$K \to \pi\pi$ and ε'
Status and Outlook

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Motivation for studying K→ππ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0 \to \pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \qquad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- In terms of isospin states: ΔI=3/2 decay to I=2 final state, amplitude A_2
  ΔI=1/2 decay to I=0 final state, amplitude A_0

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2},$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}.$$

$$\Longrightarrow \quad \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

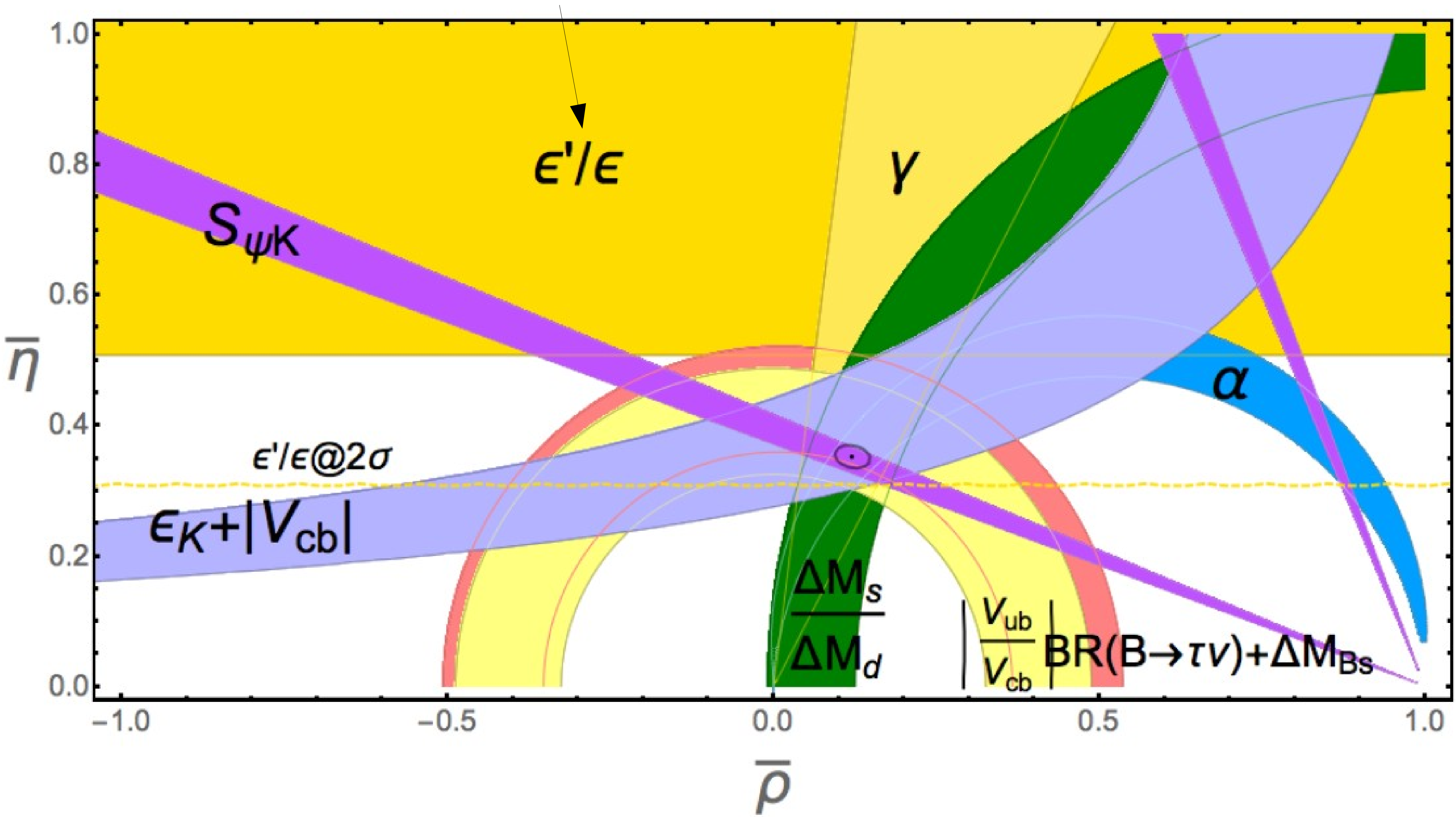
(δ_I are strong scattering phase shifts.)

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by many BSM models.

• ε' also provides a new horizontal band constraint on CKM matrix in ρ-η plane:

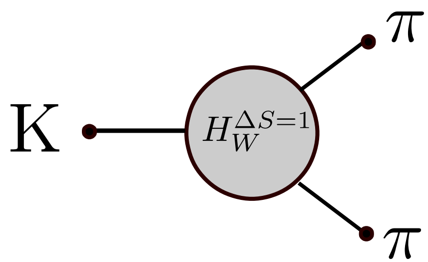
[Lehner et al arXiv:1508.01801]

new constraint from this work!



Overview of calculation

- Hadronic energy scale << M_W – use weak effective theory.
- K→ππ decays require single insertion of ΔS=1 Hamiltonian:



10 effective four-quark operators

$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

perturbative Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

LL finite-volume correction

renormalization
matrix (mixing)

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\left(z_i(\mu) + \tau y_i(\mu) \right) Z_{ij}^{\text{lat}\to\overline{\text{MS}}} M_j^{I,\,\text{lat}} \right]$$

$M_j^{I,lat} = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

- Operators must be renormalized into same scheme as Wilson coeffs
- Use RI-(S)MOM NPR and perturbatively match to MSbar at high energy scale at which PT truncation effects under control.

Key challenges of lattice calculation

- Primary challenge is to assure physical kinematics. Signal dominated by amplitude with 2 stationary pions in final state.

$$2m_\pi \approx 260 \text{ MeV} \ll m_K \approx 500 \text{ MeV}$$

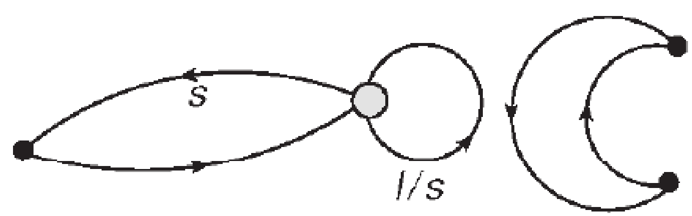
Solution: Remove stationary pion state from system by manipulating lattice spatial boundary conditions.

- Antiperiodic BCs on down-quark for A_2
- G-parity BCs on both quarks for A_0

$$p_\pi = 0 \rightarrow \pi/L$$

tune L to match E_K and $E_{\pi\pi}$

- For A_0 serious noise issue due to "disconnected diagrams"



type4

Solution: Use "all-to-all" propagators to tune source to minimize overlap with vacuum and maximize sampling for every configuration.

Despite this, we require considerably higher statistics than a typical lattice calculation.

Summary of published results

[Phys.Rev. D91 (2015) no.7, 074502]

- A_2 computed on RBC/UKQCD 64^3x128 and 48^3x96 2+1f Mobius DWF ensembles with the Iwasaki gauge action.
- $a^{-1} = 2.36$ GeV and 1.73 GeV resp - continuum limit taken.
- Statistical errors sub-percent, dominant systematic errors due to truncation of PT series in computation of renormalization and Wilson coefficients.
- 10% and 12% total errors on Re(A_2) and Im(A_2) resp.

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- A_0 computed on 32^3x64 Mobius DWF ensemble with Iwasaki+DSDR gauge action. G-parity BCs in 3 directions to give physical kinematics.
- Single, coarse lattice with $a^{-1} = 1.38$ GeV but large physical volume to control FV errors.
- 21% and 65% stat errors on Re(A_0) and Im(A_0) due to disconn. diagrams and, for Im(A_0) a strong cancellation between Q_4 and Q_6.
- Dominant, 15% systematic error is due again to PT truncation errors exacerbated by low renormalization scale 1.53 GeV.

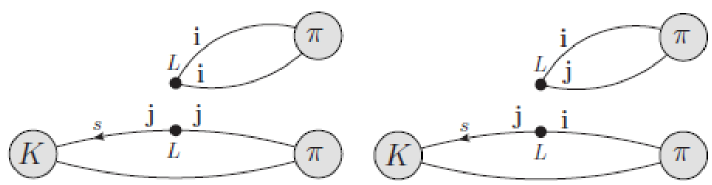
ΔI=1/2 rule

- In experiment kaons approx 450x (!) more likely to decay into I=0 pi-pi states than I=2.

$$\frac{\mathrm{Re} A_0}{\mathrm{Re} A_2} \simeq 22.5 \qquad \text{(the ΔI=1/2 rule)}$$

- Perturbative running to charm scale accounts for about a factor of 2. Where does the remaining 10x come from? New Physics?
- The answer is **low-energy *QCD*!** RBC/UKQCD [arXiv:1212.1474, arXiv:1502.00263]

Strong cancellation between the two dominant contractions

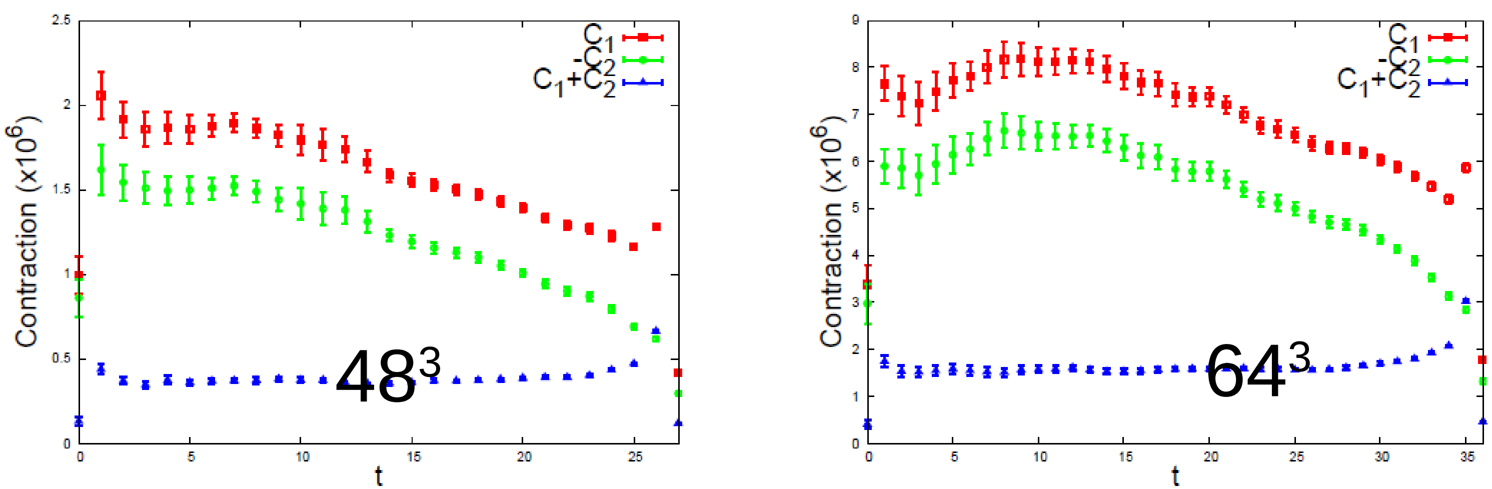


Contraction ①. Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

find $② \approx -0.7①$ heavily suppressing $\mathrm{Re}(A_2)$.

[Phys.Rev. D91 (2015) no.7, 074502]



Pure-lattice calculation

$$\frac{\mathrm{Re}(A_0)}{\mathrm{Re}(A_2)} = 31.1(11.2)$$

[$\mathrm{Re}(A_0)$ agrees with expt.]

Results for ε'

- $\mathrm{Re}(A_0)$ and $\mathrm{Re}(A_2)$ from expt.
- Lattice values for $\mathrm{Im}(A_0)$, $\mathrm{Im}(A_2)$ and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{ll} = \; 1.38(5.15)(4.43) \times 10^{-4}, & \text{(this work)} \\ 16.6(2.3) \times 10^{-4} & \text{(experiment)} \end{array}}$$

- Total error on Re(ε'/ε) is ~3x the experimental error.
- Find reasonable consistency with Standard Model (at 2.1σ level).
- Tantalizing hint of discrepancy strong motivation for continued study!
- Error is dominated by that on A_0.

Our main focuses are therefore to:

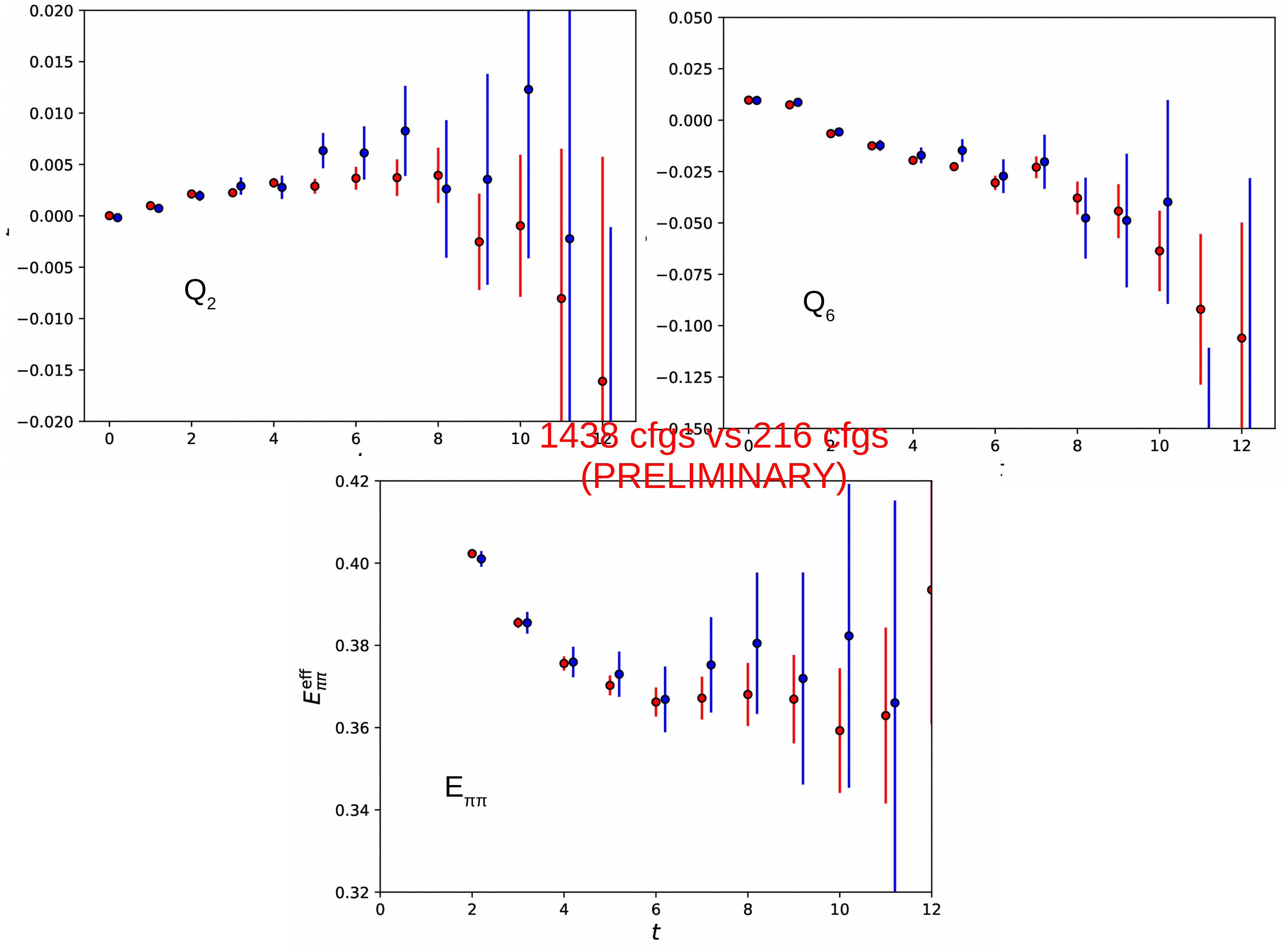
- Increase statistics on A_0 calculation, enabling improved precision and better systematic error estimation.
- Reduce dominant systematic errors, particularly NPR and finite lattice spacing.

Statistics increase

- Original goal was a 4x increase in statistics over 216 configurations used in 2015 analysis.
- 4x reduction in configuration generation time obtained via algorithmic developments (exact one-flavor implementation)
- Large-scale programme performed involving many machines:

Source	Determinant computation	Independent configs.
Blue Waters	RHMC	34+18+4+3
KEKSC	RHMC	106
BNL	RHMC	208
DiRAC	RHMC	151
KEKSC	EOFA	275+215
BNL	EOFA	245
		1259 total

- Measurements performed using IBM BG/Q machines at BNL and the Cori computer (Intel KNL) at NERSC largely complete.
- Including original data, now have 6.7x increase in statistics!


Q2
Q6
1438 cfgs vs 216 cfgs
(PRELIMINARY)
Eππ
$E^{\mathrm{eff}}_{\pi\pi}$
t

Systematic error improvements

Description	Error	Description	Error
Finite lattice spacing	12%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			27%

[RBC&UKQCD PRL 115 (2015) 21, 212001]

NPR+Wilson Coefficients

- NPR error large due to use of 1-loop PT to match to MSbar at low, 1.53 GeV renormalization scale.
- Since 2015 have improved NPR error 15% → 8% (preliminary) by increasing scale to 2.29 GeV using step-scaling procedure. [PoS LATTICE2016 (2016) 308]
- Inclusion of dim.6 gauge-invariant operator G_1 which mixes with Q_i under renormalization, effects demonstrated to be %-scale as expected.

  [G. McGlynn arxiv:1605.08807]

  Do not expect significant improvement in Wilson coeffs. error as dominated by use of PT to cross the charm threshold (1.29 GeV).
- Working on circumventing this by computing 3→4 flavor matching non-perturbatively.
- Requires $\mu \ll m_c$. At these low energies, MOM-scheme NPR severely hampered by increased mixing with tower of gauge-noninvariant operators.
- Circumvent using position-space NPR which does not require gauge fixing.

Discretization error

- Currently have results only on single lattice with coarse lattice spacing a^{-1}=1.38(1) GeV.
- Require second lattice spacing. Going to finer lattice requires more lattice sites; prohibitively expensive for current gen. computers.
- Promising alternative is to go to a coarser lattice spacing, a^{-1} ~ 1.0 GeV. Preliminary studies suggest discretization errors remain under control.

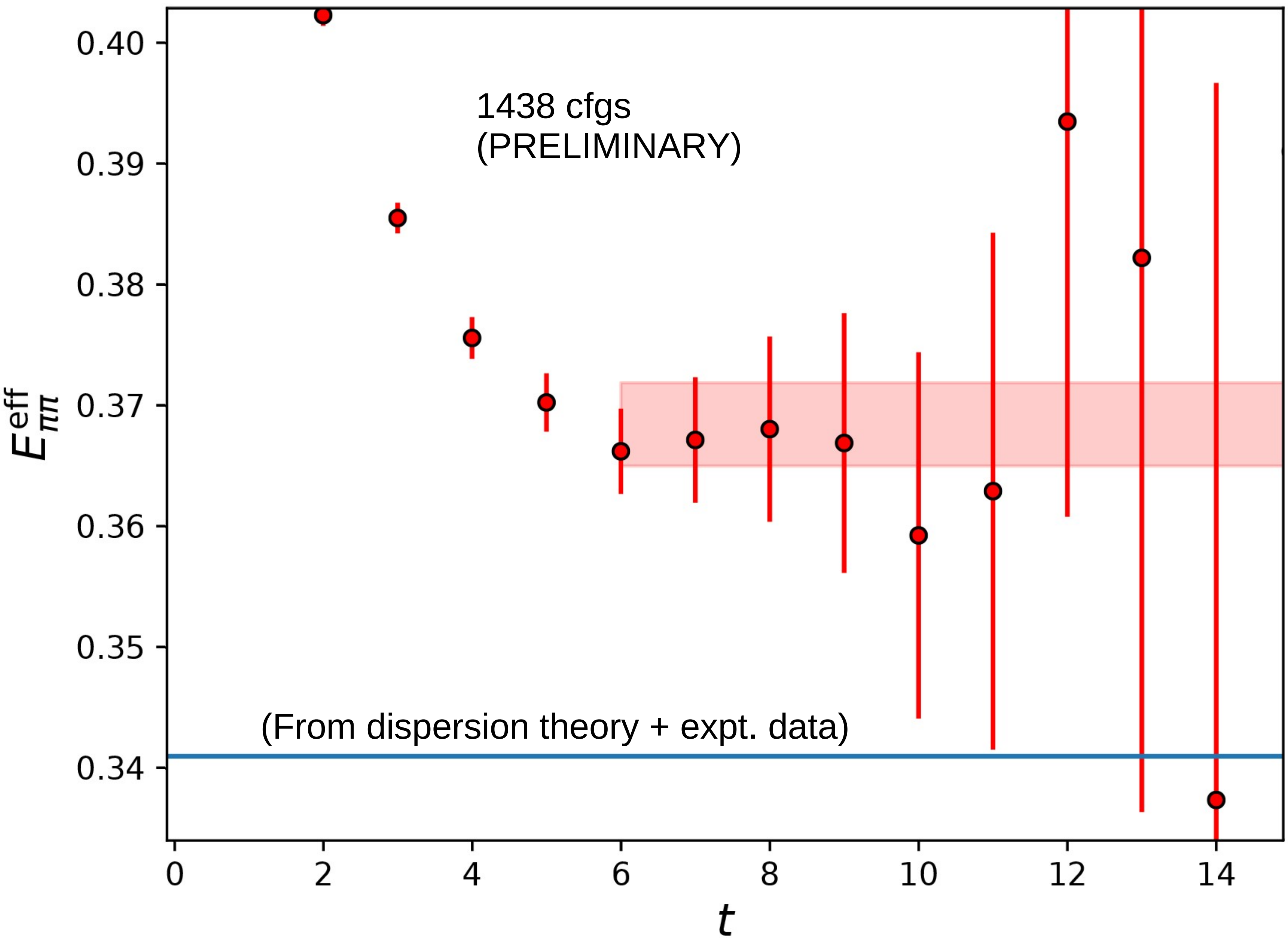
[EPJ Web Conf. 175 (2018) 02006]

Related projects on the horizon:

- Performing calculation taking advantage of modern multi-operator techniques to fit excited-state $\pi\pi$ contributions directly, without G-parity BCs.
- Laying the groundwork for non-perturbatively computing the effects of isospin breaking and electromagnetism. [EPJ Web Conf. 175 (2018) 13016]
- Study of complete, non-perturbative calculation of Wilson coefficients

[EPJ Web Conf. 175 (2018) 13014, arXiv:1711.05768]

The pi-pi puzzle



Resolving the pi-pi puzzle

- Matching dispersive result requires $E_{\pi\pi}$~470 MeV vs. our 498(11) MeV.
- Correspondingly, our phase shift δ_0=23.8(4.9)(1.2)^o is lower than the dispersive value of 38.3(1.3)^o

[RBC&UKQCD PRL 115 (2015) 21, 212001] [G.Colangelo, priv. comm.]

- Possibility of nearby excited state unresolvable from ground-state skewing fit: Indeed dispersion theory predicts excited state ~770 MeV.
- To address and to improve fitting in general, have expanded measurement programme, introducing new operators that also couple to I=0 pi-pi:
    - Added scalar bilinear operator (σ). σ-σ, σ-ππ matrix elements with a number of operator choices now included in recent fits.
    - Alongside 1s hydrogen-wavefunction pion source smearing, have added 2s form. Alternative coupling to radial excited states of pion.

- Combining these new handles on the pi-pi state and modern multi-operator methods such as GEVP will allow us to obtain the finite-volume spectrum in more detail and hopefully resolve the pipi puzzle.
- Recent, highly preliminary results support existence of excited state contamination resolvable by incorporating these new operators.

Conclusions

- 2015 calculation of ε' consistent with SM at 2.1σ – hint of tension?
- Errors dominated by A_0, roughly equal stat and sys.
- Increase of statistics by almost 7x largely complete!
- A number of programmes to address systematic errors, particularly the perturbative truncation errors in the renormalization and Wilson coeffs.
- Open puzzle is significant disagreement with I=0 pi-pi scattering phase shift obtained using expt + dispersion theory.
- Possible explanation is contamination from nearby excited state.
- Inclusion of additional operators allows use of more sophisticated methods to isolate ground state. Preliminary results hint that excited state is skewing results.

We hope to publish updated results within the next few months

Thank you!