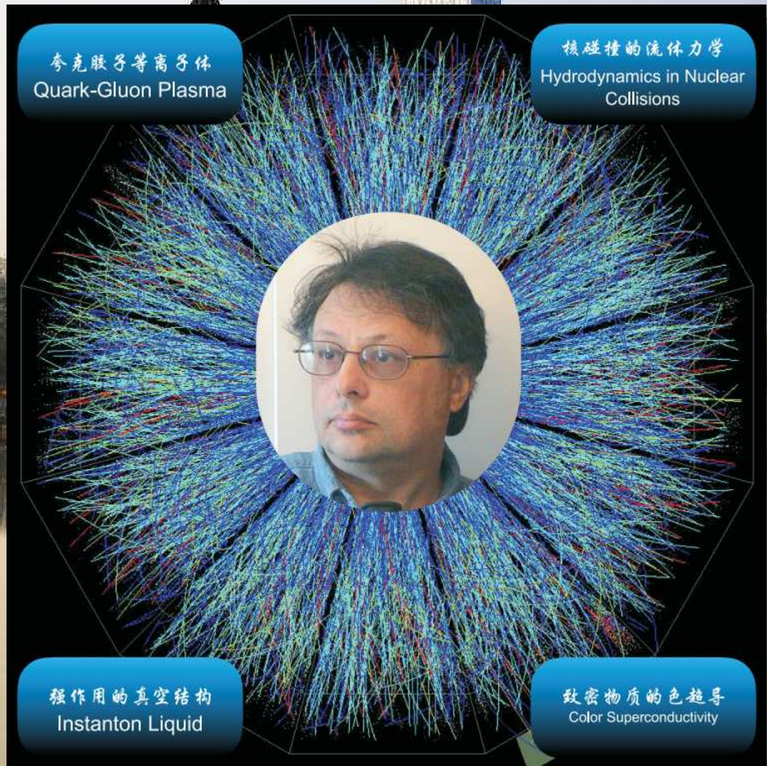
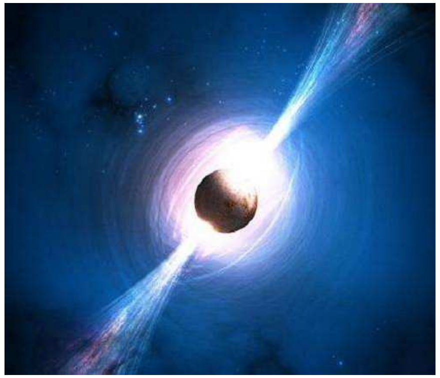
Chirality and transport phenomena in quark-gluon matter



October 10 , 2018 @ CCNU, Wuhan

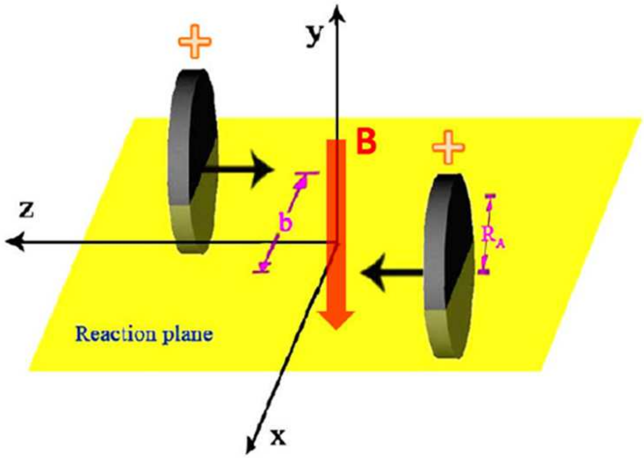
Electromagnetic (EM) field in HICs

- Strong magnetic field in nature: magnetars



$$eB \sim e \cdot 10^{14-15}\ G < \Lambda_{QCD}^2 \sim m_\pi^2$$

- Strongest magnetic field: heavy-ion collisions (HICs)



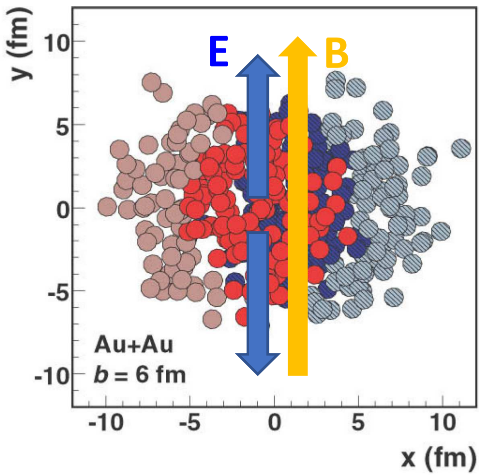
$$eB \sim e \cdot 10^{18-20}\ G \sim m_\pi^2 - 10 m_\pi^2$$

Skokov etal 2009; Deng-XGH 2012;
Blonzynski-XGH-Liao-Zhang 2012; … …

Experimental detection:
Talk by Z. Xu

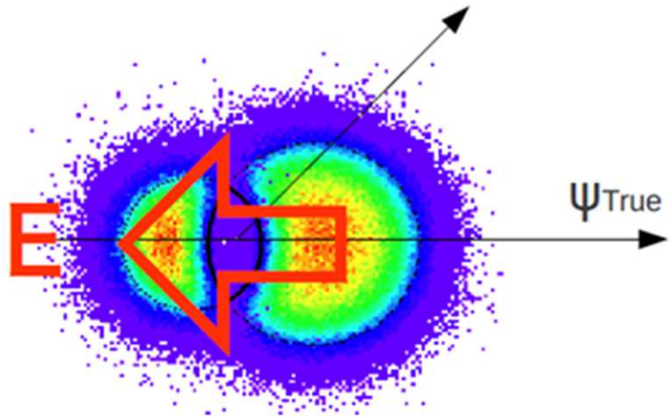
Electromagnetic (EM) field in HICs

- Strong electric field in HICs



Au + Au

$eE \sim m_\pi^2$

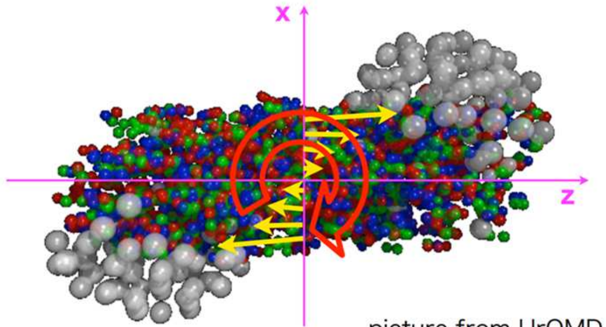


Cu + Au

- HIC: a machine that generate quark gluon matter in strong EM field

A era to study $\mathrm{QCD} \times \mathrm{QED}$: (inverse) magnetic catalysis, Schwinger mechanism, chiral magnetic effect, chiral separation effect, EM-induced directed flow, photon elliptic flow, photoproduction of hadrons, … …

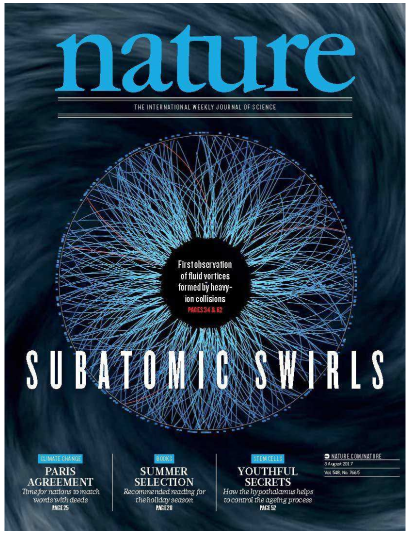
Fluid vorticity in heavy-ion collisions

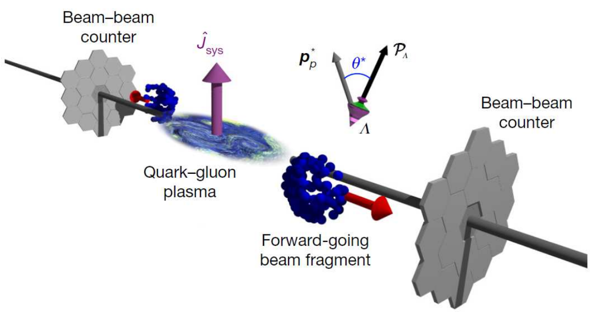


picture from UrQMD

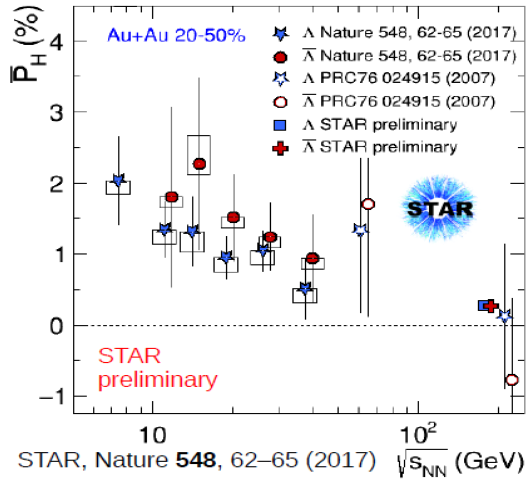
$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$$

Local angular velocity





Liang-Wang 2005



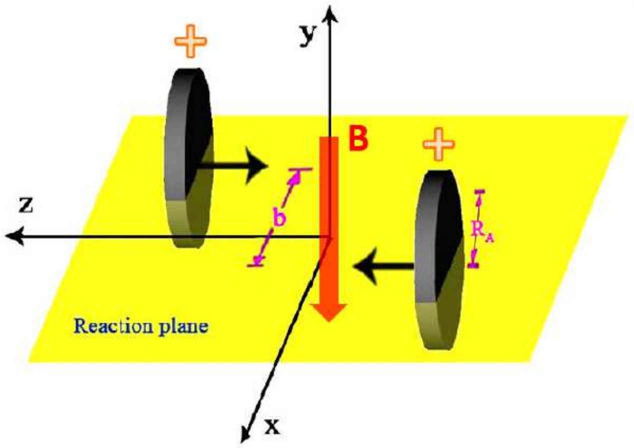
Averaged vorticity from 7.7 GeV-200 GeV: $\omega \approx (9 \pm 1) \times 10^{21} s^{-1}$ "Most vortical fluid!"

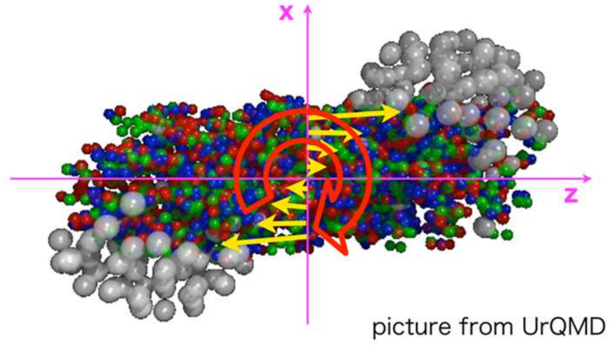
LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

The STAR Collaboration*





Strongest EM fields

Largest local rotation

What are their effects to transport properties QGP

Anomalous chiral transport

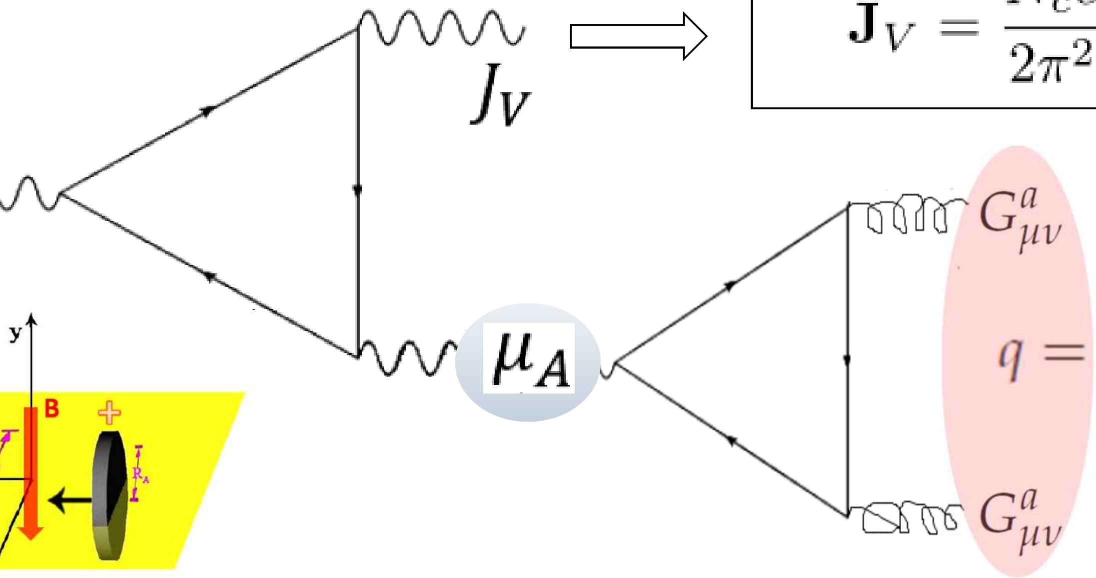
Chiral magnetic/vortical effects

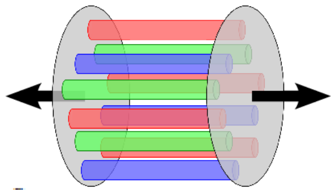
A probe of nontrivial topology of QCD using B/ω

QED VVA triangle anomaly

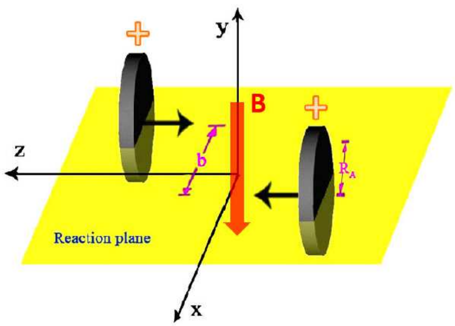
Chiral magnetic effect

$$\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B}$$

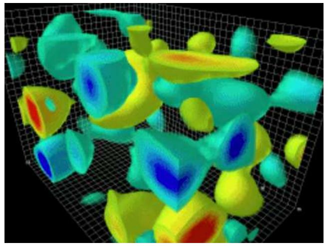




$$q = \frac{g^2}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

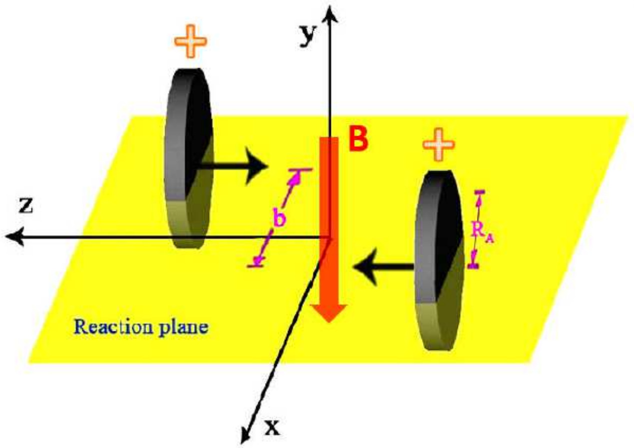


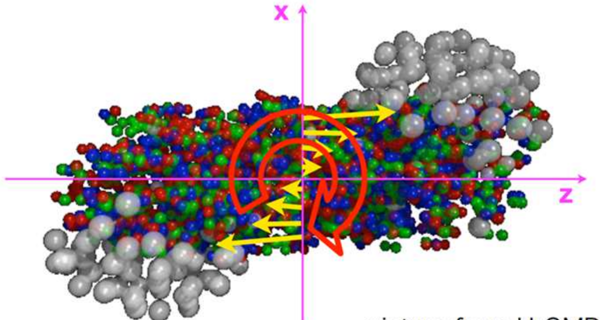
QCD VVA triangle anomaly



$eB \sim m_\pi^2 - 10 m_\pi^2$ in RHIC and LHC

Next talk by H. Huang





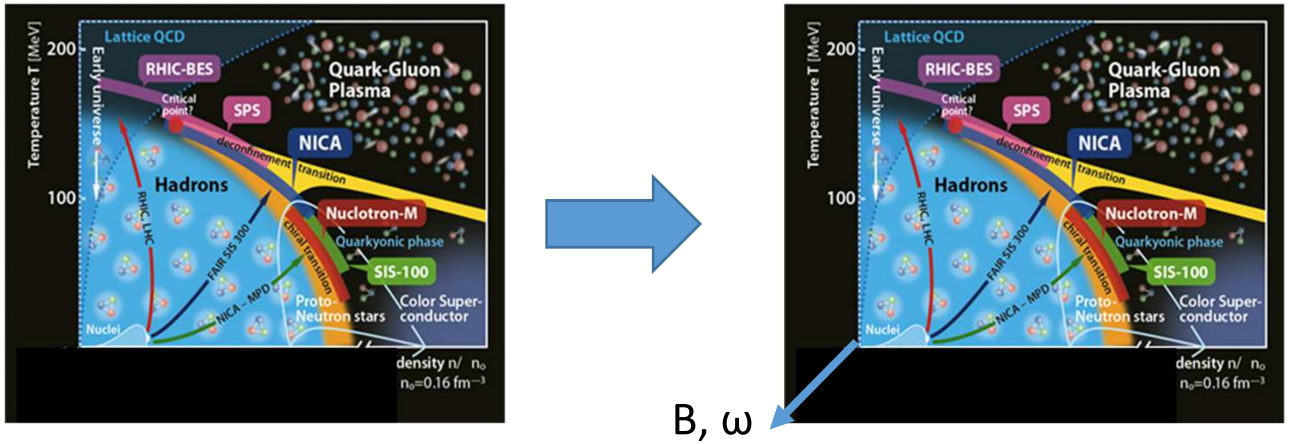
picture from UrQMD

Strongest EM fields

Largest local rotation

What are their effects to the QCD phase structure

Explore the new dimensions of the QCD phase diagram



QCD phases under electromagnetic field and rotation

Xu-Guang Huang

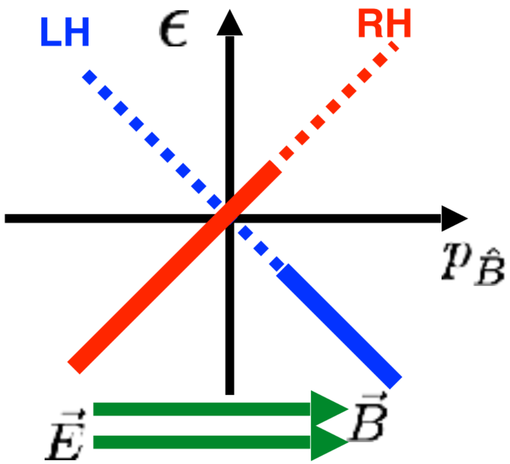
Fudan University, Shanghai

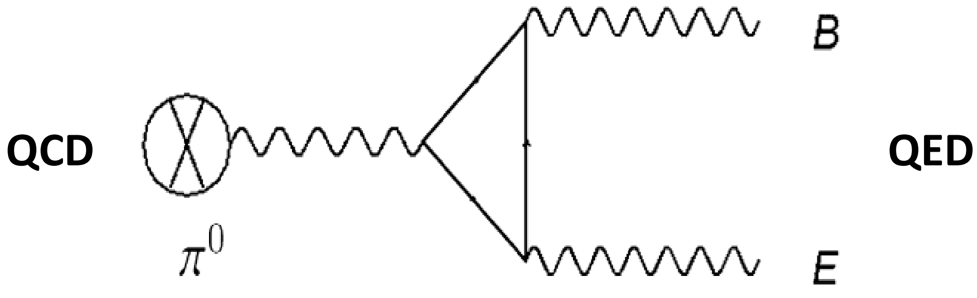
- **QCD vacuum in EM field:**
  Neutral pion condensed vacuum (G. Cao, L. Wang, P. Zhuang)
- **QCD vacuum in rotation:**
  chiral soliton lattice (K. Nishimura, N. Yamamoto)
- **Chiral condensate in rotation + magnetic field:**
  Rotational magnetic inhibition (H.L. Chen, K. Fukushima, K. Mameda)

QCD vacuum under EM field:
Neutral pion condensed vacuum

QCD vacuum under EM fields

- Triangle anomaly connects QCD and QED

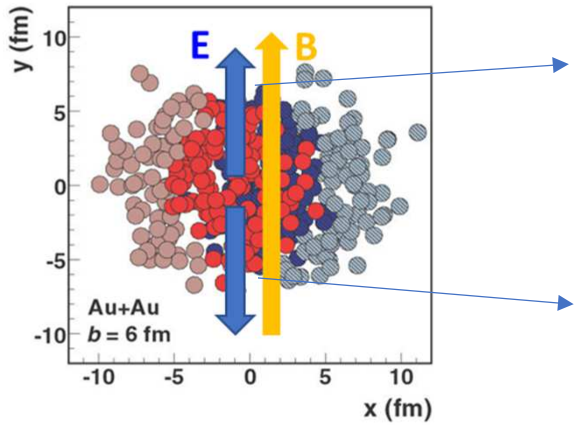




Neutral pion decay

- The inverse process of $\pi_0 \to \gamma\gamma$: $\boldsymbol{E} \cdot \boldsymbol{B} \to \langle \boldsymbol{\pi_0} \rangle$

QED anomaly induced neutral pion condensation in QCD vacuum?



$\langle \pi_0 \rangle > 0$

$\langle \pi_0 \rangle < 0$

Neutral pion condensation

- **Calculation by using hadronic degree of freedom: the chiral perturbation theory**

We start with the two-flavor ChPT described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\mathrm{WZW}},$$

where $\mathcal{L}_0$ is the usual chiral Lagrangian given by (we keep only $O(p^2)$ terms)

$$\mathcal{L}_0 = \frac{f_\pi^2}{4}\mathrm{tr}\left[D_\mu U^\dagger D^\mu U + m_\pi^2 (U + U^\dagger)\right],$$

and the Wess–Zumino–Witten term $\mathcal{L}_{\mathrm{WZW}}$ is given by

$$\mathcal{L}_{\mathrm{WZW}} = \frac{N_c}{48\pi^2} A_\mu \epsilon^{\mu\nu\alpha\beta}[\mathrm{tr}\left(Q L_\nu L_\alpha L_\beta + Q R_\nu R_\alpha R_\beta\right) - iF_{\alpha\beta} T_\nu],$$

$$T_\nu = \mathrm{tr}\left[Q^2(L_\nu + R_\nu) + \frac{1}{2}\left(QUQU^\dagger L_\nu + QU^\dagger QUR_\nu\right)\right],$$

$$L_\mu = U\partial_\mu U^\dagger,$$

$$R_\mu = \partial_\mu U^\dagger U.$$

In the above, the covariant derivative is given by

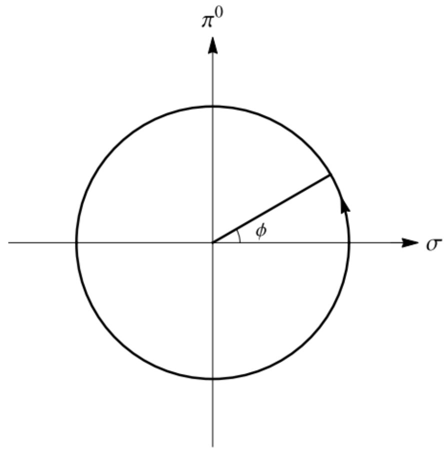
$$D_\mu U = \partial_\mu U + A_\mu [Q, U],$$

Minimizing the effective action gives:

$$\frac{\pi^0}{m^*} = \begin{cases} \dfrac{N_c}{4\pi^2 f_\pi^2 m_\pi^2}(q_u^2 - q_d^2)\boldsymbol{E}\cdot\boldsymbol{B} & \text{for } |I_2| < I_2^c \\ \mathrm{sgn}(I_2) & \text{for } |I_2| > I_2^c \end{cases}$$

$$m^* = \sqrt{(\pi^0)^2 + \sigma^2}$$

$$\sigma \sim \langle\bar{\psi}\psi\rangle \text{ and } \pi^0 \sim \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle$$



Neutral pion condensation

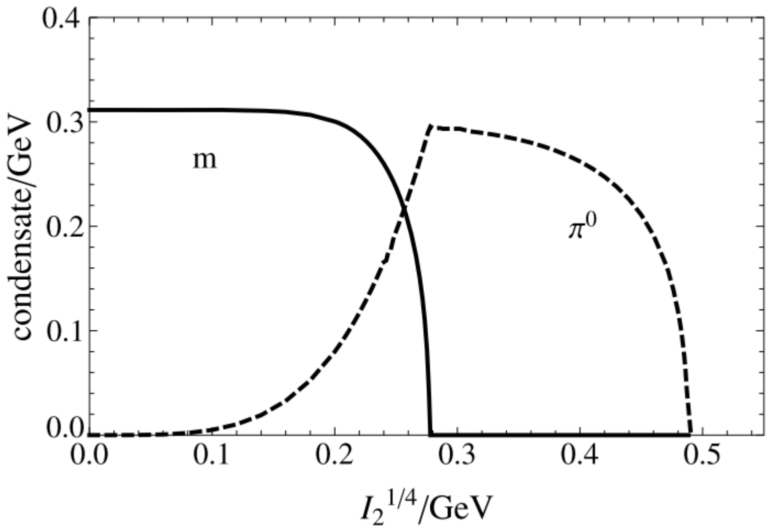
- Calculation by using quark degree of freedom: the Nambu-Jona-Lasinio model

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}(i\not{\partial} - m_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2]$$

$$\Omega = \frac{(m - m_0)^2 + (\pi^0)^2}{4G} - \frac{1}{V_4}\mathrm{Tr}\ln S^{-1}$$

$$S(x, x') = -(i\not{\partial} - m - i\pi^0\gamma_5\tau_3)^{-1}\delta^{(4)}(x - x')$$

Same result as ChPT



For E=B case, $I_2 = \boldsymbol{E}\cdot\boldsymbol{B}$

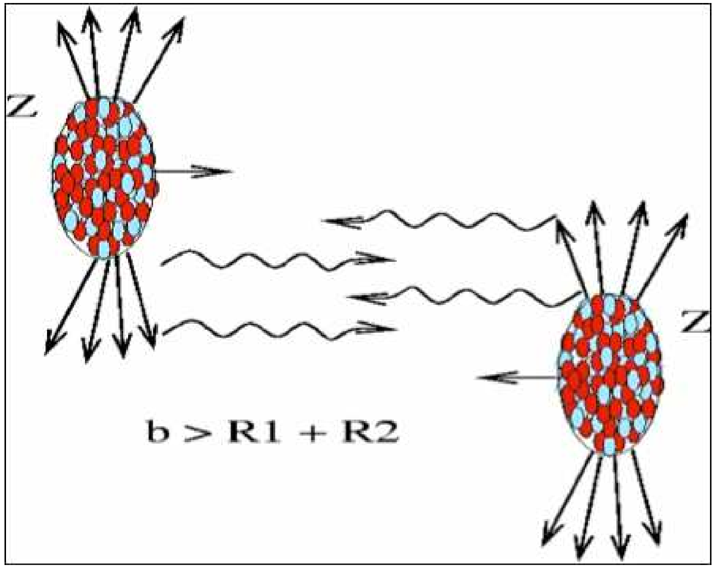
In the presence of $\boldsymbol{E}\cdot\boldsymbol{B}$, the QCD vacuum contains neutral pion condensate!

Cf: the Disoriented Chiral Condensate (DCC) which is out-of-equilibrium

Cao-XGH 2015
Wang-Cao-XGH-Zhuang 2018

Neutral pion condensation

- **Calculation by using Wigner function** **(Fang-Pang-Wang-Wang 2016; Guo-Zhuang 2017)**
- **Similar argument can apply to η condensation** **(Wang-Cao-XGH-Zhuang 2018)**
- Possible experimental signal: $\frac{\pi_0}{\pi}$ ratio in UPC



$$\frac{N_{\pi_0}}{N_{\pi^+} + N_{\pi^-} + N_{\pi^0}} > \frac{1}{3} ???$$

Cf. the detection of DCC.
Experimentally challenging.

QCD vacuum under rotation:
chiral soliton lattice

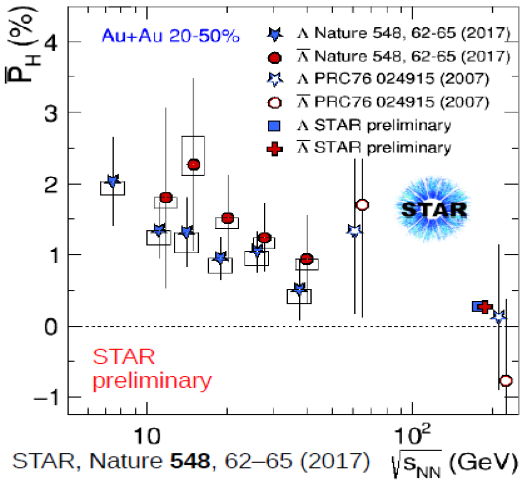
Chiral vortical effect

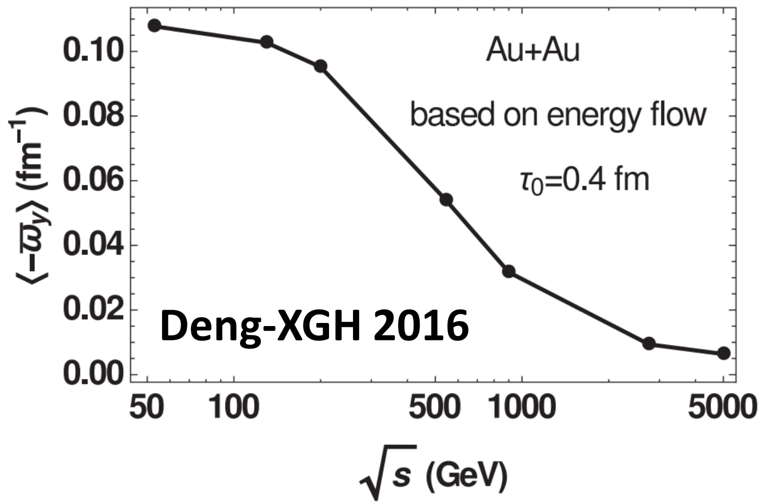
- Chiral fermions + fluid vorticity ⇒ chiral vortical effect (CVE)

(Erdmenger etal 2008; Barnerjee etal 2008, Son, Surowka 2009; Landsteiner etal 2011):

$$\vec{J}_R = \frac{1}{4\pi^2}\mu_R^2\vec{\omega} + \frac{T^2}{12}\vec{\omega}, \qquad \vec{J}_L = -\frac{1}{4\pi^2}\mu_L^2\vec{\omega} - \frac{T^2}{12}\vec{\omega}$$

Vorticity is larger at lower collision energy





Very low energy, hadronic degree of freedom, then ...

Axial chiral vortical effect

- **Axial current induced by vorticity:**

$$\boldsymbol{j}_a^5 = N_\mathrm{c}\left(d_{abc}\frac{\mu_b\mu_c}{2\pi^2} + b_a\frac{T^2}{6}\right)\boldsymbol{\Omega}\,,$$

$$d_{abc} = \frac{1}{2}\mathrm{Tr}\left[\tau_a\{\tau_b,\tau_c\}\right], \qquad b_a = \mathrm{Tr}(\tau_a)$$

- **Low-energy effective lagrangian for aCVE (anomaly matching):**

$$\mathcal{L}_\mathrm{EFT} = \frac{N_\mathrm{c}}{2f_\pi}\left(\frac{d_{abc}}{2\pi^2}\mu_b\mu_c + \frac{b_a}{6}T^2\right)\boldsymbol{\nabla}\pi_a\cdot\boldsymbol{\Omega}$$

Chiral anomaly

mixed gravitational anomaly or global anomaly?

- **Look for consequences of a CVE for low T dense matter under rotation**

A chiral soliton lattice (I)

- The Hamiltonian for the neutral pion ($\phi \equiv \pi^0/f_\pi$)

$$\mathcal{H} = \frac{f_\pi^2}{2}\left[(\partial_r\phi)^2 + \frac{1-(\Omega r)^2}{r^2}(\partial_\theta\phi)^2 + (\partial_z\phi)^2\right] + m_\pi^2 f_\pi^2(1-\cos\phi) - \frac{\mu_B\mu_I}{2\pi^2}\Omega\partial_z\phi$$

- The ground state is given by

$$\langle\partial_r\pi_0\rangle = \langle\partial_\theta\pi_0\rangle = 0 \qquad \partial_z^2\phi = m_\pi^2\sin\phi$$

- Its solution is given by zero of the Jacobi elliptic function

$$\cos\frac{\phi(\bar{z})}{2} = \mathrm{sn}(\bar{z},k) \quad \text{with} \quad \bar{z} \equiv zm_\pi/k$$

with period

$$\ell = \frac{2kK(k)}{m_\pi}$$

with $K(k)$ the 1st complete elliptic integral

XGH-Nishimura-Yamamoto 2017

A chiral soliton lattice (II)

- This is a one dimensional chiral soliton lattice. It is the ground state when

$$|\Omega| \geq \Omega_{\rm CSL} \equiv \frac{8\pi m_\pi f_\pi^2}{\mu_{\rm B}|\mu_{\rm I}|}$$

- Each lattice cell carries topological charges

$$\frac{J_z}{A} = \frac{\mu_{\rm B}\mu_{\rm I}}{\pi}, \quad \frac{N_{\rm B}}{A} = \frac{\mu_{\rm I}\Omega}{\pi}, \quad \frac{N_{\rm I}}{A} = \frac{\mu_{\rm B}\Omega}{\pi}$$

with crossed correlation between baryon and isospin

- The energy density

$$\frac{\mathcal{E}_{\rm tot}}{V} = 2m_\pi^2 f_\pi^2 \left(1 - \frac{1}{k^2}\right) < 0$$

XGH-Nishimura-Yamamoto 2017

Topological Barnett-Einstein-de Haas effect

- **Introducing also the magnetic field (chiral limit)**

$$\mathcal{H} = \frac{1}{2}(\boldsymbol{\nabla}\pi_0)^2 - \frac{\mu_\mathrm{B}}{4\pi^2 f_\pi}\boldsymbol{\nabla}\pi_0 \cdot (2\mu_\mathrm{I}\boldsymbol{\Omega} + \boldsymbol{B})$$

- **Integrating out pion**

$$\mathcal{H}_\mathrm{mix} = -\frac{\mu_\mathrm{B}^2\mu_\mathrm{I}}{8\pi^4 f_\pi^2}\boldsymbol{\Omega}\cdot\boldsymbol{B}$$

- **This induces cross-correlated response between rotation and magnetic field**

$$\boldsymbol{j} = \chi_{jB}\boldsymbol{B}$$ ~ Einstein-de Haas effect

$$\boldsymbol{m} = \chi_{m\Omega}\boldsymbol{\Omega}$$ ~ Barnett effect

with $\chi_{jB} = \chi_{m\Omega} = \dfrac{\mu_\mathrm{B}^2\mu_\mathrm{I}}{8\pi^4 f_\pi^2}$

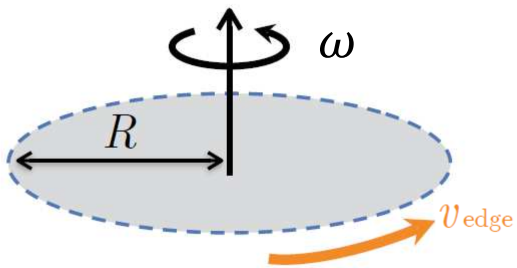
ChSB under both B field and rotation:
rotational magnetic inhibition

Analogy between rotation and density

- **Hamiltonian: rotation vs chemical potential**

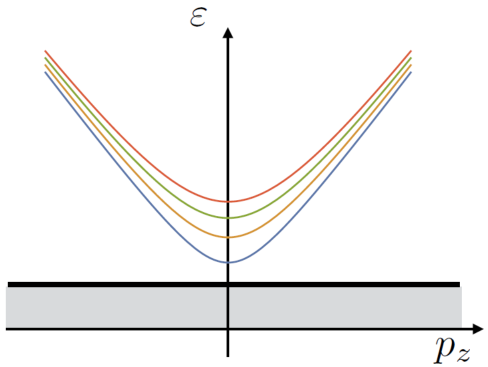
$$H_{\mathrm{rot}} = H - \omega J_z \qquad\qquad H_{\mu} = H - \mu N$$

This indicates ωJ_z plays similar role as chemical potential term μN. However … …



Causality: $v_{\mathrm{edge}} = \omega R < 1$

Rotating system must be finite!



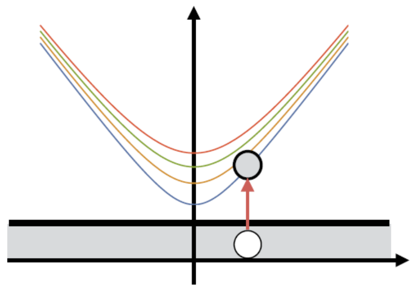
Excitation gap due to finite size: J_z/R
Effective chemical pot.: $\omega J_z < J_z/R$

Pure rotation does not excite any modes.
(Ebihara-Fukushima-Mameda 2017)

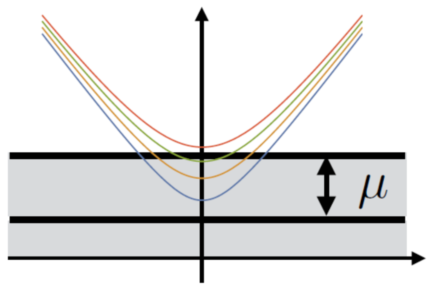
Figures drawn by Mameda

Analogy between rotation and density

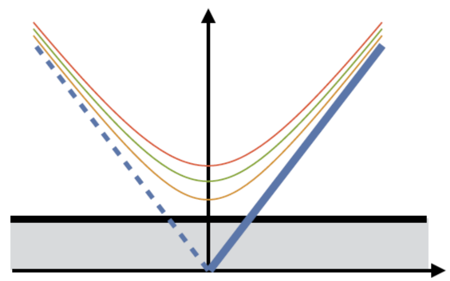
- To see rotational effect we need $T, \mu, B, \ldots \ldots$



T: Jiang-Liao 2016; Chernodub-Gongyo 2017

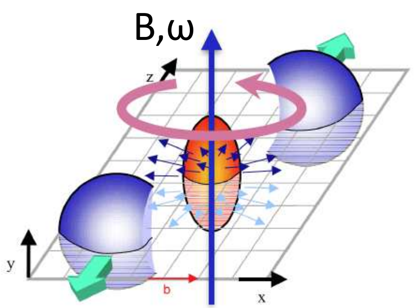


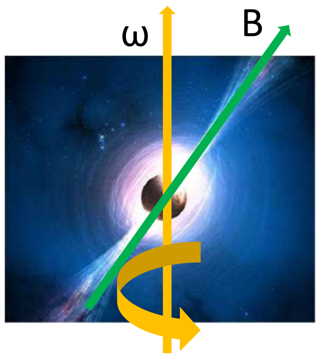
μ: XGH-Nishimura-Yamamoto 2017



B: Chen-Fukushima-XGH-Mameda 2015

- Indeed, rotation is commonly associated with B



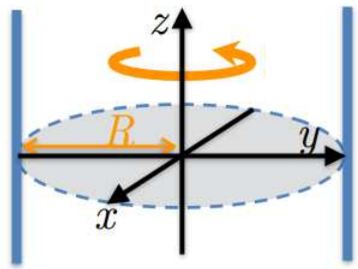


We consider the interplay btw B and ω

Dirac fermion in rotation and B field

- The spectrum can be obtained from the Dirac Eq.

$$[i\gamma^{\mu}(\nabla_{\mu} + \Gamma_{\mu}) - m]\psi = 0$$



$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Consider the situation where

$$1/\sqrt{eB} \ll R \leq 1/\Omega$$

so that we can take the quasi-thermodynamic limit

- The spectrum reads

$$[E + \Omega(\ell + s_z)]^2 = p_z^2 + (2n + 1 - 2s_z)eB + m^2$$

Angular momentum **Landau levels**

Dirac fermion in rotation and B field

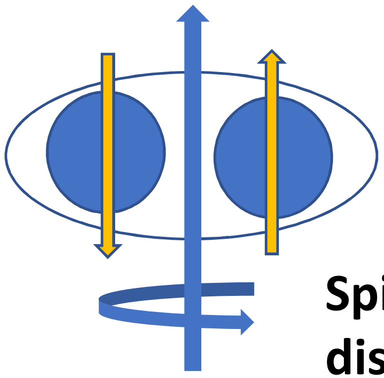
- **The enhancement of IR density of state**

$$\text{Magnetic field}: \int \frac{dp_x dp_y}{(2\pi)^2} \to \frac{eB}{2\pi}\sum_{n=0}^{\infty},$$

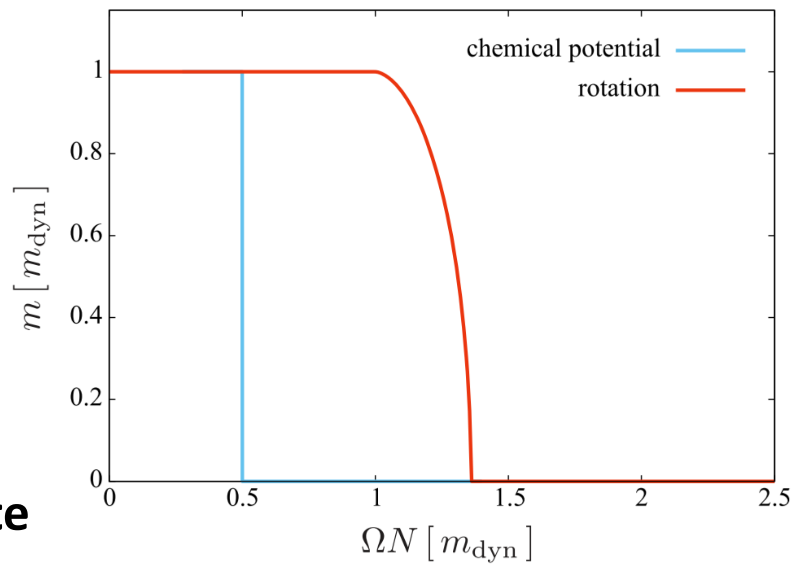
$$\text{Magnetic field} + \text{Rotation}: \int \frac{dp_x dp_y}{(2\pi)^2} \to \frac{1}{S}\sum_{n=0}^{\infty}\sum_{\ell=-n}^{N-n}$$

- **Consider a four fermion interaction**

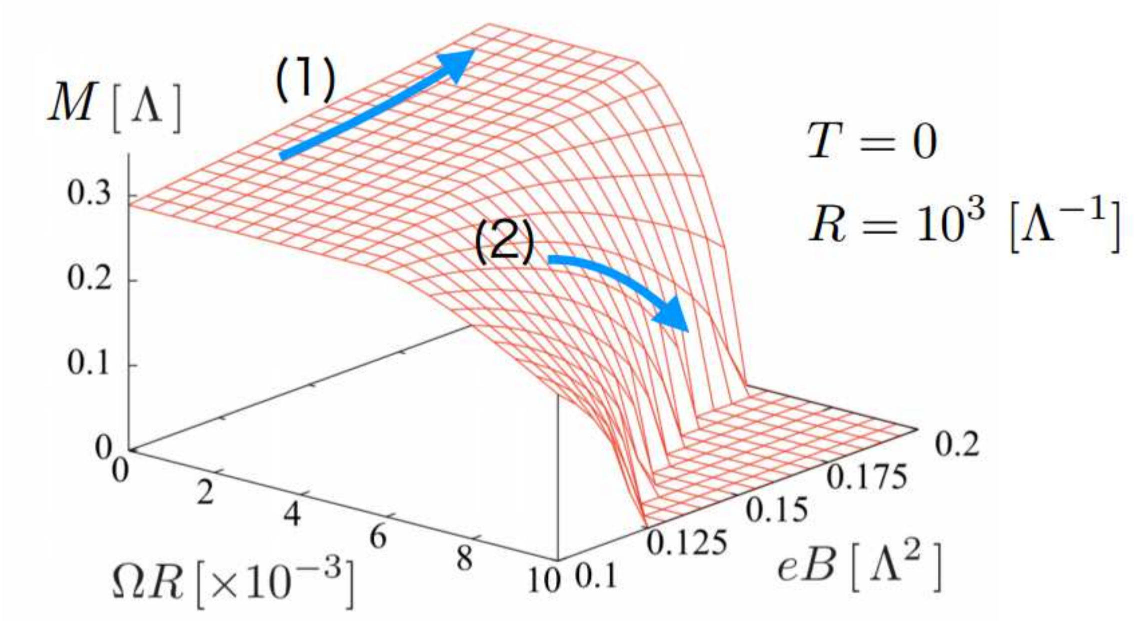
$$\frac{G}{2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}\gamma_5\psi)^2\right]$$



Spin-0 condensate disfavored



Dirac fermion in rotation and B field



(1) eB increases → M increases   Magnetic Catalysis

(2) eB increases → M decreases   Inverse of MC

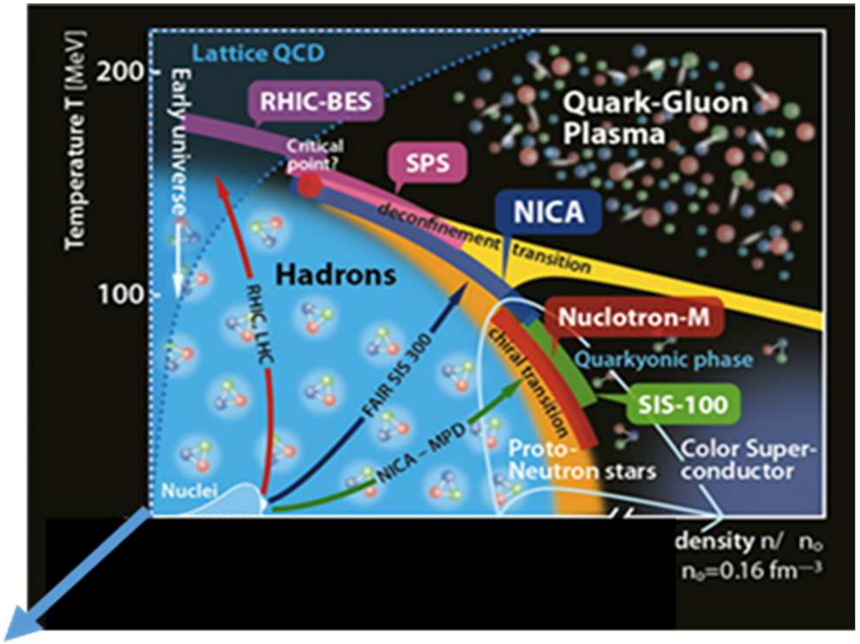
'Rotational magnetic inhibition'

Chen-Fukushima-XGH-Mameda 2015

See also: Jiang-Liao 2016; Chernodub-Gongyo 2016; Liu-Zahed 2017; Chen-Fukushima-XGH-Mameda 2017; Wang-Wei-Li-Huang 2018 …

Summary

New dimension of QCD phase diagram



EM , ω

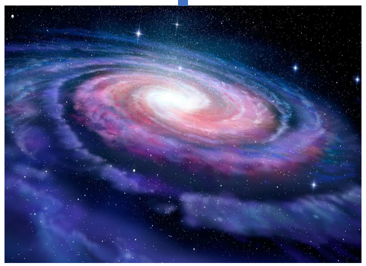
Magnetic catalysis at T=0;
Inverse magnetic catalysis at Tc;
Neutral pion condensation;
Chiral soliton lattice in B field and/or in rotation;
Suppression of spin-0 condensate due to rotation;
Rotational magnetic inhibition;
… …

Thank you!

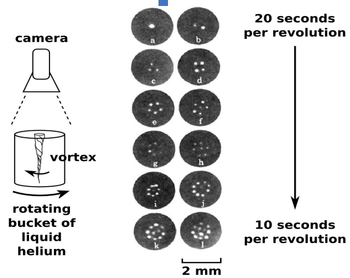
Fluid vorticity in heavy-ion collisions

- Vortices: common phenomena in fluids across a very broad hierarchy of scales

$10^{21}m$ — $10^{2}-10^{-2}m$ — $10^{-5}-10^{-8}m$ — $10^{-15}m$

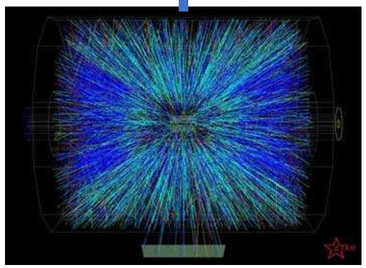


Rotating galaxies



Superfluid helium

Tornados, ocean vortices, …



Quark gluon matter

Neutral pion condensation

- Vafa-Witten theorem: no parity odd condensate in QCD vacuum if theta term vanishes. But in E field … …
- 2 flavor quarks, Dirac operator,

$$\mathcal{D} = \gamma_\mu(\partial_\mu - ig\mathcal{A}_\mu) + Q\gamma_4 A_0 + iQ\gamma_i A_i + M$$

- Dirac positivity and thus Vafa-Witten theorem is violated in E field

Important consequences: **Schwinger mechanism**, **sign problem**, **pion condensation in vacuum**

Note that in B field, Dirac positivity is NOT violated. So in B field, no parity odd or vector-like condensates can occur in QCD vacuum

Outline

- **QCD vacuum in EM field:**
  Neutral pion condensed vacuum (G. Cao, L. Wang, P. Zhuang)

- **QCD vacuum in rotation:**
  chiral soliton lattice (K. Nishimura, N. Yamamoto)

- **Chiral condensate in rotation + magnetic field:**
  Rotational magnetic inhibition (H.L. Chen, K. Fukushima, K. Mameda)

- **Summary**