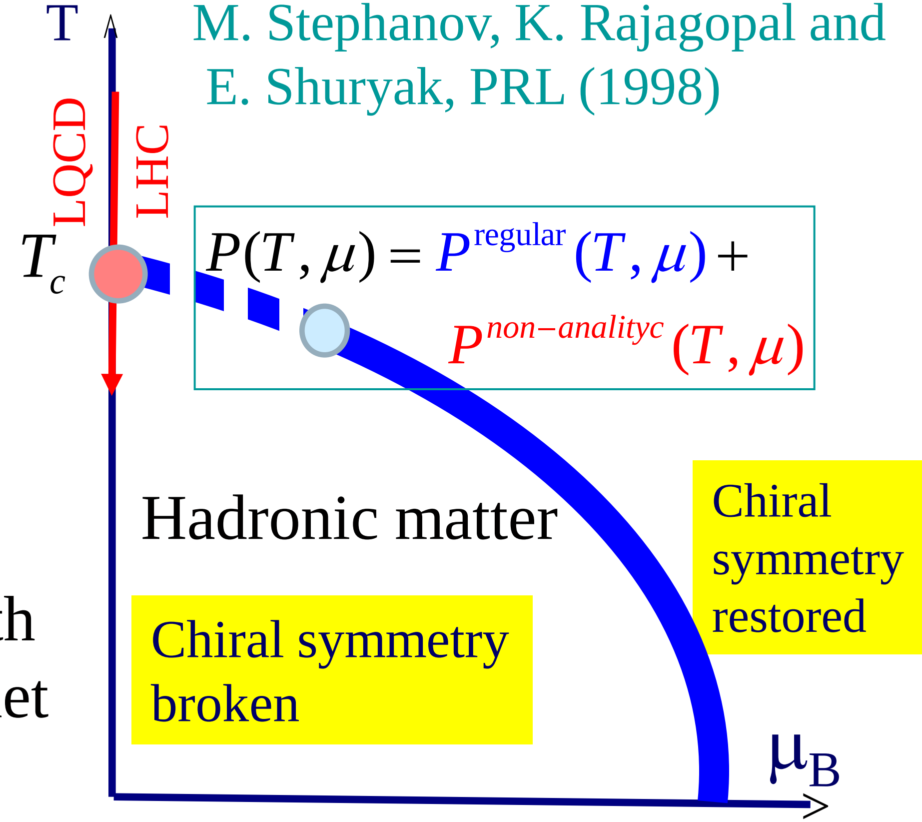
Chiral criticality from fluctuations of baryon density and its Fourier expansion coefficient

Krzysztof Redlich (Uni Wroclaw)

- Modelling regular part of pressure in hadronic phase: S-matrix approach:
  - charge-baryon correlations in LQCD
- Fluctuations of net-baryon charge:
  - probing chiral criticality systematics:

    FRG-PNJL model versus STAR data
  - decoding phase structure of QCD with a Fourier expansion coefficients of net baryon density



in collaboration with: Gabor Almasi, Bengt Friman, Pok Man Lo, Kenji Morita, Chihiro Sasaki: Anton Andronic, Peter Braun-Munzinger, Johanna Stachel

Statistical operator of HRG provides good approximation of QCD thermodynamics in hadronic phase

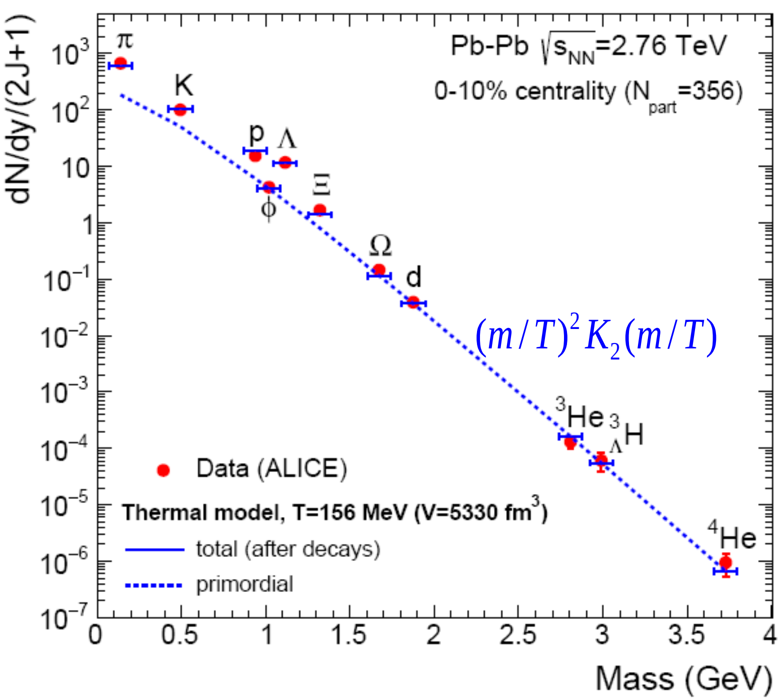
Hadron Resonace Gas (HRG):

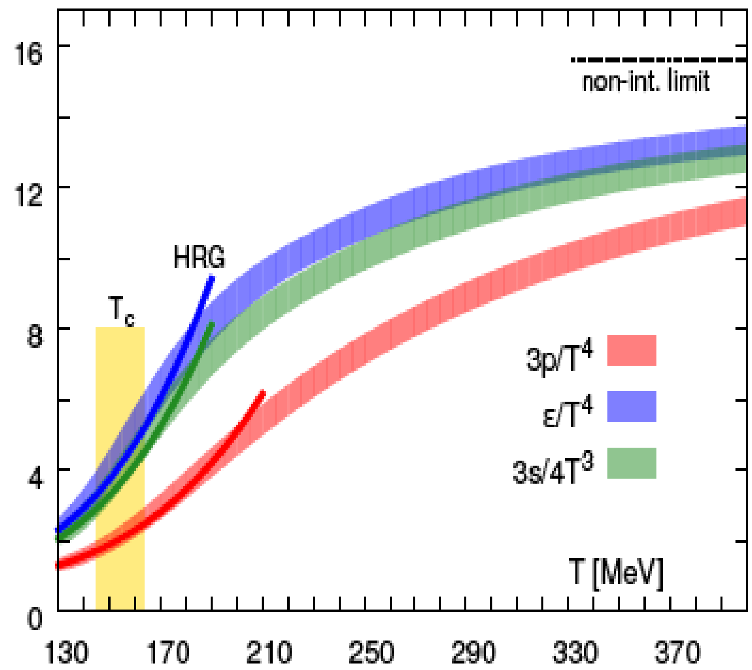
$$P^{regular}(T,\vec{\mu}) = \sum_H P_H^{id} + \sum_R P_R^{id}$$

- Good description of particle yields data and EqS from LQCD

A. Andronic, P. Braun-Munzinger, J. Stachel & K.R. Nature, (2018)

A. Bazavov et al. HotQCD Coll. Phys.Rev. D90 (2014) 094503





See also: P. Alba et al. arXiv:1711.05207

- HRG provides 1st approximation of QCD free energy in hadronic phase,

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

A. Asakawa at. al.

S. Ejiri et al.,…

M. Stephanov et al.,

K. Rajagopal,

E. Schuryak

B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &

S. Mukherjee et al.,

C. Ratti et al.

P. Braun-Munzinger et al.

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2}=\frac{\partial^2(\hat{P})}{\partial(\hat{\mu}_N)^2} \qquad \frac{\chi_{NM}}{T^2}=\frac{\partial^2(\hat{P})}{\partial\hat{\mu}_N\partial\hat{\mu}_M}$$

$$N=N_q-N_{-q},\quad N,M=(B,S,Q),\quad \hat{\mu}=\mu/T,\quad \hat{P}=P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2}=\frac{1}{VT^3}(<N^2>-<N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n>=\sum_N N^n\mathrm{P}(N)$$

Consider special case:

$$< N_q > \equiv N_q \quad =>$$

Charge carrying by particles $q = \pm 1$

- Charge and anti-charge uncorrelated and Poisson distributed, then
- $\mathrm{P}(N)$ the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{N_q}{N_{-q}} \right)^{N/2} I_N(2\sqrt{N_q N_{-q}}) \exp[-(N_q + N_{-q})]$$

- Then, the susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

expressed by yields of particles and antiparticles carrying the conserved charge |q|.

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

- The probability distribution

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$$< S_{-q} > \equiv S_{-q}$$

$$q = \pm 1, \pm 2, \pm 3$$

$$P(S) = \left(\frac{S_1}{S_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}\left(S_n + S_{\bar{n}}\right)\right]$$

$$\sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(\frac{S_3}{S_{\bar{3}}}\right)^{\frac{k}{2}} I_k\left(2\sqrt{S_3 S_{\bar{3}}}\right) \left(\frac{S_2}{S_{\bar{2}}}\right)^{\frac{i}{2}} I_i\left(2\sqrt{S_2 S_{\bar{2}}}\right)$$

$$\left(\frac{S_1}{S_{\bar{1}}}\right)^{-i-\frac{3k}{2}} I_{2i+3k-S}\left(2\sqrt{S_1 S_{\bar{1}}}\right)$$

Fluctuations

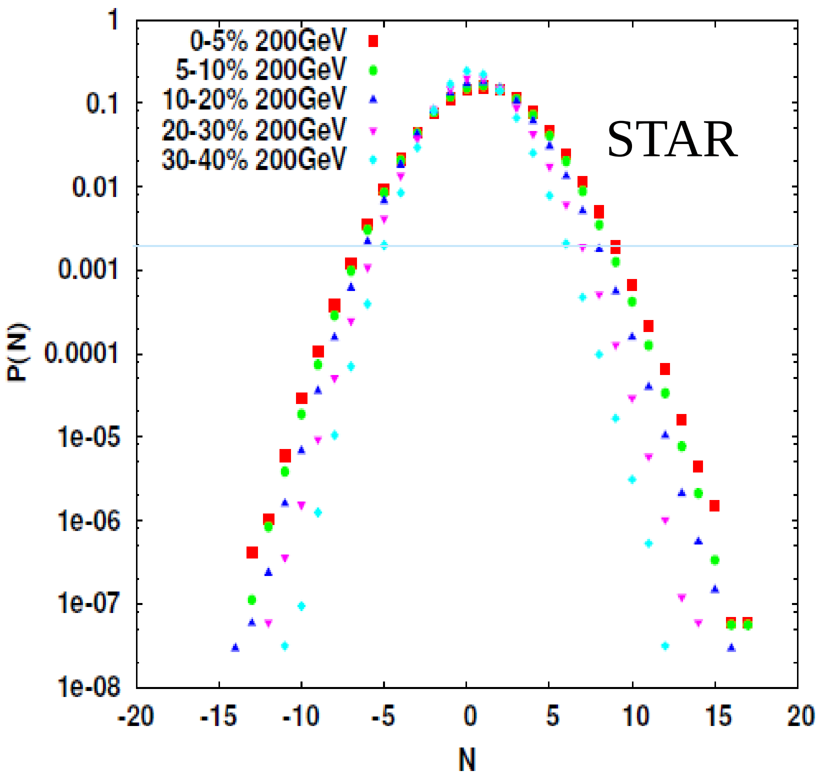
$$\boxed{\frac{\chi_S}{T^2} = \frac{1}{VT^3} \sum_{n=1}^{|q|} n^2 \left(\langle S_n \rangle + \langle S_{-n} \rangle\right)}$$

Correlations

$$\boxed{\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3} \sum_{m=-q_M}^{q_M} \sum_{n=-q_N}^{q_N} nm \langle S_{n,m} \rangle}$$

$\langle S_{n,m} \rangle$ is the mean number of particles carrying charge $N = n$ and $M = m$

Variance at 200 GeV AA central coll. at RHIC

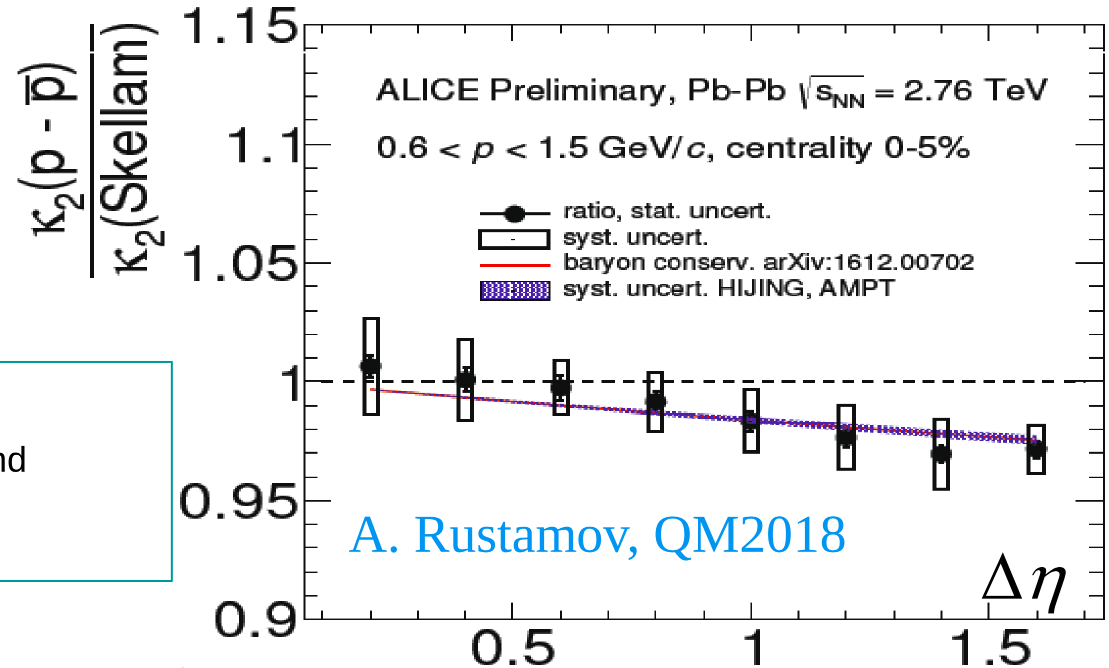


STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\frac{< p > + < \bar{p} >}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- ALICE data consistent with Skellam $\Delta\eta < 1$



- Skellam distribution is a good approximation to calculate the 2nd order charge fluctuations in HIC

The influence of baryon number conservation: P. Braun-Munzinger, A. Rustamov, J. Stachel. Nucl Phys. A960 (2017) 114

Variance at 200 GeV AA central coll. at RHIC

K. Morita, B. Friman and K.R.
Phys.Lett. B741 (2015) 178

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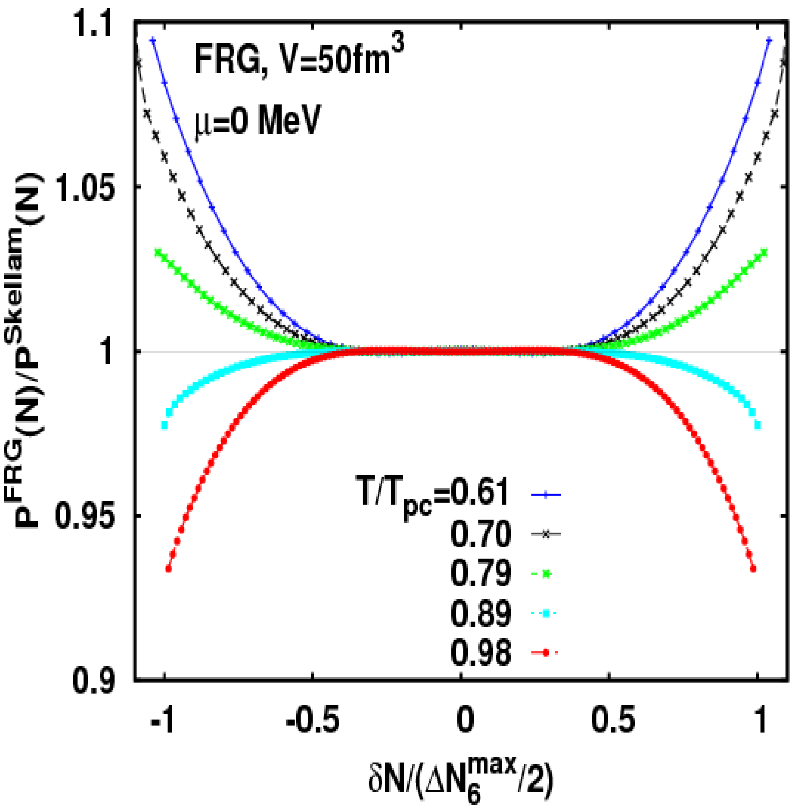

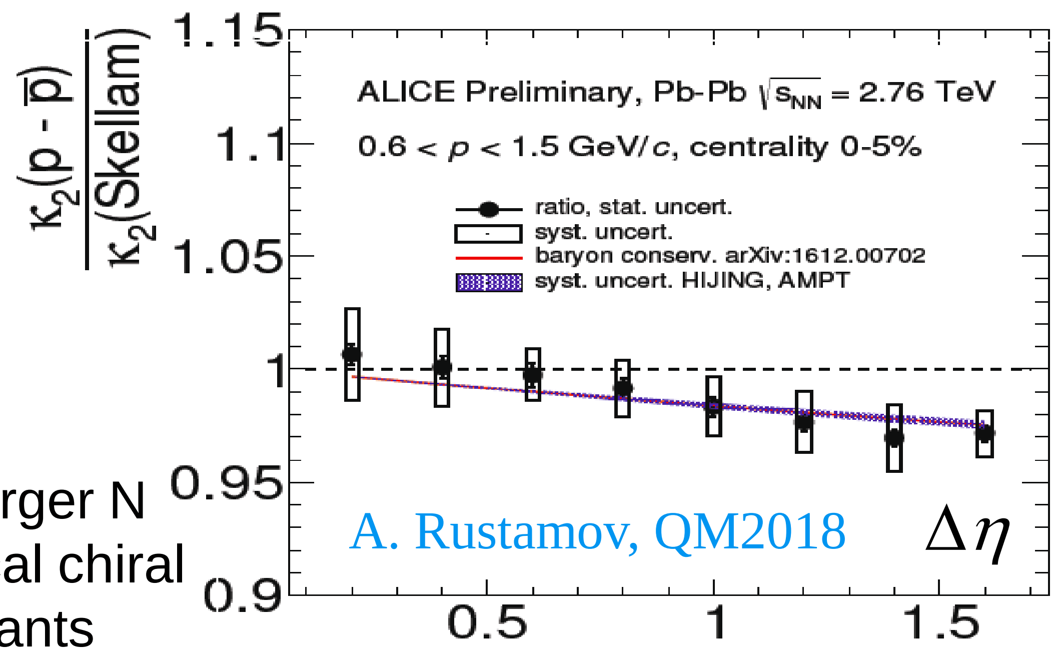

- Shrinking of Skellam distr. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

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K. Morita, B. Friman and K.R. Phys.Lett. B741 (2015) 178

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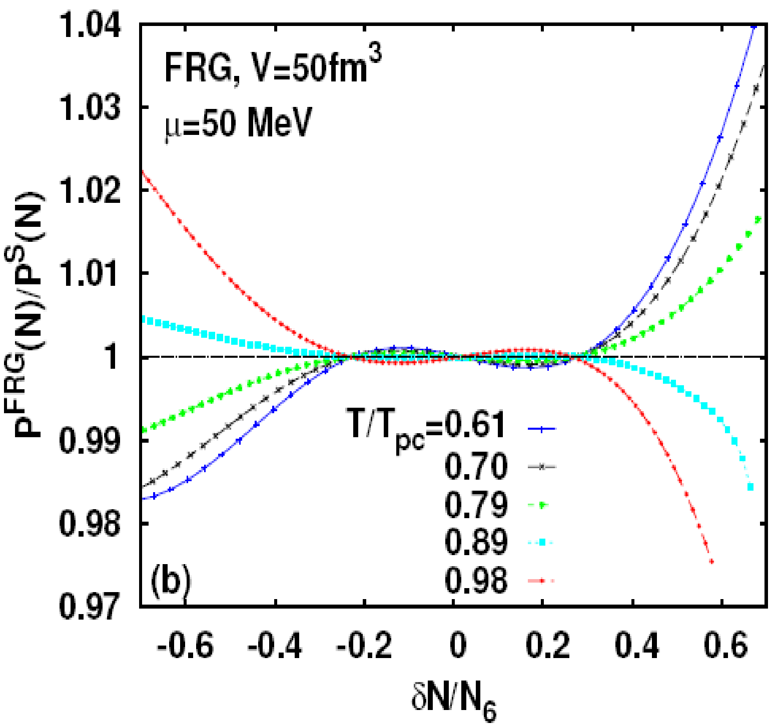

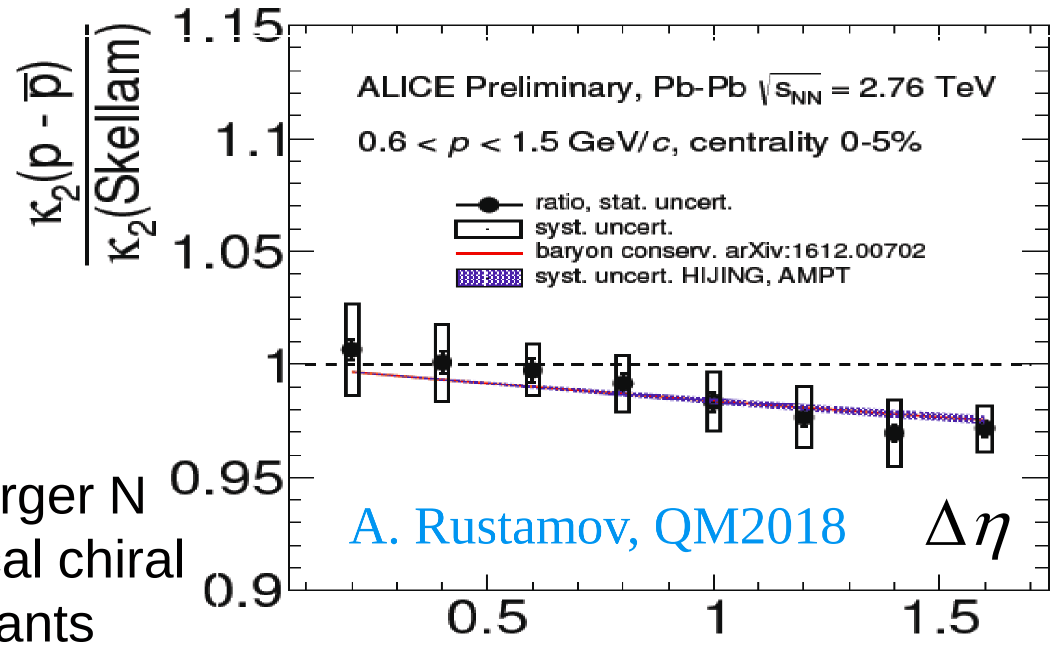

A. Rustamov, QM2018

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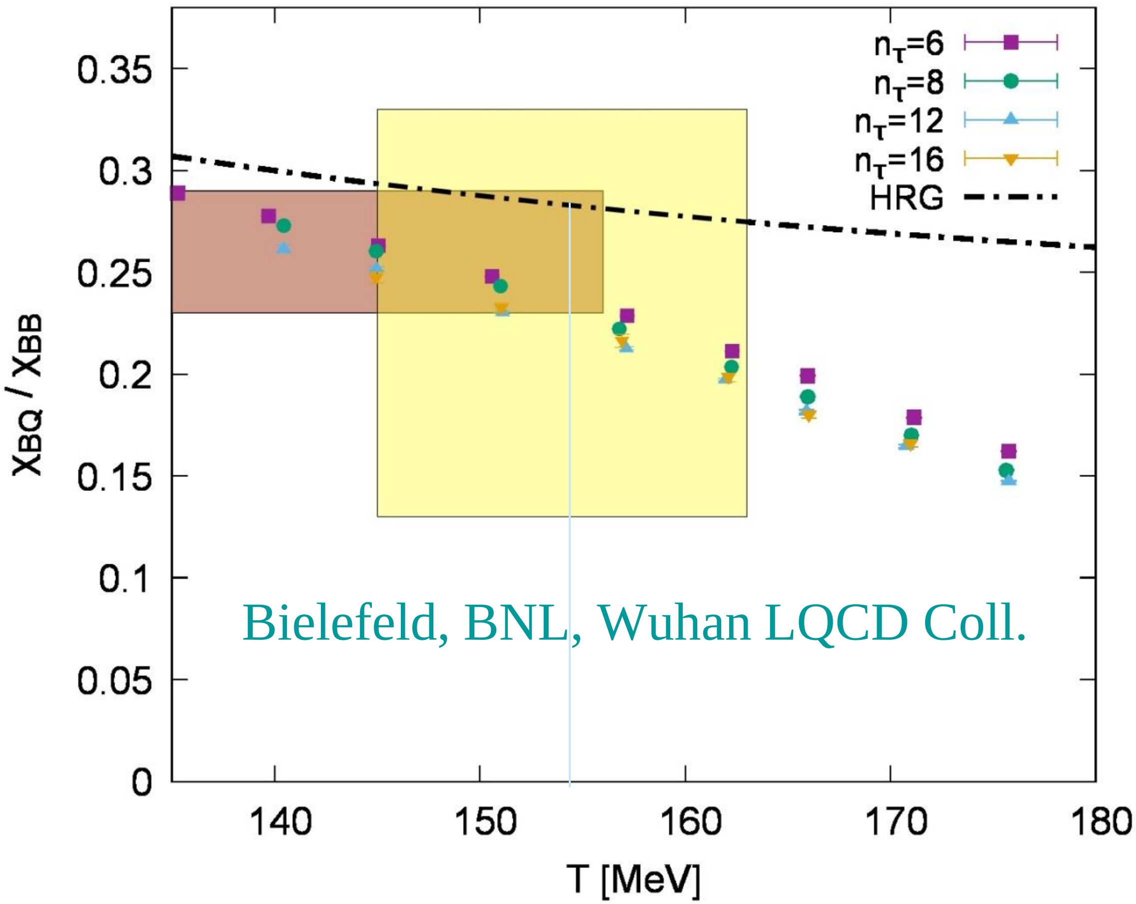
Constraining the upper value of the chemical freeze-out temperature at the LHC



$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|)/2$$

Isospin symmetry

- Considering the ratio

$$\frac{<(\delta B)(\delta Q)>}{<(\delta B)^2>} = \frac{\chi_{BQ}}{\chi_B}$$

- From the comparison of 2nd order fluctuations and correlations in the charge baryon sector in HRG stronger then in LQCD

Implementing interactions via resonance contribution can be too crude assumption

HRG in the S-MATRIX APPROACH

Pressure of an interacting, $\pi + N \Leftrightarrow \pi + N$ hadron gas in an equilibrium

$$\boxed{P(T) \approx P_{\pi}^{id} + P_{N}^{id} + P_{\pi N}^{\text{int}}}$$

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P^{\text{int}} = \sum_{I,j} \int_{m_{th}}^{\infty} dM \ B_j^I(M) P^{id}(T, M)$$

$$\Downarrow$$

$$B_j^I(M) = \frac{1}{\pi} \frac{d}{dM} \delta_j^I(M)$$

$$\downarrow \qquad\qquad \downarrow$$

Effective weight function *Scattering phase shift*

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187, 345 (1969)
R.Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718.
W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

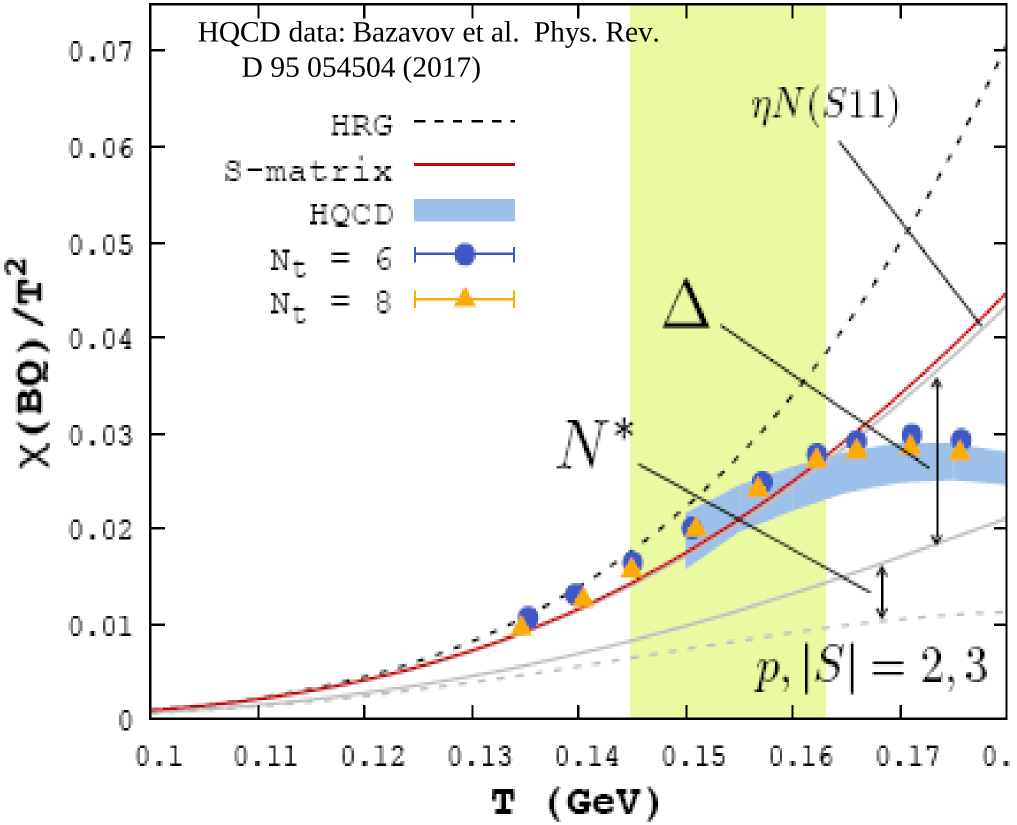
- Interactions driven by narrow resonance of mass M_R

$$B(M) = \delta(M^2 - M_R^2) \quad \Rightarrow \quad P^{\text{int}} = P^{id}(T, M_R) \ \Rightarrow HRG$$

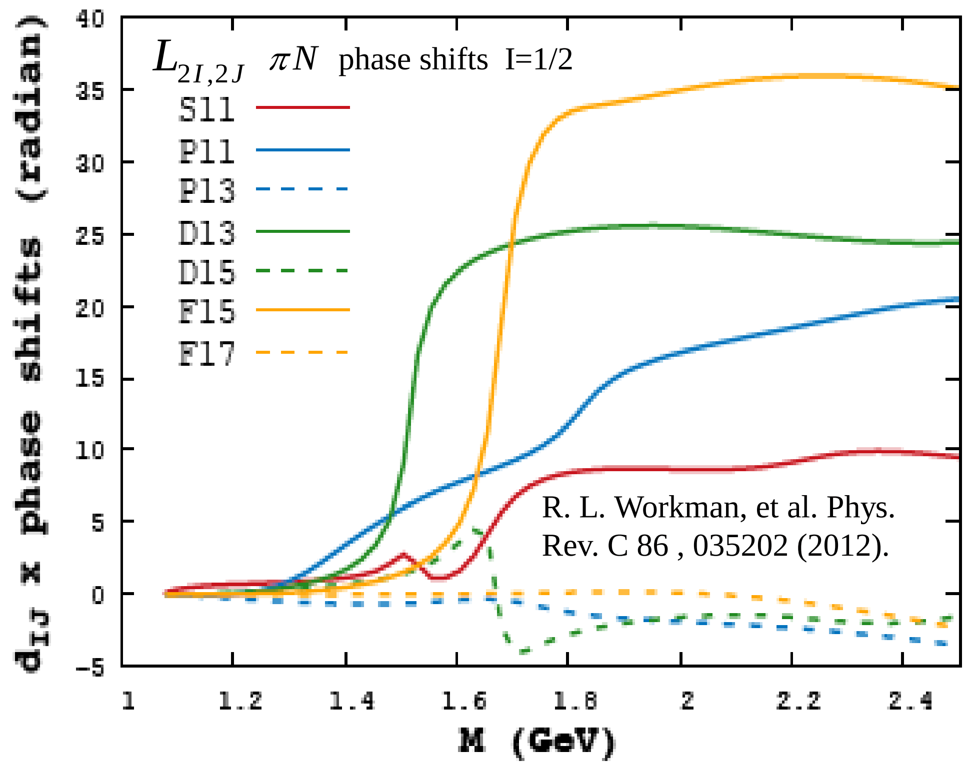
- For non-resonance interactions or for broad resonances the HRG is too crude approximation and $P^{\text{int}}(T)$ should be linked to the phase shifts

Probing non-strange baryon sector in πN- system

Pok Man Lo, B. Friman, C. Sasaki & K.R. , Phys.Lett. B778 (2018)



$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|)/2$$

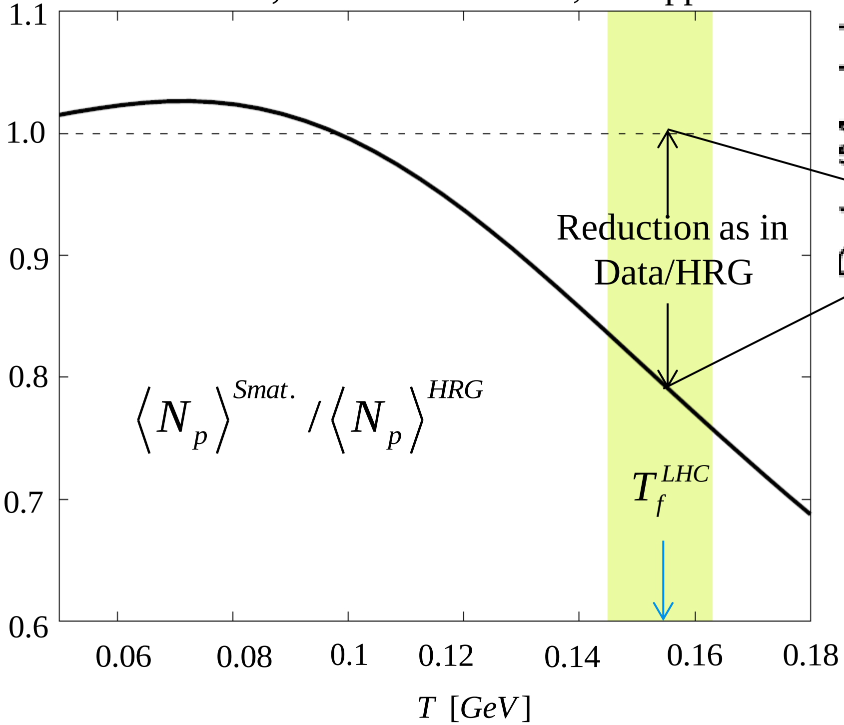


$$\Delta\chi_{BQ} \approx \sum_{I_z, j, B} d_j BQ \int dM \int d^3p \frac{1}{T} \frac{d\delta_j^I}{dM} \times e^{-\beta\sqrt{p^2+M^2}} (1+e^{-\beta\sqrt{p^2+M^2}})^{-2}$$

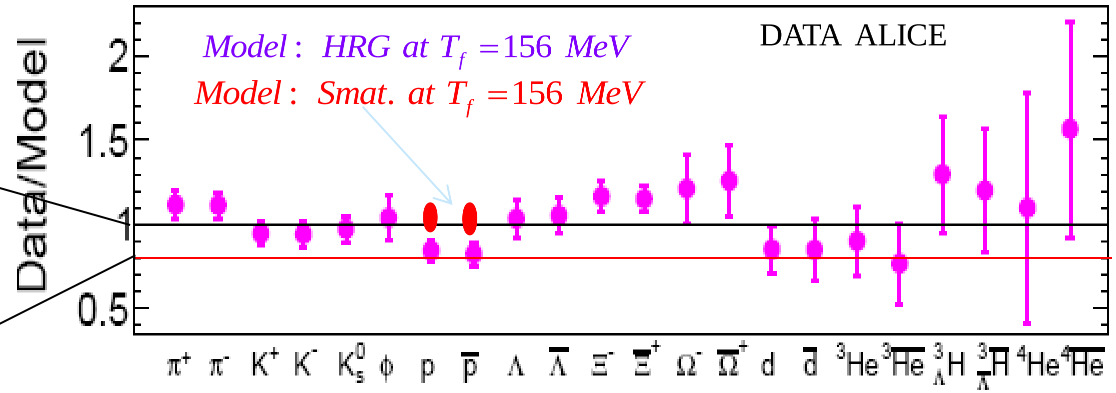
- Considering contributions of all πN $\delta_j^{I=(1/2),\ (3/2)}$ (N^*, Δ^* resonances) to χ_{BQ} within S-matrix approach, reduces the HRG predictions towards the LQCD in the chiral crossover $0.15 < T < 0.16\ GeV$

Phenomenological consequences: proton production yields

A. Andronic, P. Braun-Munzinger, B. Friman, Pok Man Lo, J. Stachel & K.R., too appear.

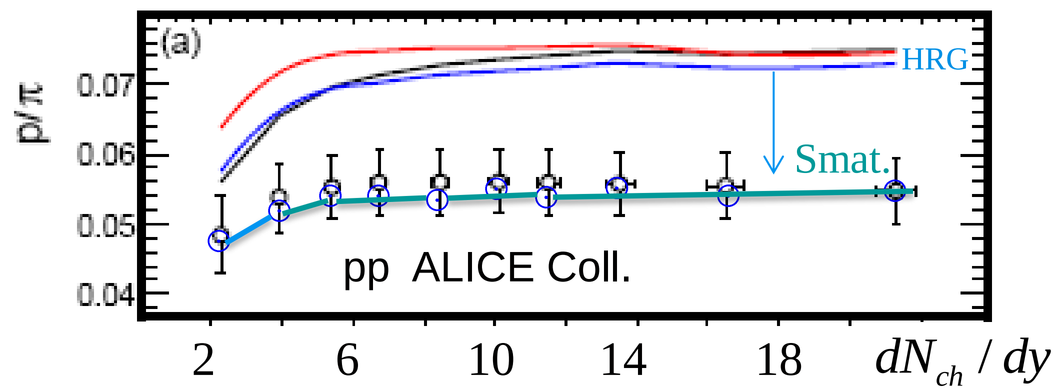


HRG: A. Andronic, P. Braun-Munzinger, J. Stachel & K.R.



- Yields of protons in AA collisions at LHC is consistent with S-matrix result within 1σ

HRG: N. Sharma, J. Cleymans, B. Hippolite, arXiv: 1803.05409



- Yields of protons in the S-matrix is suppressed relative to HRG

For further consequences of smat. See also: P. Huovinen, P. Petreczky Phys. Lett. B77 (2018) P. Huovinen, poster QM2018

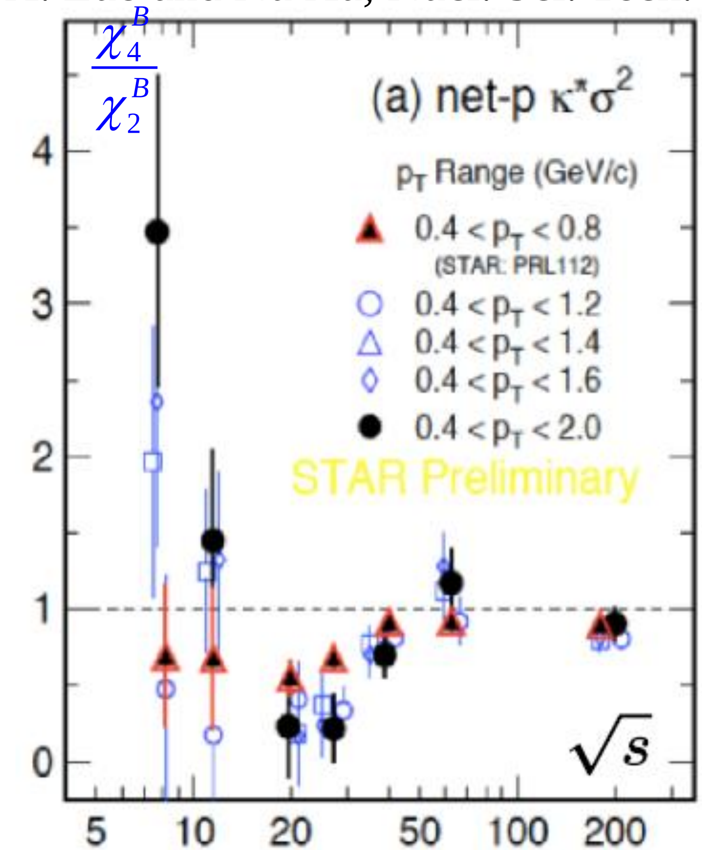
- S-matrix results well consistent with pp data

Net-baryon fluctuations as a probe of chiral criticality

G. Almasi, B. Friman & K.R, Phys. Rev. D96 (2017) 014027

X. Luo et al. (2015), STAR Coll.

X. Luo and Nu Xu, Nucl. Sci. Tech. (17)



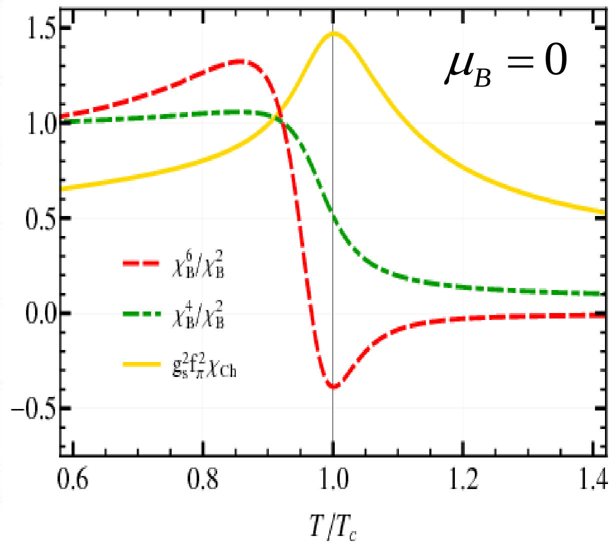
- An excellent observable of chiral criticality

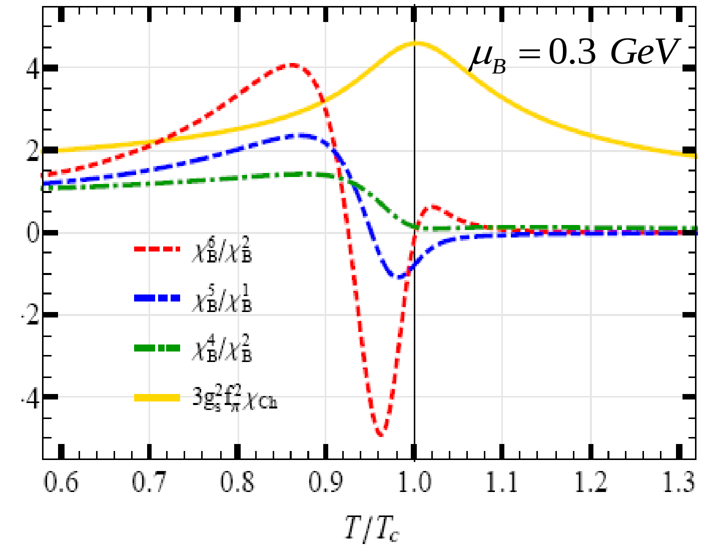
  S. Ejiri, F. Karsch & K.R. Phys.Lett. B633 (2006) 275

  S. Ejiri et al., Nucl.Phys.Proc.Suppl. 140 (2005) 505 , Phys.Rev. D71 (2005) 054508

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} \quad \text{and} \quad R^{n,m} = \frac{\chi_n^B}{\chi_m^B}$$

- Modelling chiral properties of QCD in PNJL model within FRG approach.



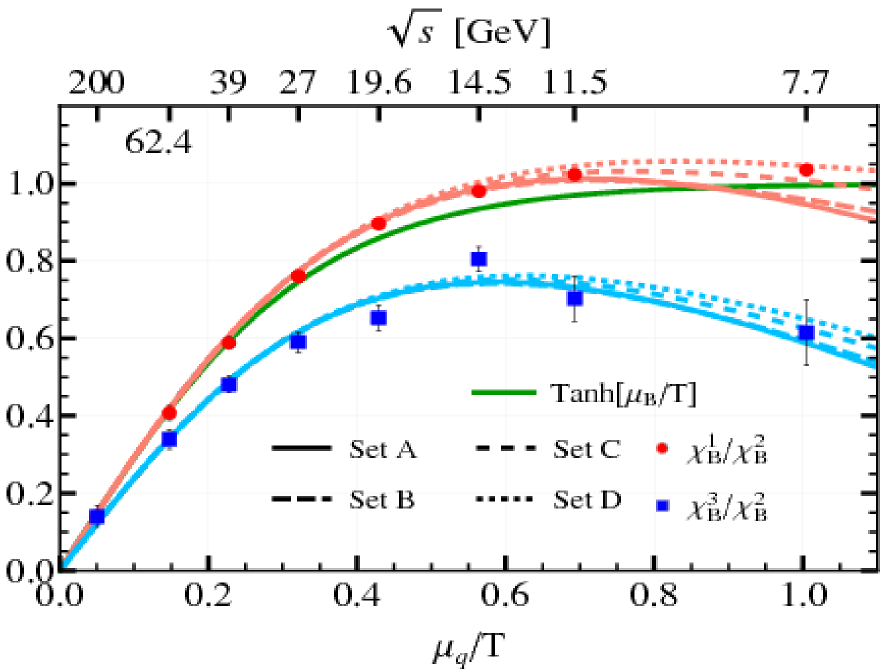


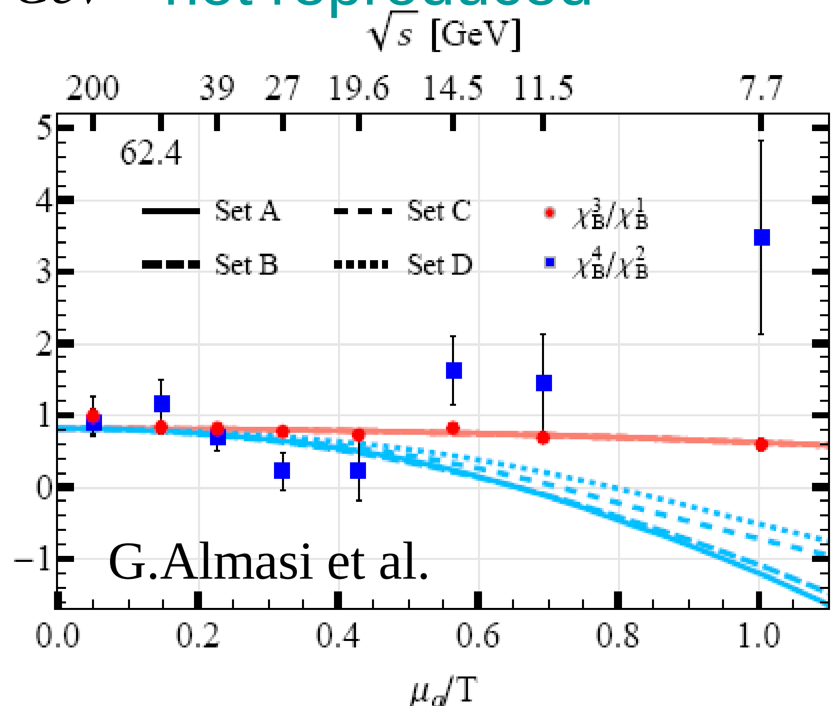
Are the above deviations an indication of the chiral criticality and the existence of the CEP?

Consider systematics of $R^{n,m}$ in relation to STAR data

Self - consistent freeze-out and STAR data

- Freeze-out line in $(T,\mu)-$ plain is fixed by χ_B^3/χ_B^1 to data
- Ratio $\chi_B^1/\chi_B^2 \approx \tanh(\mu/T)$ => further evidence of equilibrium and thermalisation at $7\ GeV \le \sqrt{s} < 5\ TeV$
- Ratio $\chi_B^1/\chi_B^2 \ne \chi_B^3/\chi_B^2$ expected due to critical chiral dynamics
- Enhancement of χ_B^4/χ_B^2 at $\sqrt{s} < 20\ GeV$ not reproduced

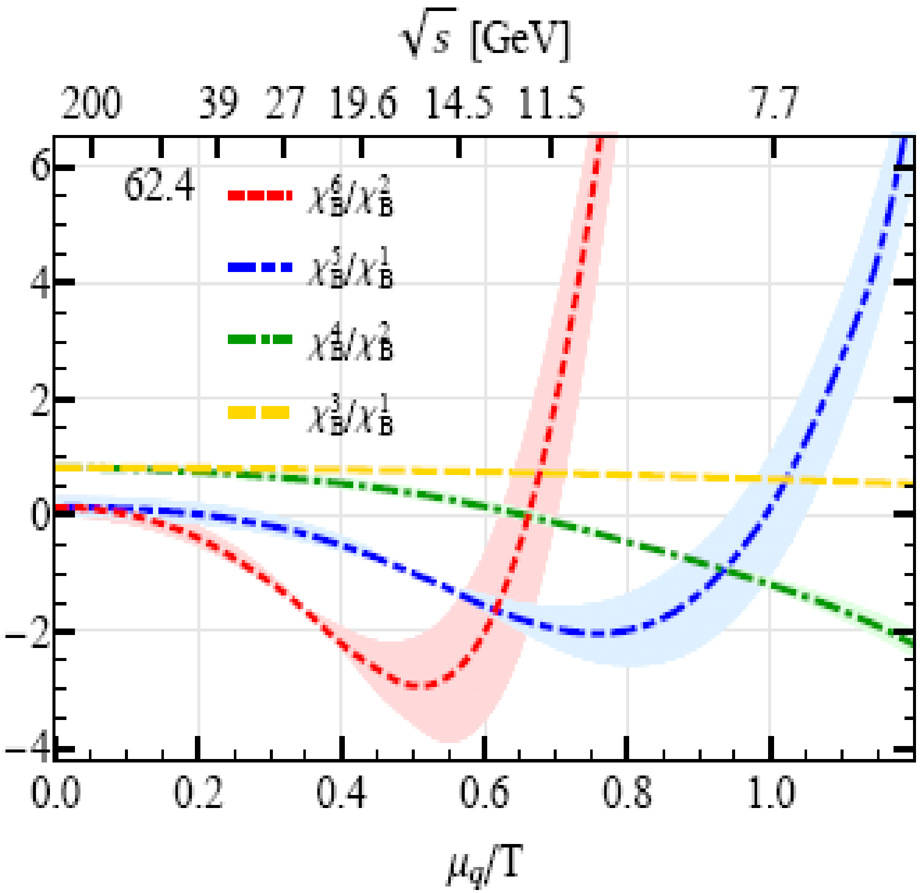




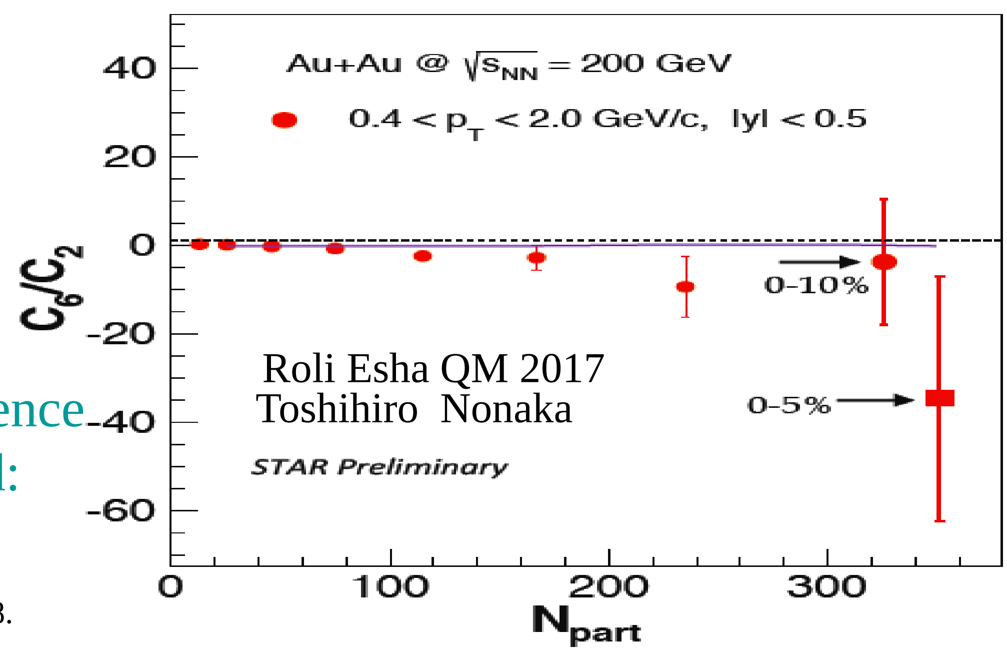
Similar conclusions as in the previous comparison of LQCD results with STAR data:

Frithjof Karsch J. Phys. Conf. Ser. 779, 012015 (2017)

Higher order cumulants - energy dependence



- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC
- At freeze-out , the ratio $\chi_B^6 / \chi_B^2 \approx 0$ in agreement with preliminary STAR data, albeit within still very large error



However, to make final conclusions the influence of non-critical fluctuations must be analyzed:

See e.g. P. Braun-Munzinger, A. Rustamov and J. Stachel Nucl. Phys. A 960, 114 (2017),
A. Bzdak, V. Koch, V. Skokov, Eur.Phys.J. C77 (2017) 288.

Fourier coefficients of $\chi_B^1(T,\mu)$ and chiral criticality

G. Almasi, B. Friman, P.M. Lo, K. Morita & K.R. arXiv: **1805.04441**

- Considering the Fourier series expansion* of baryon density

$$\chi_B^1(T,\mu) = \sum_{k=1}^{\infty} b_k(T)\sinh(k\mu) \qquad \text{with} \qquad \boxed{b_k(T) = \frac{2}{\pi}\int_0^{\pi} d\theta[\mathrm{Im}\,\chi_B^1(T,i\theta)]\sin(k\theta)}$$

and $\mu = (\mu/T)$, $\theta = \mathrm{Im}\,\mu$

- At $\mu = 0$,the susceptibility $\chi_B^n(T)$ expressed by Fourier coefficients

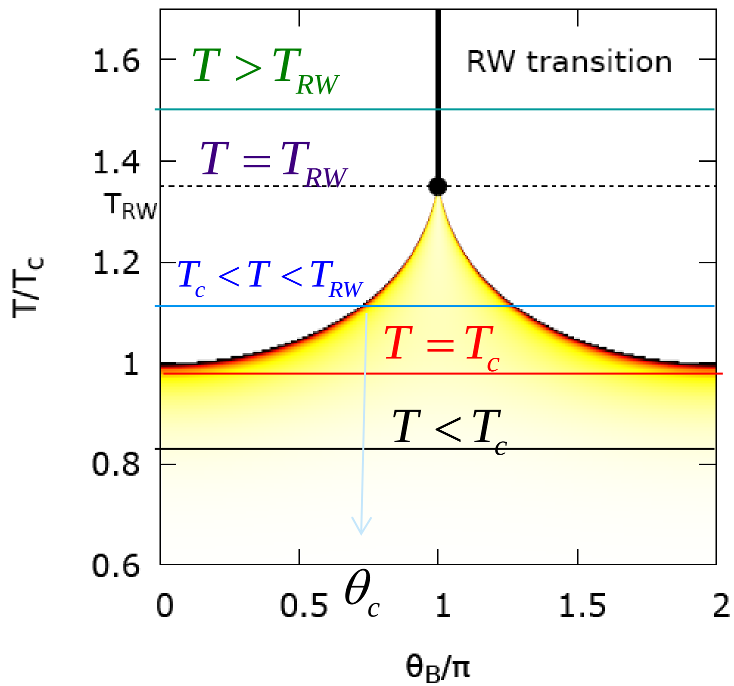
$$\chi_B^n(T,\mu) = \sum_{k=1}^{\infty} b_k(T)\frac{\partial^{n-1}}{\partial\mu^{n-1}}\sinh(k\mu), \qquad \text{thus} \qquad \boxed{\chi_B^n(T,\mu=0) = \sum_{k=1}^{\infty} k^{2n-1} b_k(T)}$$

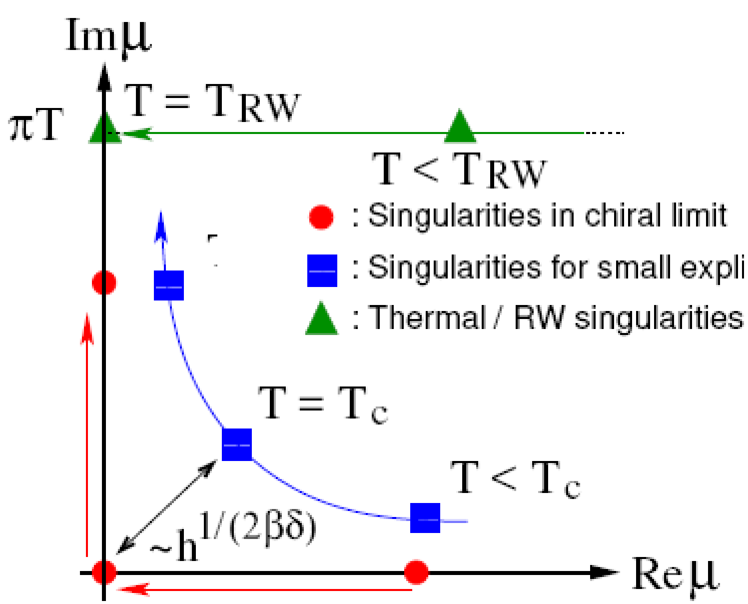
- Since $b_k(T)$ are carrying information on chiral criticality, thus their $T-$ and $k-$ dependence must inform about phase transition

* The first four $b_k(T)$ obtained recently in LQCD: V. Vovchenko, A. Pasztor, Z. Fodor, S. D. Katz, and H. Stoecker, Phys. Lett. B **775**, 71 (2017).

* see also K. Kashiwa and A. Ohnishi (2017) hep-1712.06220, for $b_k(T)$ properties related with deconfinement transition

Chiral limit: scaling of $b_k(T)$





- $T < T_c$, $P(T,\mu)$ dominated by

$P(T,\mu) \approx f(m)\cosh(\mu)$, thus

exponential damping $\Rightarrow$ $b_k(T) \approx (-1)^{k+1} \dfrac{e^{-km}}{k^{3/2}}$

- $T = T_c$, $P(T,\mu)$ dominated by $P^{non-analytic}$

$$\chi_1^B \approx \theta \left| \frac{T-T_c}{T_c} - \kappa\theta^2 \right|^{1-\alpha} \Rightarrow b_k(T_c) \approx k^{2\alpha-4}$$

- $T_c < T < T_{RW}$

$\Rightarrow$ $b_k(T > T_c) \approx k^{\alpha-2} \sin(k\theta_c - \alpha\pi/2)$

- $T = T_{RW}$, $P^{\sin gular}$ in Z(2) universality class

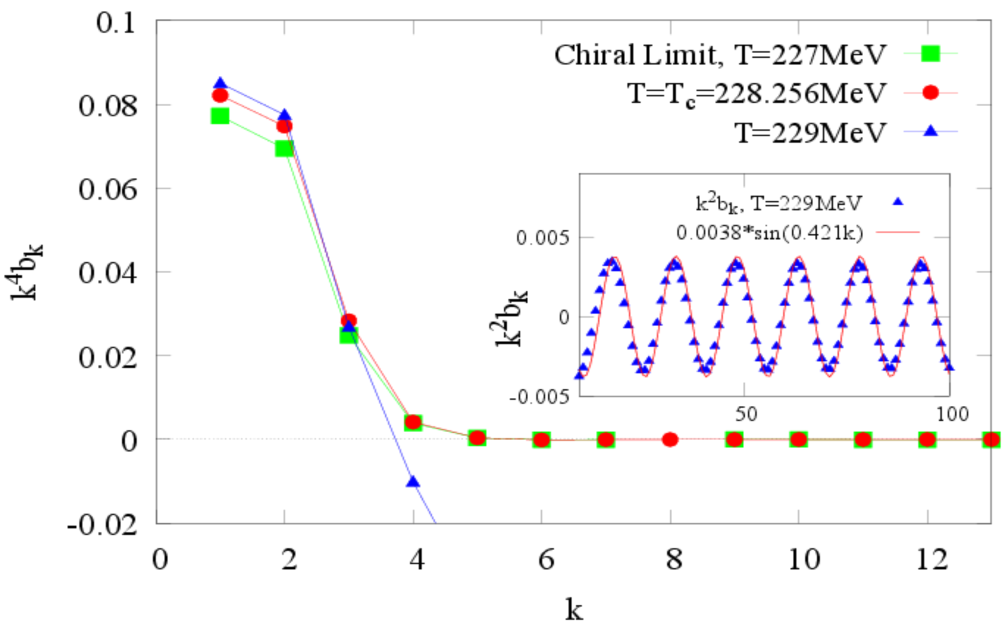
$$\chi_1^B \approx (\pi - \theta_B)^{1/\delta} \Rightarrow b_k(T_{RW}) \approx (-1)^{k-1} k^{-1-1/\delta}$$

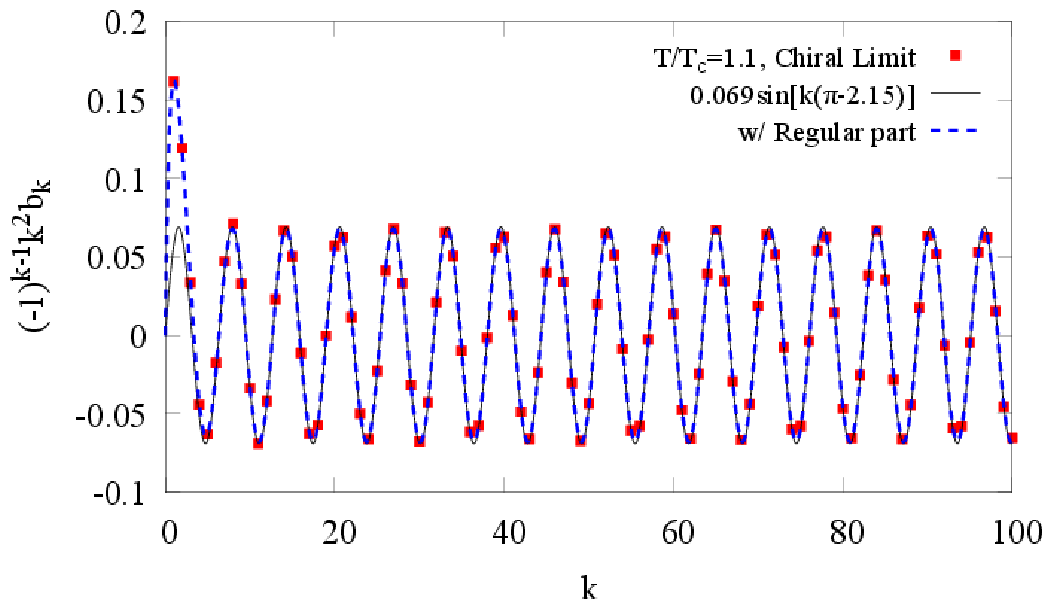
- $T > T_{RW}$, 1st order transition at $\theta = \pi$.

$\Rightarrow$ $b_k(T) \approx (-1)^{k-1} k^{-1}$

Scaling of Fourier coefficients: PNJL MF-results

- In the chiral limit, i.e. $m_\pi = 0$, the phase transition is signaled by oscillations of $b_k(T)$ just above T_c

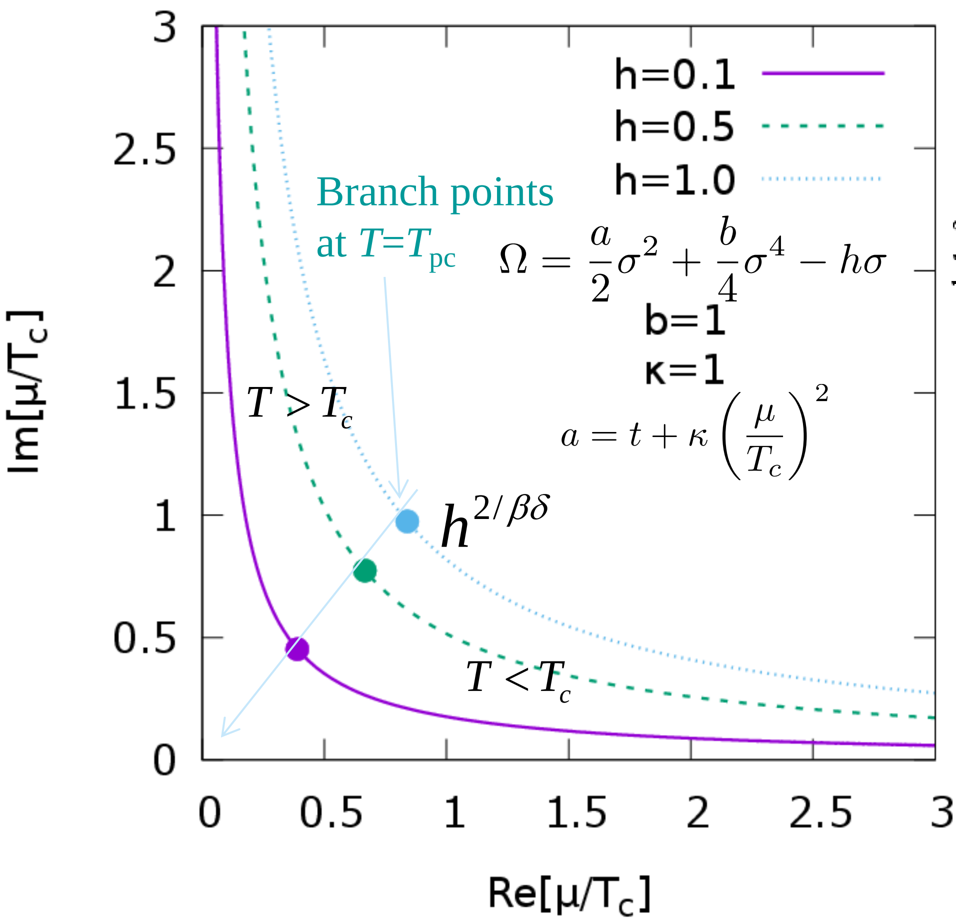


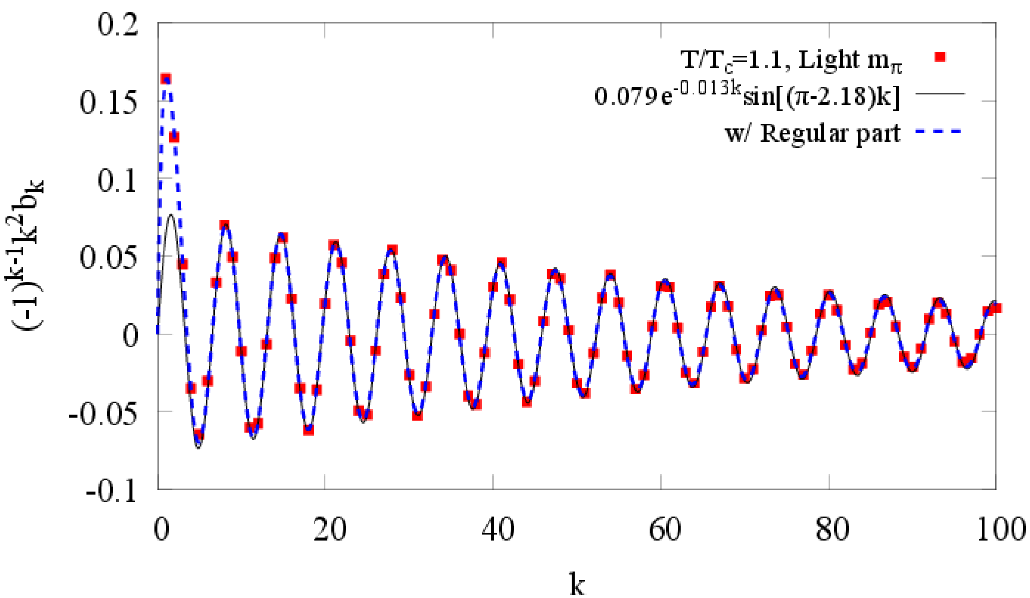


$$b_k(T > T_c) = A(T)\ k^{\alpha-2} \sin(k\theta_c - \alpha\pi/2)$$

Fourier coefficient at finite pion mass

- For $h \neq 0$ the singularity moves to the complex chemical potential plain resulting in an additional dumping of oscillations



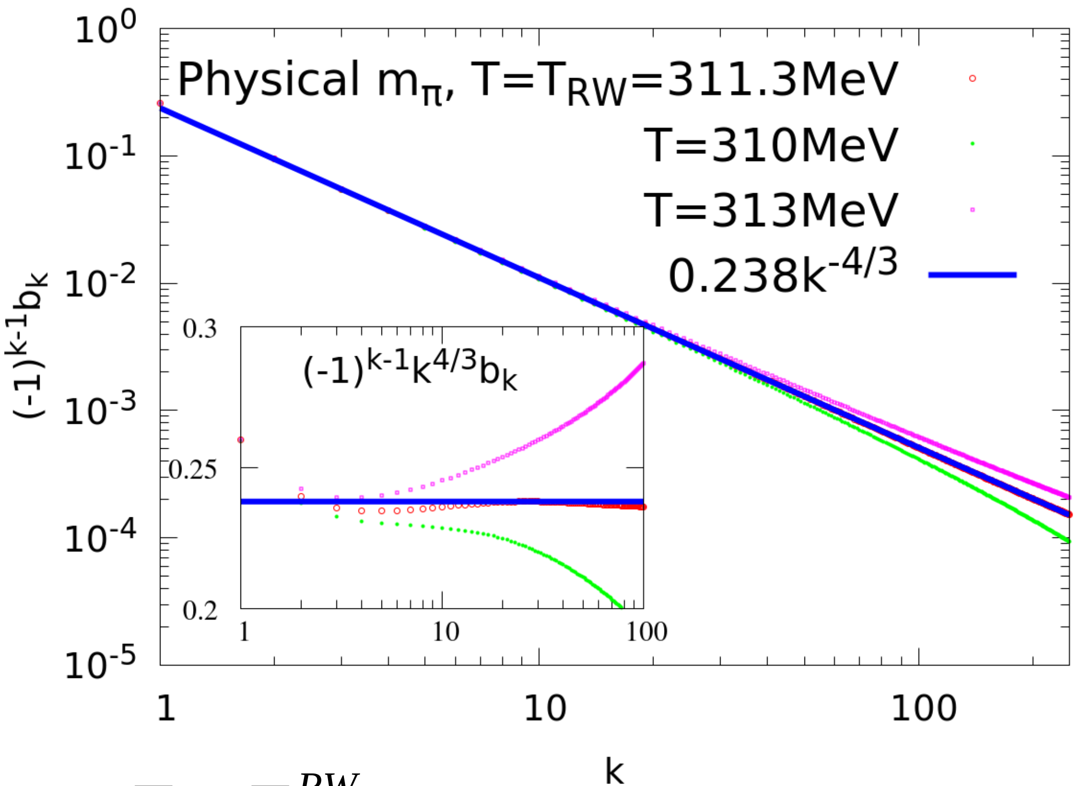


$$b_k \simeq k^{-2} e^{-k\,\mathrm{Re}\,\mu_c(m_\pi,T)} \sin(k\theta_c)$$

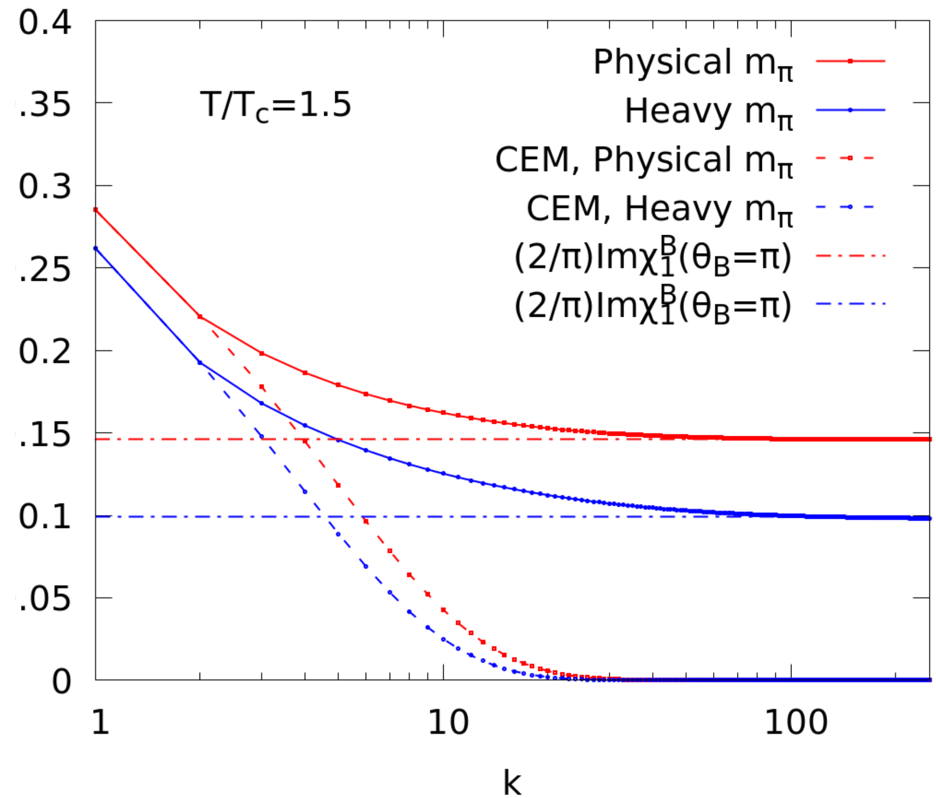
Scaling of Fourier coefficients: PNJL MF-results

K. Morita et al.

Scaling at the Roberge-Weiss transition



Beyond the Roberge-Weiss transition



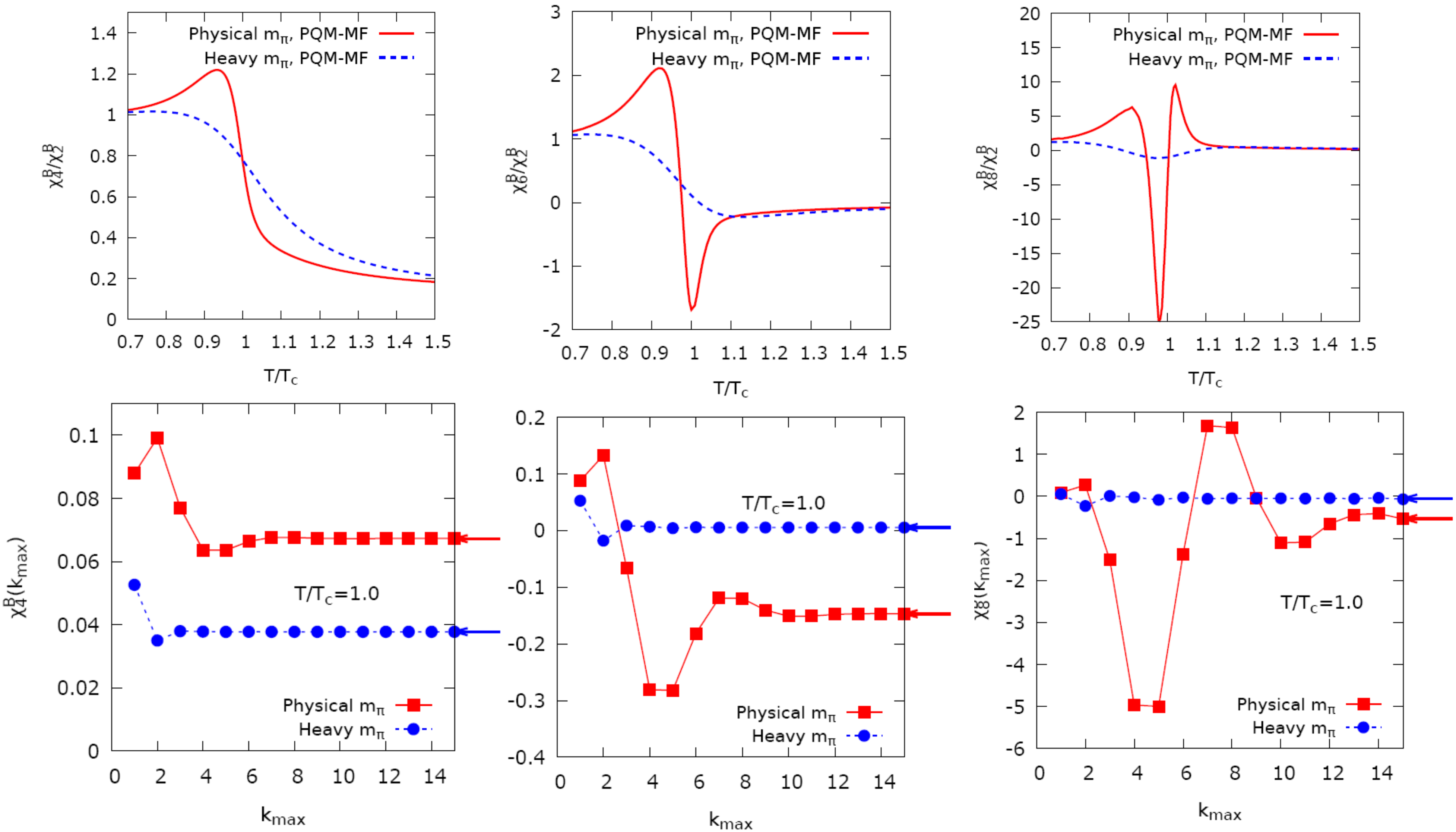
$$T = T^{RW}$$

$$T > T^{RW}$$

$$\boxed{b_k \approx \frac{(-1)^{k+1}}{k^{1+\frac{1}{\delta}}}}$$

$$\boxed{b_k \approx \frac{2}{\pi}\,\mathrm{Im}\,\chi_B^1(\theta=\pi)\times\frac{(-1)^{k+1}}{k}}$$

Reconstructing: $\chi_B^n(T,\mu=0)=\sum_{k=1}^{k_{max}} k^{2n-1} b_k(T)$ with fixed k_{max}



- hallo

Conclusions:

- The S-matrix approach to hadron gas thermodynamics with empirical scattering phase shifts provides consistent description of LQCD results on (electric charge)-baryon correlations in the chiral crossover, and the proton production yields in AA and pp collisions at the LHC and LHC data
- Systematics of net-proton number fluctuations at $\sqrt{s} > 20\ GeV$ measured by STAR in HIC at RHIC is consistent with the expectation, that they are influenced by the critical chiral dynamics and deconfinement,

  however, possible contributions to fluctuation observables from effects not related to critical phenomena have to be understood
- The Fourier expansion coefficients of baryon density exhibit rich structure to probe the QCD phase diagram and chiral criticality in the complex chemical potential