# **Special moments….**

### **Forty Years of Quark-Gluon Plasma**

Wuhan, China

October 10-11, 2018



**Jean-Paul Blaizot, IPhT-Saclay**

## **Special moments….**

# **Hiking with Edward**











# **Arguing with Edward…**





**'Special moments' can also provide simple tools to understand the emergence of hydrodynamical behavior in expanding quark-gluon plasmas**

### How much entropy is produced in strongly coupled Quark-Gluon Plasma (sQGP) by dissipative effects?

M.Lublinsky and E.Shuryak

Department of Physics and Astronomy, State University of New York, Stony Brook NY 11794-3800, USA (Dated: June 14, 2013)

We argue that estimates of dissipative effects based on the first-order hydrodynamics with shear viscosity are potentially misleading because higher order terms in the gradient expansion of the dissipative part of the stress tensor tend to reduce them. Using recently obtained sound dispersion relation in thermal  $\mathcal{N}=4$  supersymmetric plasma, we calculate the resummed effect of these high order terms for Bjorken expansion appropriate to RHIC/LHC collisions. A reduction of entropy production is found to be substantial, up to an order of magnitude.

**The hydrodynamic description of matter produced in heavy ion collisions works amazingly well !…**

> **even in situations where, a priori, it should not … (e.g. in presence of strong gradients)**

Fluid behavior requires (some degree of) local equilibration (='thermalization'). How is this achieved?

**Usual picture:**

- **• microscopic degrees of freedom relax quickly towards local equilibrium**
- **• long wavelength modes, associated to conservation laws, relax on longer time scales**

**Thermalization**

### **Two main issues**

**i) relative populations of different momentum modes** 

**ii) isotropy of momentum distribution**



m

 $P_V$ 

**Main topic for the rest of this talk**



when

 $P_{Z}$ 

 $\mathsf{p}_{\mathsf{X}}$ 

**Longitudinal expansion hinders isotropization**  $\overline{\phantom{a}}$ expaincion hinders isotropization τ/τ<sup>R</sup>

**The fast expansion of the matter along the collision axis tends to drive the momentum distribution to a very flat distribution along the z direction** FIG. 16. Red: *T*<sup>0</sup> = 0*.*8, ⌧<sup>0</sup> = 0*.*05; Green: *T*<sup>0</sup> = 0*.*1, ⌧<sup>0</sup> = 8*.*16; Blue: *T*<sup>0</sup> = 0*.*8, ⌧<sup>0</sup> = 2*.*34. Note that *T*0⌧*R*(*T*0)=5⌘*/s* = 5 F<sub>1</sub> The fest expension of the metter elene the

> **Translates into the**  existence of two **different pressures**





**(longitudinal) (transverse)**



**Anisotropy relaxes slowly, like a 'collective' variable associated to a conservation law**

> **Hydrodynamic behavior may emerge before local isotropization if achieved**

> > **"Hydrodynamization"**

**First hints came from holographic descriptions**



### **Holographic description of a boost invariant plasma** VI. THE HYDRODYNAMIC FIXED POINT

(Heller, Janik, Witaszczyk, [1103.3452])



Viscous hydro can cope with partial thermalization, and large differences between longitudinal and transverse pressures The hydrodynamic attractor. When we combine the two e $\mathcal{A}$ differences between longitudinal and transverse pressures

## The gradient expansion is divergent

$$
f \equiv \frac{2}{3} + \frac{\mathcal{R}}{6} = \sum_{n=0}^{\infty} f_n w^{-n} \qquad f_n \sim n!
$$

 $f_n$  has been calculated up to n=240 (!) (Heller, Janik, Witaszczyk , 2013)

### In the simple situation where  $\alpha$  is the expansion of motion become the expansion of motion become  $\alpha$ **Sophisticated resummation yields a 'transseries'**

@*L*<sup>0</sup> @⌧ = 0*,* **(Heller, Spalinski , 2015)**  $I$ In the simple situation where  $S$   $\sim$   $Q$   $\left(\mu$   $\right)$   $\sim$   $Q$   $\left(\pi$ )

$$
f = \sum_{n=0}^{\infty} f_n w^{-n} + c \ e^{-\frac{3}{2C_{\tau\Pi}} w} \left( w^{\frac{C_{\eta} - 2C_{\lambda_1}}{C_{\tau\Pi}}} \sum_{n=0}^{\infty} f_n^{(1)} w^{-n} \right) + \dots
$$

Similar features are observed in kine resurgence, which fixes Im(c). **(Heller, Kurkela, Spalinski , Svensson, 2016)**The energy density remains constant, while the anisotropy of the momentum distribution is washed out. **Sim** @*L*<sup>1</sup>  $\mathbf{m}$ ar Similar features are observed in kinetic theory

### $S$ imple kinetic equation = 0*,* ˜*f*(*p*?*, pz,* ⌧ ) = ˜*f*0(*p*?*, pz*⌧*/*⌧0)=e⌧0*/*⌧*<sup>R</sup> <sup>f</sup>*0(*p*?*, pz*⌧*/*⌧0)*.* (16)

• Relaxation time approximation

$$
\left[\partial_{\tau} - \frac{p_z}{\tau} \partial_{p_z}\right] f(\mathbf{p}/T) = -\frac{f(\mathbf{p}, \tau) - f_{\text{eq}}(p, \tau)}{\tau_R}
$$

**(derivative at constant**  $p_z \tau$ **)**  $\mathbf{C}$ 

> **•** Free streaming (e.g. in absence of collisions) **COLLET DISTRIBUTION** e⌧ */*⌧*<sup>R</sup> <sup>f</sup>*eq ✓q *p*2 ? + (*pz*⌧*/*⌧ <sup>0</sup> )2 **•Free streaming (e.g. in absence of collisions)**  $\overline{a}$

 $f(t,\boldsymbol{p}) = f_0(\boldsymbol{p}_\perp,p_z t/t_0)$  $\mathbf{S}(\mathbf{y}, \mathbf{y})$  sum of the general free streaming solution plus the particular solution of the full equation. is a particular solution of the equation for ˜*f* which vanishes for ⌧ = ⌧0. The solution in Eq. (14) then follows as the

**• Solved long ago by Baym** [PLB 138 (1984) 18] energy integration and using the matching condition, Eq. (10), one derives and using the matching condition, Eq. (10), one derives an equation for the matching condition, equation for the matching condition for the matchin

$$
\epsilon(\tau) = e^{-(\tau-\tau_0)/\theta} \epsilon^{(0)}(\tau) + e^{-\tau/\theta} \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_R} e^{\tau'/\tau_R} \frac{\tau'}{\tau} \epsilon(\tau') h(\tau'/\tau)
$$
  
(free streaming)  

$$
h(x) \equiv \int_0^1 d\mu \sqrt{1-\mu^2+\mu^2 x^2}
$$

**(angular integral)**

 $\overline{\phantom{a}}$ 

### **Here come the special moments** (JPB, Li Yan, 2017, 2018) 017, 2018)<br>' A. The moments *L<sup>n</sup>* Here *p*<sub>2</sub>*n*(*p*<sub>2</sub>*n*)*f*(*p*<sub>2</sub>*p*<sub>2</sub>*n*)*f*(*p*<sub>2</sub>*n*)*f*(*p*<sub>2</sub>*p*<sub>2</sub>*p*<sub>2</sub>*n*)*f*(*p*<sub>2</sub>*p*<sub>2</sub>*n*)</sub>*f*(*p*<sub>2</sub>*n*)</sub>*f*(*p*<sub>2</sub>*n*)</sub>*f*(*p*<sub>2</sub>*n*)</sub>*f*(*p*<sub>2</sub>*n*)</sub>*f*(*p*<sub>2</sub>*n*)

**Special moments whereal is a Legendre polynomial of order 2** *p*<sub>z</sub>

 $p_z = p \cos \theta$ where *P*2*<sup>n</sup>* is a Legendre polynomial of order 2*n*, and cos ✓ = *pz/p*. Recall that where *P2n* is a Legendre polynomial of order 2<sub>n, and</sub> cos ∠ *p*<sub>z</sub>*/p*. Recall that  $\alpha$  *pz/*<sub>*p*</sub>. Recall that  $\alpha$ 

$$
\mathcal{L}_n \equiv \int_p p^2 P_{2n}(\cos \theta) f(\mathbf{p}) \qquad P_0(z) = 1 \qquad P_2(z) = \frac{1}{2}(3z^2 - 1)
$$

$$
p^{2}P_{2n}(\cos \theta) f(\mathbf{p}) \qquad P_{0}(z) = 1 \qquad P_{2}(z) = \frac{1}{2}(3z^{2} - 1)
$$

#### m*e*nts ?  $\mathbf{B}$  is the relations of  $\mathbf{B}$  $\mathbf{r}$  and  $\mathbf{r}$ . **Why moments ?**

- *P*<sup>1</sup>, *P*<sub>2</sub>  $\rightarrow$  **P**<sup>2</sup>  $\rightarrow$  **P**<sup>2</sup>  $\rightarrow$  *P*<sup>2</sup>  $\rightarrow$  *P*<sup>2</sup>  $\rightarrow$  *P*<sup>2</sup>  $\rightarrow$  *P*<sup>2</sup>  $\mathsf{r}$ of the system of the system with Biorget and order with Biorget and order with a consequence of the instruments o<br>There is too much information in the distribution function For an expanding system with Bjorken geometry, order moments vanish as a consequence of the invariance of the i<br>The invariance of the invariance of th • There is too much information in the distribution function *L*<sup>0</sup> = "*, L*<sup>1</sup> = *P<sup>L</sup> P<sup>T</sup> .* (4)
- on the state of the second the second second the second of the state of the second of the state of the state o<br>The state of the st of the distribution function under parity (or under reflection with respect to the *z* = 0 plane, i.e. *p<sup>z</sup>* ! *p<sup>z</sup>* and • We want to focus on the angular degrees of freedom we want The moments *L<sup>n</sup>* of higher order are associated to finer structures of the momentum anisotropy of the distribution **•** We want to focus on the angular degrees of freedom **•** *L* of the momentum anisotropy of the momentum and momentum anisotropy of the momentum and momentum and momentum and momentum and momentum and momentum and momen

The energy momentum tensor is described by first two moments

$$
T^{\mu\nu} = \int_p f(p)p^{\mu}p^{\nu} \qquad \mathcal{L}_0 = \varepsilon \qquad \mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T
$$



## **Two antagonistic agents**



**drives the momentum distribution toward a flat distribution in the longitudinal direction** 



**drive the momentum distribution toward an isotropic distribution** 





#### Coupled equations for the moments (4*<sup>n</sup>* 1)(4*<sup>n</sup>* + 1) ' *n*  $\overline{\mathbf{v}}$ ts By using the recursion relations among the Legendre coupled equations for the mome finite) set of coupled equations of coupled equations of coupled equations of coupled equations of coupled equ<br>The coupled equations of coupled experimental experimental experimental experimental experimental experimental here. Note that all the *L<sup>n</sup>* have the same dimension. Coupled equations for the mom polynomials, we can recast Eq. (1) into the following (in-*L<sup>n</sup>* ⌘ *p*  $p_{21}$ *p*  $\int$ <sup>2</sup> $\int$ <sup>2</sup>

$$
\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau} \left[ a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1} \right] - \frac{\mathcal{L}_n}{\tau_R} \quad (n \ge 1)
$$
\n(Free streaming)

\n
$$
\frac{\partial \mathcal{L}_0}{\partial \tau} = -\frac{1}{\tau} \left[ a_0 \mathcal{L}_0 + c_0 \mathcal{L}_1 \right] \qquad a_0 = 4/3, \ c_0 = 2/3
$$

 $\mathcal{L}_{1}^{\prime}$ 

L1/

 $\overline{a}$ 

 $\boldsymbol{\mathcal{L}}$  $\bm{\mathsf{O}}$ 

 $\overline{\phantom{0}}$ 

$$
\mathcal{L}_0 = \varepsilon \qquad \mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T
$$

*a<sup>n</sup>* =  $a_n, b_n, c_n$  are positions  $a_n, b_n, c_n$ coefficients  $a_n, b_n, c_n$  are pure numbers icients  $a_n, b_n, c_n$ icients  $a_n, b_n, c_n$  are p (2*n* 1)2*n*(2*n* + 2) (4*<sup>n</sup>* 1)(4*<sup>n</sup>* + 1) *,* • The coefficients  $a_n, b_n, c_n$  are pure numbers

- $comp$ etition between expansior etition between expansion and collisions is made *(* FIG. 1. Comparison of the *L*-moment equations obtained *c<sup>n</sup>* = Note that the energy-momentum tensor is given by the energy is given by the energy is given by the second by t **• The competition between expansion and collisions is made obvious**
	- entirely determined by the free streaming part of the king part of the kiexternal of counted linear equations *p f*(*p*)*<i>p*<sub>*n*</sub>*p*<sub>*n*</sub>*<i>p*<sup>*n*</sup></sup>*p*<sup>*n*</sup>*<i>p*<sup>*n*</sup></del>*<i>p*<sup>*n*</sup></del></sup>*p<sup><i>n*</sup>*<i>p***<sup>***n***</sup></del>***p***<sup>***n***</sup><b>***<i>p<i><i>p*<sup>*n*</sup></del> *p* **• Interesting system of coupled linear equations** *f*(*p*)*p<sup>µ</sup>p*⌫ (5)
- nergence of hydrodynamics is transparent: equations for a lowest moments the energy density, but only the moments entirely determined by the free streaming part of the ki-· Emergence of hydrodynamics is transparent: equations for **a** the lowest moments the moments of the
	- wides much insight on verious versions of viscous hydrog *a* vides much insignt on various versions or viscous ny  $t_{\rm eff}$  is not hard to see that the resultinics a stable solution possesses a stability of the stability of the solution at large time, in the stability ovides much insight on various versions of viscous hydra *a<sup>n</sup>* = *b<sup>n</sup>* = *c<sup>n</sup>* = 0, the moments evolve according to  $t$  collision to  $t$ **• Provides much insight on various versions of viscous hydrodynamics**

## Free streaming solution

$$
\mathcal{L}_n^{(0)}(t)=\frac{t_0}{t}\epsilon_0\,\mathcal{F}_n\left(\frac{t_0}{t}\right)
$$



Poor convergence: all moments are important at late time **Poor convergence: all moments are important at late time** 



#### **Free streaming in the general results obtained in the perspective of Simple truncations work** In order to analyze this system of equations, we set *t* ⌘ log ⌧*/*⌧0, consider *L* infinite dimensional space), and write  $\mathcal{M}$  sumple truncations work of the next times, and the next truncation involves the two moments  $\mathcal{F}$  is only the t through its interaction with the next truncation with the fixed point behavior  $\mathcal{C}(\mathcal{C})$  . The next truncation is a  $\mathcal{C}(\mathcal{C})$  . The corresponding equations reading equations  $\mathcal{C}(\mathcal{C})$  . The corresponding equ

$$
\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau} \left[ a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1} \right]
$$
\n
$$
\frac{\partial \mathcal{L}}{\partial \tau} = -M \vec{\mathcal{L}}
$$
\nno only the first two moments one acts

\n
$$
(t = \ln(\tau/\tau_0))
$$

**Example 19 Keeping only the first two moments one gets**  $\mathbb{R}^n$  and  $\mathbb{R}^n$  and  $\mathbb{R}^n$  and  $\mathbb{R}^n$  and  $\mathbb{R}^n$  are  $\mathbb{R}^n$  and  $\mathbb{R}^n$  and  $\mathbb{R}^n$  and  $\mathbb{R}^n$  are  $\mathbb{R}^n$  and  $\mathbb{R}^n$  are **Keeping only the first two moments one gets**

$$
\frac{\partial}{\partial t}\left(\begin{array}{c} \mathcal{L}_0 \\ \mathcal{L}_1 \end{array}\right)=-\left(\begin{array}{cc} \frac{4}{3} & \frac{2}{3} \\ \frac{8}{15} & \frac{38}{21} \end{array}\right)\left(\begin{array}{c} \mathcal{L}_0 \\ \mathcal{L}_1 \end{array}\right)
$$

@*tL*<sup>~</sup> <sup>=</sup> *ML*~*,* (127)  $Tw<sub>o</sub>$  eige Two eigenmodes  $\lambda_0=0.929366$   $\lambda_1=2.21349$ 

### **Truncations are reasonably accurate** the control of th<br>The control of the control of t <sup>2</sup> ) = (0*.*75744*,* 1). Note that the eigenvalues are positive so that the two modes are damped.



### **Free streaming fixed point**   $Free\ strenning\ fixed point$

independently one constant of the coupled linear equations into a single pon linear eigenvalue is stable while that associated with the largest eigenvalue is unstable. The stability of the fixed points is best analyzed from the equation for  $g(0, \mu)$  which, in the absence of interaction, in the a **One can transform the coupled linear equations into a single non linear differential equation**

$$
\tau \frac{\mathrm{d}g_0}{\mathrm{d}\tau} + g_0^2 + (a_0 + a_1)g_0 + a_0a_1 - c_0b_1 = 0, \qquad g_n(\tau) \equiv \tau \partial_\tau \ln \mathcal{L}_n
$$

#### which we rewrite , and with a slight abuse of  $\mathcal{A}$  slight abuse of  $\mathcal{A}$ **Write this as**

$$
\tau \frac{dg_0}{d\tau} = \beta(g_0) \qquad \qquad \beta(g_0) = -g_0^2 - (a_0 + a_1)g_0 - a_0a_1 + c_0b_1
$$



## Including collisions Simple truncations work well 2 (drop all moments beyond a certain n)



## **First few moments are relevant**



moments of free streaming solution

damping of higher moments by collisions

#### , simple model can be constructed A simple model can be constructed

$$
\partial_{\tau} \mathcal{L}_0 = -\frac{1}{\tau} \left( a_0 \mathcal{L}_0 + c_0 \mathcal{L}_1 \right)
$$
 ("Almost" viscous hydrodynamics...  

$$
\partial_{\tau} \mathcal{L}_1 = -\frac{1}{\tau} \left( b_1 \mathcal{L}_0 + a_1 \mathcal{L}_1 \right) - \frac{1}{\tau_R} \mathcal{L}_1
$$

simple equatio *dt*<sup>0</sup>  $\blacksquare$ *e*(*tt*<sup>0</sup> )*/*⌧rel apture *F*<br>This simple equations capture much of the physics and illuminates the analytic features of the transition to viscous by features of the transition to viscous hy This simple equations capture much of the physics and illuminates the analytic drives the system towards the system towards the system of the syste some familiar viscous de la production de<br>En 1990, est de la production de la produc **features of the transition to viscous hydrodynamics**

### $\tau$  and system of the system of  $\mathcal{L}$  ved  $\tau$ Consider the system of equations truncated at the first two moments **The hydrodynamic fixed point**

$$
\partial_{\tau}\mathcal{L}_0 = -\frac{1}{\tau}\left(a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right) \qquad \partial_{\tau}\mathcal{L}_1 = -\frac{1}{\tau}\left(b_1\mathcal{L}_0 + a_1\mathcal{L}_1\right) - \frac{1}{\tau_R}\mathcal{L}_1
$$

**Example te damping of first moment and complete damping of first moment** 

$$
\tau \partial_{\tau} \mathcal{L}_0 = -a_0 \mathcal{L}_0 = -(4/3) \mathcal{L}_0
$$
 (hydro fixed point)

#### **1**<br>
1 **g**<sub>0</sub>  $\mathfrak{h}$ **g2** + *g***2**  $\blacksquare$  *g***2 +** *Modified equation of motion* <sup>0</sup> + (*a*<sup>0</sup> + *a*<sup>1</sup> + *w*)*g*<sup>0</sup> + *wa*<sup>0</sup> + *a*0*a*<sup>1</sup> *b*1*c*<sup>0</sup> = 0*,* ⌧*RT* = Cste*,* (196)

$$
wg'_0 + g_0^2 + (a_0 + a_1 + w)g_0 + wa_0 + a_0a_1 - b_1c_0 = 0, \quad \tau_R = \text{Cste}
$$
  

$$
\frac{dg_0}{d \ln w} = \beta(g_0, w)
$$



### **The transition from free streaming to hydrodynamics ( Attractor solution )**

**Early and late times are controlled by the free streaming and the hydrodynamic fixed points, respectively**



#### **Renormalization of the viscosity** Renormalization of ⌘*/s*. Alternatively, the e↵ects zenormalization of the viscosity  $20 \times (2000) + 1$ <sup>1</sup> *c*0)<sup>2</sup>⇧<sup>2</sup>. It can be verified that rection induction of the viscosity

tion of higher moments can be viewed as a renormalisation of the equation *L*<sub>1</sub>. The moments can be viewed as a renormansation for rest moments. i.e. of the viscous hydrodynamical equation cous correction derived in Ref. [17]. Obviously, it would The effect of higher moments can be viewed as a renormalisation of the equation  $\qquad$ west moments, i.e. of the viscous hydrodynamical equations large *n*, ↵(0) *<sup>n</sup>* / *<sup>n</sup>*!, and the series of the ↵(0) moments, i.e. of the<br> $\frac{1}{2}$ *.* (14) **.** (1 **for the lowest moments, i.e. of the viscous hydrodynamical equations**

$$
\partial_{\tau}\mathcal{L}_1 = -\frac{1}{\tau}\left(a_1\mathcal{L}_1 + b_1\mathcal{L}_0\right) - \left[1 + \frac{c_1\tau_R}{\tau}\frac{\mathcal{L}_2}{\mathcal{L}_1}\right]\frac{\mathcal{L}_1}{\tau_R}
$$

isation of relaxation time, **cancel** renormalisation of relaxation time, **a multiplication time** viscosity at early time

$$
\tau_R \to Z_{\eta/s} \tau_R
$$

$$
\eta = \frac{4}{15} \tau_R \epsilon
$$

**in Suggestion of such an effect:**  $0.1$  The strains of the point of such an effect:  $0.1$  The point of the point.

. The dimensionless ratio of the higher moments can be treated as a renormaliza-**viscosity at early time** Sizeable reduction of the effective **Sizeable** obtained iteratively. The quantity *Z*⌘*/s* in Eq. (13) then



**Variants of viscous hydrodynamics** A. Leading order viscous hydro. Navier Stokes A. Leading order viscous hydro. Navier Stokes The equation for viscous hydro in leading order reads (see e.g. [18]) ⇧ *,* ⇧ <sup>=</sup> <sup>4</sup>⌘ @⌧ ✏ <sup>=</sup> <sup>4</sup> ✏ + ⇧ *,* ⇧ <sup>=</sup> <sup>4</sup>⌘ 3⌧ *,* (377) @⌧ ✏ <sup>=</sup> <sup>4</sup> 3 ✏ + ⇧ ⌧ *,* ⇧ <sup>=</sup> <sup>4</sup>⌘ *,* (377) @⌧ ✏ <sup>=</sup> <sup>4</sup> 3 ⌧ + <sup>3</sup>⌧ <sup>2</sup> *.* (378) This equation has problems (causality, instability). The improvement due to Israel Steward consists in writing an ✓ ◆ @⌧⇧ <sup>=</sup> ⇧ ⌧⇡ 3⌧ ⌧⇡ <sup>=</sup> <sup>1</sup> ⌧⇡ Note that is ⌧⇡ is chosen very small, then ⇧ relaxes rapidly to the leading order value ⇧ = 4⌘*/*3⌧ *.*

Different approximate ways to solve the equations for the first two moments The equation for viscous hydrogeneous hydrogeneous  $\mathcal{E}_{\mathcal{A}}$  $\overline{I}$  $\overline{a}$ <br>we the equations for the first two moments that is **Navier Stokes** pproximate ways to solve the equations for the first two moments.  $\mathbb{R}^n$  $\frac{1}{2}$  $\epsilon$  of motion for a new parameter  $\epsilon$  that for rate is fixed by a new parameter called  $\epsilon$ . We have in leading  $\epsilon$ tter  $\mathbf{r}$  $\frac{4r}{3}$ The equations for the equations of the equations of the equations of the equations of the same of the equations of the same of *.* (379) Note that is chosen very small, then we small, then  $\alpha$  relaxes rapidly to the leading order value  $\alpha$  . Then  $\alpha$ 

**EXAMPLE 5 TOKES**  
\n
$$
\partial_{\tau} \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau} + \frac{\Pi}{\tau} \qquad \qquad \Pi = \frac{4\eta}{3\tau} \qquad \qquad \partial_{\tau} \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau} + \frac{4\eta}{3\tau^2} \qquad \qquad \Pi = -c_0 \mathcal{L}_1
$$

 $T_{\rm{final}}$  . Choward (causality, instability). The improvement due to Israel Steward consists in writing and  $T_{\rm{final}}$  $\epsilon$  that for motion for motion for  $\epsilon$  is fixed by a new parameter called  $\epsilon$ . We have in leading  $\epsilon$  is fixed by a new parameter called  $\epsilon$ . We have in leading  $\epsilon$ Muelle<mark>r-Israel-Steward</mark>  $\epsilon$  that for  $\epsilon$  that forces  $\epsilon$  to relax. The rate is fixed by a new parameter called  $\epsilon$ . We have in leading  $\epsilon$ .|إ .<br>د ه Steward  $\mathcal{L}$ 

$$
\partial_{\tau} \Pi = -\frac{\Pi}{\tau_{\pi}} + \frac{4\eta}{3\tau \tau_{\pi}} = -\frac{1}{\tau_{\pi}} \left( \Pi - \frac{4\eta}{3\tau} \right)
$$

**Second order hydro (DNMR)** and the second order that the second ord Note that is ⌧⇡ is chosen very small, then ⇧ relaxes rapidly to the leading order value ⇧ = 4⌘*/*3⌧ *.*

$$
\partial_{\tau} \Pi = \frac{4}{3} \frac{\eta}{\tau \tau_{\pi}} - \beta_{\pi \pi} \frac{\Pi}{\tau} - \frac{\Pi}{\tau_{\pi}} \qquad \beta_{\pi \pi} = \frac{38}{21} = a_1 \qquad \tau_{\pi} = \tau_R
$$
  
\n**same as**  $\frac{\partial \mathcal{L}_1}{\partial \tau} = -b_1 \frac{\epsilon}{\tau} - a_1 \frac{\mathcal{L}_1}{\tau} - \frac{\mathcal{L}_1}{\tau_R} \qquad \text{provided} \qquad \frac{4}{3} \frac{\eta}{\tau \tau_{\pi}} = c_0 b_1 \frac{\epsilon}{\tau}$   
\nwhich holds in leading order if  $\tau_{\pi} = \tau_R$ 

 $f_{\text{nondom}}$   $f_{\text{nondom}}$ Following **[**1709.0664]  $\sum_{i=1}^{n}$ Similar analysis can be made for BRSSS hydro (full second order, conformal), or third equation of motion for motion for motion for rate is fixed by a new parameter called  $\alpha$ <sup>21</sup> <sup>=</sup> *<sup>a</sup>*1*,* ⌧⇡ <sup>=</sup> ⌧*R,* (381) 3 *,* ⌧⇡ <sup>=</sup> <sup>6</sup> 0 de for RDSSS hydro (full second of order (Jaiswal).  $5$ SS hyd ✏ ro (full seco nd order, conformal), or third which, after multiplication by  $c_0$  reads of  $c_0$ These results are to be compared with our equation for *L*<sup>1</sup> @*L*<sup>1</sup> ✏ : made fc ✏ ⌧ *<sup>a</sup>*<sup>1</sup> SS hy Similar analysis can be made for BRSSS hydro (full second order, conformal), or third **order (Jaiswal).** 

Niemi, Molr 3 r, Risc enicol, Niemi, Molnar, Rischke <mark>(2</mark>0 . (380). (38 e<mark>nicol, N</mark> iemi, Molnai *,* (383) @⌧⇧ = *c*0*b*<sup>1</sup> ✏ which, after multiplication by *c*<sup>0</sup> reads [DNMR= Denicol, Niemi, Molnar, Rischke (2012)]

tschke, Son, Starinets,  $\overline{a}$ ⌘ ‴, , tarine<sup>.</sup> s, Stepl *.* (380) where it would be while in MIS  $\frac{1}{2}$ *,*  $\frac{1}{2}$ which, and **c a**<br>200 ✏ ⇧  $\frac{1}{2}$ [BRSSS= Baier, Romatschke, Son, Starinets, Stephanov (2008)]

## **Conclusions**

In high energy collisions, the longitudinal expansion prevents the system to reach full isotropy in a short time (expansion plays a role somewhat similar to a conservation law…)

However strong anisotropy does not hinder the emergence of (viscous) hydrodynamic behavior

A simple picture based on special set of moments of the distribution functions provides much insight into the mathematical structure of viscous hydrodynamics of expanding (boost invariant) systems

Strong reduction of the viscosity at early times due to out of equilibrium effects (coupling to higher moments)

Coupled equations for the first few (two) moments could be a convenient alternative to viscous hydrodynamics