Modelling and optimization of a PWFA-LC stage

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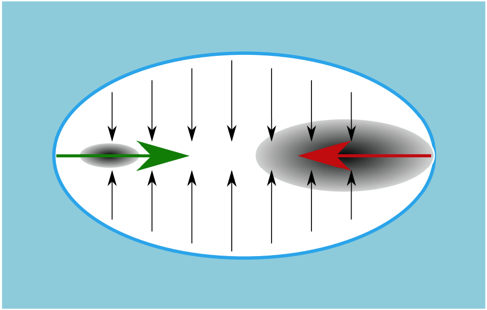
Why PWFA?

Advantages:

- Can sustain enormous gradients on the order of tens of GV/m [3].
- Focusing everywhere inside the bubble.



Transverse instabilities

Challenges, for instance:

- Transverse wakefields
  - Field generated by a driving particle's interaction with the acc. cavity due to misalignment.
  - Unstable oscillations.
  - CLIC: $7\,\mathrm{V/pC/mm/m}$ [4].
  - PWFA simulations: $5 \cdot 10^7\,\mathrm{V/pC/mm/m}$.
  - Limits the beam charge.
- Crucial to understand in order to develop mitigation techniques.

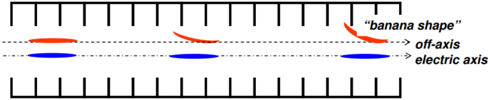


Figure: Off-axis beam kicked towards the cavity walls, S. Di Mitri, USPAS 2015.

- Mitigation methods exist.
- Need
  - systematic parameter studies.
  - study of emittance growth through many stages to verify the effectiveness of suppression.

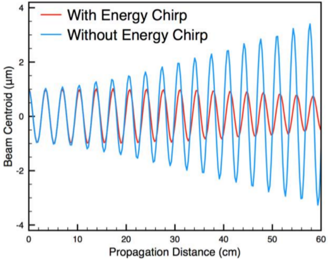


Figure: FACET-II parameters, 10% energy spread, W. An.

Related work

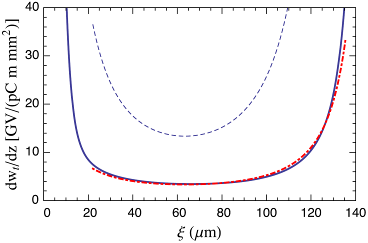
Intra-beam transverse wake of trailing beam

- G. Stupakov [7]:

$$\frac{\mathrm{d}w_\mathrm{t}}{\mathrm{d}z} = \frac{8}{(r_\mathrm{b}(\xi) + 0.75k_\mathrm{p}^{-1})^4}$$

- V. Lebedev et al. [6]:

$$W_\perp(\xi, \xi_2) \approx \frac{8(\xi - \xi_2)}{r_\mathrm{b}(\xi) r_\mathrm{b}^3(\xi_2)} \Theta(\xi - \xi_2)$$



- Expressing the hosing instability in terms of a wakefunction will allow a more global parameter optimization.

Section 2

Wakefield Modelling

- Find an appropriate wakefunction for PWFA.
- Benchmarked with modified FACET-II parameters.
- Compared the transverse wakefield

$$\mathcal{W}_\perp(\xi) = \int_{\xi_\mathrm{H}}^{\xi} W_\perp(\xi'-\xi)\lambda(\xi')X(\xi')\,\mathrm{d}\xi', \tag{1}$$

  to the directly calculated $\mathcal{W}_\mathrm{QP}$ using the fields from the QuickPIC simulation results.

$X(\xi)$: mean transverse offset of a slice at ξ.
$\lambda(\xi)$: longitudinal charge distribution.
ξ_H: longitudinal coordinate of beam head.

	Drive beam	Trailing beam
γ	195690	195690
N_B [10^{10}]	1.0	0.333
X_0 [µm]	0	3.7575
σ_x [µm]	2.05	2.05
σ_y [µm]	2.05	2.05
σ_z [µm]	12.77	6.38

Table: Beam parameters used in the simulation. Q is the charge per particle, m is the mass per particle, γ is the initial Lorentz factor, N_B is the total number of particles, X_0 is the transverse offset, from the ξ-axis, and the various σ's give the beam dimension along the x, y and z-direction.

QuickPIC open source

- Fully parallelized, fully relativistic, three-dimensional quasi-static PIC code.
- Quasi-static approximation.
- Reduces computation time with 2-3 OM.
- Agrees well with full PIC codes for problems of interest.

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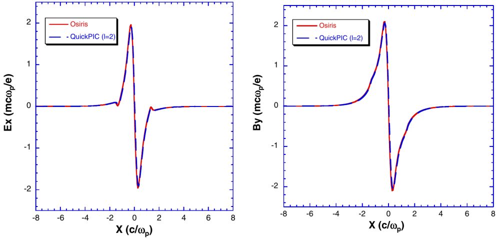


Figure: Radial electric and azimuthal magnetic fields comparisons for electron drive beam. [5]

Parametric wakefunction

$$W_\perp(\xi' - \xi) = \frac{2}{\pi \varepsilon_0 a^{*4}} (\xi' - \xi) \Theta(\xi' - \xi) \qquad (2)$$

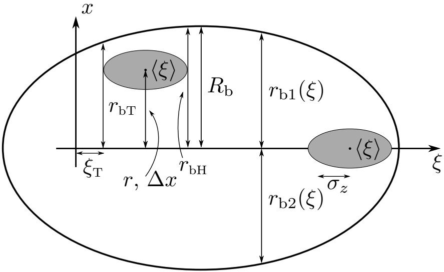


Figure: The driving and trailing beams shown together with the plasma bubble and various scales used in the calculations.

- Originally proposed for metal structures [2].
- Structure iris a in plasma?
- Best choice: $a^* = r_\mathrm{bT} + k_\mathrm{p}^{-1}$. Similar approach used by Stupakov [7].

Parametric wakefunction

$$W_{\perp}(\xi' - \xi) = \frac{2}{\pi \varepsilon_0 a^{*4}} (\xi' - \xi) \Theta(\xi' - \xi) \tag{2}$$

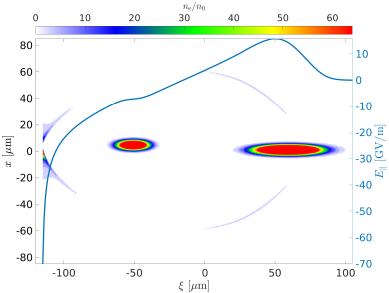


Figure: The beam and plasma electron density per unit per unit initial plasma density and the axial electric field.

- Plasma density $n_0 = 4.0 \cdot 10^{16}\,\mathrm{cm}^{-3}$.
- Area of interest: $\pm 3\sigma_{z\mathrm{WB}}$ from the center of the witness beam.

Results

- Comparison of the theoretical and simulated wakes.
- Relative error

$$\Delta = \left| \frac{\mathcal{W}_\perp - \mathcal{W}_{\mathrm{QP}}}{\langle \mathcal{W}_{\mathrm{QP}} \rangle} \right|$$

  - $\langle ... \rangle$: fields directly extracted from QuickPIC and averaged over the area of interest.

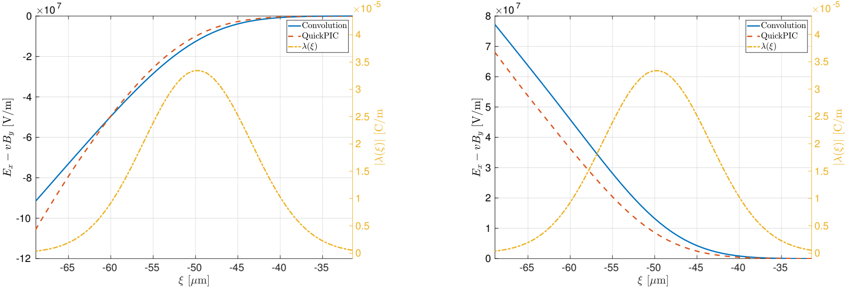


Figure: $s = 0$, $\Delta = 0.0965$.

Figure: $s = 1.1\,\mathrm{m}$, $\Delta = 0.253$.

Results

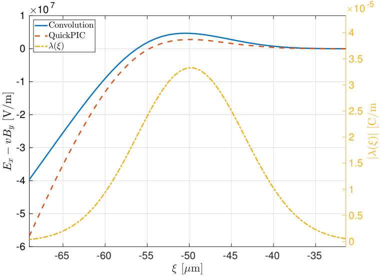


Figure: $s = 2.2$, $\Delta = 0.397$.

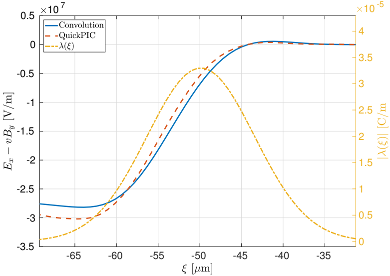


Figure: $s = 3.3\,\mathrm{m}$, $\Delta = 0.0776$

- Not perfect, but gives decent agreement.
- Is used in the numerical model.

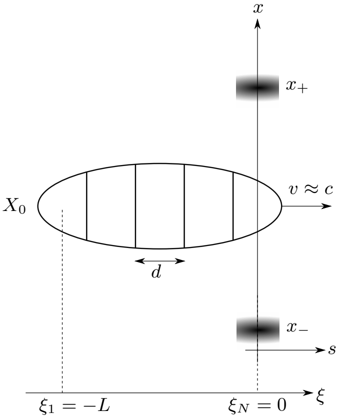
Section 3

Simple Quasi-Static Numerical Model

Preparations

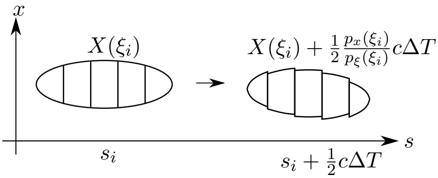
- Inspired by D. Schulte's model, and developed together with D. Schulte and E. Adli.
- Long plasma too heavy in QuickPIC.
- Beam of length L divided into N slices with thickness d.
- Longitudinal position of slices: $\xi = [\xi_1, \xi_2, ..., \xi_N]$
- Initial offset X_0.
- Offset of each beam slice: $X(\xi) = X(\xi_i) = [X_1, X_2, ..., X_N]$

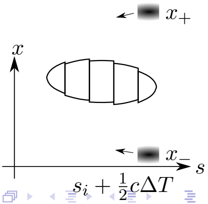
- Plasma element transverse position: $x_\pm$



Procedure

- Leapfrog integration, drift-kick-drift.
- Quasi-static approximation.
- Propagate the beam half a time step and update $X(\xi_i)$.
- Plasma-beam interaction by "scanning" the plasma slices backwards along the beam.
- $F_x(\xi_i)$ on the beam slices determined by $W_\perp(\xi' - \xi_i, a^*)$ and $X(\xi_i)$.
- Kick the beam longitudinally and transversely.
- Propagate the beam half a time step and update $X(\xi_i)$.





Benchmarking

Comparison with QuickPIC

- Mean transverse offset of beam slices located 0 – 2 rms beam length σ_z behind the beam center VS. propagation distance.
- Modified FACET-II parameters for stable propagation through one plasma cell.

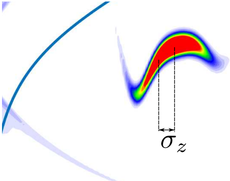


Figure: From QuickPIC.

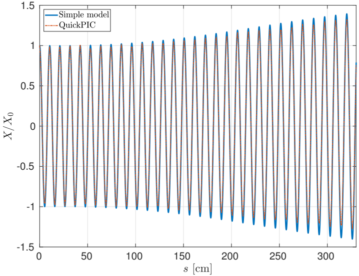


Figure: Beam center.

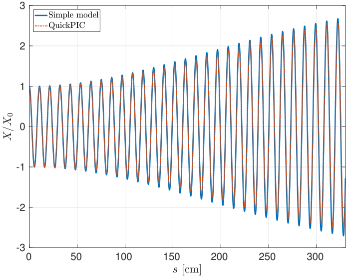


Figure: One σ_z behind beam center.

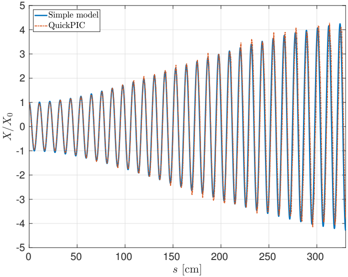


Figure: $2\sigma_z$ behind beam center.

Procedure for the study of PWFA parameters

- Focused on beam length and charge.
- SNOWMASS parameters by E. Adli et al. [1]:

	Drive beam	Trailing beam
γ	48924	48924
N_B $[10^{10}]$	2.0	1.0
σ_x [µm]	0.69	0.69
σ_y [µm]	0.69	0.69
σ_z [µm]	40	20

- $n_0 = 2 \cdot 10^{16}\,\mathrm{cm}^{-3}$, $\Delta z = 187\,\mu\mathrm{m}$.

Simplifications/assumptions:

1. Perfect drive beam.
2. Gradient profile remain unchanged (extracted from QuickPIC simulations).
3. The beam length is scaled proportionally to the beam charge.

Procedure for the study of PWFA parameters

Procedure:

1. Start with an offset beam.
2. Calculate the initial RMS amplitude

$$\Lambda_0 = \sum_i \left[\left(\frac{X_i}{\sigma_x} \right)^2 + \left(\frac{X_i'}{\sigma_{x'}} \right)^2 \right]. \tag{3}$$

3. After propagating beam through 60 plasma cells: calculate the final RMS amplitude Λ.
4. Check the stability Λ/Λ_0.
5. Loop through different beam charges, adjust the length and check the stability.

Final relative RMS energy spread and efficiency:

$$\frac{\sigma_{\mathscr{E}}}{\langle \mathscr{E} \rangle} = \frac{1}{\langle \mathscr{E} \rangle} \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\mathscr{E}_i - \langle \mathscr{E} \rangle)^2} \qquad \eta = \frac{E_{\mathrm{dec}}^{\mathrm{max}}}{\langle E_{\mathrm{acc}} \rangle} \frac{N_{\mathrm{WB}}}{N_{\mathrm{DB}}} = T \frac{N_{\mathrm{WB}}}{N_{\mathrm{DB}}}. \tag{4}$$

Results

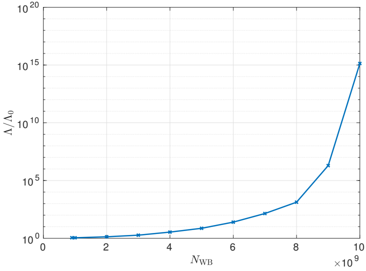


Figure: Amplification of Λ_0 VS. N_{DB}.

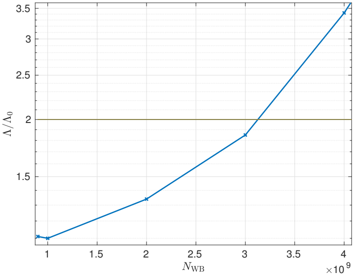
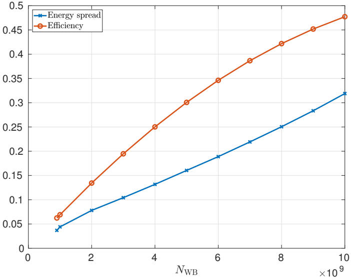
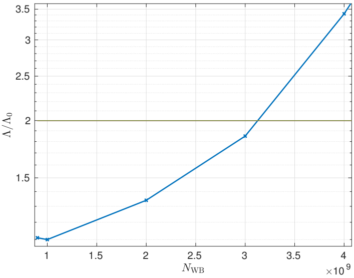


Figure: Zoomed into the lhs.

- What Λ/Λ_0 is acceptable?
- $\Lambda/\Lambda_0 = 2$ was chosen.
- Already 100% luminosity loss.
- Limit: $\underline{N_{\mathrm{DB}} \lesssim 3 \cdot 10^9,\ \sigma_z \lesssim 6\,\mu\mathrm{m}.}$

Results





- What Λ/Λ_0 is acceptable?
- $\Lambda/\Lambda_0 = 2$ was chosen.
- Already 100% luminosity loss.
- Limit: $N_{\mathrm{DB}} \lesssim 3 \cdot 10^9$, $\sigma_z \lesssim 6\,\mu\mathrm{m}$.
- $\eta \approx 20\%$, $\sigma_{\mathcal{E}}/\langle \mathcal{E} \rangle \approx 10\%$.

Summary

- Acc. gradient several o.m. larger than conventional NC acc. structures may be achieved with PWFA.
- Transverse wakefields have to be understood and mitigated.
- Proposed the wakefunction:

$$W_{\perp}(\xi' - \xi) = \frac{2}{\pi \varepsilon_0 a^{*4}} (\xi' - \xi) \Theta(\xi' - \xi)$$

- Compared against QuickPIC simulations.
- Simple quasi-static model.
- Simple PWFA parameter study attempting in finding a rough limit for a stable beam.

Make accelerators small again!

E. Adli, J-P. Delahaye, S. J. Gessner, M. J. Hogan, T. Raubenheimer, W. An, C. Joshi, and W. Mori.
A beam driven plasma-wakefield linear collider: From higgs factory to multi-tev.
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