HF Workshop, Wuhan, 8-11 October 2018

Probing QGP Transport Properties via Heavy-Flavor Langevin Dynamics

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Mostly based on: PRC.98.014909; PRC.98.034914

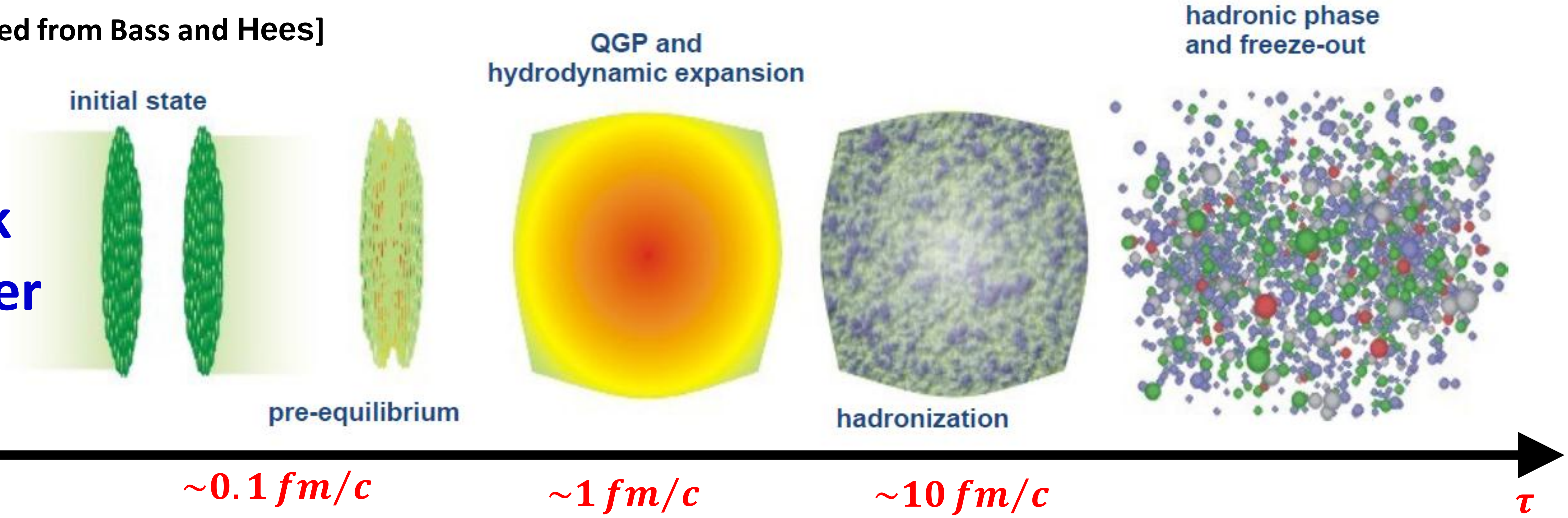
Outline

- **Introduction**
  - ✓ Heavy quarks as probes of QGP
- **Hybrid modeling of HQ evolution**
  - ✓ Initialization: spatial- and momentum-space configuration
  - ✓ HQ diffusion in QGP: Langevin transport approach
  - ✓ "Dual" hadronizaiton: fragmentation + heavy-light coalescence
- **Results**
  - ✓ Nuclear modification factor R_{AA}
  - ✓ Elliptic flow coefficient v_2
  - ✓ Relative azimuthal angle correlation $dN/d\phi$
- **Summary & Outlook**

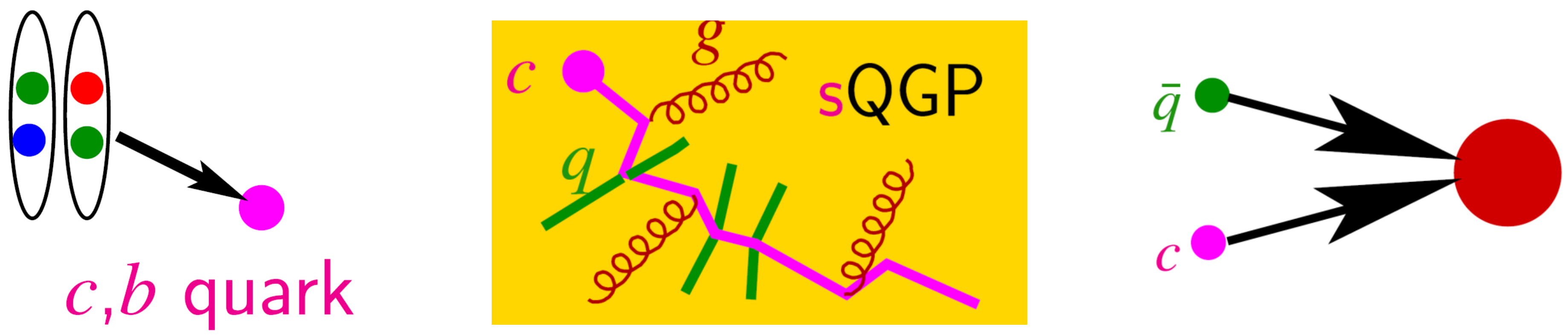
Heavy quarks (charm & bottom) as probes of Quark-Gluon Plasma

[Adopted from Bass and Hees]

Bulk matter



Heavy quarks



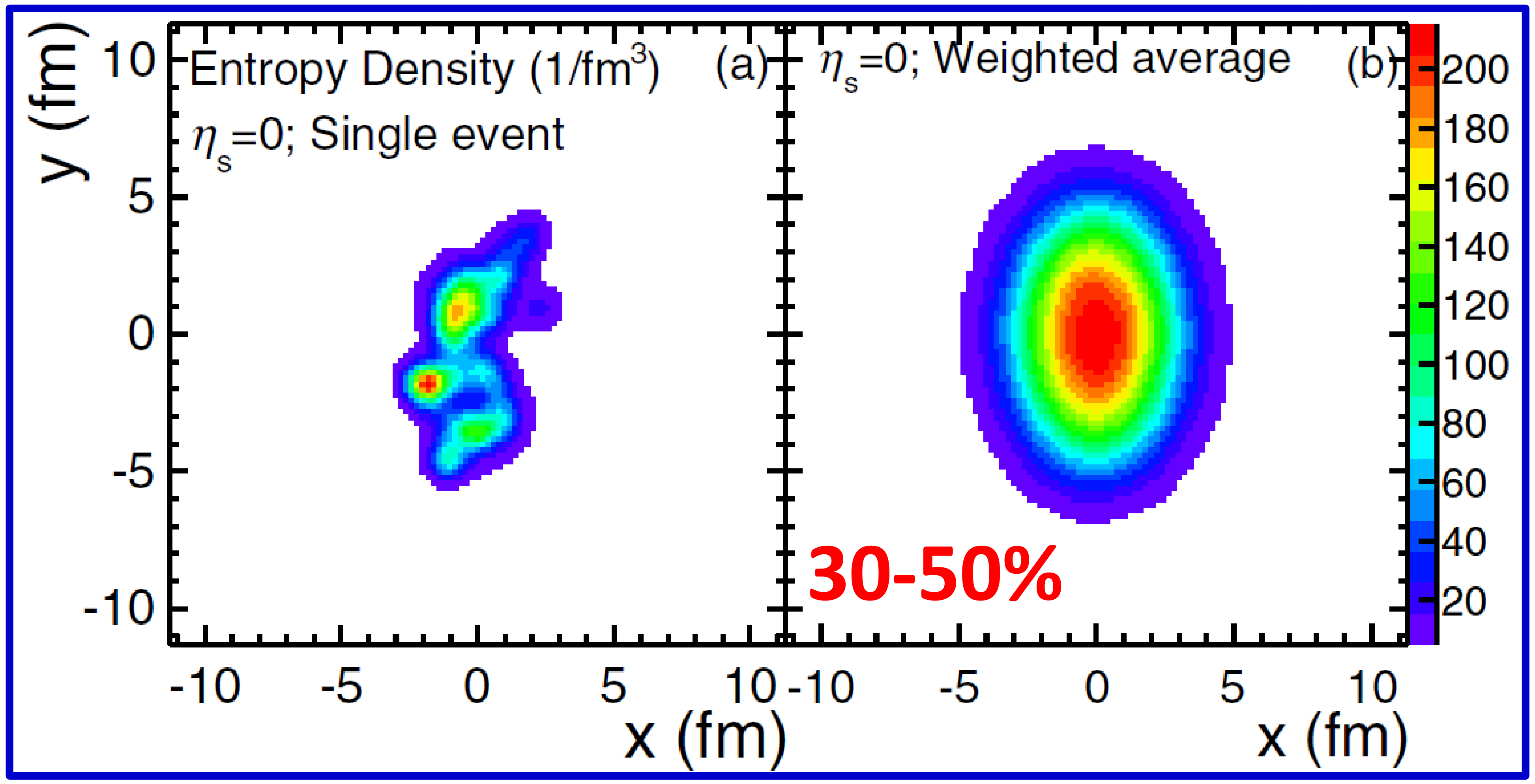
[1803.03824
1809.07894
1809.10734]

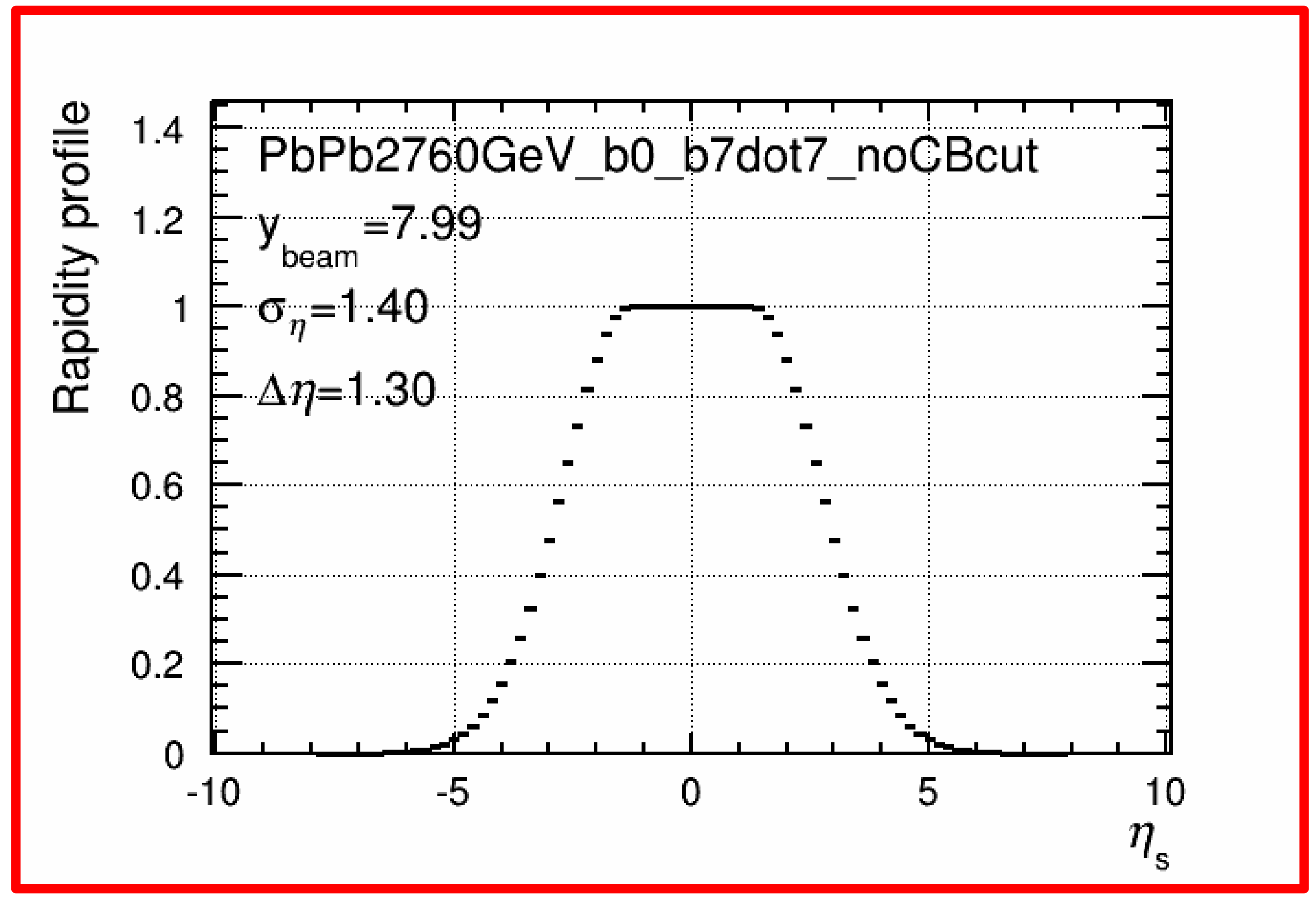
- Heavy quarks (HQs) initial hard production is well described by pQCD+PDFs
- HQ rescattering in QGP via **Langevin approach**: interactions encoded in the drag and diffusion coefficients
- Hadronization into D/B mesons via fragmentation and heavy-light coalescence mechanisms

A realistic study requires developing a multi-step model

Initial spatial-space configuration

$$s(\vec{r}_\perp, \eta_s, \tau_0) \equiv s_0(\vec{r}_\perp, \eta_s = 0) \cdot \rho(\eta_s)$$

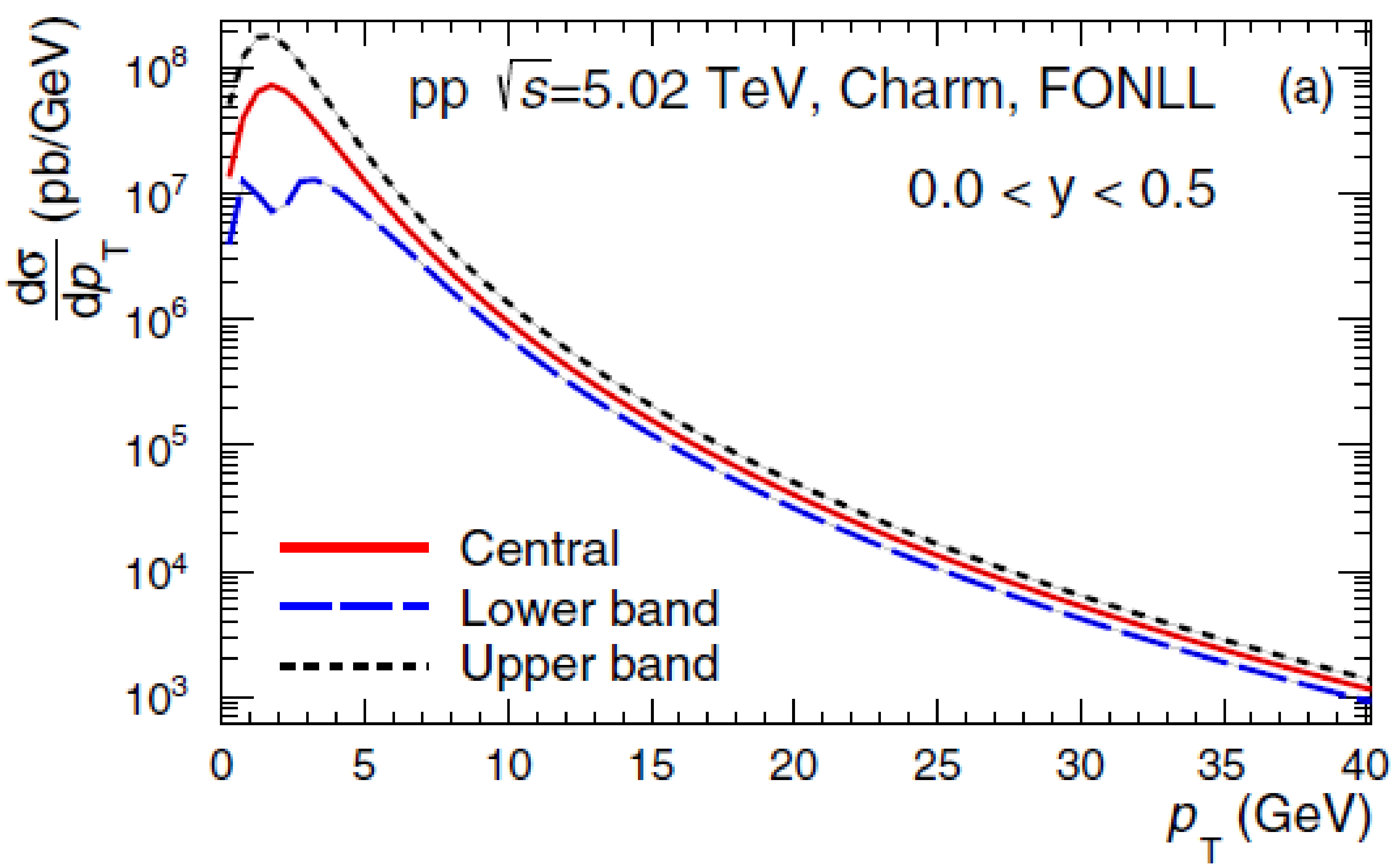




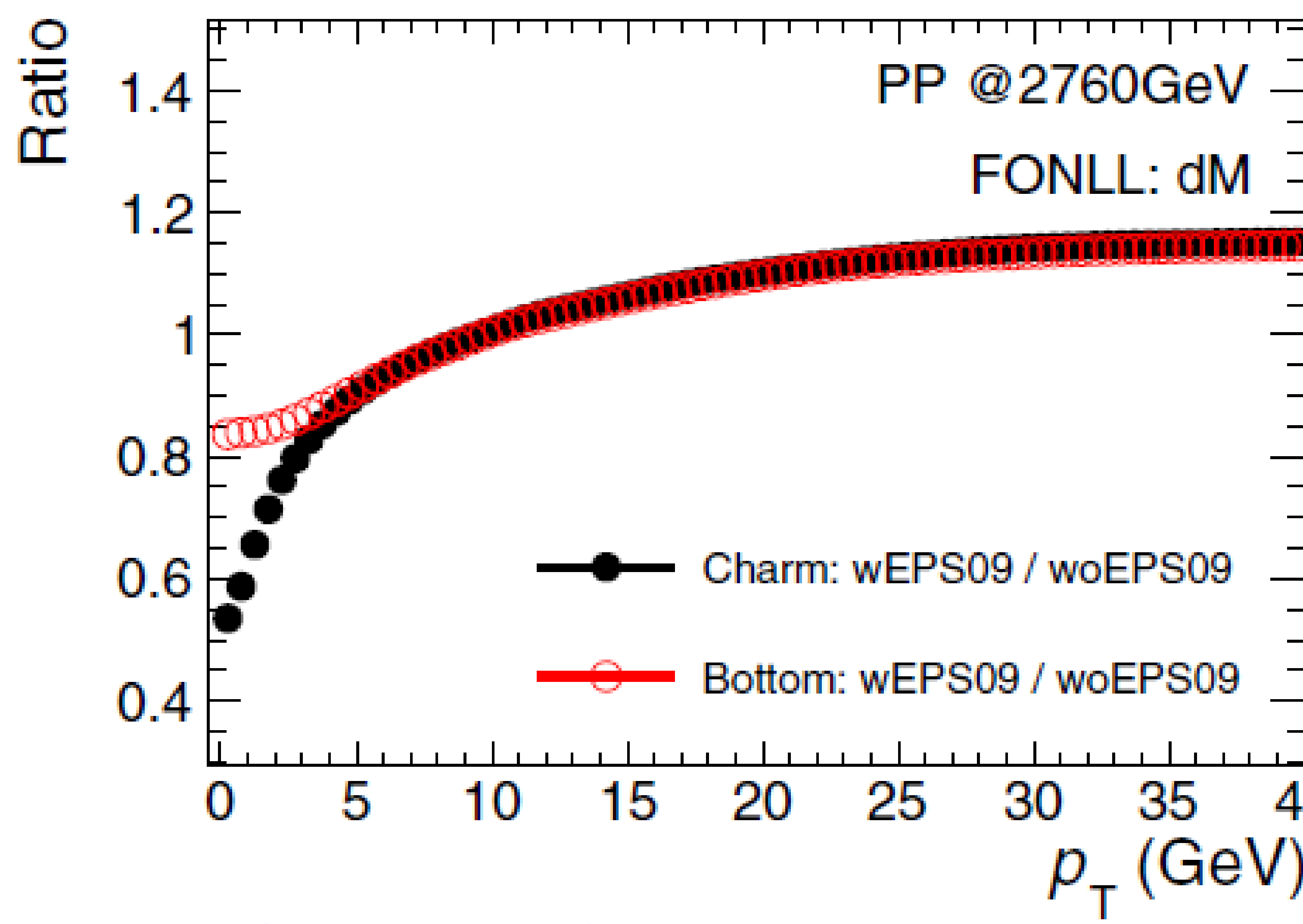
- Transverse profile: Glauber-based model, i.e. SuperMC
- Longitudinal behavior: a data-inspired phenomenological function, i.e. Gaussian fall-off
- Event-averaged smooth initial transverse profile adopted to avoid the full event-by-event hydrodynamic simulation

[SuperMC: Comput. Phys. Commun. 199, 61 (2016). Hirano, PRC 65, 011901(R) (2001).]

Initial momentum-space configuration



FONLL predictions



w/ and w/o Pb nuclear PDFs

- Sampled according to FONLL calculations
- Assuming the initial back-to-back azimuthal correlation in nucleus-nucleus collisions
- EPS09 NLO parametrization used for various nucleus PDFs

[FONLL: JHEP 05, 007 (1998); 03, 006 (2001); 10, 137 (2012). EPS09: JHEP 04, 065 (2009)]

Langevin dynamics (1/2)

- Langevin transport equation

[Rapp and Hees, Quark-Gluon Plasma 4; Beraudo *et. al.*, NPA 831, 59-90 (2009); He *et.al.*, PRE 88, 032138 (2013)]

$$\frac{dp_i}{dt} = F_i^{Drag} + F_i^{Diff}$$

✓ drag force: $F_i^{Drag} = -\eta_D(\vec{p}, T) \cdot p$

✓ thermal random force: $F_i^{Diff} = \frac{1}{\sqrt{dt}} C_{ij}(t, \vec{p} + \xi d\vec{p}, T) \cdot \rho_i$

➢ momentum argument of the covariance matrix $C_{ij}(t, \vec{p} + \xi d\vec{p}, T)$ where $\xi = 0$ for pre-point Ito, $\xi = 1/2$ for mid-point and $\xi = 1$ for post-point discretization scheme of the stochastic integral;

➢ represent C_{ij} as

$$C^{ij}(\vec{p}) = \sqrt{\kappa_L(\vec{p}, T)}\, \hat{p}^i \hat{p}^j + \sqrt{\kappa_T(\vec{p}, T)}\, (\delta^{ij} - \hat{p}^i \hat{p}^j),$$ thus,

$$\eta_D = \frac{\kappa_L}{2TE} + (\xi - 1)\frac{1}{2p}\frac{\partial \kappa_L}{\partial p} + \frac{d-1}{2p^2}[\xi(\sqrt{\kappa_T} + \sqrt{\kappa_L})^2 - (3\xi - 1)\kappa_T - (\xi + 1)\kappa_L]$$

Langevin dynamics (2/2)

- Modified Langevin equation

[Cao *et. al.* PRC 88, 044907 (2013); 1809.07894; Xu *et.al.*, 1809.10734]

$$\frac{dp_i}{dt} = F_i^{Drag} + F_i^{Diff} + F_i^{Gluon}$$

✓ recoil force from gluon radiation

$$F_i^{Gluon} = -dp_i^{Gluon}/dt$$

✓ gluon distribution taken from Higher Twist calculation

$$\frac{dN_g}{dxdk_T^2dt} = \frac{2\alpha_s(k_T)C_A}{\pi k_T^4} \cdot P(x) \cdot \hat{q}_q \cdot \left(\frac{k_T^2}{k_T^2 + x^2 m_Q^2}\right)^4 \cdot sin^2\left(\frac{t - t_i}{2\tau_f}\right)$$

[Guo and Wang, PRL 85, 3591 (2000); Majumder *et.al.*, PRD 85, 014023 (2012); Zhang *et.al.*, PRL 93, 072301 (2004); Qin and Muller, PRL 106, 162302 (2011)]

- splitting function: $P(x) = (x^2 - 2x + 2)/x$;
- gluon radiation time: $\tau_f = [2xE_Q(1-x)]/(k_T^2 + x^2 m_Q^2)$;
- additional cutoff $\omega \geq \pi T$ applied to balance the gluon radiation and the inverse absorption.

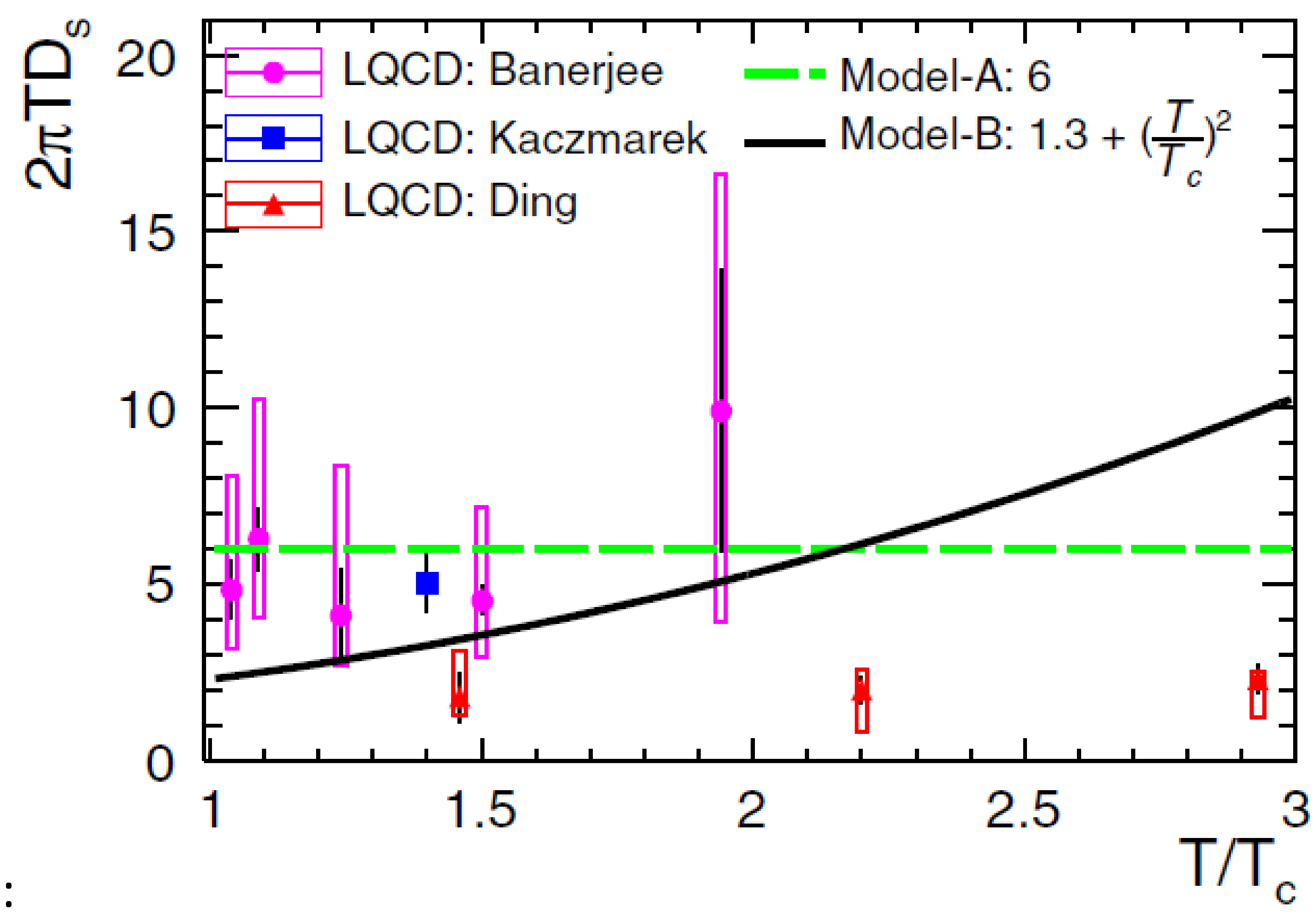
"Minimal model" as our first step

- Strategies
  - ✓ assuming an isotropic momentum space diffusion: $\kappa_T = \kappa_L = \kappa$;
  - ✓ additionally, taking the post-point scheme ($\xi = 1$) or the momentum independent behavior of κ: $\partial\kappa / \partial p = 0$;
  - ✓ therefore, the Einstein relation reads $\boldsymbol{\eta_D = \kappa/(2TE)}$.
- The only adjustable parameter is the spatial diffusion coefficient D_s

$$\begin{cases} \eta_D = \dfrac{1}{2\pi T D_s} \dfrac{2\pi T^2}{E} \\ \\ \kappa = \dfrac{1}{2\pi T D_s} 4\pi T^3 \end{cases}$$

and the transport coefficient

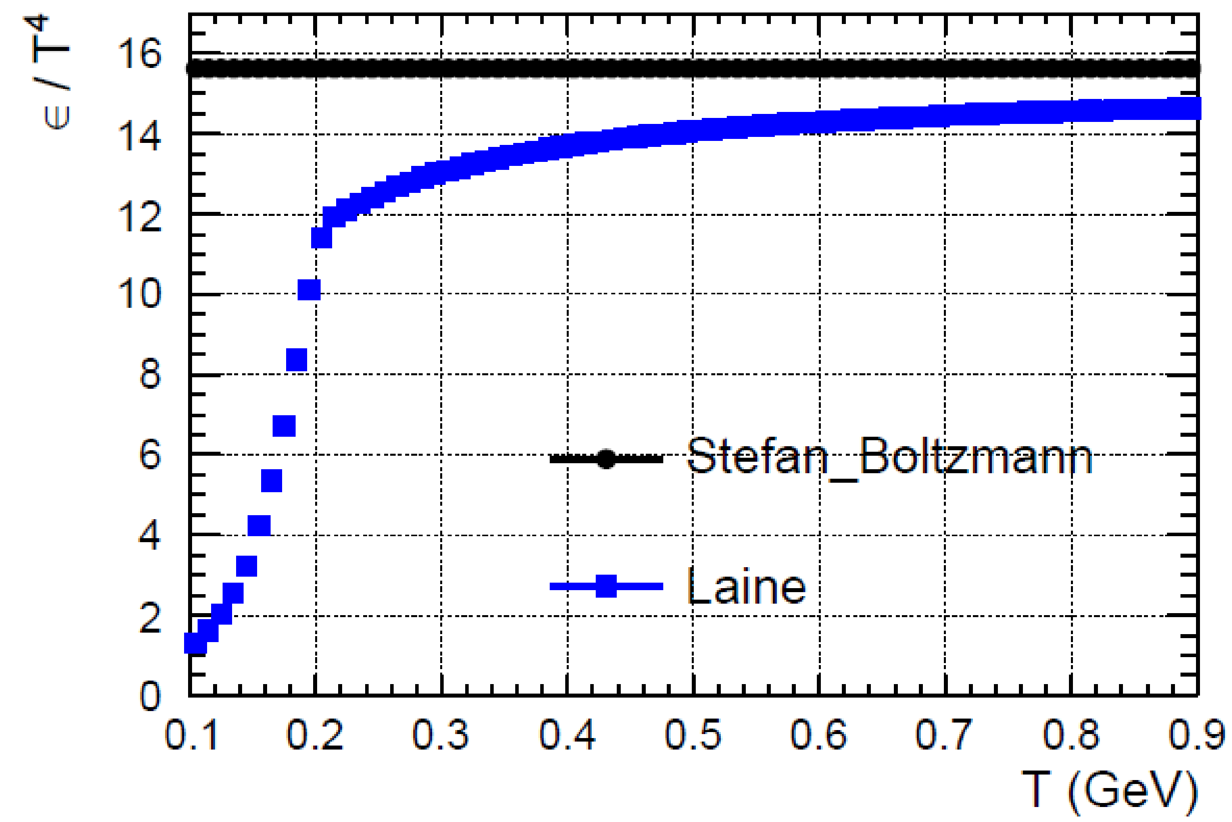
$$\hat{q}_q = \frac{2}{v}\kappa \approx 2\kappa$$



[Similar approaches:
Moore *et. al.* PRC 71, 064904 (2005); Akamatsu *et. al.* PRC 79, 054907 (2009);
Cao *et. al.*, PRC 92, 024907 (2015);
Aarts *et.al.* EPJA 53, 93 (2017); Chatterjee and Bożek, PRL 120, 192301 (2018)]

Model the expanding fireball

- **(3+1)-dimensional relativistic viscous hydrodynamics based on the HLLE algorithm**
  - ✓ starting timescale $\tau_0 = 0.6\, fm/c$
  - ✓ shear viscosity $\frac{\eta}{s} = \frac{1}{4\pi}$
  - ✓ Equation of State (EoS): lattice QCD calculations



- **Hydro outputs**
  - ✓ $u^\mu(\tau, x, y, \eta_s)$: used to perform the boost to the local rest frame of the fluid cell;
  - ✓ $T(\tau, x, y, \eta_s)$: set the value of the transport coefficients;

- **Hadronization**
  - ✓ $T_c = 165$ MeV
  - ✓ instantaneous approach (i.e. Cornelius) → isothermal freeze-out
  - ✓ momentum spectra of various hadron species obtained via Cooper-Frye formalism

[vHLLE: Comput. Phys. Commun. 185, 3016 (2014). Laine: PRD 73, 085009 (2006).
Cornelius: EPJA 48, 171 (2012). Cooper-Frye: PRD 10, 186 (1974).]

Hadronization via heavy-light coalescence (1/2)

- Instantaneous approach utilized
- Momentum spectrum of **mesons** (M) formed from the coalescence of **heavy quark** (Q) and **anti-light-quark ($\bar{q}$)** is then given by [Han *et.al.*, PRC 93, 045207 (2016)]

$$\frac{dN_M}{d^3\vec{p}_M} = g_M \int d^3\vec{x}_Q d^3\vec{p}_Q\, d^3\vec{x}_{\bar{q}} d^3\vec{p}_{\bar{q}} f_Q(\vec{x}_Q,\vec{p}_Q) f_{\bar{q}}(\vec{x}_{\bar{q}},\vec{p}_{\bar{q}}) \cdot \overline{W}_M^{(n)}(\vec{y}_M,\vec{k}_M)\delta^3(\vec{p}_M-\vec{p}_Q-\vec{p}_{\bar{q}})$$

where the overlap integral of the Wigner function of the $Q\bar{q}$ pair and the **meson in n excited state**

$$\overline{W}_M^{(n)}(\vec{y}_M,\vec{k}_M) = \int \frac{d^3\vec{x}'_Q d^3\vec{p}'_Q}{(2\pi)^3} \frac{d^3\vec{x}'_{\bar{q}} d^3\vec{p}'_{\bar{q}}}{(2\pi)^3} W_Q(\vec{x}'_Q,\vec{p}'_Q) W_{\bar{q}}(\vec{x}'_{\bar{q}},\vec{p}'_{\bar{q}}) W_M^{(n)}(\vec{x}'_M,\vec{p}'_M) = \frac{\Theta^n}{n!} e^{-\Theta}$$

✓ parton (meson) wave function behaves the Gaussian wave packet (simple harmonic oscillator)

$$\Theta = \frac{1}{2}\left(\frac{\vec{y}_M^2}{\sigma_M^2} + \sigma_M^2 \vec{k}_M^2\right)$$

$$\sigma_M^2 = \begin{cases} \dfrac{2}{3}\dfrac{(e_Q+e_{\bar{q}})(m_Q+m_{\bar{q}})^2}{e_Q m_{\bar{q}}^2 + e_{\bar{q}} m_Q^2} < r_M^2 > & (n=0) \\[2ex] \dfrac{2}{5}\dfrac{(e_Q+e_{\bar{q}})(m_Q+m_{\bar{q}})^2}{e_Q m_{\bar{q}}^2 + e_{\bar{q}} m_Q^2} < r_M^2 > & (n=1) \end{cases}$$

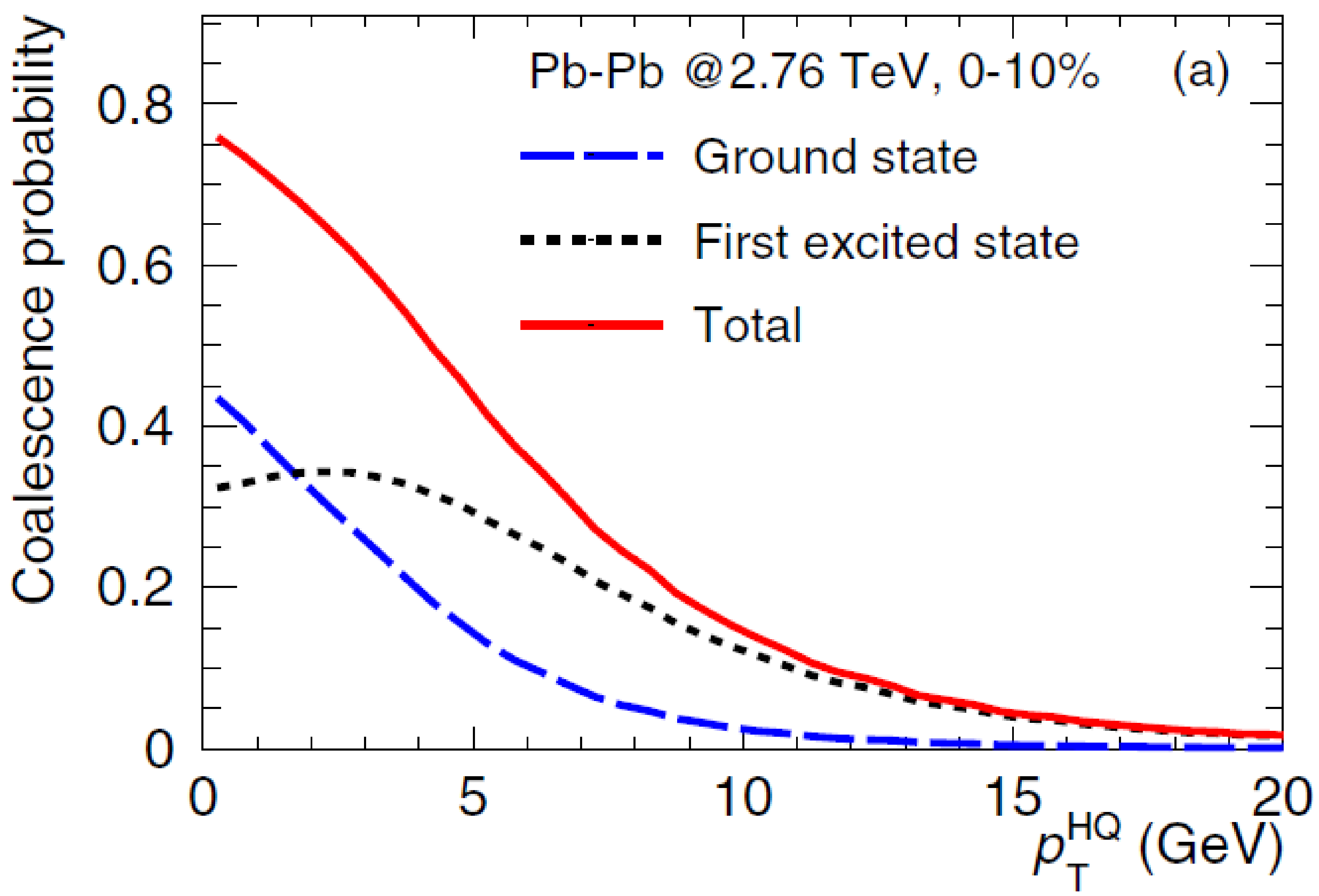
$$< r_M^2 > \approx (0.9\ fm)^2$$

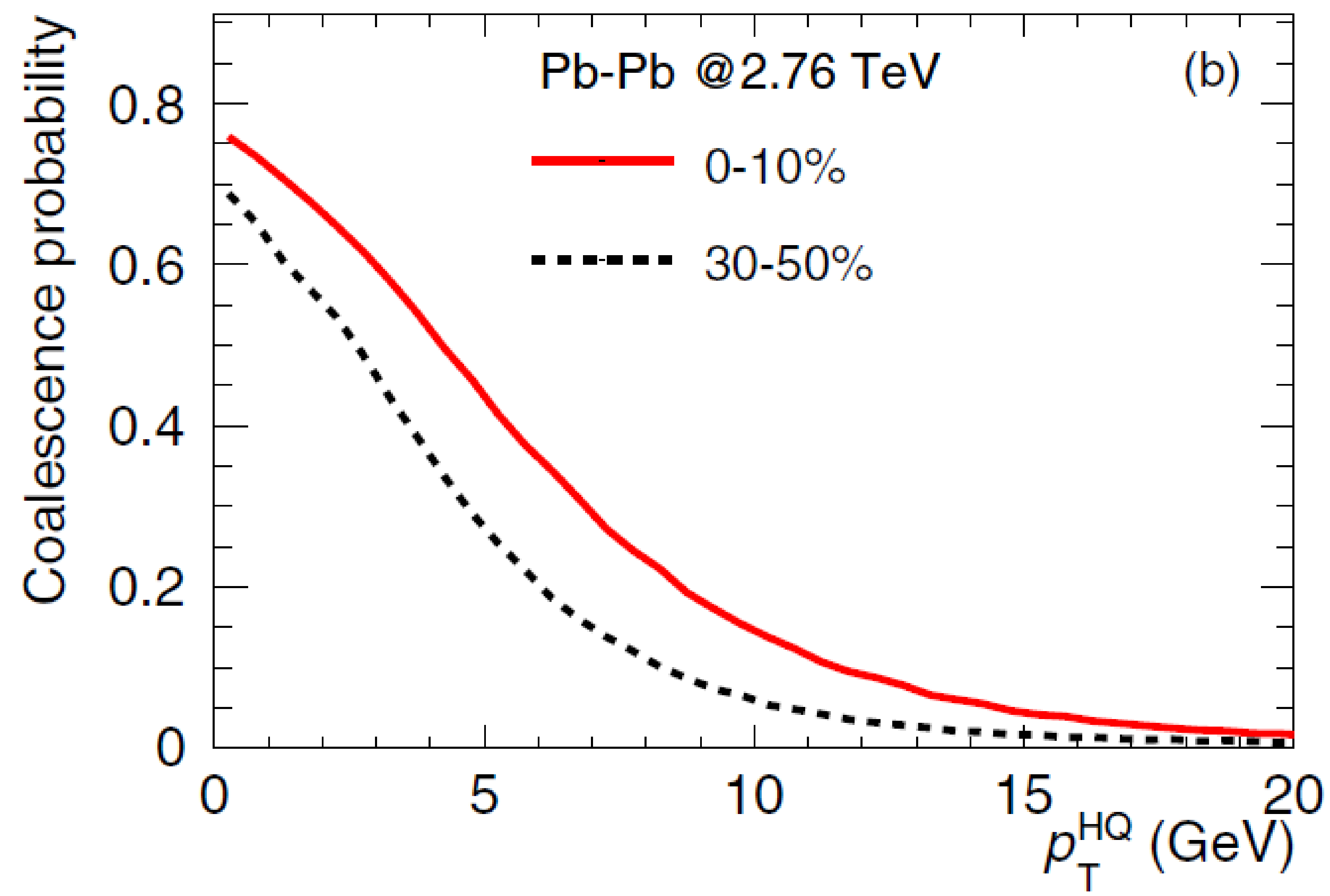
$$m_c = 1.5\ \mathrm{GeV}$$

$$m_u = m_d = 300\ \mathrm{MeV}$$

$$m_s = 475\ \mathrm{MeV}$$

Hadronization via heavy-light coalescence (2/2)

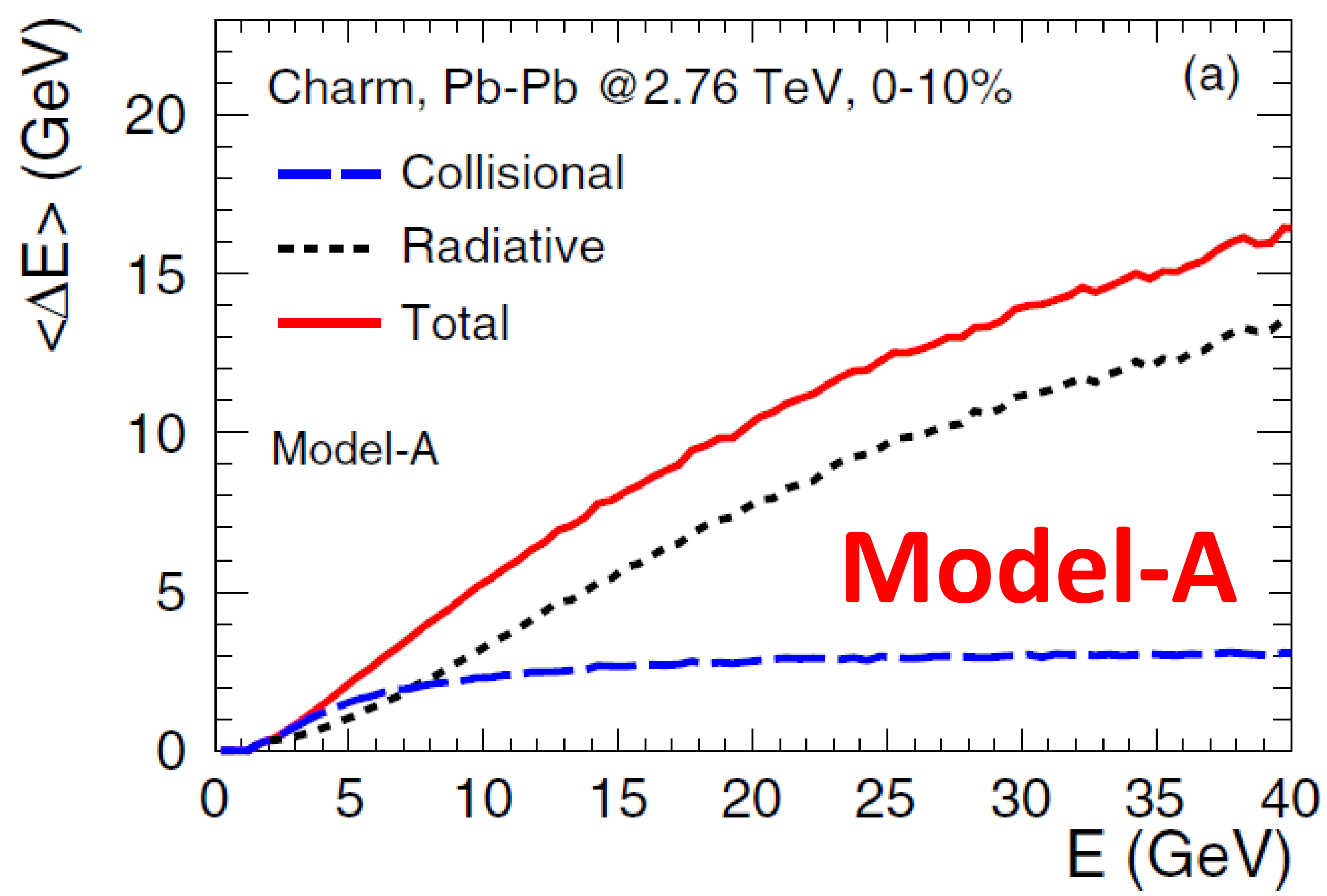


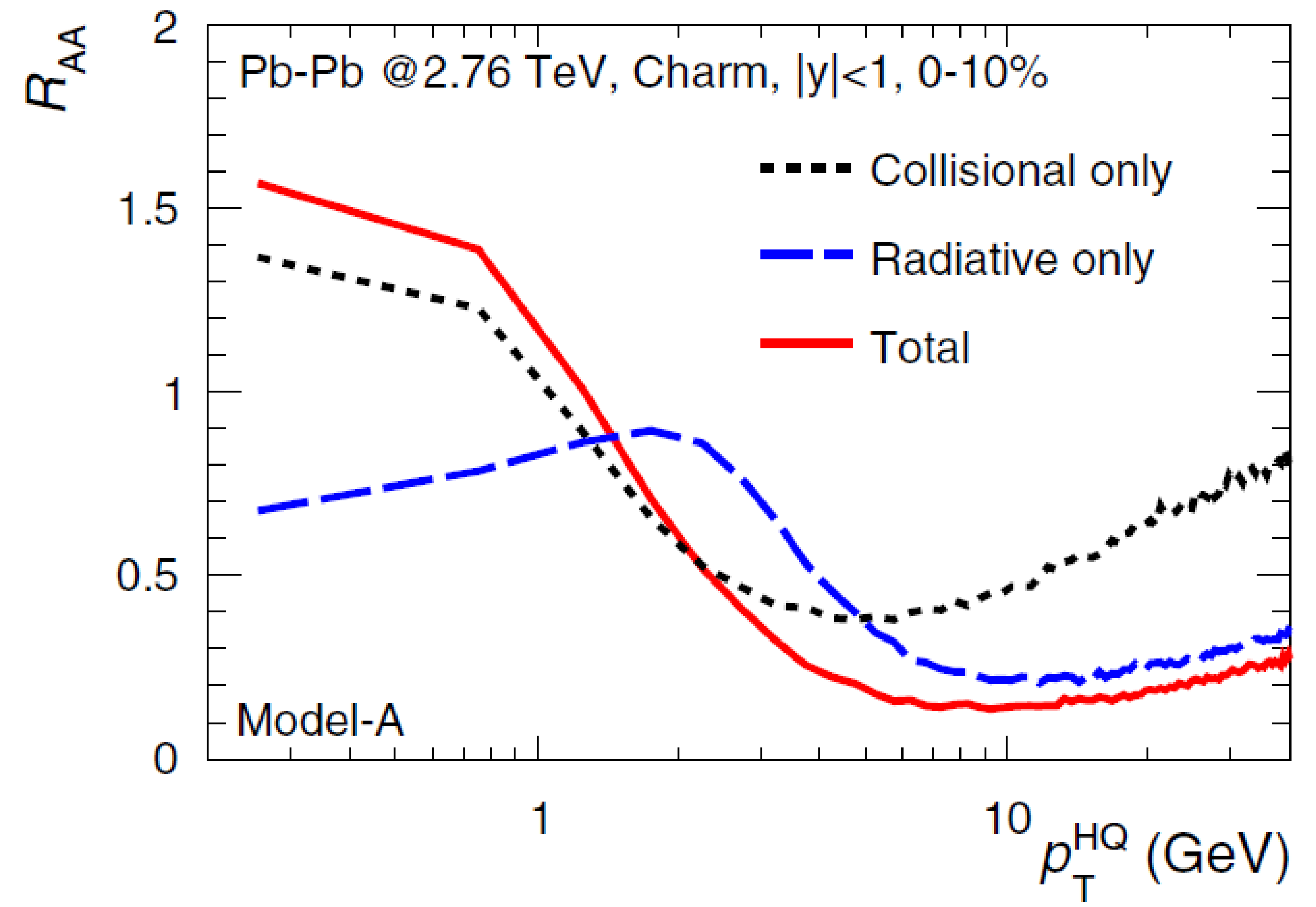


- Coalescence into ground states $\overline{W}_M^{(0)}(\vec{y}, \overline{k})$ has maximum probability at $p_T \approx 0$, and decreases toward high p_T (due to the difficulty of finding a coalescence partner in this region)
- Coalescence into first excited state $\overline{W}_M^{(1)}(\vec{y}, \overline{k})$ shows a slightly increasing behavior for $p_T \lesssim 3\ GeV/c$ → energetic charm (anti-)quarks are needed to form D mesons in the highly excited states ?
- Larger coalescence probability for more central collisions → the coalescence partner density is larger in **0-10%** than in **30-50%**, resulting in a larger probability to form heavy-light combinations

Charm quark energy loss: radiative vs. collisional

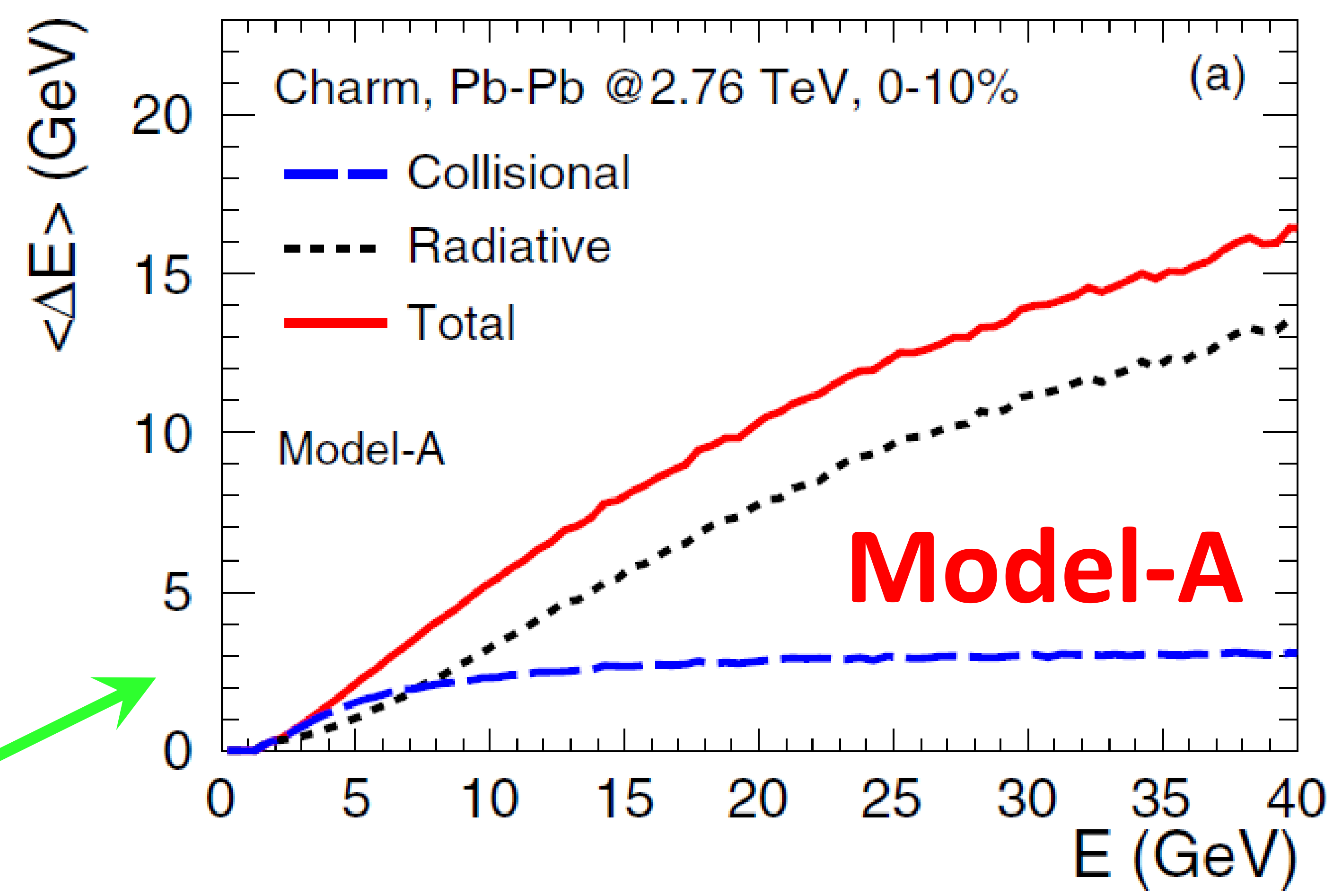
- Radiative energy loss is dominated at high energy, while the collisional energy loss is significant at low energy → reflected in R_{AA}

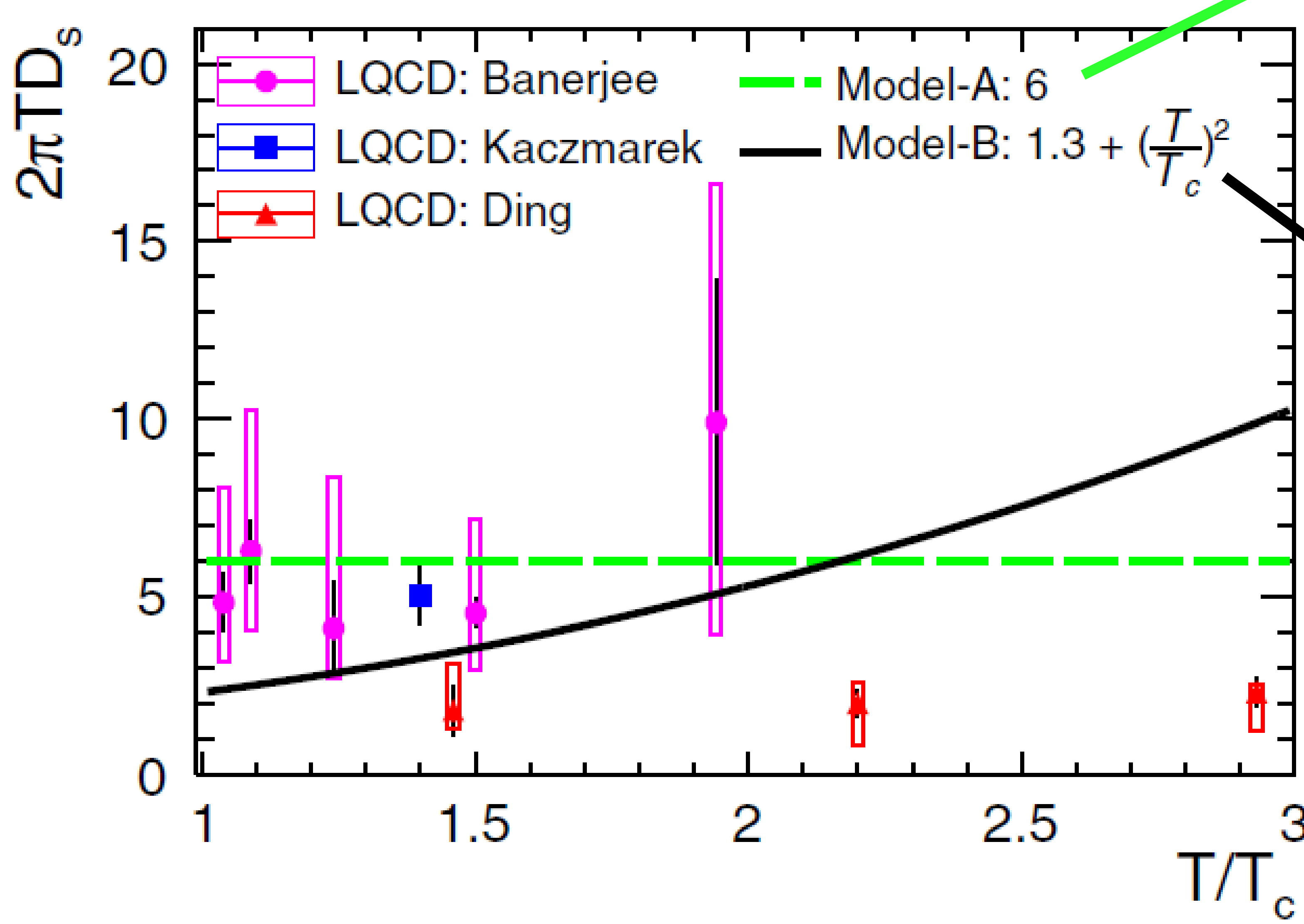


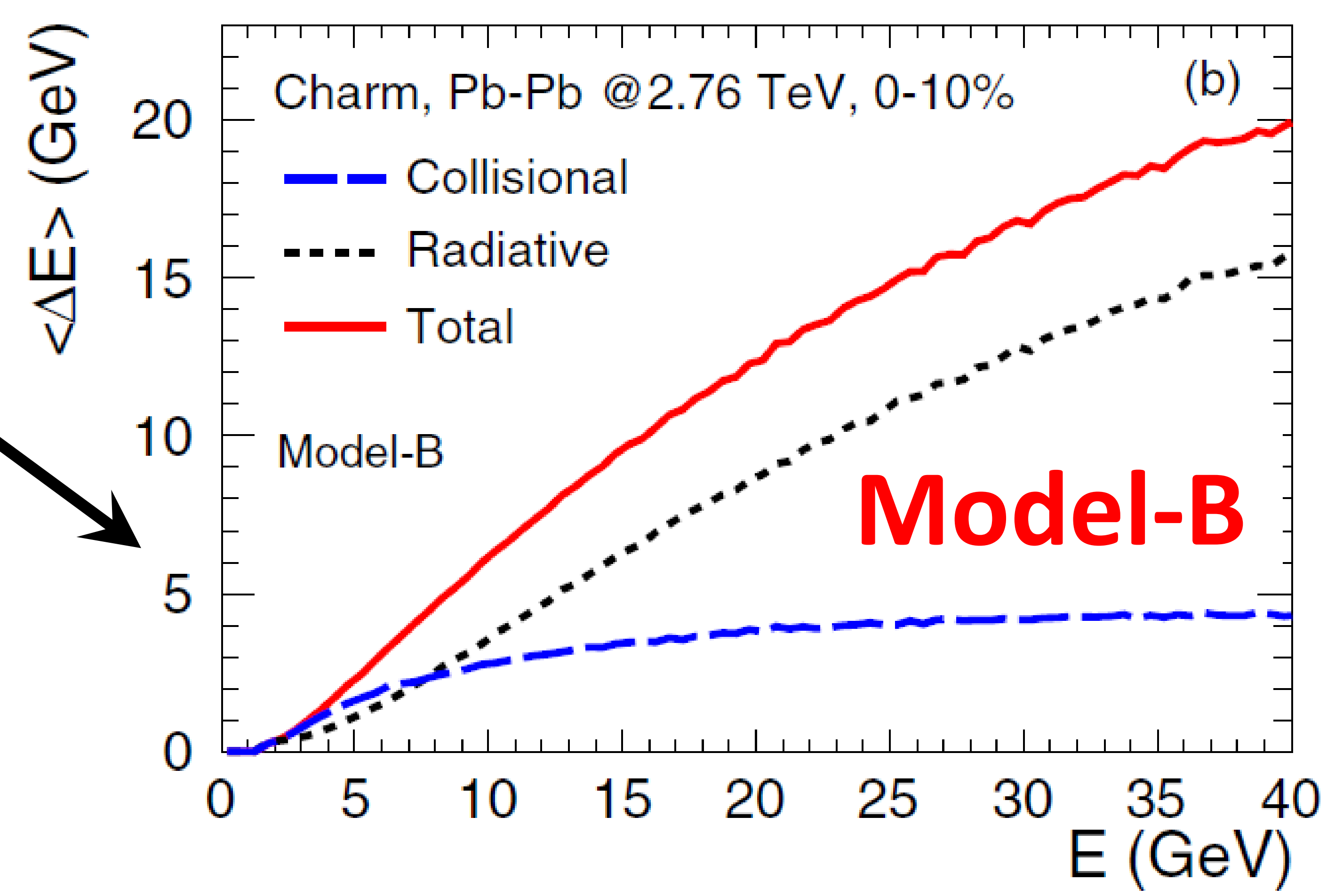


Charm quark energy loss: radiative vs. collisional

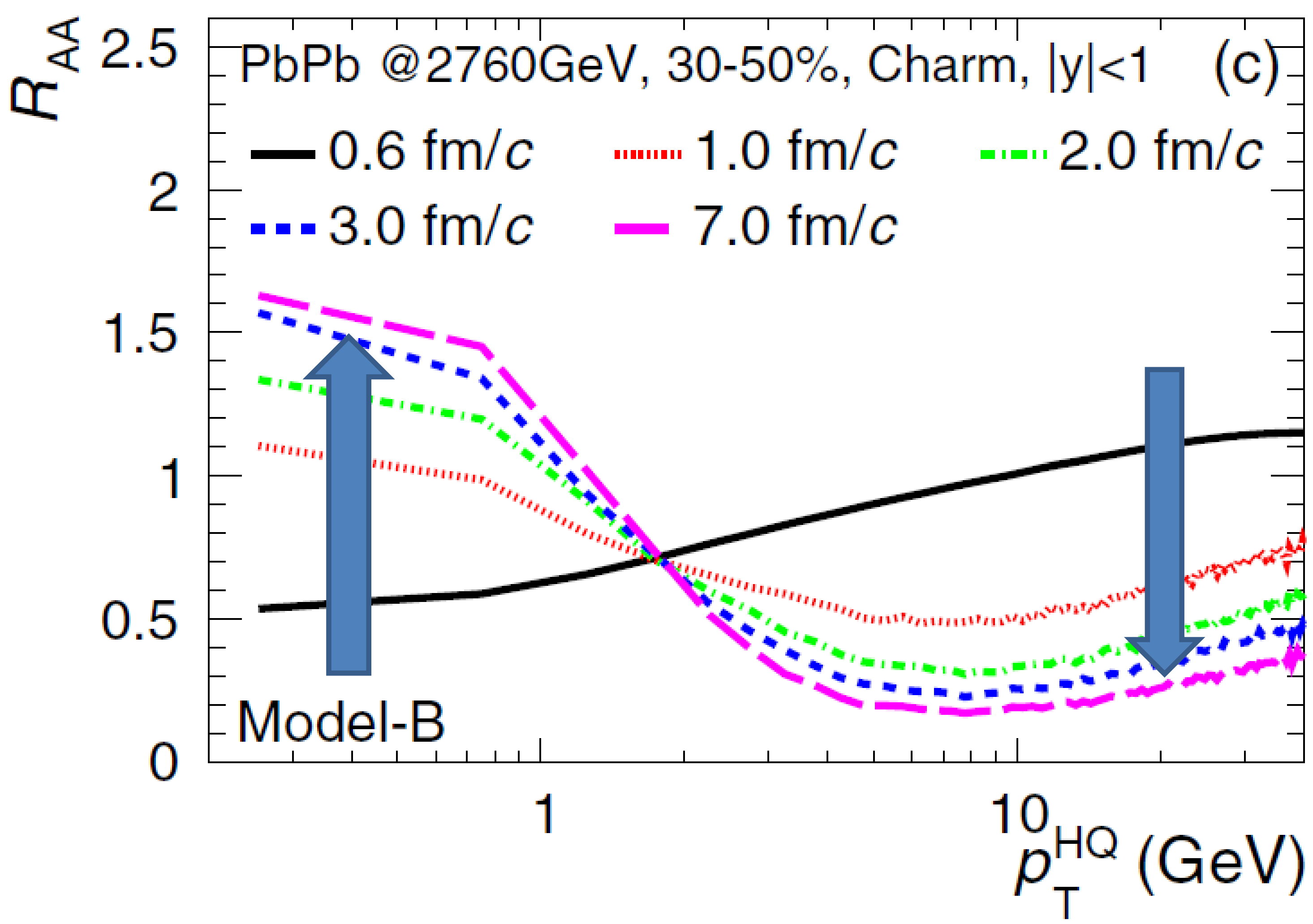
- Radiative energy loss is dominated at high energy, while the collisional energy loss is significant at low energy → reflected in R_{AA}
- Multiple elastic scatterings are dominated by the drag term → larger drag force with model-B → stronger interaction strength between HQ and medium parton → HQ lose more energy with model-B approach

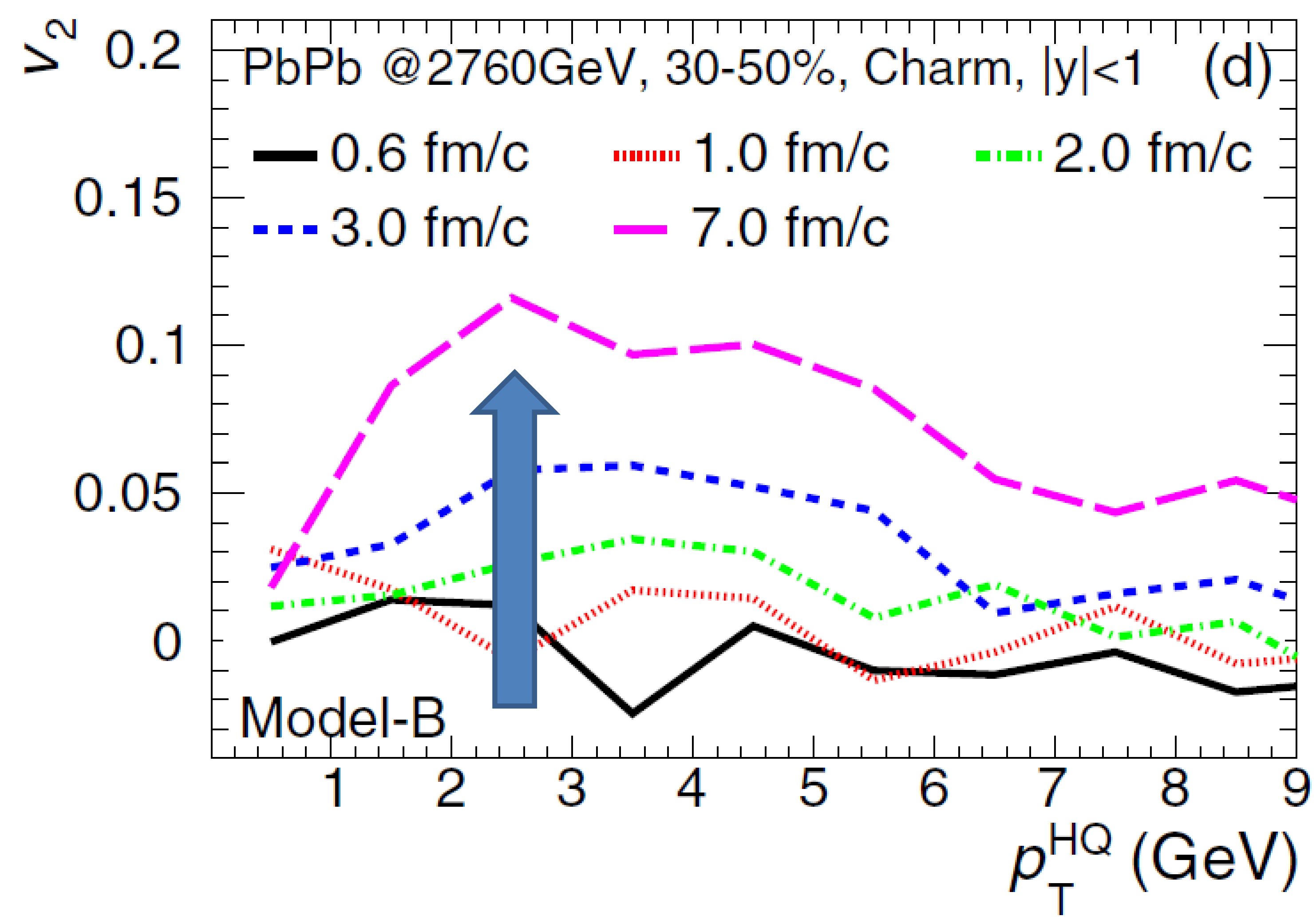






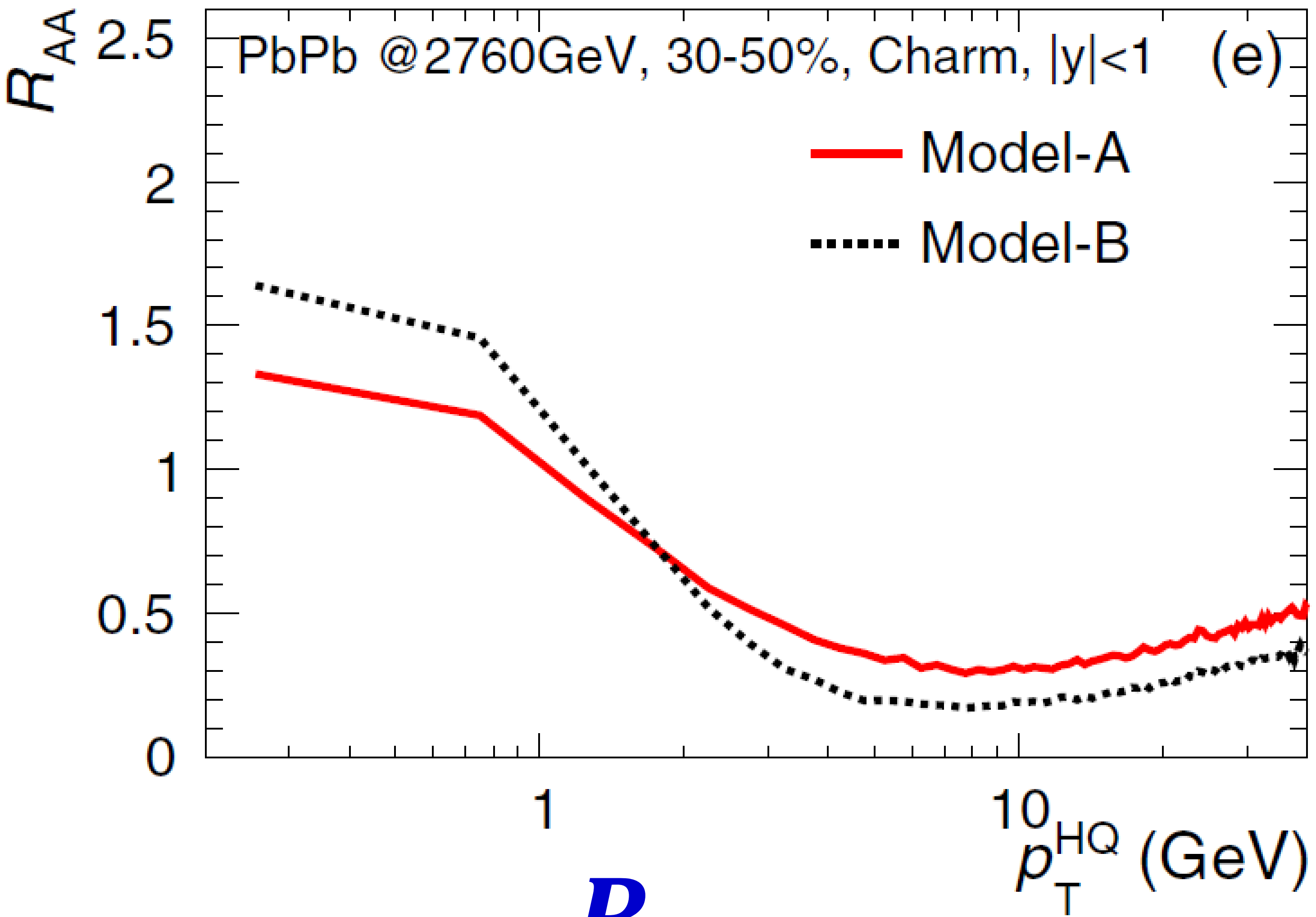
Time evolution of charm quark R_{AA} and v_2



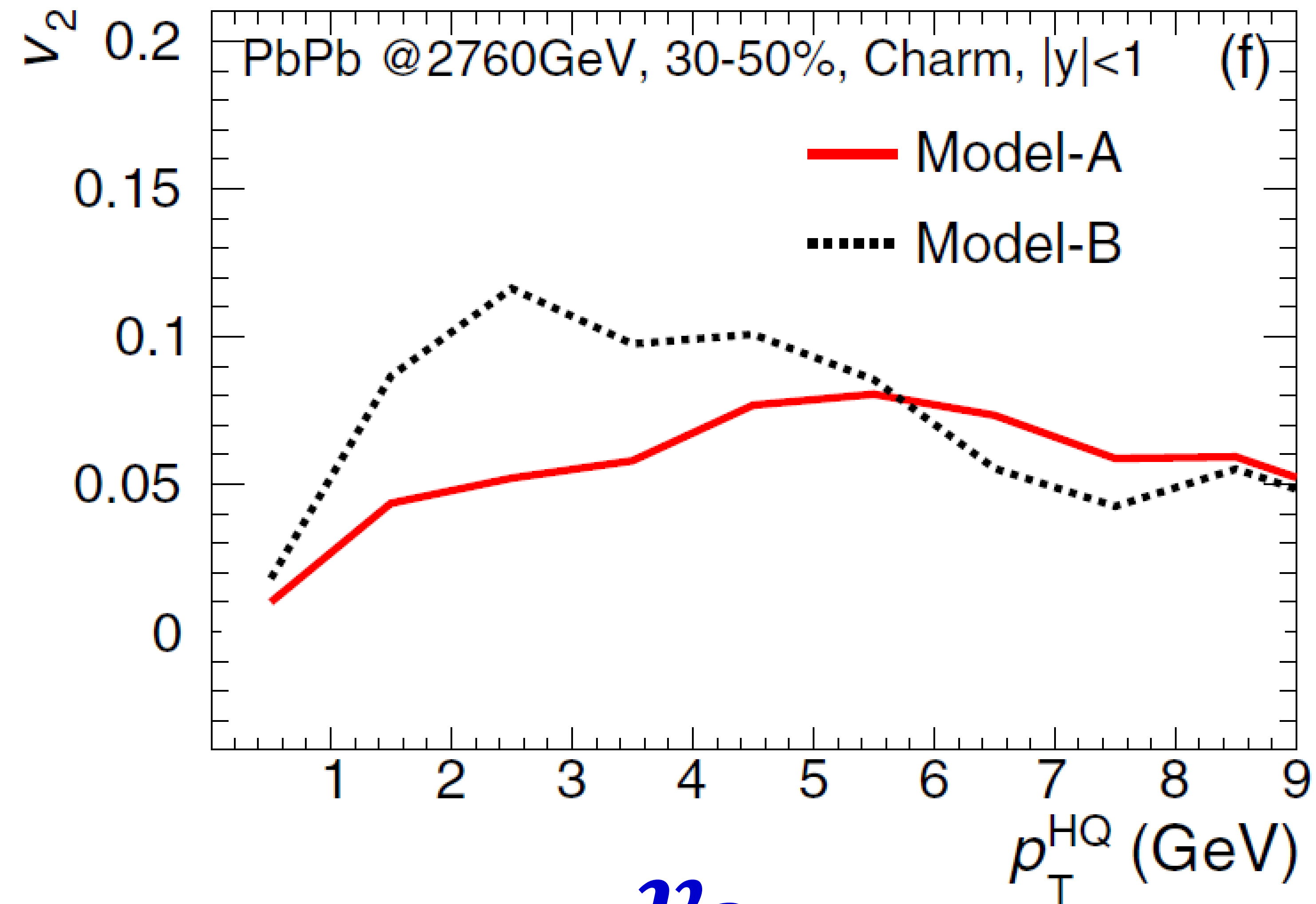


- R_{AA} (v_2) develops during the early (late) stage of the evolution
  - ✓ R_{AA} modification is less pronounced after $\sim\tau = 7\ fm/c$;
  - ✓ v_2 develops mostly at late times, reaching the maximum at $\sim\tau = 7\ fm/c$;
- Competition between the initial drag and the subsequent collective effect tends to restrict the time dependence of R_{AA}

T-dependence of spatial diffusion coefficient: charm quarks



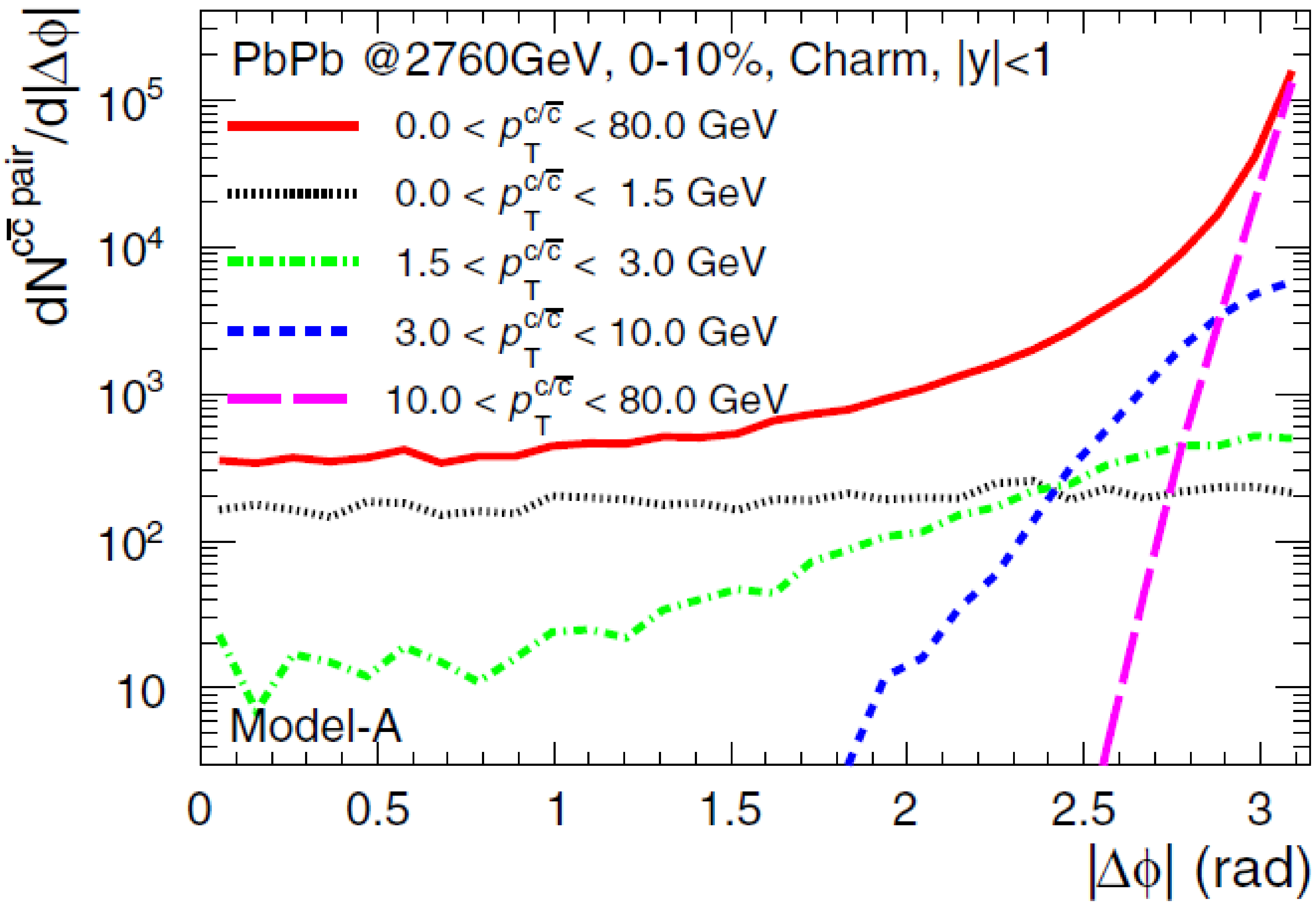
R_{AA}



v_2

- Charm quarks lose more energy with **model-B** approach, i.e. temperature dependent $2\pi T D_s$
  $\rightarrow$ the relevant R_{AA} is smaller (larger) at high (low) p_{T}
- Moreover, v_2 is significantly enhanced at intermediate p_{T} ($2 \lesssim p_{\mathrm{T}} \lesssim 4$ GeV)

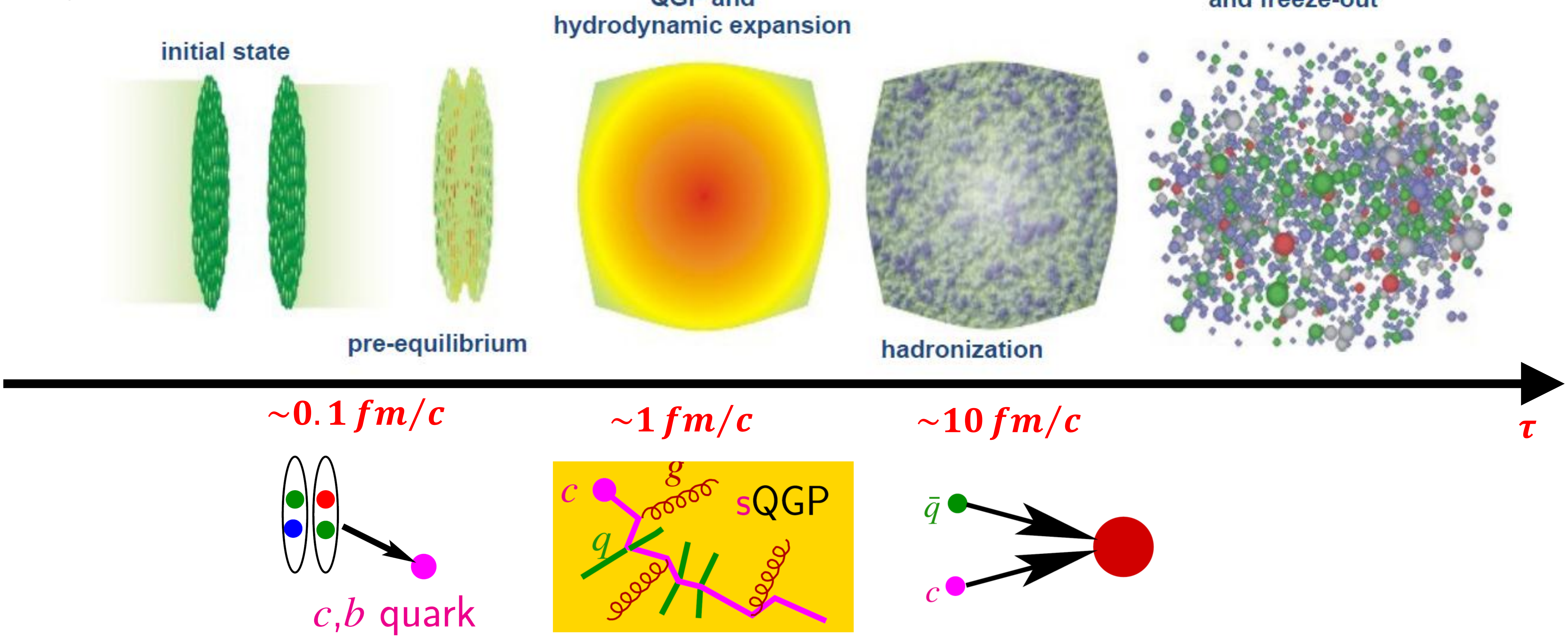
Correlation in relative azimuthal angle



- Broadening behavior found at parton level
  - ✓ initially back-to-back properties are largely washed out for $c\bar{c}$ pairs with small initial p_{T};
  - ✓ The broadening tends to decrease with increasing p_{T} → larger survival probability toward high p_{T};
- Similar behavior observed at hadron level

Short summary of the hybrid model

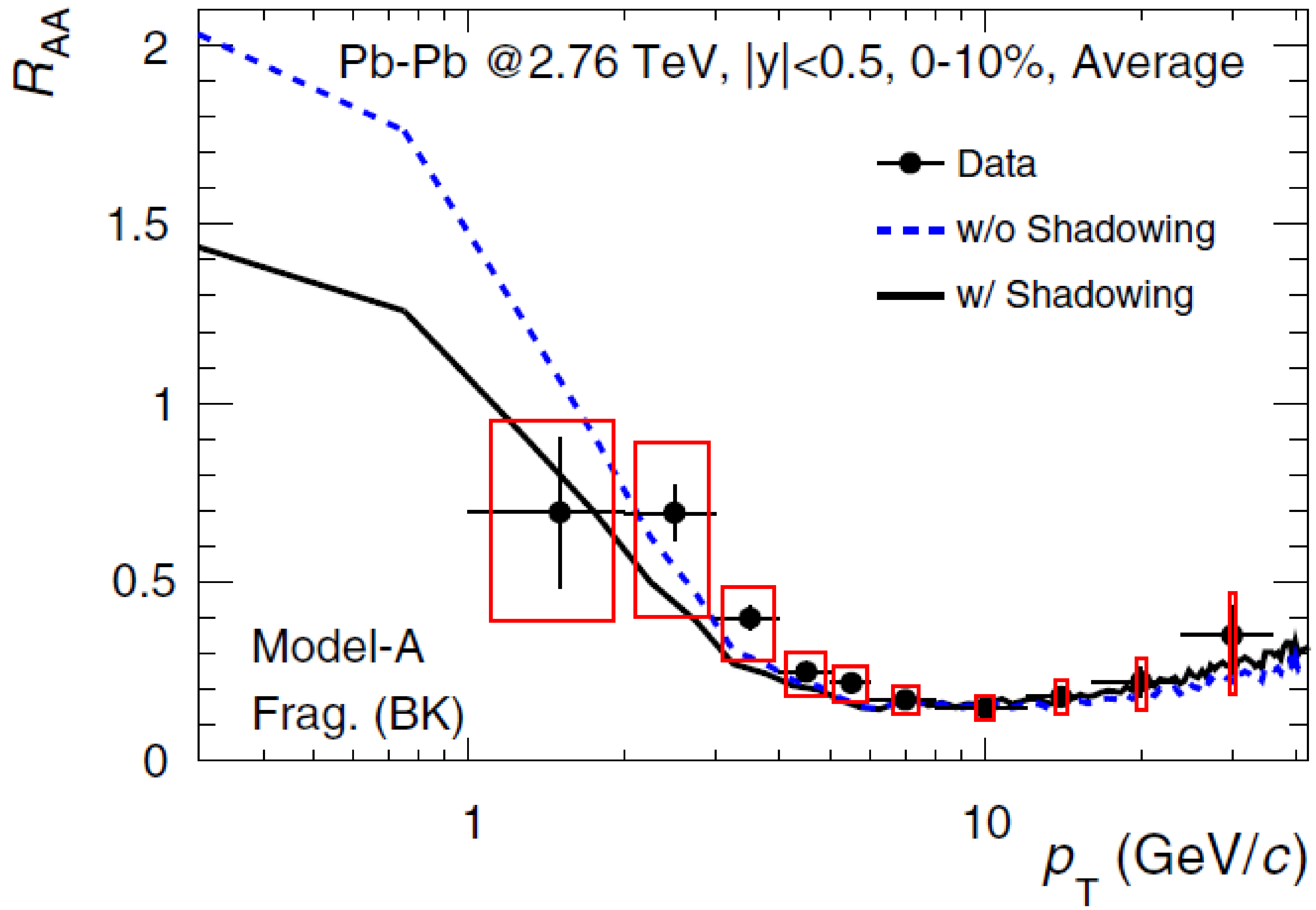
[Adopted from Bass and Hees]



Bulk matter	Glauber model (SuperMC)	(3+1)D viscous hydro (vHLLE)	Cooper-Frye (iSS)
Heavy quark	Glauber for spatial and FONLL+EPS09 for momentum	Langevin Transport Equation modified for collisional+radiation	"Dual" hadronization model: fragmentation + coalescence

[SuperMC, iSS: Comput. Phys. Commun. 199, 61 (2016). vHLLE: Comput. Phys. Commun. 185, 3016 (2014) .
FONLL: JHEP 05, 007 (1998); 03, 006 (2001); 10, 137 (2012). EPS09: JHEP 04, 065 (2009)]

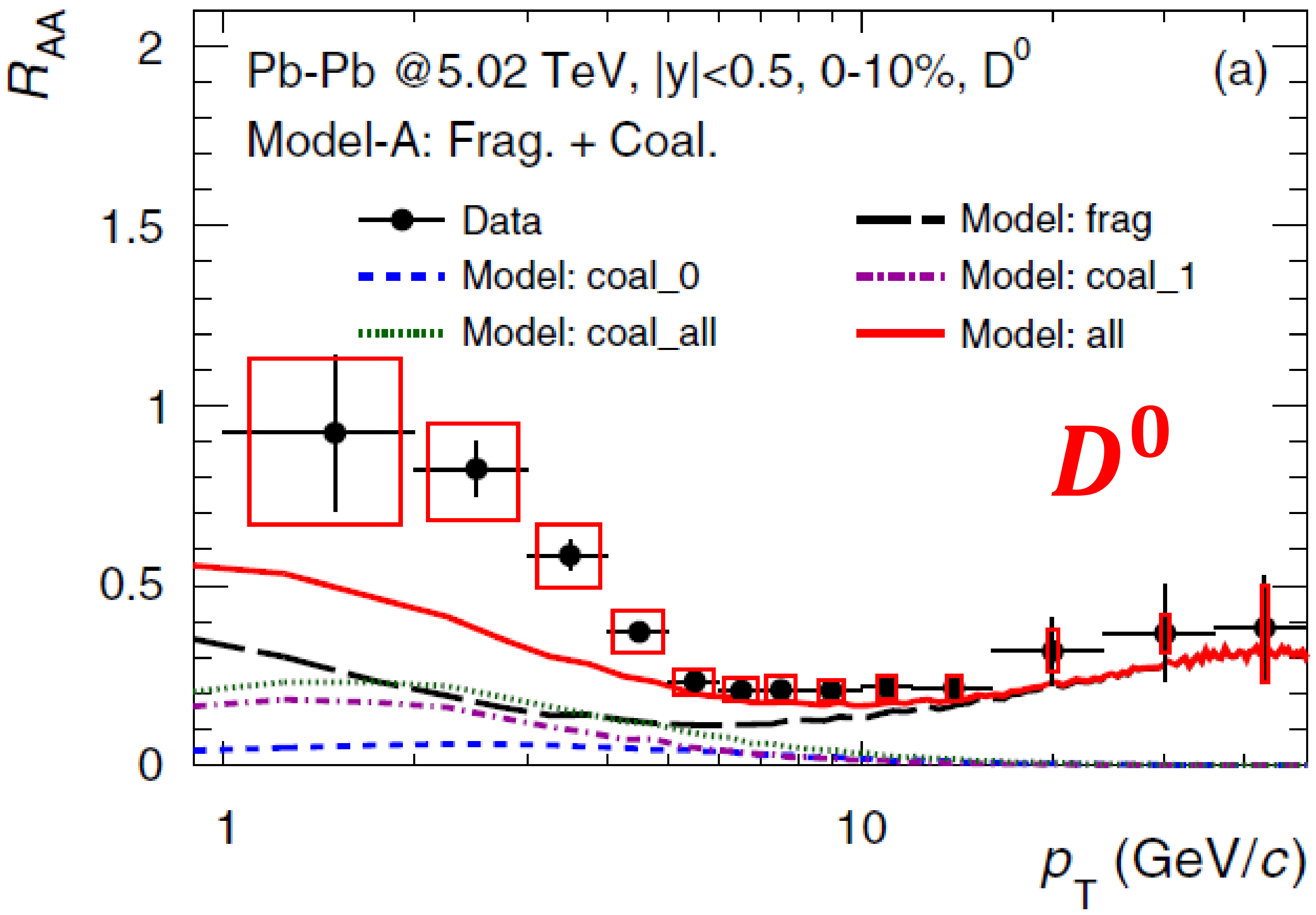
Impact of the nuclear shadowing effect

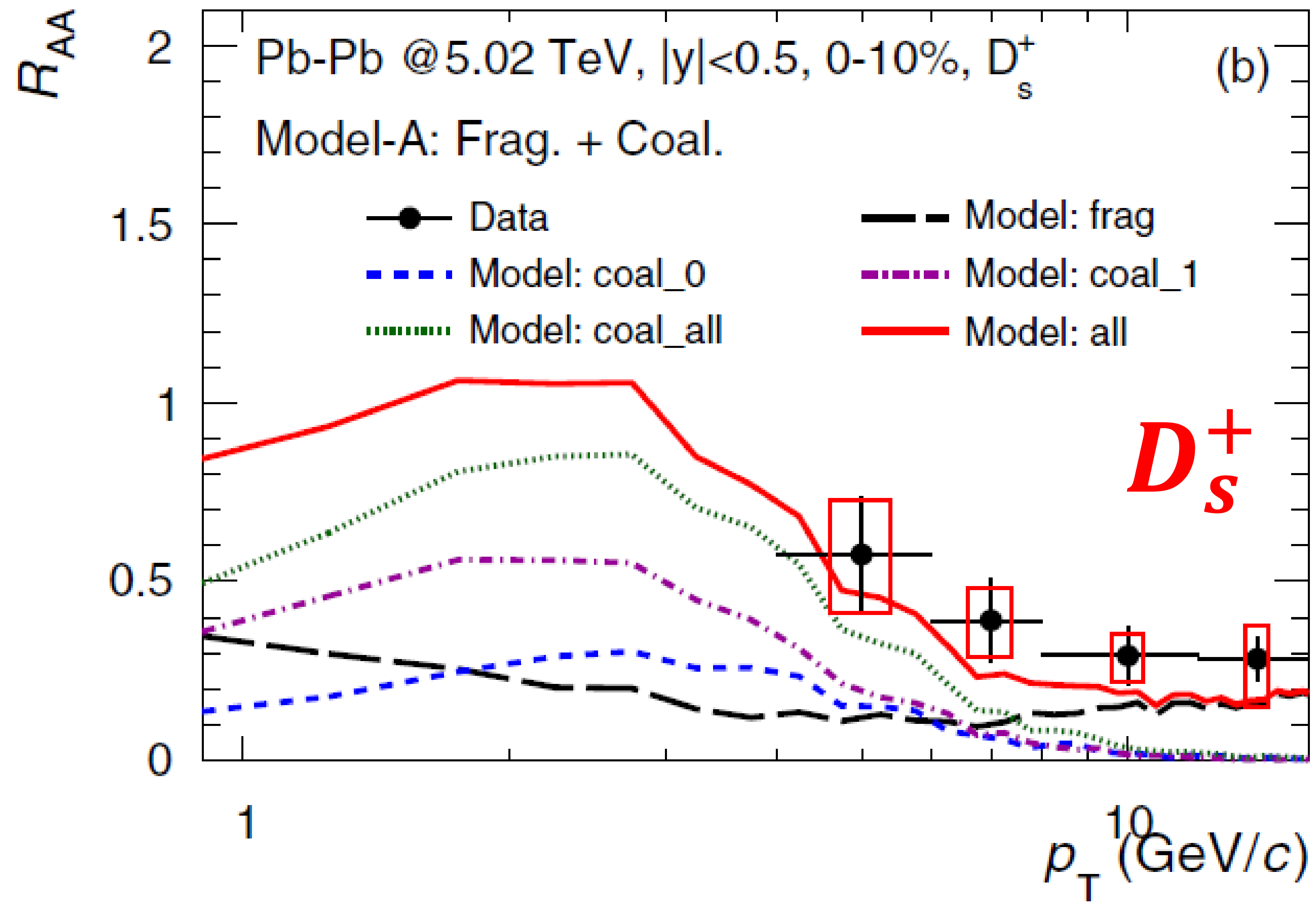


- As expected, results w/ shadowing is suppressed at low p_{T}, while it is slightly enhanced at high p_{T}
- Measurements with higher precision are needed to quantify this effect, in particular in the intermediate-p_{T} region

[Data: JHEP 03, 081 (2016)]

Hadronization: fragmentation vs. coalescence

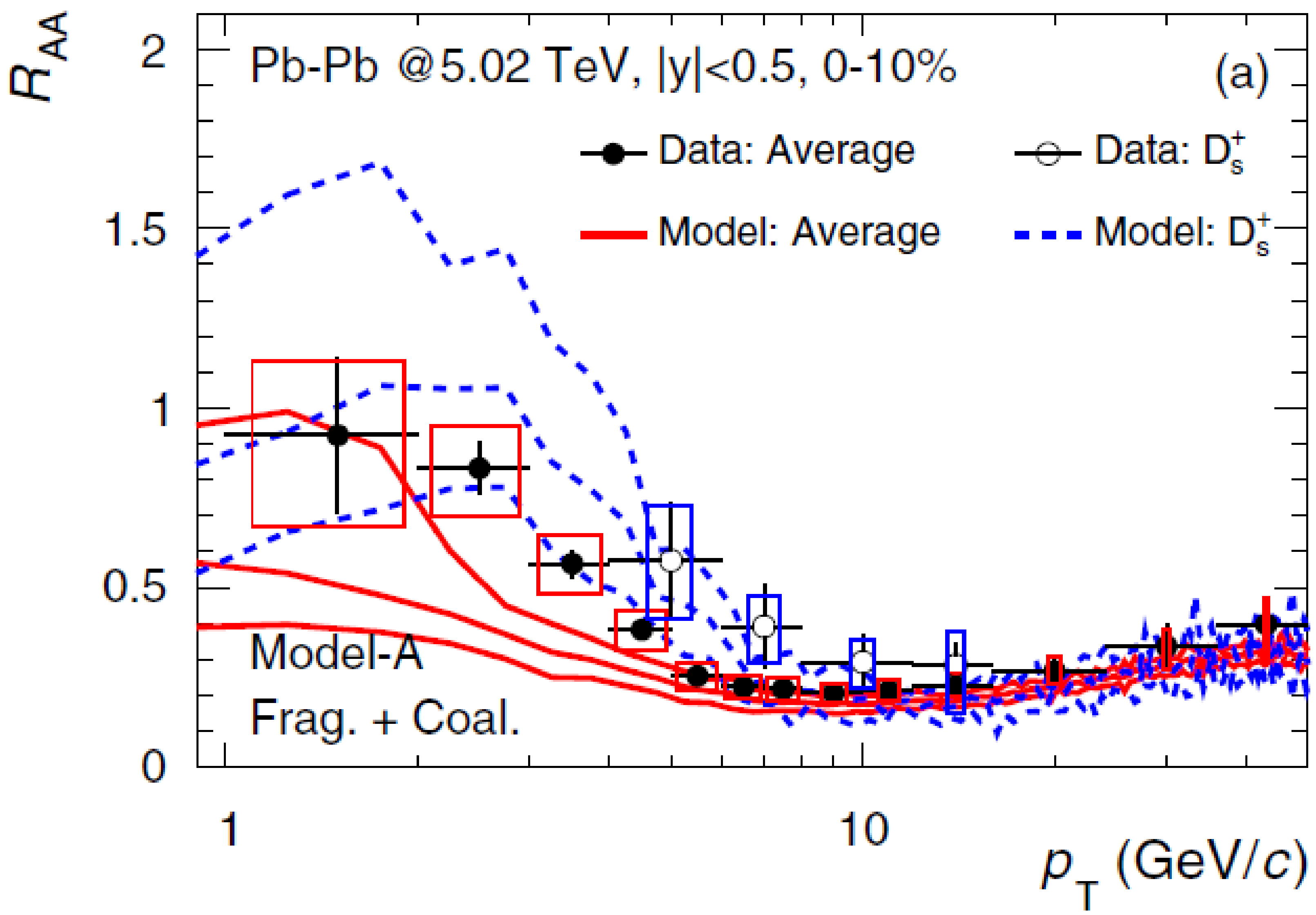


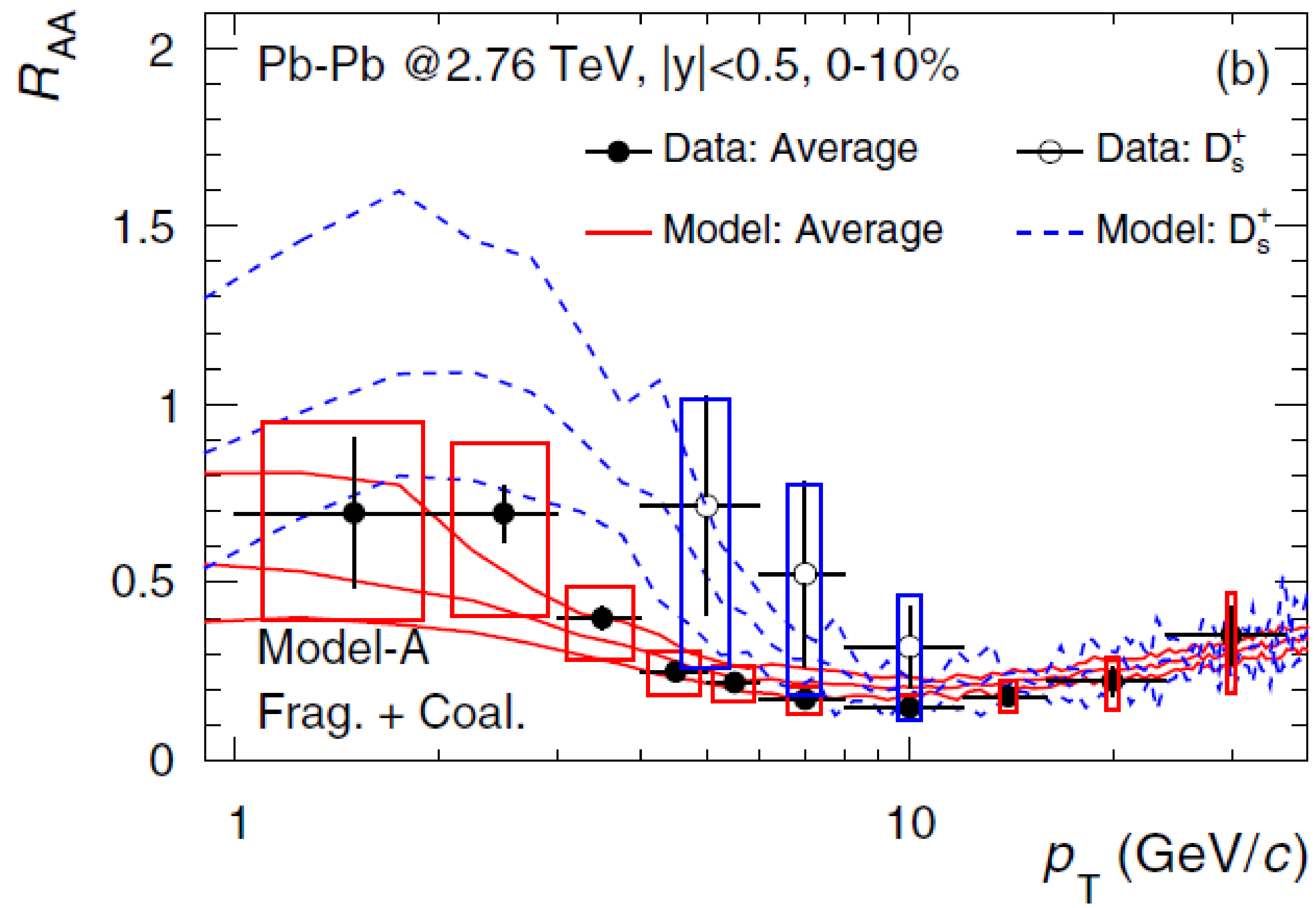


- Fragmentation is dominant at hight p_{T}, and the coalescence contribution is significant at moderate p_{T}, where the first excited states contribution is larger than that of ground state
- Heavy-light coalescence effect is more pronounced for charm-strange meson D_s^+
- Further x-check: $R_{\mathrm{AA}}(D^0)$ is close to $R_{\mathrm{AA}}(D_s^+)$ by considering alone the fragmentation
- With model-A approach, measurements can be well described by the *central predictions* in the range $p_{\mathrm{T}} \gtrsim 5\ \mathrm{GeV}/c$

[Data:1804.9083]

$R_{AA}(nonstrange)$ vs. $R_{AA}(D_s^+)$



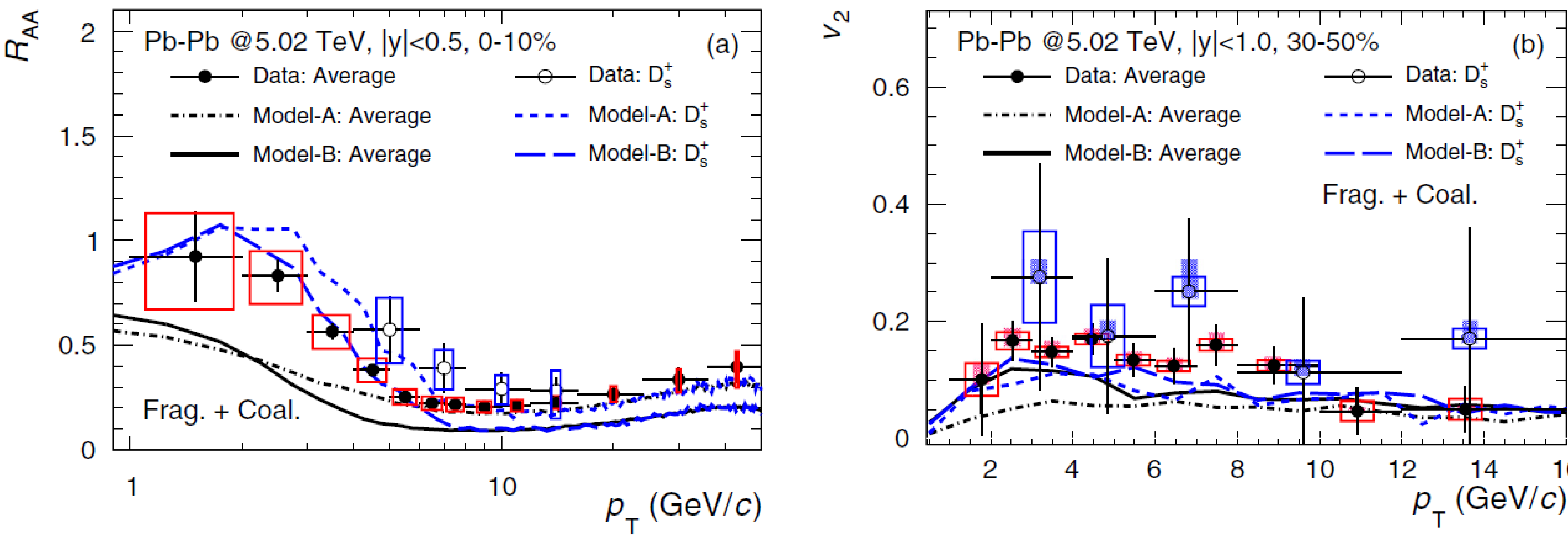


Pb-Pb @5.02 TeV **Pb-Pb @2.76 TeV**

- Systematic uncertainty on FONLL calculations dominates at $p_\mathrm{T} \lesssim 3\,\mathrm{GeV}/c$, while the one on nuclear shadowing is compatible with FONLL at higher p_T
- $R_{AA}(nonstrange)$ *and* $R_{AA}(D_s^+)$ are similar in the range $p_\mathrm{T} \gtrsim 6\ \mathrm{GeV}/c\ \ (p_\mathrm{T} \gg m_c)$, while the latter one is systematically larger at $2 \lesssim p_\mathrm{T} \lesssim 5\,\mathrm{GeV}/c$ → enhancement of D_s^+ production compared with the nonstrange one → coalescence effect

[**5.02TeV**: 1804.9083. **2.76 TeV**: JHEP 03, 081 (2016)]

T-dependence of spatial diffusion coefficient: D mesons



Model-A: $2\pi T D_s = const.$
Model-B: $2\pi T D_s = 1.3 + (T/T_c)^2$

- Similar behaviors found at parton and hadron level
- R_{AA} favors model-A while the measured v_2 prefer model-B

  → still true for the available measurements performed at RHIC

  → it is necessary to consider the temperature/momentum dependence of $2\pi T D_s$ to simultaneously describe the D meson R_{AA} and v_2 [Xu *et. al.*, PRC 97, 014907 (2018)]

[R_{AA}: 1804.9083. v_2: PRL 120, 102301 (2018)]

Summary

- **Hybrid model built to extract QGP transport properties via HF Langevin dynamics**
  - ✓ **initial conditions**
    - ➢ spatial space: Glauber-based model (⊥) + data-inspired parametrization (∥)
    - ➢ momentum space: pQCD calculations
  - ✓ **Langevin transport**
    - ➢ drag and diffusion coefficients obtained via a "minimum model": isotropic momentum space diffusion; post-point scheme; $2\pi TD_s$ from phenomenological fit to lQCD
    - ➢ gluon radiation included according to HT calculations
    - ➢ underlying temperature and flow velocity filed quantified by a (3+1)D viscous hydro
  - ✓ **"dual" hadronizaiton**
    - ➢ universal fragmentation functions
    - ➢ heavy-light coalescence

Model-A: $\boldsymbol{2\pi TD_s = const.}$
Model-B: $\boldsymbol{2\pi TD_s = 1.3 + (T/T_c)^2}$

- **Charm quark in-medium energy loss mechanisms: radiative vs. collisional**
  - ✓ radiative component is dominant at high energy, while the collisional contribution is significant at low energy
- **Charm quark hadronization: fragmentation vs. coalescence**
  - ✓ fragmentation is dominant at high p_T, while the coalescence is significant at moderate p_T, resulting in an enhancement of D_s^+ yield in this region

R_AA favors model-A's assumption for the dependence of $2\pi TD_s$ on temperature, while the measured v_2 prefer model-B → further improvement needed in the determination of drag and diffusion coefficients

[Catania: PLB 747, 260-264 (2015)]

Outlook

- **CUJET3 + Langevin joint model → well advanced**
  - ✓ **idea & strategy**
    - ➢ CUJET3 predictions on open heavy-flavor will be extended from hard to soft
    - ➢ drag and diffusion coefficients in Langevin transport equation can be rewritten in terms of $\hat{q}$, which is available from CUJET3 calculations
  - ✓ **key questions to-be further discussed** [Xin, 1810.00996]
    - ➢ heavy quark in-medium energy loss mechanisms: radiative vs. collisional
    - ➢ heavy quark hadronization: fragmentation vs. coalescence
    - ➢ moreover, heavy quark transport model: Langevin vs. Boltzmann

[**CUJET3**: 1808.05461; JHEP 02, 169 (2016); NPA 956, 617-620 (2016); CPL 32, 092501 (2015)]

Thank you all for the attention !

Backup

Heavy Quarks (charm and beauty) as probes

- Heavy quarks produced in the initial hard scatterings at short time scale
  ($\tau \sim 1/m_Q \sim 0.05 - 0.15\ fm/c < \tau_{\mathrm{QGP}}$)
  - ✓ production in QGP expected to be negligible

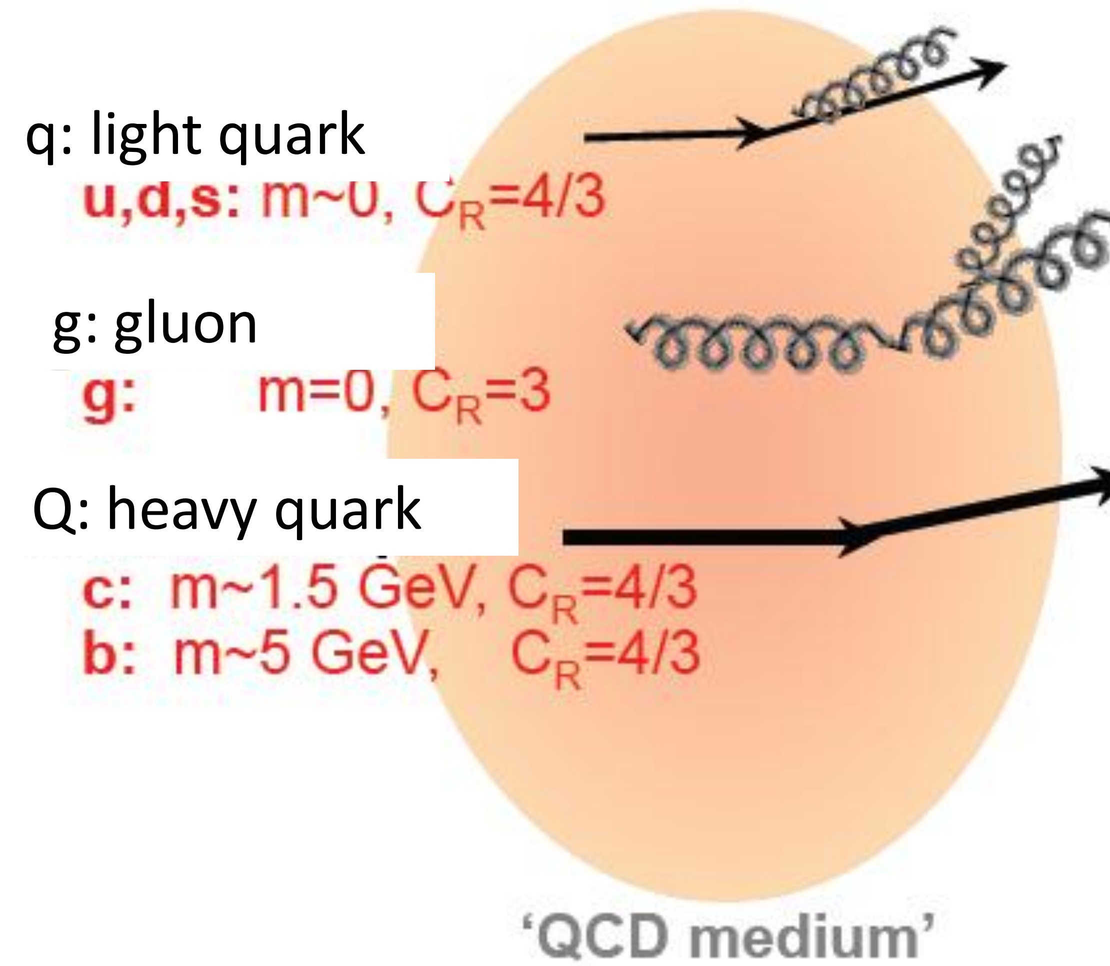
  → experience the "full evolution" of the QCD medium
- Energy loss via (in)elastic interaction with the medium constituents
  - ✓ gluon radiation (inelastic) and collisional (elastic)
  - ✓ interplay between them
- Open heavy flavours (D, B…) probe medium density
  - ✓ medium-induced gluon radiation: ΔE is proportional to C_{R}, m_{Q}, $\hat{q}$ and L

    → C_{R}: color charge dependence of ΔE

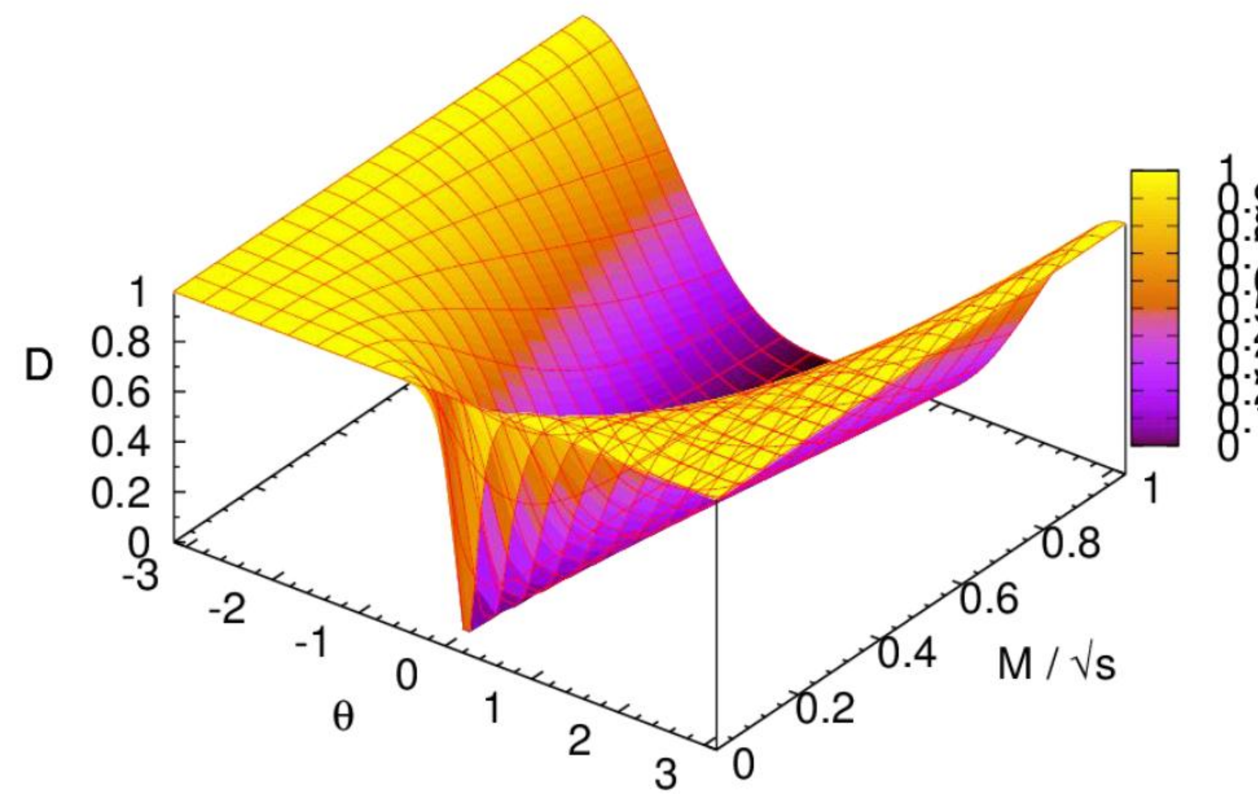
    → m_{Q}: mass dependence of ΔE

    $\Delta E_{\mathrm{g}} > \Delta E_{\mathrm{q}} > \Delta E_{\mathrm{c}} > \Delta E_{\mathrm{b}}$ expected

[for illustration]



Dead-cone effect for radiative energy loss (gluon radiation probability suppressed by D)



[J. Uphoff, QM2014]

Fragmentation models (1/3)

- Lund model modified for HQ (with the default param.)

hep-ph/0603175

$$f(z) \propto \frac{1}{z^{1+r_Q b m_Q^2}} z^{a_\alpha} \left(\frac{1-z}{z}\right)^{a_\beta} \exp\left(-\frac{b m_\perp^2}{z}\right)$$

- The famous Peterson frag. fun. (with the default param.)

PRD. 27. 105

$$f(z) \propto \frac{1}{z\left(1-\frac{1}{z}-\frac{\epsilon_Q}{1-z}\right)^2}$$

- The Braaten/FONLL style

hep-ph/9409316

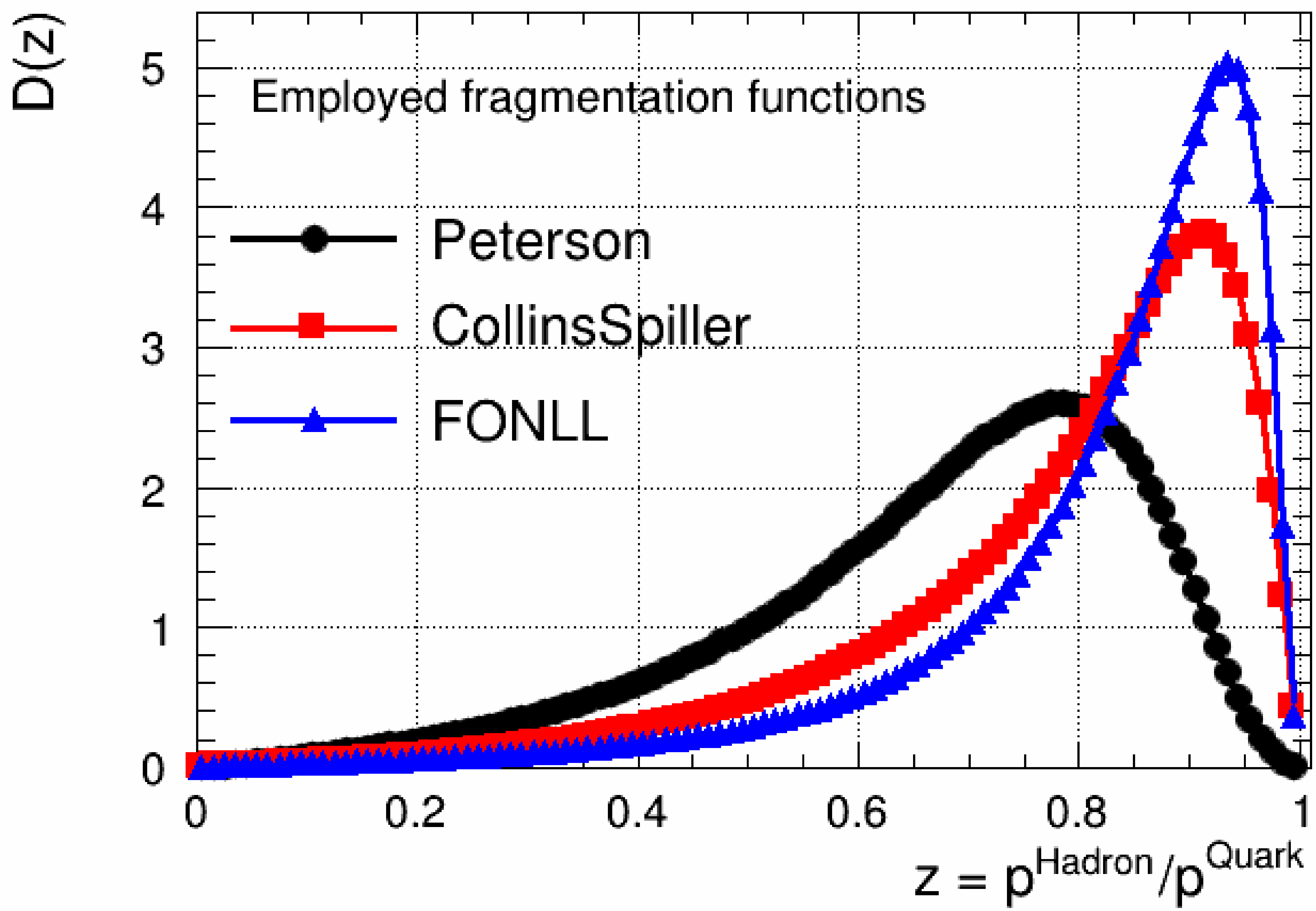
$$\begin{aligned} D_{Q\to P}(z) &= N\frac{rz(1-z)^2}{(1-(1-r)z)^6}\left[6-18(1-2r)z+(21-74r+68r^2)z^2\right. \\ &\quad \left.-2(1-r)(6-19r+18r^2)z^3+3(1-r)^2(1-2r+2r^2)z^4\right] \\ D_{Q\to V}(z) &= 3N\frac{rz(1-z)^2}{(1-(1-r)z)^6}\left[2-2(3-2r)z+3(3-2r+4r^2)z^2\right. \\ &\quad \left.-2(1-r)(4-r+2r^2)z^3+(1-r)^2(3-2r+2r^2)z^4\right] . \end{aligned}$$

- The Collins-Spiller style (with the default param.)

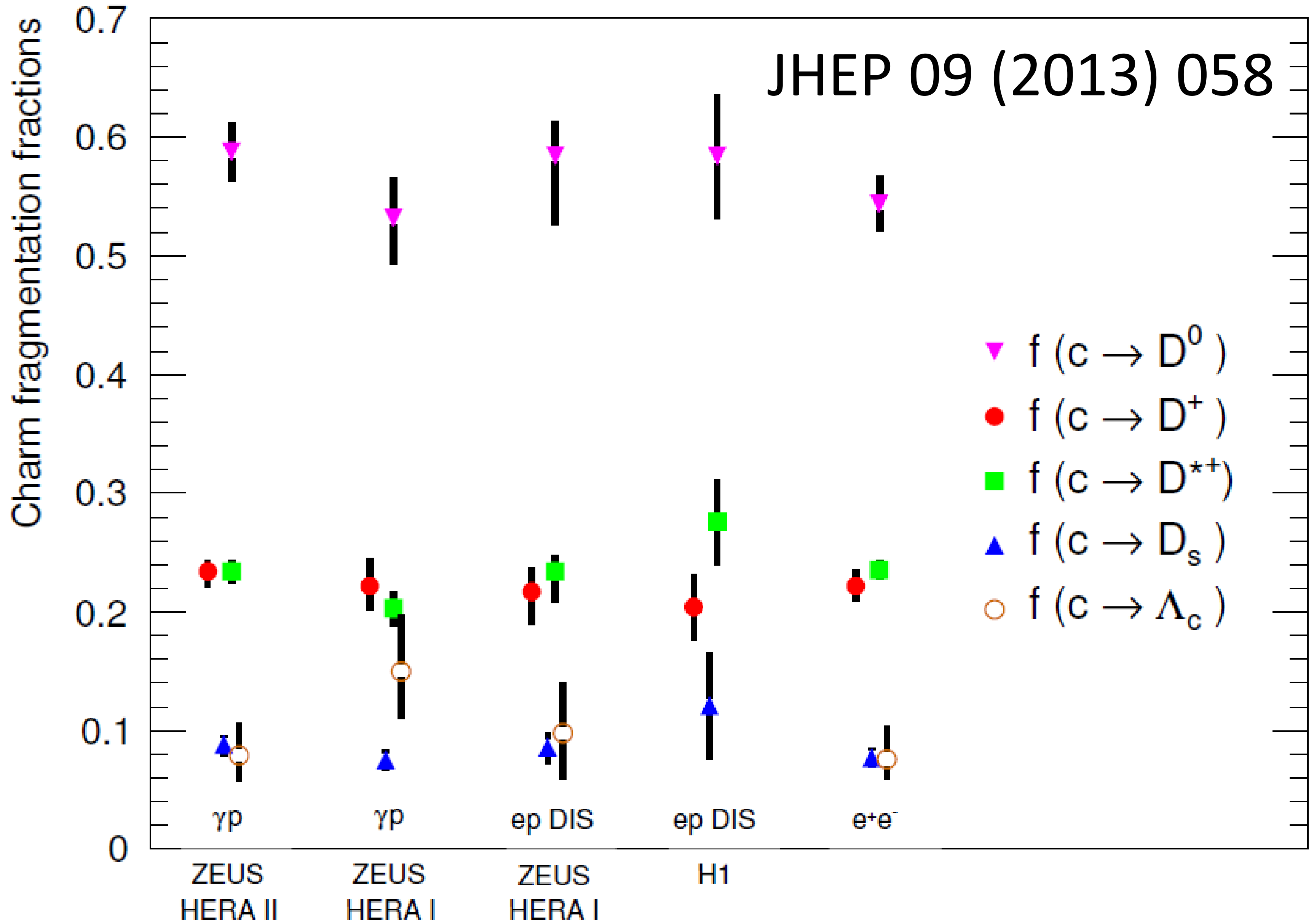
JPG. 11. p. 1289

$$f(z) \propto \left[\frac{1-z}{z}+\epsilon_Q\frac{2-z}{1-z}\right]\cdot\left[\frac{1+z}{1-1/z-\varepsilon_Q/(1-z)}\right]^2$$

Fragmentation models (2/3)



Fragmentation models (3/3)



- The fractions of the produced HF-hadrons
  - ✓ Weighted average for the final results;
  - ✓ D^0(0.566), D^+(0.227), D^{*+}(0.230), D_s^+(0.081), Λ_C(0.080);
- The energy-momentum of HF-hadron
  - ✓ Same momentum direction between HF and HF-hadron.

Flavor determination of the coalescence partners

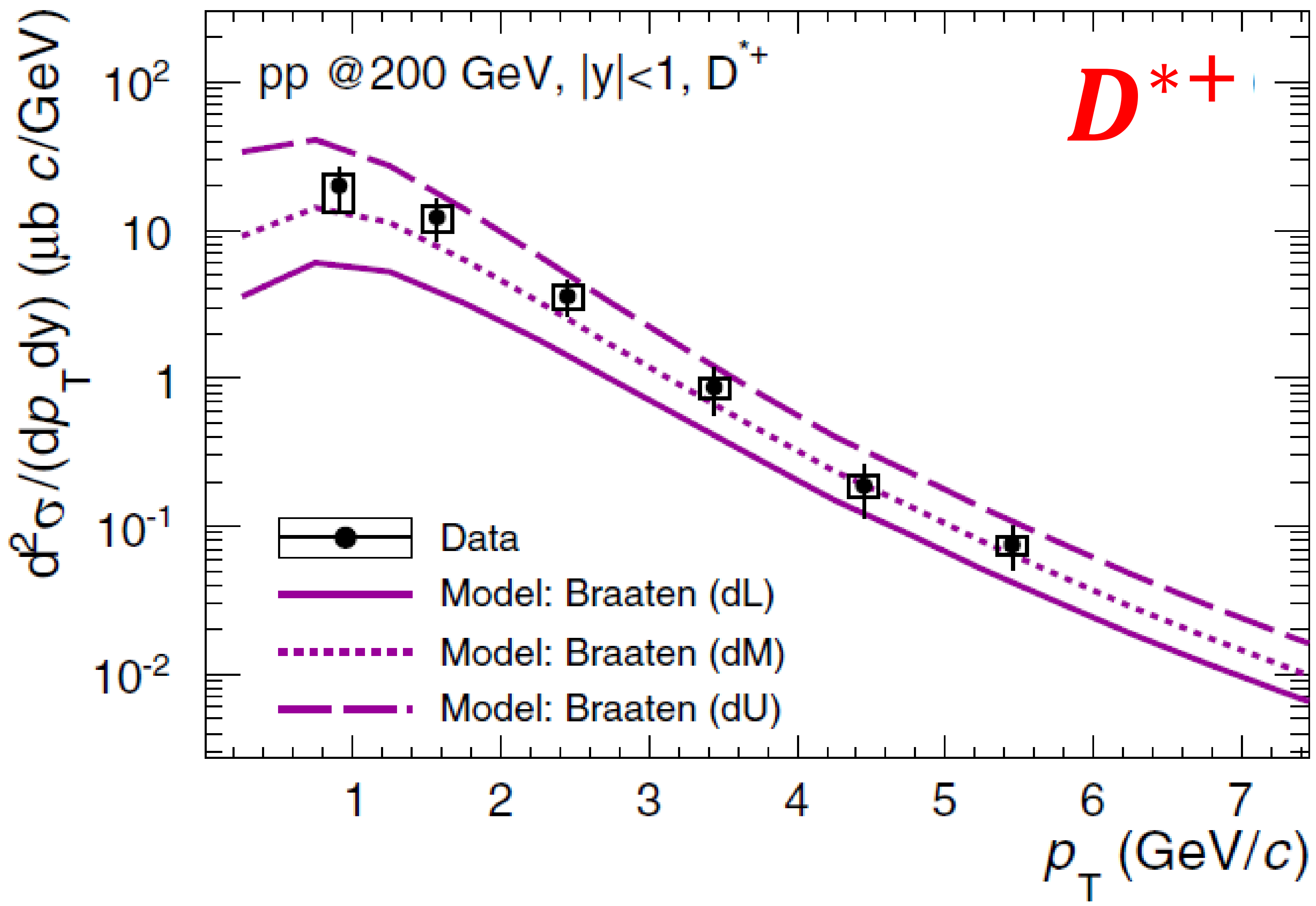
Note that the coalescence candidates are sampled among various light (anti)quarks, which are assumed to thermalize inside the QGP. Therefore, we utilize the Fermi-Dirac approach, $f_q(\vec{p}) \propto 1/\exp\{\sqrt{\vec{p}^2 + m_q^2}/T_c + 1\}$, to describe its density distribution, where m_q is the light (anti)quark mass and $T_c =$ 165 MeV is the critical temperature. The flavor of the light (anti)quark is determined according to the integrated parton density $\rho = \int_{-\infty}^{\infty} d^3\vec{p}\, f_q(\vec{p})$. For instance, $\rho_{u/d} = 0.18\ \mathrm{fm}^{-3}$ for $u/\bar{u}$ and $d/\bar{d}$ quarks and $\rho_s = 0.10\ \mathrm{fm}^{-3}$ for $s/\bar{s}$ quarks, resulting in the relative ratio $\rho_u : \rho_d : \rho_s \approx 1 : 1 : 0.5$, which is kept during the sampling procedure.

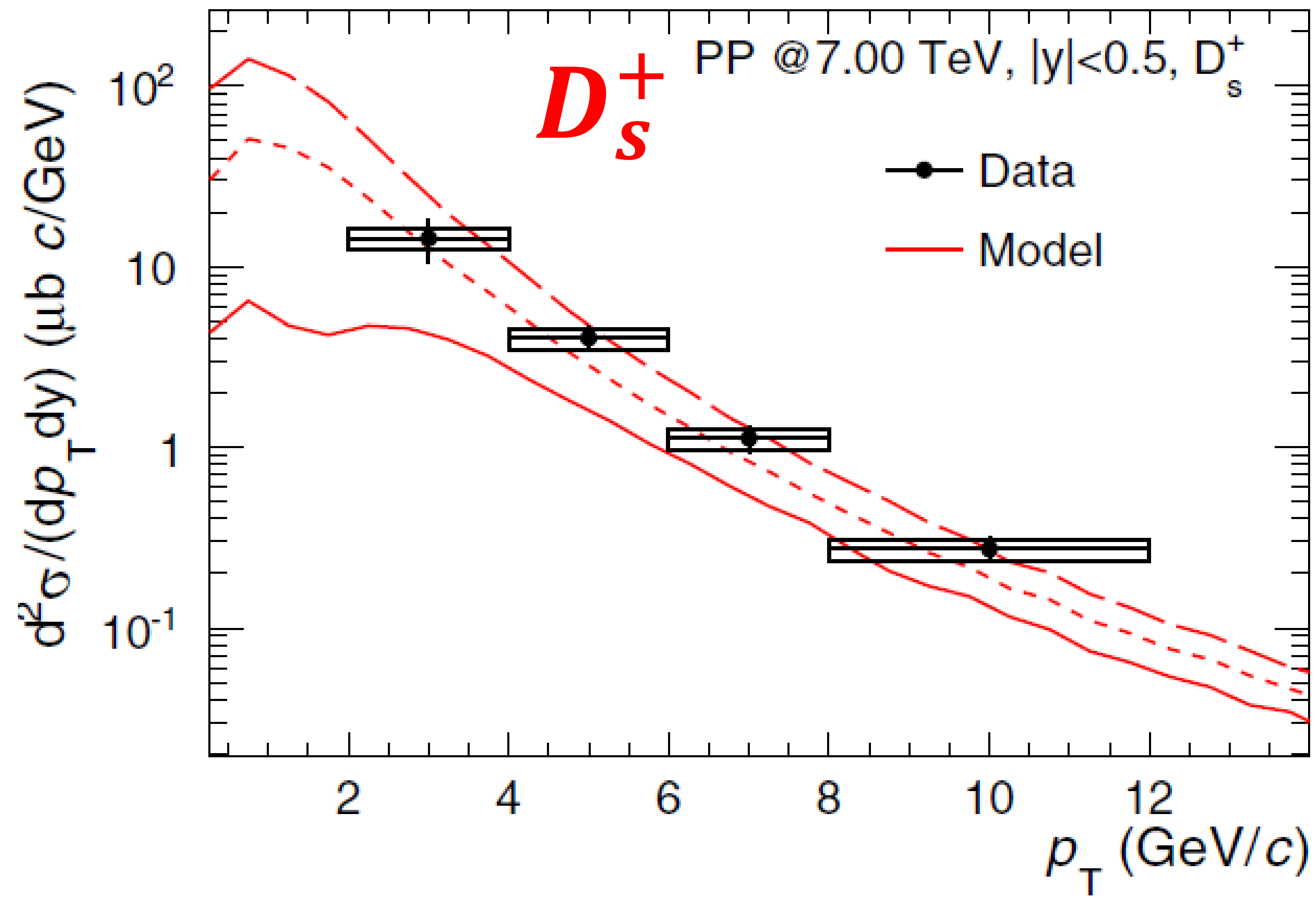
Systematic uncertainties

The initial charm quark spectra are determined by the FONLL calculations [24], as well as the corresponding central values obtained by setting $\mu_R = \mu_F = \mu_0 \equiv \sqrt{p_T^2 + m_c^2}$, where, μ_R (μ_F) is the renormalization (factorization) scale; m_c denotes the heavy quark mass, and its central value is $m_c = 1.5$ GeV. The relevant uncertainties are estimated via a conservative approach [43]: $\mu_0/2 < \mu_R,\ \mu_F < 2\mu_0, \mu_R/2 < \mu_F < 2\mu_R$ and $1.3 < m_c < 1.7$ GeV.

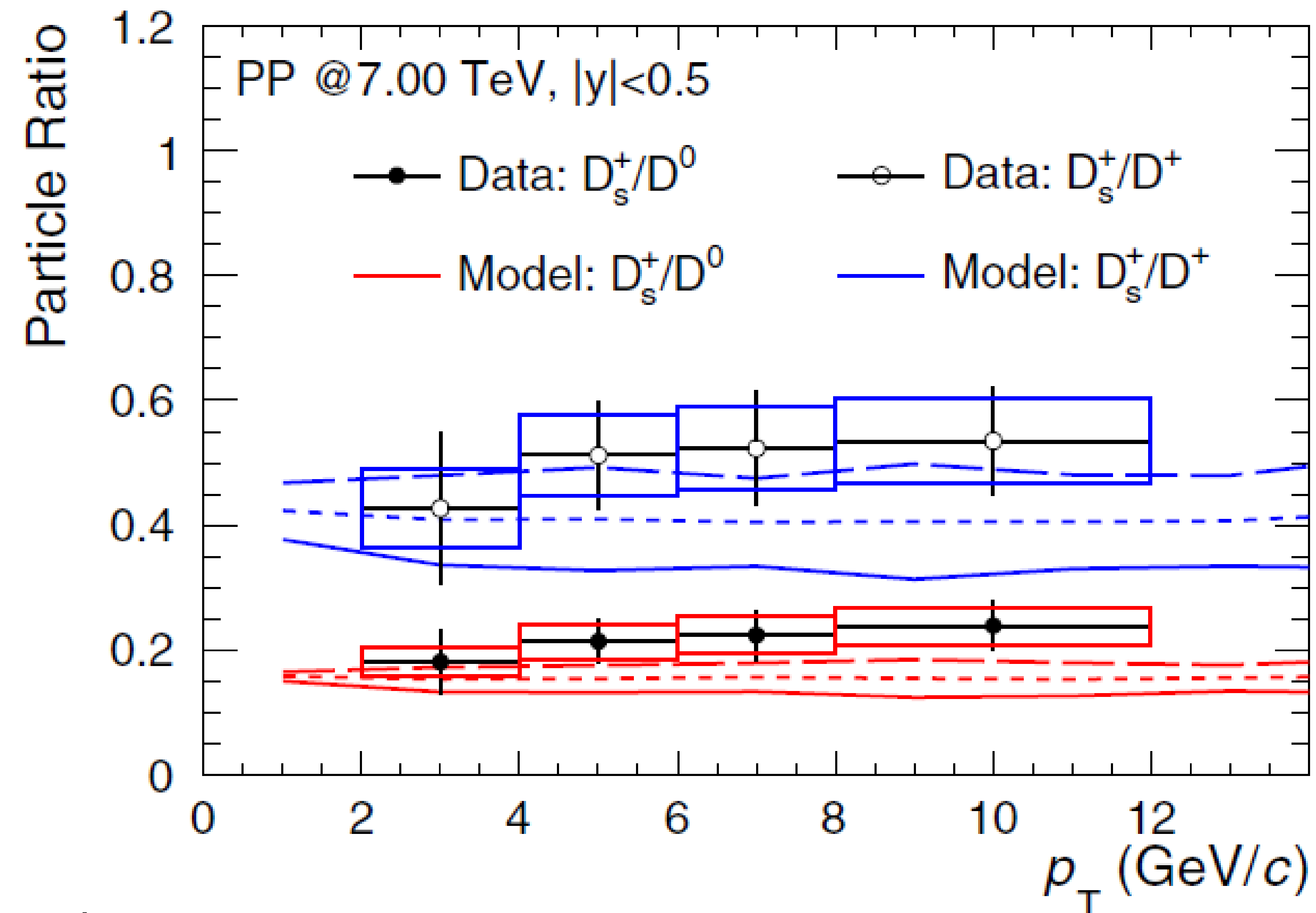
The uncertainty on nuclear shadowing is estimated according to the various nuclear parton distribution function (nPDF) sets in the EPS09NLO parametrization, which are obtained by tuning the fit parameters to reproduce the available measurements [27]. In this work, we employ the nPDF sets up to $k = 7$. See Eqs. (2.12) and (2.13) in Ref. [27] for details.

Production cross section in pp collisions





- Within uncertainties, the measurements can be reproduced by the model predictions
- Phenomenological fragmentation function is validated



[D^{*+}: PRD 86, 072013 (2012).D_s^+: EPJC 77, 550 (2017)]