  (assume not significant NP effects at tree level).

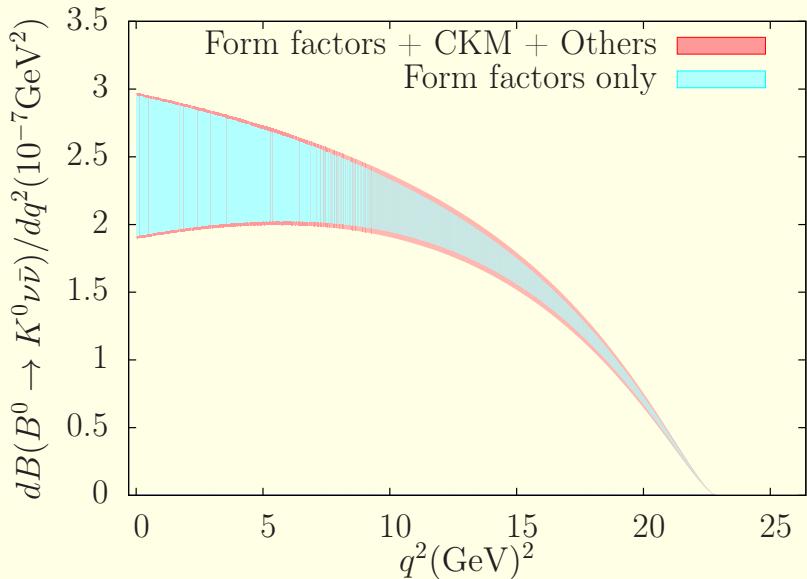
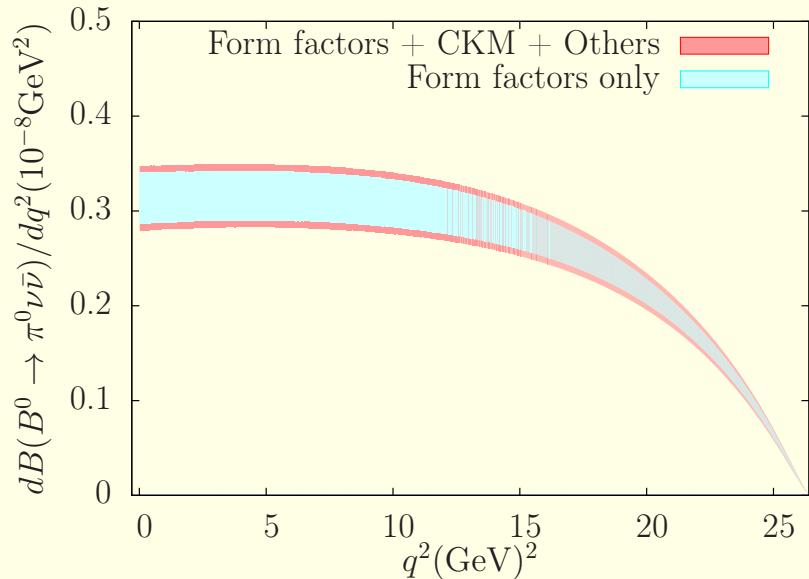


[D. Du et al.](#) 1510.02349 SM prediction for  $R_\pi = \frac{\mathcal{B}(B \rightarrow \pi \tau \nu_\tau)}{\mathcal{B}(B \rightarrow \pi \ell \nu)} = 0.641(17)$ .

Expected to be measured at [Belle-II](#)

# Rare semileptonic $B$ decays to $\nu\bar{\nu}$ states

D. Du et al. 1510.02349 with FNAL/MILC form factors



Predictions for both neutral and charged channels: complementary information (also  $V_{td,ts}$ )

- \* Theoretically clean (no problem with charm LD contributions)
- \* Difficult to measure experimentally, Belle-II expected precision  $\sim 10\%$  for  $B \rightarrow K$

$$\mathcal{B}(B^0 \rightarrow \pi^0 \nu\bar{\nu}) \cdot 10^7 = 0.668(41)(49)(16)$$

$$\mathcal{B}(B^0 \rightarrow K^0 \nu\bar{\nu}) \cdot 10^7 = 40.1(2.2)(4.3)(0.9)$$

$$\mathcal{B}(B^+ \rightarrow \pi^+ \nu\bar{\nu}) \cdot 10^6 = 9.62(1)(92); \mathcal{B}(B^+ \rightarrow K^+ \nu\bar{\nu}) \cdot 10^6 = 4.94(52)(6)$$

# Summary and outlook

## Charged-current $b$ decays

# LQCD calculations for complete set of form factors needed for  $R(D)$  (several lattice collaborations) errors commesurate with experiment

- \* Expect new results with reduced errors ( $N_f = 2 + 1 + 1$  and physical quark masses) soon FNAL/MILC,HPQCD,LNAL/SNU

# LQCD form factor for  $B \rightarrow D^*$  only at zero recoil (several coll.), but new results for complete set of form factor at all values of  $\omega$  soon

- \* Settle the value of  $|V_{cb}|$ ?
- \* Provide an improved determination of  $R(D^*)$
- \* Also, new results soon at zero recoil: LNAL/SNU

# LQCD predictions (and more work on-going) for  $R(D_s)$ ,  $R(\Lambda_b \rightarrow \Lambda_c)$

# On-going work for  $R(B_c \rightarrow J/\psi)$ ,  $R(\Lambda_b \rightarrow \Lambda_c^*)$

# Summary and outlook

## Neutral-current $b$ decays

# LQCD calculations for complete set of form factors needed for  $R_K$  (several lattice collaborations)

\* New results with reduced errors ( $N_f = 2 + 1 + 1$  and physical quark masses) before the end of the year (also for  $B \rightarrow \pi$  and  $B_s \rightarrow K$ )

# Form factors for  $R_{K^*}$  (also for  $B_s \rightarrow \phi$ ) available at low recoil with stable  $K^*(\phi)$

\* Theoretical framework for semileptonic  $B$  decays to vector meson final states exists [Briceño et al 1406.5965](#), [Agadjanov et al 1605.03386](#)

\*\* Pilot studies of form factors for  $B_s \rightarrow K^* \ell \nu$ ,  $B \rightarrow K^* \ell \ell, \dots$  underway

\* On-going work for  $B_s \rightarrow \phi$

# LQCD predictions for related quantities:  $R_\pi$ ,  
 $\mathcal{B}(B \rightarrow K(\pi)\nu\bar{\nu})$ ,  $B_s \rightarrow K\ell\nu, \dots$

# First complete calculation of  $\Lambda_b \rightarrow \Lambda$  form factors (some tension with experiment)

# Summary and outlook

**Goal:** Semileptonic decay form factors with (sub)percent level precision

- \* Growing number of results on ensembles of configurations with physical quark masses
  - \*\* Including charm on the sea ✓
- \* Subdominant effects: isospin breaking effects ✓, structure-dependent QED corrections ...
- \* Future: Use the same (relativistic) action for all heavy and light quarks
- \* Extend range in  $q^2$