Views of Confinement, Deconfinement, and Inbetween

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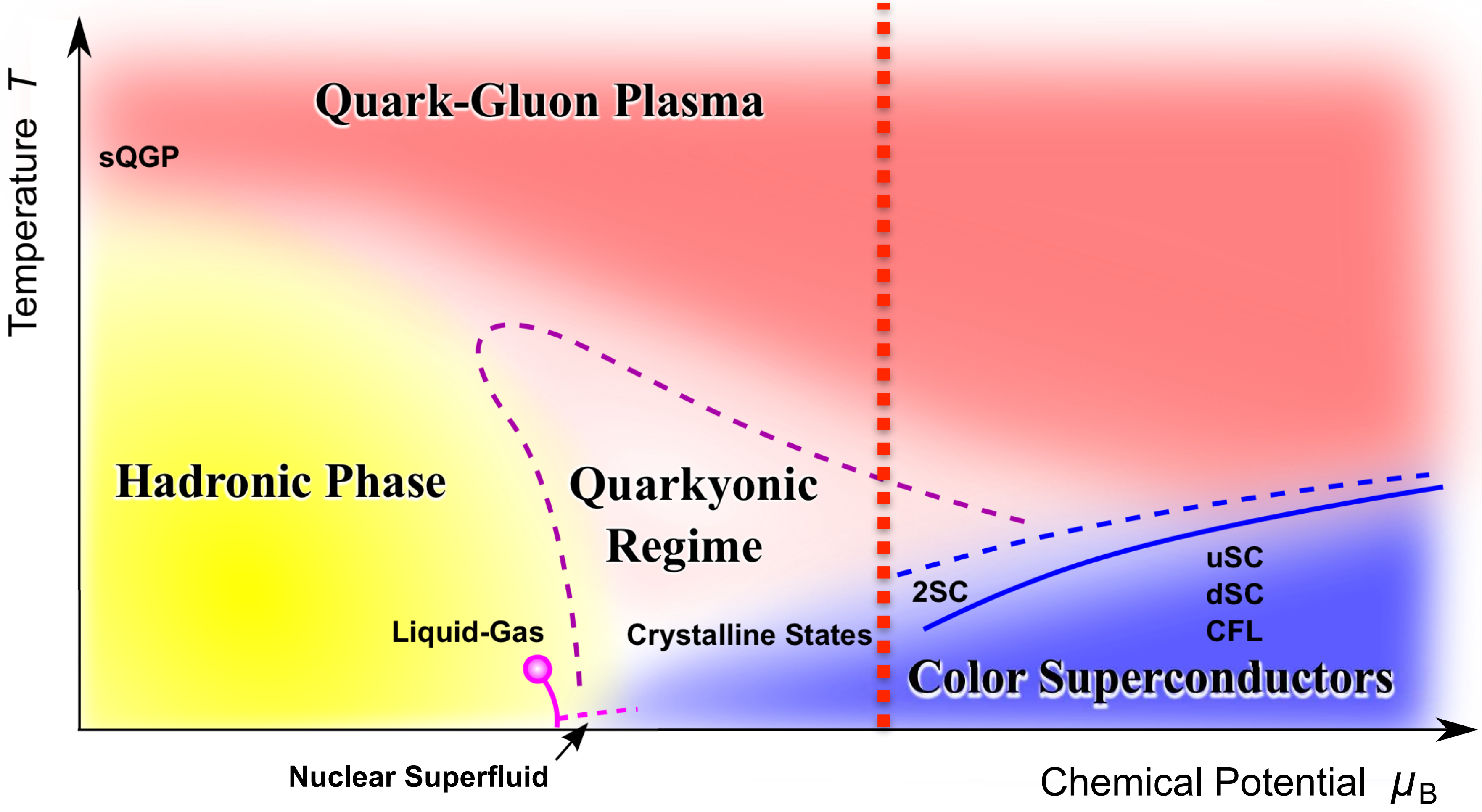
Congratulations!

This Talk

Question to be addressed

1st-order transition of "deconfinement" ?



Question to be addressed

1808.04827 [hep-th] **Very interesting paper, but…**

Anyonic particle-vortex statistics and the nature of dense quark matter

Aleksey Cherman,[1,*] Srimoyee Sen,[1,†] and Laurence G. Yaffe[2,‡]

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We show that $\mathbb{Z}_3$-valued particle-vortex braiding phases are present in high density quark matter. Certain mesonic and baryonic excitations, in the presence of a superfluid vortex, have orbital angular momentum quantized in units of $\hbar/3$. Such non-local topological features can distinguish phases whose realizations of global symmetries, as probed by local order parameters, are identical. If $\mathbb{Z}_3$ braiding phases and angular momentum fractionalization are absent in lower density hadronic matter, as is widely expected, then the quark matter and hadronic matter regimes of dense QCD must be separated by at least one phase transition.

A counter example of quark-hadron duality?

Unbelievable… (but hard to falsify it…)

Extreme QCD Matter

Ultimate goal of theory

Confinement of Quarks and Gluons

Controllable parameter (T, μ, B, ω) to approach confinement from (deconfined) extreme matter

(Examples)

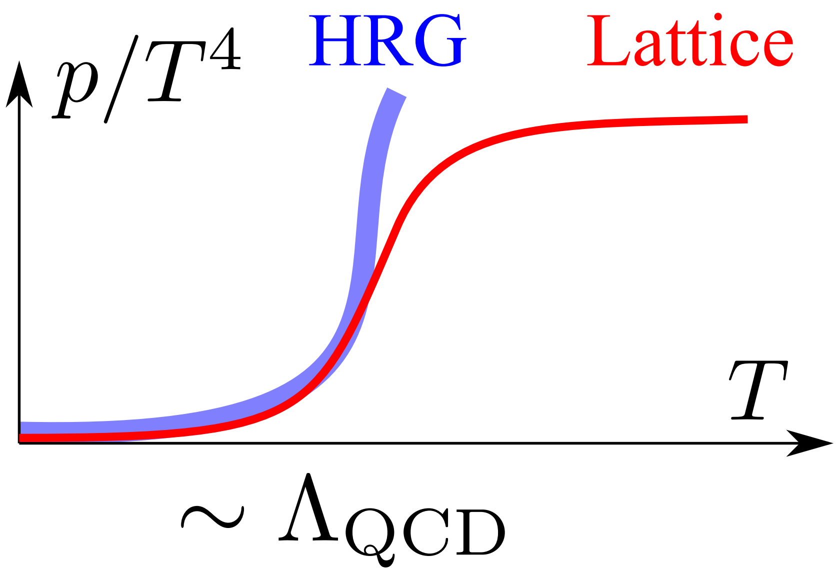
KvBLL instantons (dyons)

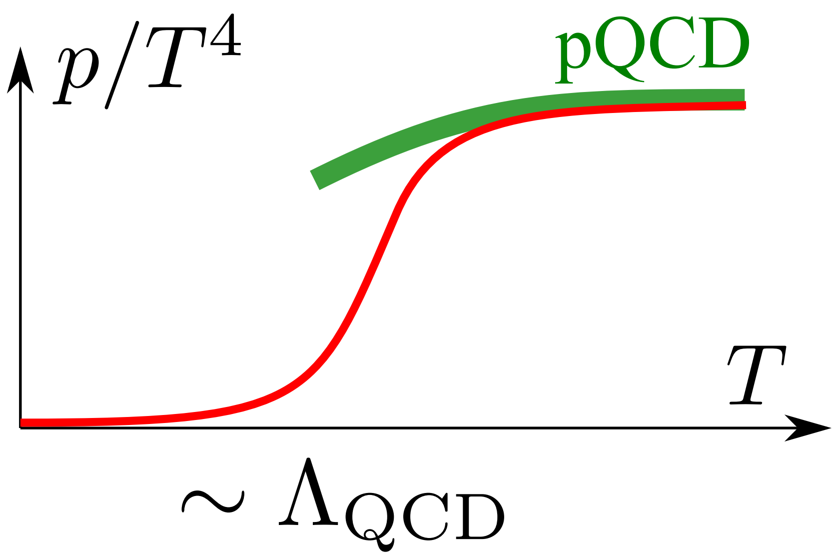
QCD-like theories on $S^1 \times R^3$ with adjoint fermions

Quark-hadron continuity at high baryon density

Extreme QCD Matter

[As I emphasized at QM2016]





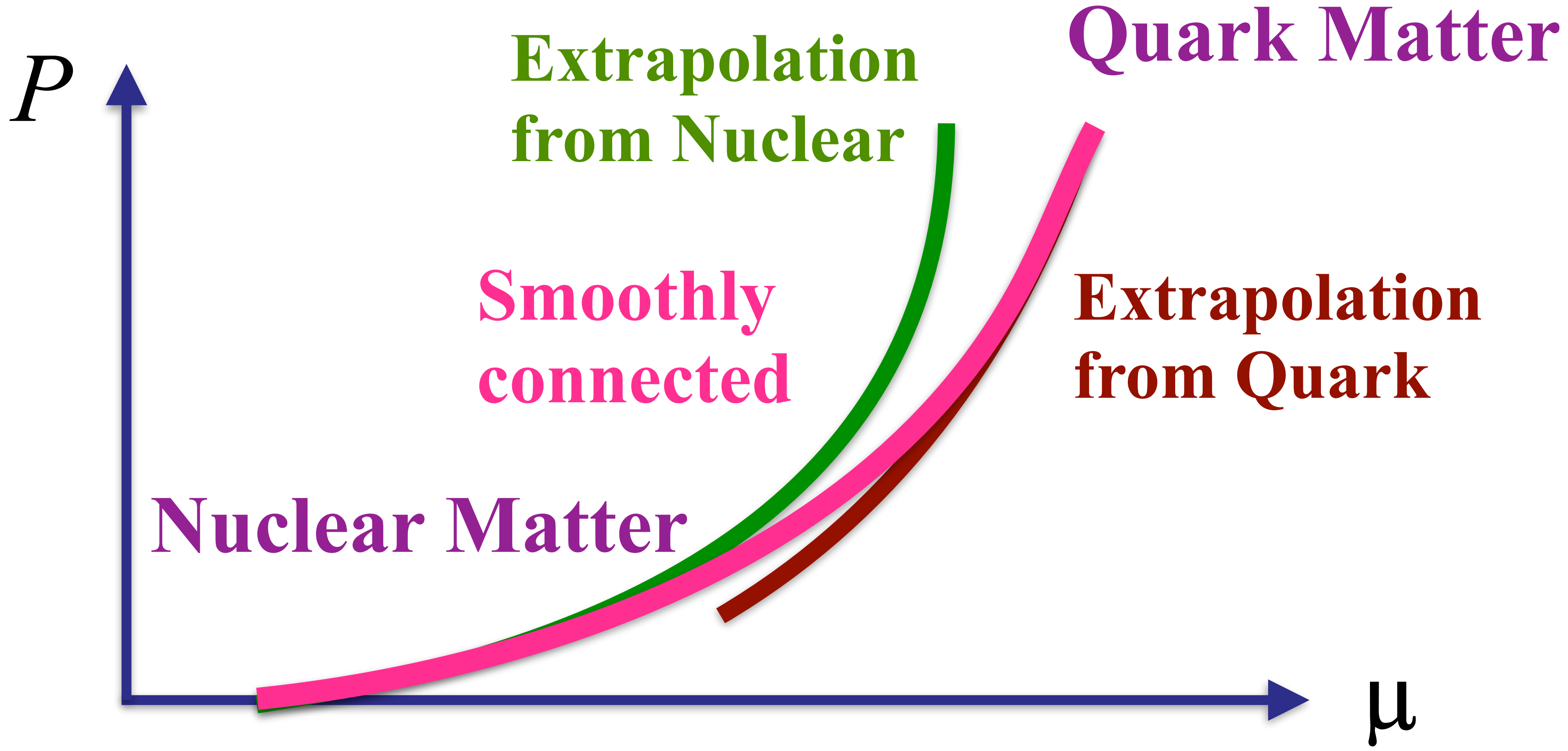
Confinement — Deconfinement

"Inbetween"

Quark-Hadron Duality / sQGP / Quarkyonic

Extreme QCD Matter

Quark-Hadron Duality



Main Player of This Talk

hep-ph/9711396

Diquark Bose Condensates in High Density Matter and Instantons

R. Rapp[1], T. Schäfer[2], E.V. Shuryak[1] and M. Velkovsky[3]
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Instanton-induced interaction leads to large diquark cond.

Diquarks **(Anselmino *et al*. 1992)**

It is too simple to regard a diquark as a point particle with the quantum numbers of two quarks. More generally, *a diquark is any system of two quarks considered collectively*. For example, a two-quark correlation in a hadron containing more than two quarks is a diquark. A diquark in its ground state has positive parity, and may be an axial vector (spin 1) or a scalar (spin 0). An axial vector diquark is often called a vector for short.

"Good" Diquark

Diquark makes no phase transition!

hep-ph/9811473

Continuity of Quark and Hadron Matter

Thomas Schäfer[1]

and

Frank Wilczek[2]

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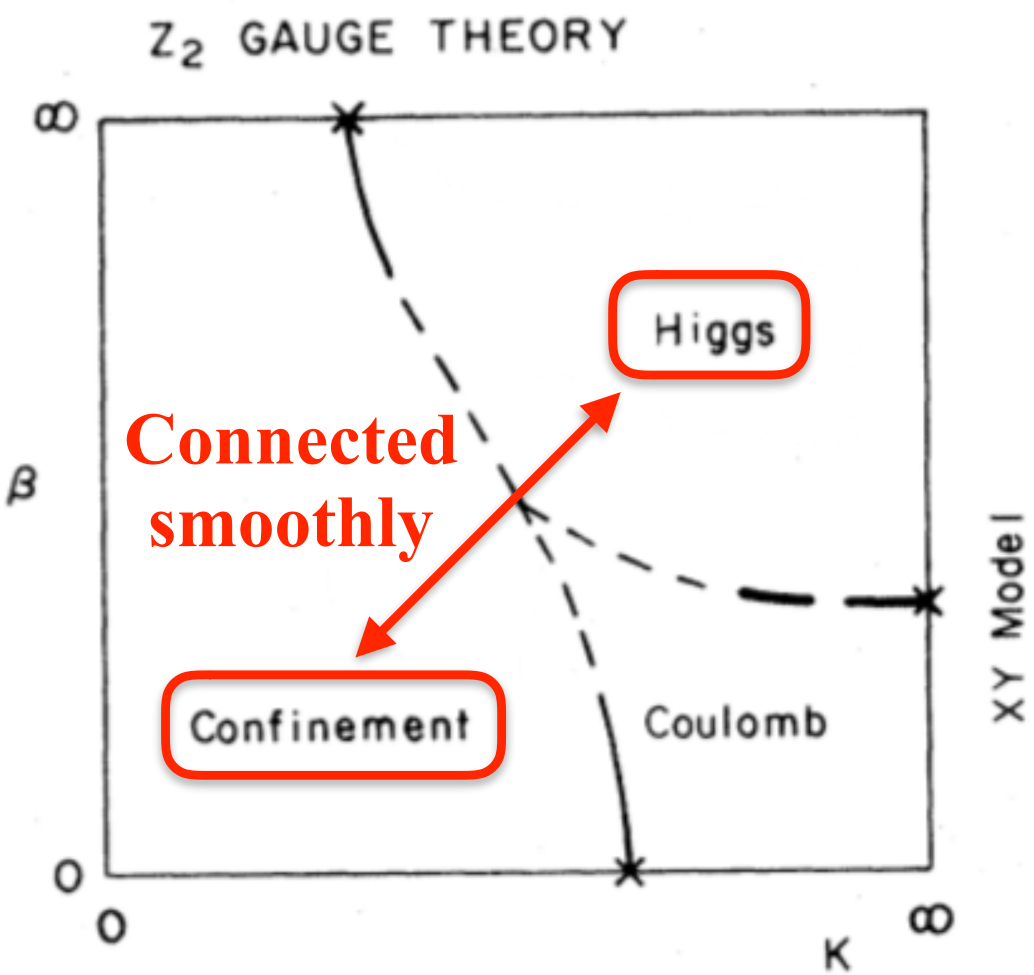
Diquark makes no phase transition!

Fradkin-Shenker (1979)



Diquark makes no phase transition!

Photon-Gluon Mixing

Diquarks have both electric and QCD charges

Modified electric charge $$\tilde{Q} = Q + \frac{1}{\sqrt{3}} T_8$$

Electric charge **Color charge**

Quarks carry integer $\tilde{e}$ with diquark condensates just like baryons in the hadronic phase !

One realization of the quark-hadron continuity

Diquark makes no phase transition!

Heuristic view to understand the same physics

Hadronic Matter	**Quark Matter**
Baryons 8+1 (low-lying)	**Quarks 3color × 3flavor = 9**

Flavor Triplet

Condensate

Excitation

All the condensates and excitations have correspondence

8 vector mesons	**8 gluons**
8 pseudo-scalar mesons	**8 tetra-quark mesons**

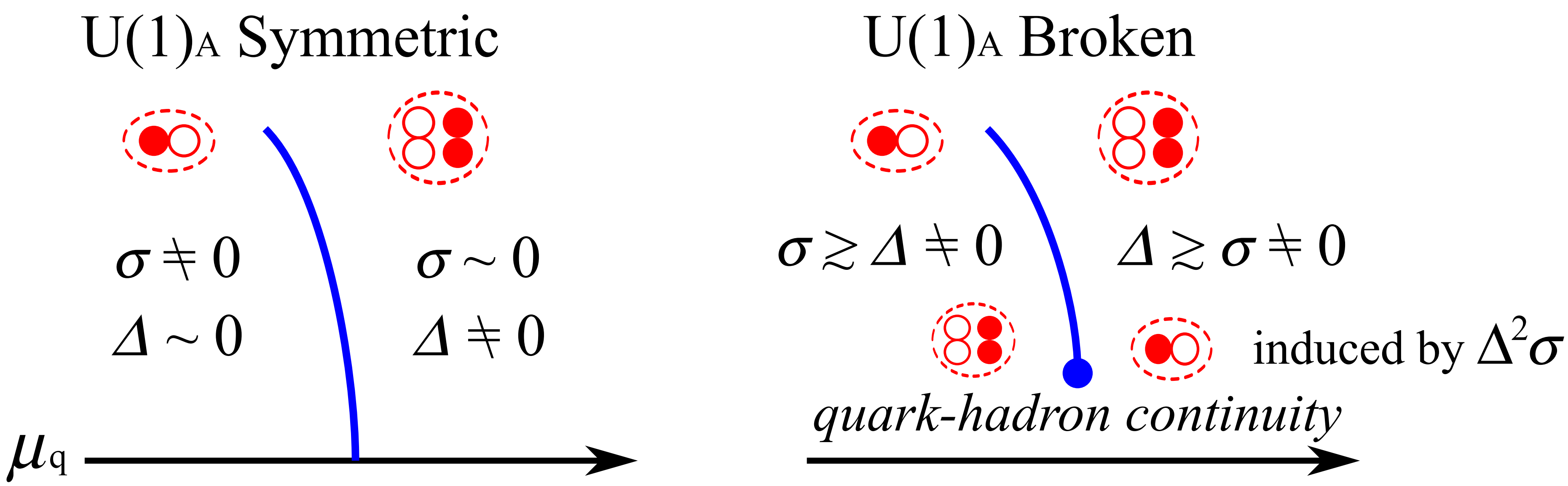
Diquark makes no phase transition!

Another view to understand the same physics

Instanton-induced interaction

$$\det[\bar{\psi}(1 \pm \gamma_5)\psi] \;\sim\; \Delta^2 \sigma$$

Linear in terms of the chiral condensate σ !



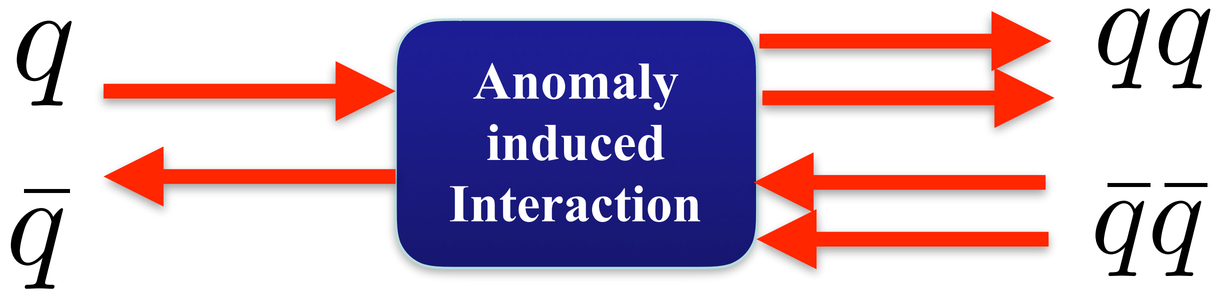
Diquark makes no phase transition!

Yet another view to understand the same physics

Hadronic Phase : Chiral broken by the chiral condensate.

Quark Phase : Chiral broken by the diquark condensate.

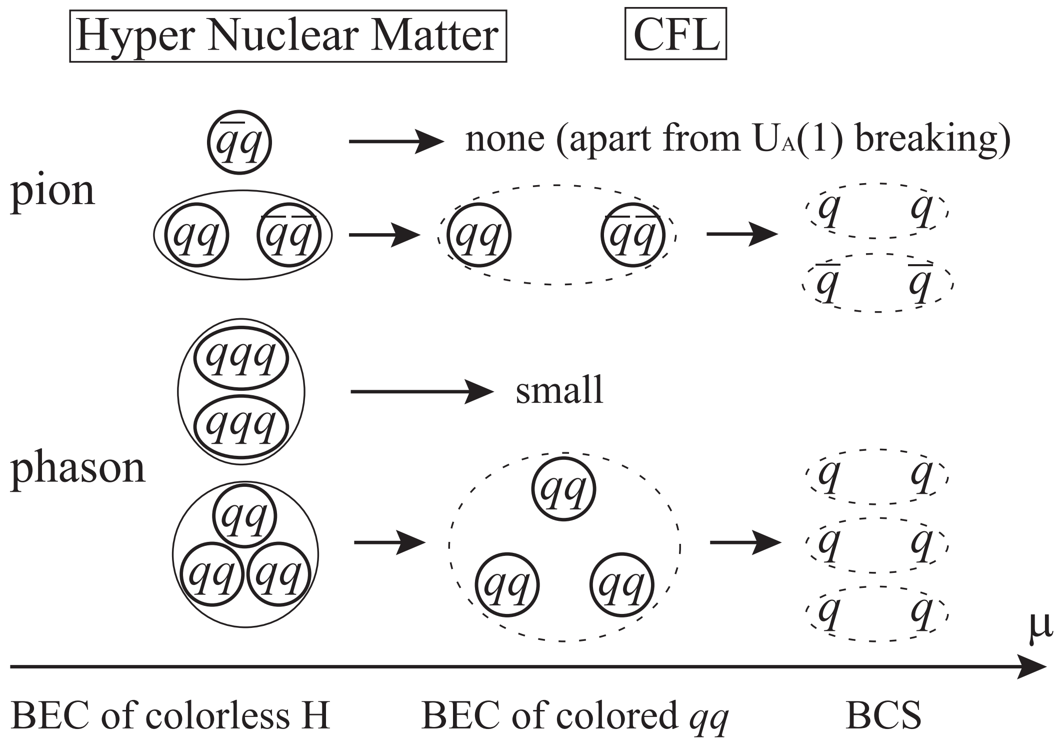
Breaking patters is indistinguishable (due to anomaly)



Ordinary meson-like **Tetra-quark meson-like**

Diquark makes no phase transition!

Fukushima (2002)

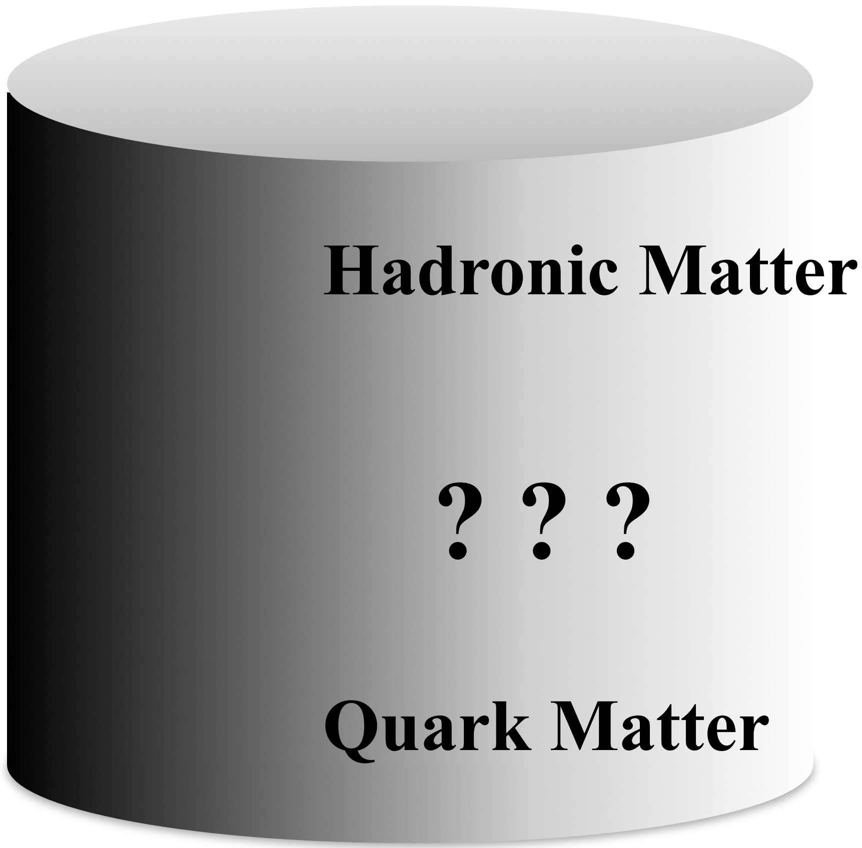


mentioning…

qualitatively. If our scenario is realized, then it would be quite interesting to see how the nature of H (or χ_H) affects the internal structure of the vortices in a superfluid along the lines of [47].

Thinking Experiment

Questioned by Gordon Baym 15 years later…



Pour quarks into your "bucket"
Upper part : Hadronic Matter
Lower part : Quark Matter

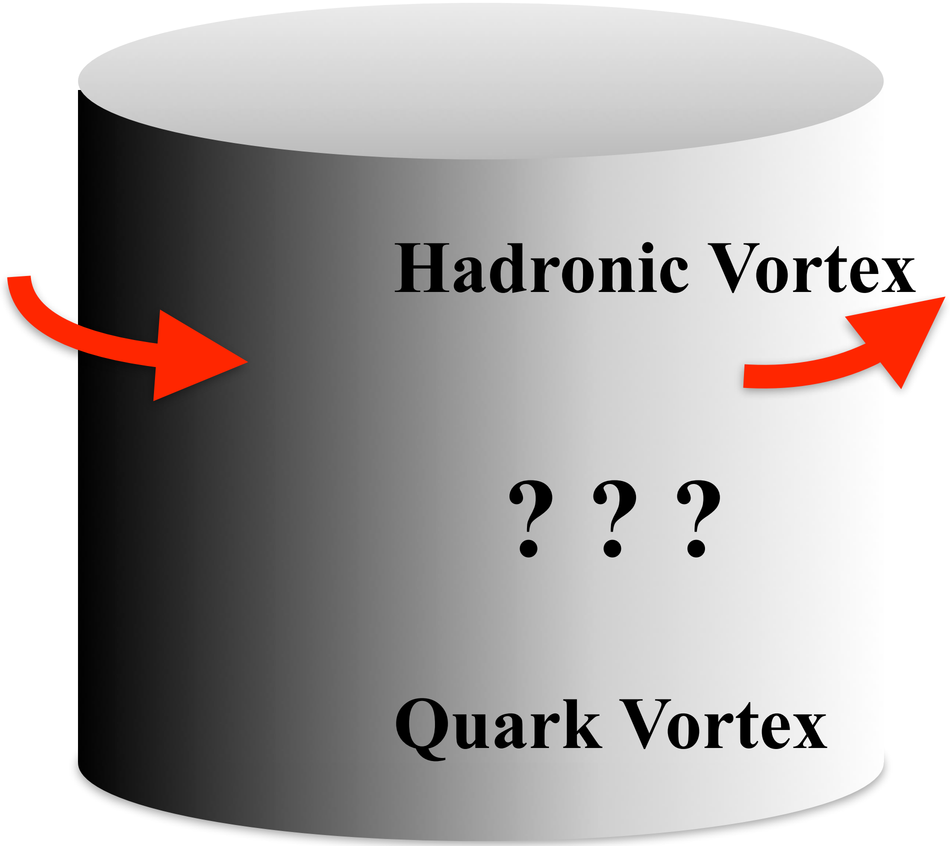
Is there any interface?

Not necessarily!

**We already know the ans:
Quark-Hadron Continuity**

Thinking Experiment

Questioned by Gordon Baym 15 years later…



Rotate the bucket filled with quarks

Upper part : Hadronic Vortex

Lower part : Quark Vortex

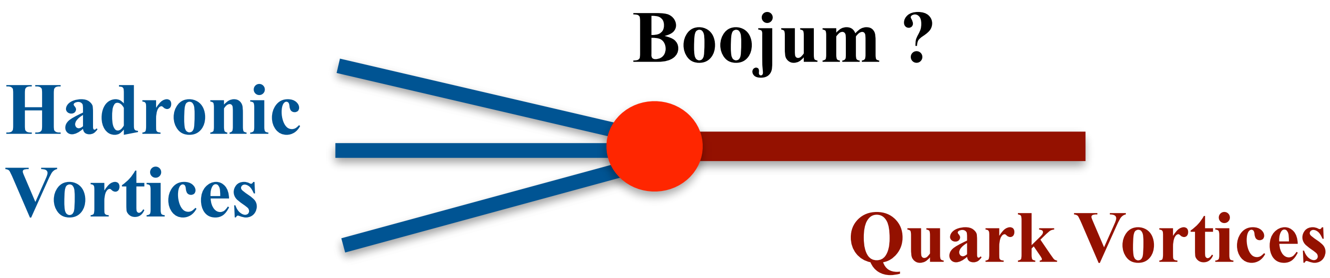
How can they be connected?

Puzzle ?

Circulation $$C = \oint_C \vec{v} \cdot d\vec{\ell} = 2\pi \frac{\nu}{\mu}$$

Winding number ν should match.

Circulation differs between nuclear and quark matter?
(Difference comes from chemical potentials)



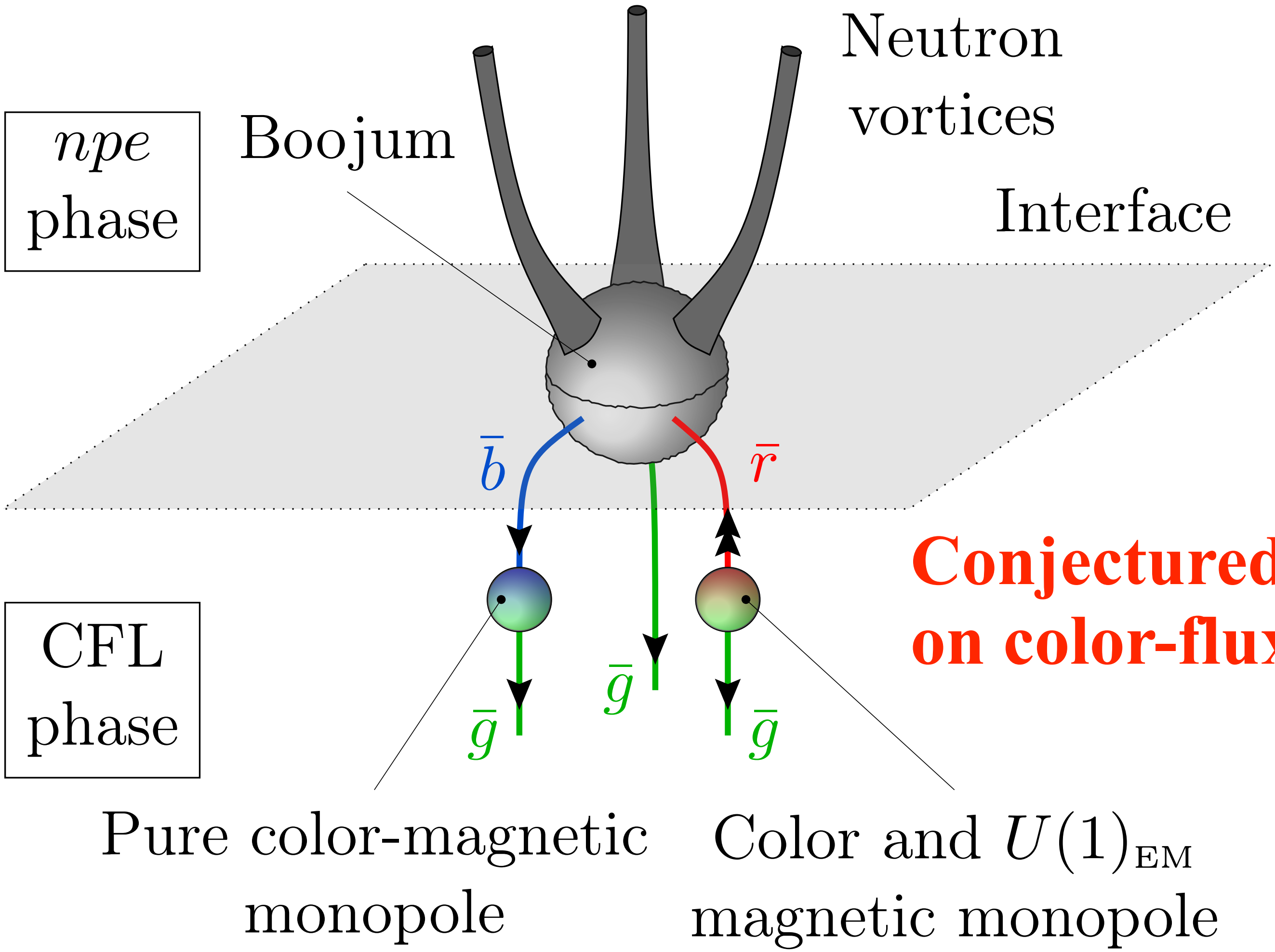
Puzzle ?

Boojum

Wikipedia

Non-Abelian CFL Vortex

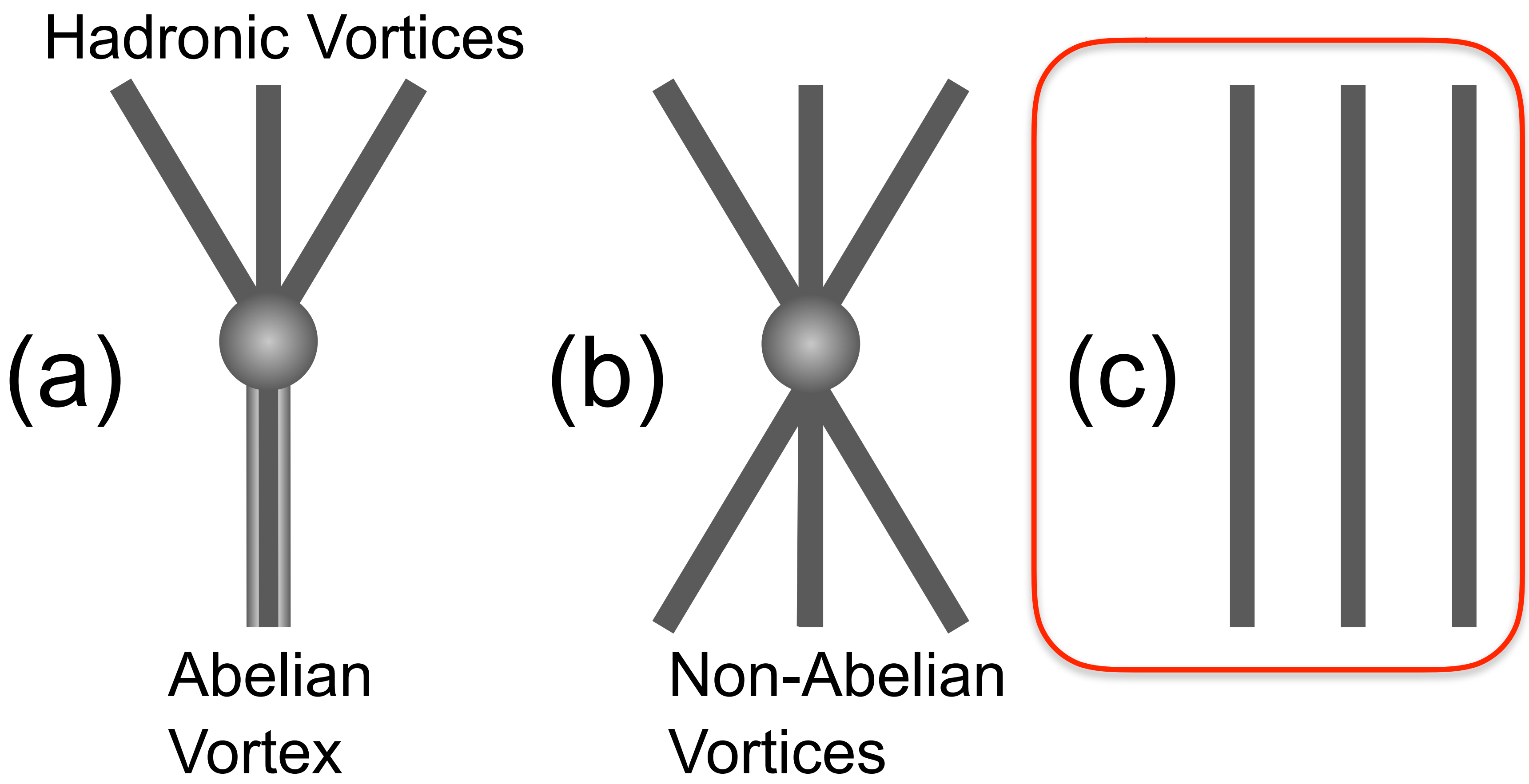
Cipiriani-Vinci-Nitta (2012)



Conjectured based on color-flux conservation

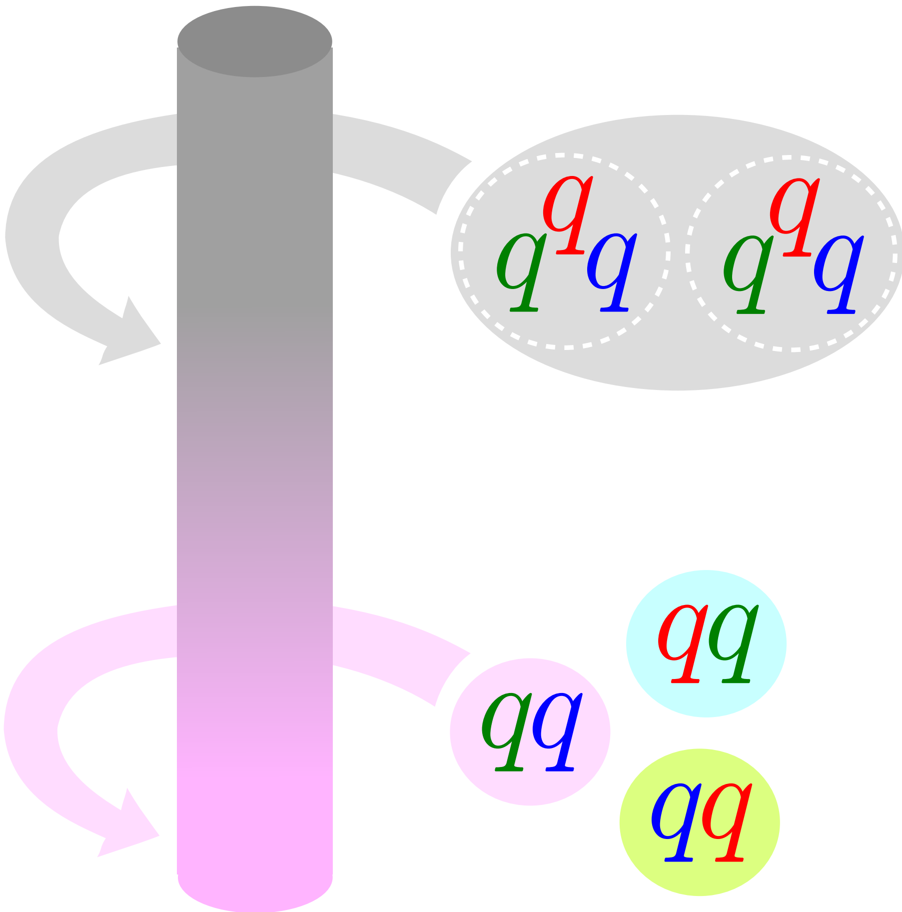
Our Solution

Non-Abelian CFL vortices ~ Hadronic vortices



Our Solution

Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)



Some More Details

Gap matrix in color-flavor space

$$\Phi = \begin{pmatrix} \Phi^{\bar{r}\bar{u}} & 0 & 0 \\ 0 & \Phi^{\bar{g}\bar{d}} & 0 \\ 0 & 0 & \Phi^{\bar{b}\bar{s}} \end{pmatrix}$$

Some More Details

Abelian CFL vortex

$$\Phi^{\mathrm{A}} = \Delta_{\mathrm{CFL}}\, e^{i\nu_{\mathrm{A}}\varphi} \begin{pmatrix} f(r) & 0 & 0 \\ 0 & f(r) & 0 \\ 0 & 0 & f(r) \end{pmatrix}$$

Non-Abelian CFL vortex

$$\Phi^{(1)} = \Delta_{\mathrm{CFL}} \begin{pmatrix} e^{i\nu_1\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}$$

Some More Details

Why Non-Abelian ?

$$\Phi^{(1)} = \Delta_{\rm CFL} e^{\frac{i}{3}\nu_1\varphi} \begin{pmatrix} e^{\frac{2i}{3}\nu_1\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) \end{pmatrix}$$

Abelian Phase (Global Vortex)

Non-Abelian Phase (T_3 and T_8) (Gauged Vortex)

$$A_\varphi^{(1)} = -\frac{\nu_1}{g_c r}\left[1 - h(r)\right] \begin{pmatrix} -\frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Non-Abelian vortex carries Non-Abelian Magnetic Flux

Some More Details

What we (Alford *et al.*) discussed:

$$\hat{\Upsilon}^{ijk}(\vec{r}) \equiv \frac{1}{6}\epsilon_{\alpha\beta\gamma}\hat{\Phi}^{\alpha i}\hat{\Phi}^{\beta j}\hat{\Phi}^{\gamma k}$$

$$= \frac{1}{3}\epsilon^{kmn}(C\gamma_5)_{ab}\,\hat{B}^{i\,a}_{m}\,\hat{B}^{j\,b}_{n}$$

6 quark objects = 3 diquarks = 2 baryons

Quantum numbers match

Non-Abelian vortices = Flavor singlet + Non-singlets

$(\sim \Lambda\Lambda)$

Some More Details

What Cherman *et al*. discussed:

$$\Phi^{(1)} = \Delta_{\mathrm{CFL}} e^{\frac{i}{3}\nu_1\varphi} \begin{pmatrix} e^{\frac{2i}{3}\nu_1\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\nu_1\varphi} g(r) \end{pmatrix}$$

$$A_\varphi^{(1)} = -\frac{\nu_1}{g_c r}\left[1 - h(r)\right] \begin{pmatrix} -\frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

Calculate the Wilson loop $\langle W_3(C)\rangle / \langle W_0(C)\rangle$

This measures the non-Abelian magnetic flux

Some More Details

What Cherman *et al*. discussed:

Abelian phase is irrelevant (no gauge potential)

If C is large enough, $f \to 1$, $g \to 1$, $h \to 0$

$$\langle W_3(C) \rangle \sim \mathrm{tr} \begin{pmatrix} e^{-\frac{4\pi i}{3}\nu_1} & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}\nu_1} & 0 \\ 0 & 0 & e^{\frac{2\pi i}{3}\nu_1} \end{pmatrix}$$

$$\sim e^{\frac{2\pi i}{3}\nu_1}$$

Center element of the non-Abelian magnetic flux appears (making the vortices "anyons")

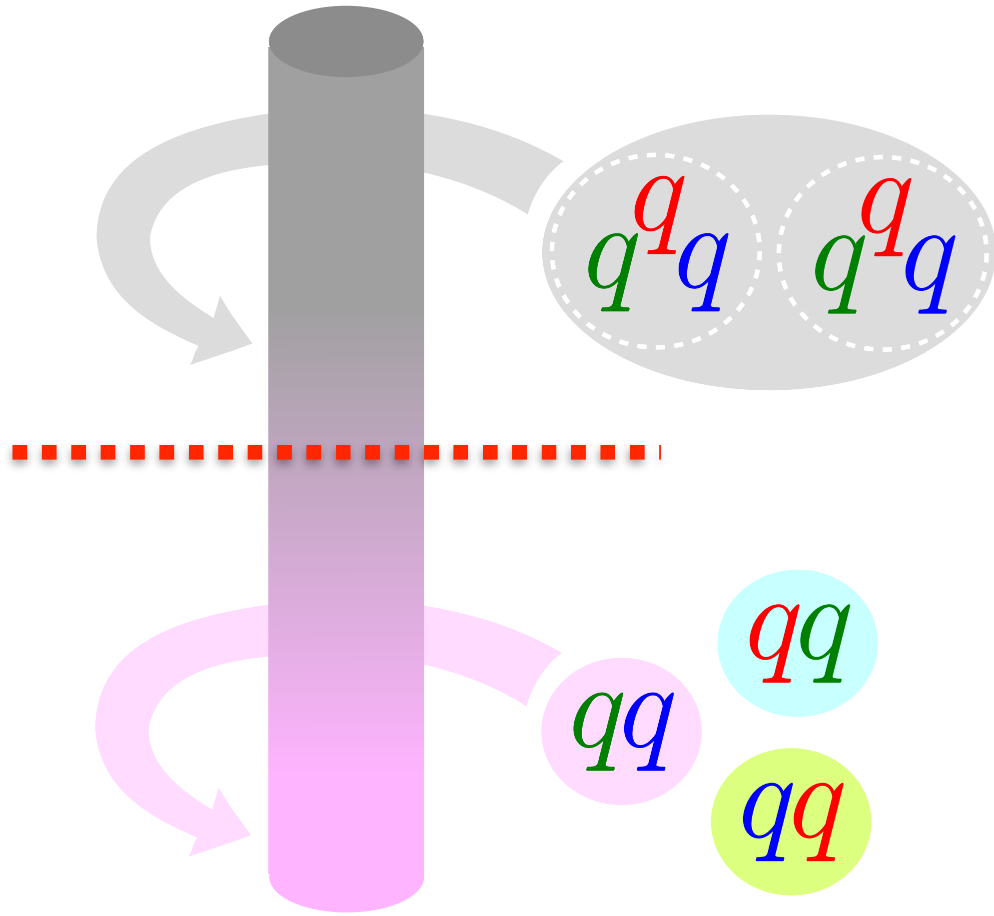
Some More Details

What Cherman *et al.* discussed:

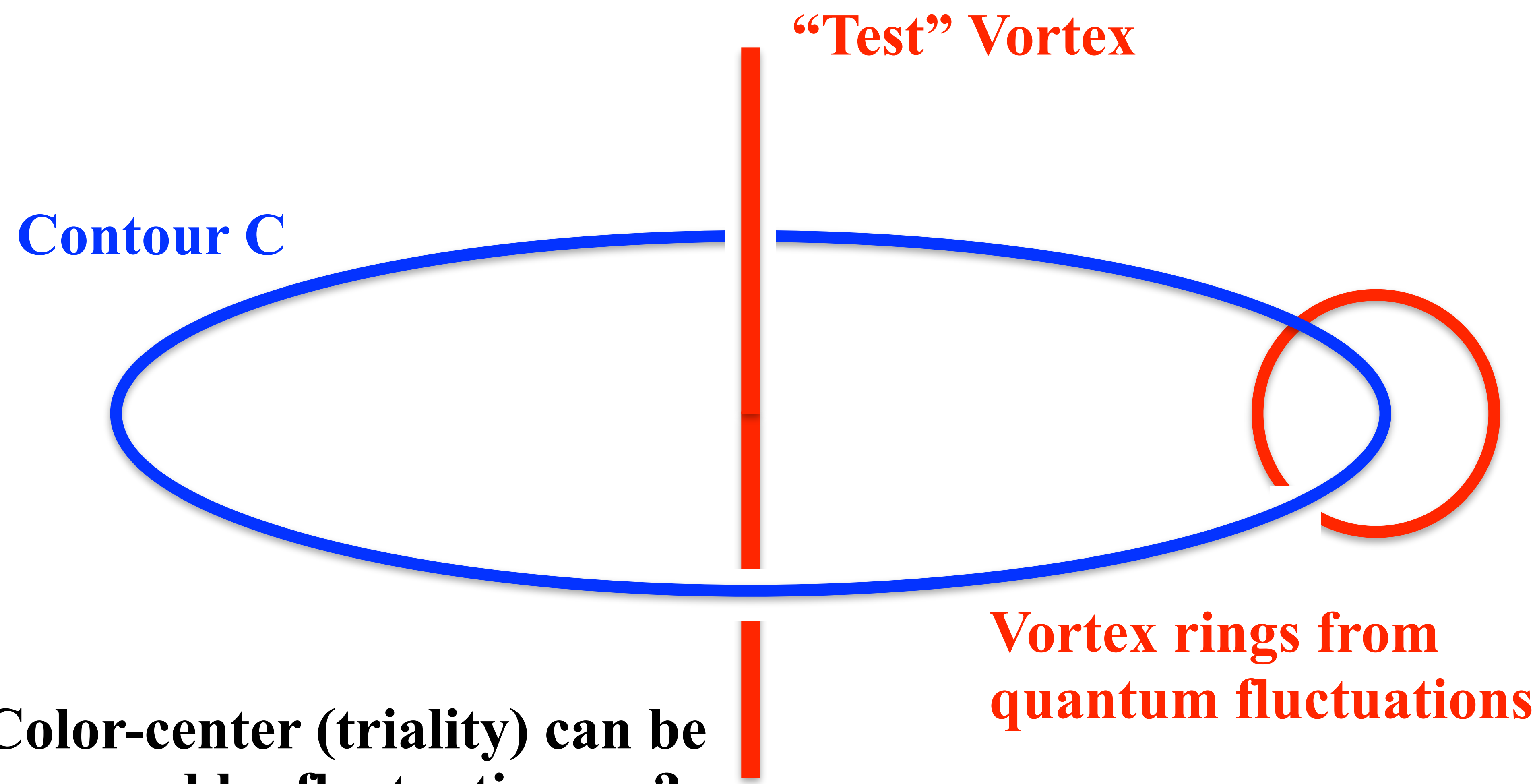
No color-center flux

There must be a phase transition !

Color-center flux (gauge invariant!)

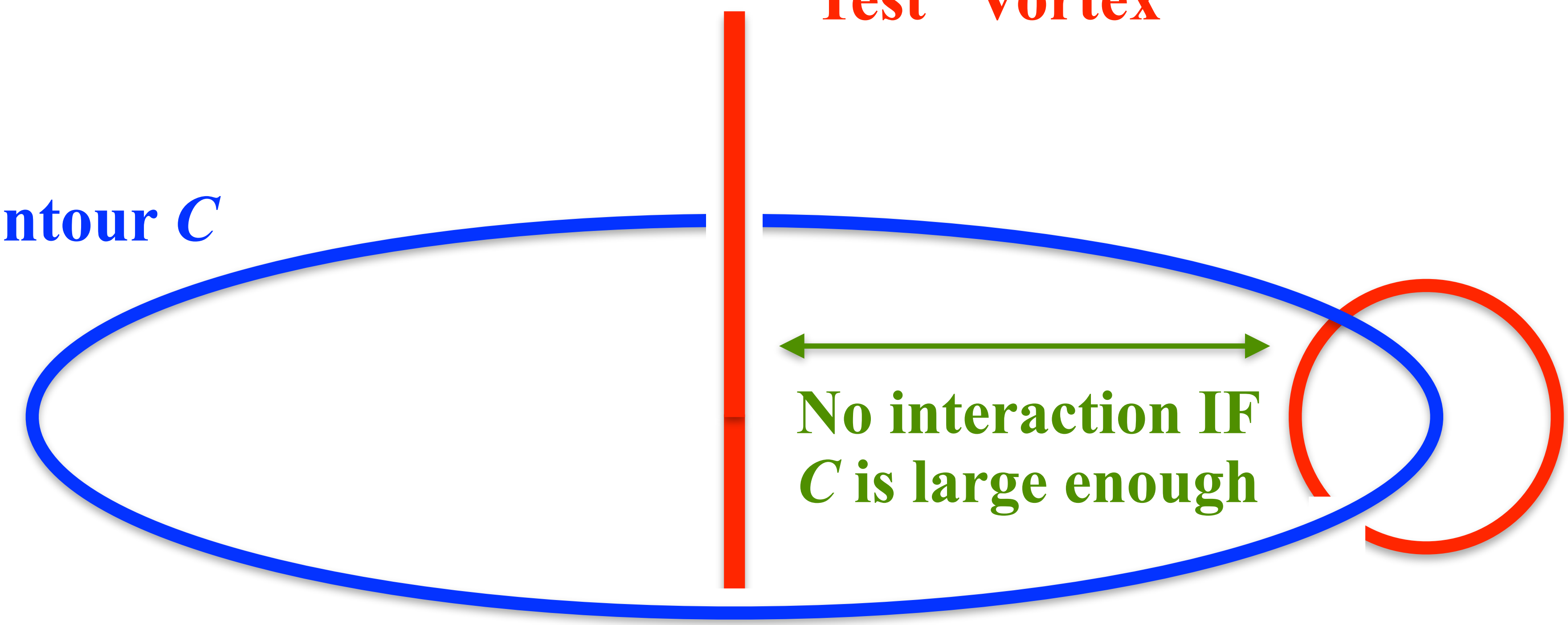


Is this really true ?



Color-center (triality) can be screened by fluctuations…?

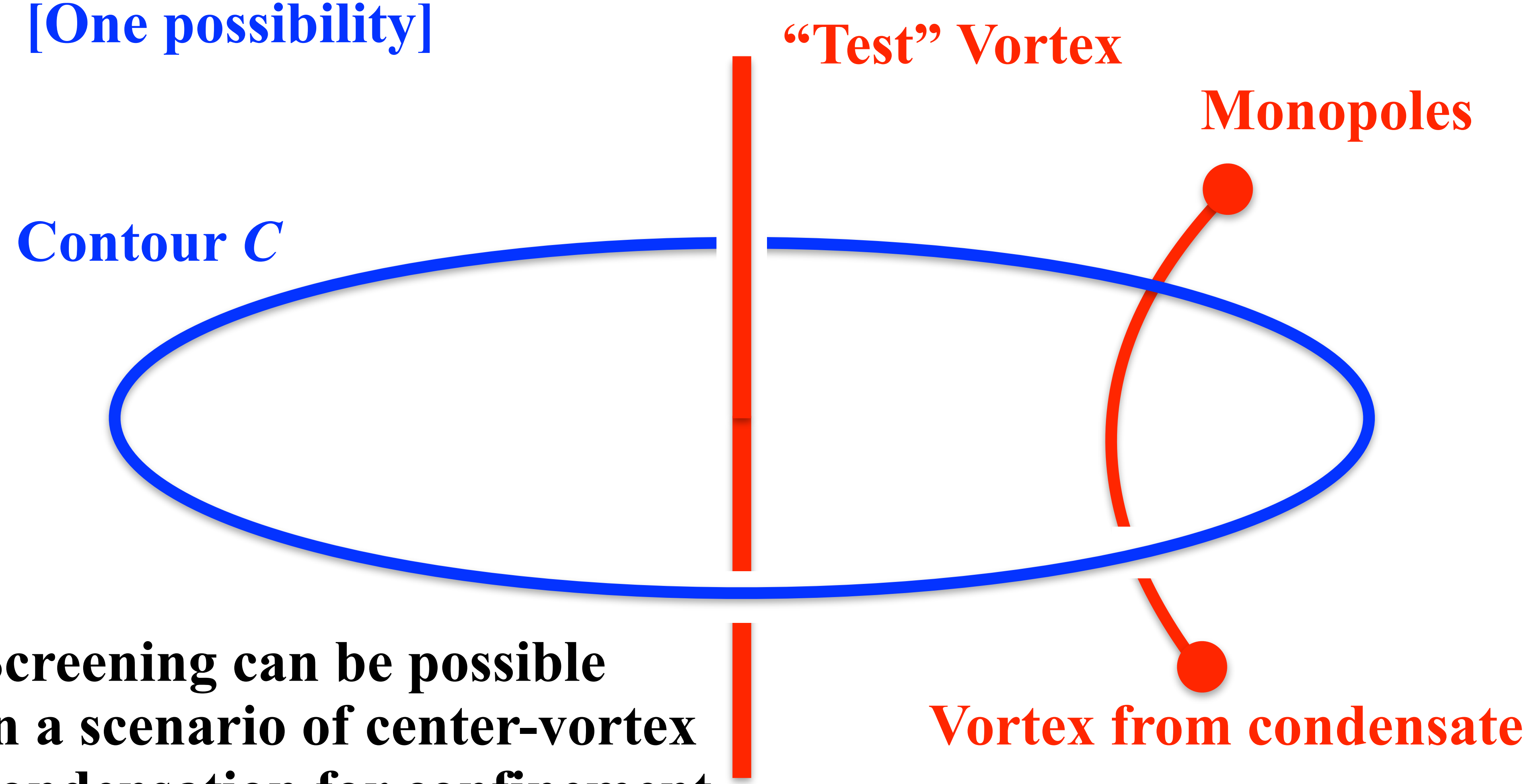
Is this really true ?



Gauged vortices have short-ranged interactions
Fluctuations → normalization

Is this really true ?

[One possibility]



Screening can be possible in a scenario of center-vortex condensation for confinement

Is this really true ?

Similar subject studied in cond-mat/0111192

Vortices with fractional flux
in two-gap superconductors and in extended Faddeev model

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Yes, a phase transition occurs between weak and strong vortex fluctuation regimes

If so, Cherman *et al.* may be right…?

Is this really true ?

A comment:

The idea has some similarity to "canonical ensemble w.r.t. the triality" to define an order parameter.

If the quark number is restricted to multiple of N_c (which is possible on lattice), in a *finite* volute, a 1st-order phase transition is seen.

Why could the idea work for vortices, though not quarks?

Summary

- Quark-hadron duality at high baryon density is still a subtle problem.

- Vortex makes the problem quite complicated because they may carry (gauge-invariant!) color magnetic flux that cannot penetrate into the hadronic phase.

- Our understanding of (de)confinement is challenged by vortex continuity problem.