LIVING WITHOUT SUPERSYMMETRY

Philip D. Mannheim

University of Connecticut

Seminar at Discrete 2018

November 2018

Living Without Supersymmetry – the Conformal Alternative and a Dynamical Higgs Boson, J. Phys. G 44, 115003 (2017). (arXiv:1506.01399 [hep-ph])

Mass Generation, the Cosmological Constant Problem, Conformal Symmetry, and the Higgs Boson, Prog. Part. Nucl. Phys. 94, 125 (2017). (arXiv:1610:08907 [hep-ph])

1 INTRODUCTION

The assumption of a supersymmetry between bosons and fermions has been found capable of addressing many key issues in particle physics and gravity.

(1) In flat space physics an interplay between bosons and fermions can render logarithmically divergent Feynman diagrams finite.

(2) An interplay between bosons and fermions can cancel the perturbative quadratic divergence that an elementary Higgs scalar field would possess (the hierarchy problem).

(3) The existence of fermionic supersymmetry generators allows one to evade the Coleman-Mandula theorem that forbids the combining of spacetime and bosonic internal symmetry generators in a common Lie algebra.

(4) With the inclusion of supersymmetry one can potentially achieve a unification of the coupling constants of $SU(3) \times SU(2)_{\mathrm{L}} \times U(1)$ at a grand unified energy scale.

(5) In the presence of gravity an interplay between bosons and fermions can cancel the quartic divergence in the vacuum energy.

(6) Cancellation of perturbative infinities can also be found in supergravity, the local version of supersymmetry.

(7) With supersymmetry one can construct a consistent candidate quantum theory of gravity, string theory, which permits a possible unification of all of the fundamental forces and a metrication (geometrization) of them.

(8) Finally, with supersymmetry one has a prime candidate for dark matter.

Despite this quite extensive theoretical inventory, so far no sign of any superparticles.

Now while one can always push superparticle masses above current experimental capabilities, there is a tension since need a superparticle close enough in mass to Higgs boson in order to resolve the hierarchy problem.

The situation is disquieting enough that one should at least contemplate whether it might be possible to dispense with supersymmetry altogether.

If one is to consider doing so however, then one must seek an alternative that has the potential to also achieve its key successes. The best option is if it is also based on a symmetry, so consider **conformal symmetry**.

On the gravity side, consider conformal gravity. The conformal gravity theory is based on the action

$$I_{\mathrm{W}} = -\alpha_g \int d^4x (-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \tag{1}$$

where

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \tfrac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \tfrac{1}{6}R^{\alpha}{}_{\alpha}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}),$$

is the Weyl conformal tensor and α_g is dimensionless. The action is the unique action that is invariant under the local scaling

$$g_{\mu\nu}(x) \to e^{2\beta(x)} g_{\mu\nu}(x),$$

with

$$C^{\lambda}{}_{\mu\nu\kappa} \to C^{\lambda}{}_{\mu\nu\kappa},$$

and with all derivatives of $\beta(x)$ dropping out.

On the particle physics side we consider scaling with anomalous dimensions (conformal bootstrap). For the Abelian gluon model this can be achieved by the renormalization group $\beta(\alpha)$ having a fixed point at some α, or by considering the quenched approximation in which the photon is not dressed, this being equivalent to $\beta(\alpha)$ being zero for any α.

In either case the asymptotic solution to the Schwinger-Dyson equation for the fermion propagator is of the form (Johnson, Baker, Wiley 1961)

$$\tilde{S}^{-1}(p) \;=\; \not{p} - m\left(\frac{-p^2 - i\epsilon}{m^2}\right)^{\gamma_\theta(\alpha)/2} + i\epsilon. \tag{2}$$

where $\gamma_\theta(\alpha)$ is the anomalous part of the dimension $d_\theta(\alpha)$ of the mass operator $\bar{\psi}\psi$, according to

$$d_\theta(\alpha) = 3 + \gamma_\theta(\alpha). \tag{3}$$

The Higgs self-energy quadratic divergence hierarchy problem is currently the most pressing concern for supersymmetry.

Problem does not exist if Higgs boson is dynamical composite rather than elementary. This is important to know anyway. We will see that Higgs width can serve as a diagnostic.

If action is scale invariant then no $-\mu^2\phi^2$ term allowed. Symmetry breaking must then be dynamical, with $\bar{\psi}\psi$ acquiring a non-vanishing vacuum expectation value. In such a case the Higgs mass is automatically of the same order as the fermion mass that is induced by dynamical symmetry breaking.

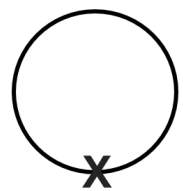
We thus need a calculational scheme to implement dynamical symmetry breaking. We shall augment the chiral symmetric Nambu-Jona-Lasinio four-fermion model by dressing its point vertices with the Johnson-Baker-Wiley scaling form for the insertion of $\bar{\psi}\psi$ into the inverse Fermion propagator:

$$\tilde{\Gamma}_{\mathrm{S}}(p,p,0,m) = \left(\frac{-p^2 - i\epsilon}{m^2}\right)^{\gamma_\theta(\alpha)/2}, \tag{4}$$

We find that vacuum breaks if $\gamma_\theta(\alpha) = -1$ (Mannheim 1974, 1975), dynamical Goldstone and Higgs bosons being generated (Mannheim 2017), and with the four fermion theory then becoming renormalizable to all orders (Mannheim 2017).

2 THE NAMBU-JONA-LASINIO (NJL) CHIRAL FOUR-FERMION MODEL

2.1 Quick Review of the NJL Model as a Mean-Field Theory in Hartree-Fock Approximation



Introduce mass term with m as a trial parameter and note $m^2/2g$ term

$$\begin{aligned} I_{\rm NJL} &= \int d^4x \left[i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{g}{2}[\bar{\psi}\psi]^2 - \frac{g}{2}[\bar{\psi}i\gamma_5\psi]^2 \right] \\ &= \int d^4x \left[i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + \frac{m^2}{2g} \right] + \int d^4x \left[-\frac{g}{2}\left(\bar{\psi}\psi - \frac{m}{g}\right)^2 - \frac{g}{2}\left(\bar{\psi}i\gamma_5\psi\right)^2 \right] \\ I_{\rm NJL} &= I_{\rm MF} + I_{\rm RI}, \quad \text{mean field plus residual interaction} \end{aligned} \tag{5}$$

Hartree-Fock approximation

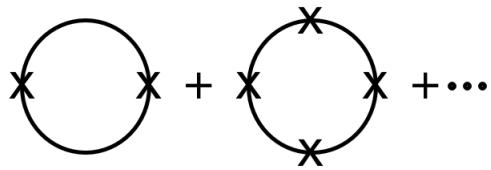
$$\langle\Omega_{\rm m}| \left[\bar{\psi}\psi - \frac{m}{g}\right]^2 |\Omega_{\rm m}\rangle = \langle\Omega_{\rm m}| \left[\bar{\psi}\psi - \frac{m}{g}\right] |\Omega_{\rm m}\rangle^2 = 0, \tag{6}$$

$$\langle\Omega_{\rm m}|\bar{\psi}\psi|\Omega_{\rm m}\rangle = -i\int \frac{d^4k}{(2\pi)^4} {\rm Tr}\left[\frac{1}{\not{p} - m + i\epsilon}\right] = \frac{m}{g}, \tag{7}$$

Satisfied by self-consistent M, and defines g^{-1}

$$-\frac{M\Lambda^2}{4\pi^2} + \frac{M^3}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) = \frac{M}{g}. \tag{8}$$

2.2 Vacuum Energy



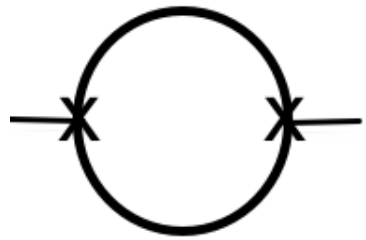
$$
\begin{aligned}
\epsilon(m) &= i\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\,\ln\left[\frac{\not{p}-m+i\epsilon}{\not{p}+i\epsilon}\right] \\
&= -\frac{m^2\Lambda^2}{8\pi^2}+\frac{m^4}{16\pi^2}\ln\left(\frac{\Lambda^2}{m^2}\right)+\frac{m^4}{32\pi^2}
\end{aligned}
\tag{9}
$$

is quadratically divergent.

$$
\begin{aligned}
\tilde{\epsilon}(m) &= \epsilon(m)-\frac{m^2}{g} \\
&= \frac{m^4}{16\pi^2}\ln\left(\frac{\Lambda^2}{m^2}\right)-\frac{m^2M^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right)+\frac{m^4}{32\pi^2}.
\end{aligned}
\tag{10}
$$

is only log divergent, with double-well potential emerging, but still cutoff dependent.

2.3 The Collective Tachyon Modes when the Fermion is Massless



$$\begin{aligned}
\Pi_{\rm S}(q^2) &= \int d^4x e^{ip\cdot x}\langle\Omega|T(\bar\psi(x)\psi(x)\bar\psi(0)\psi(0))|\Omega\rangle = -i\int\frac{d^4p}{(2\pi)^4}{\rm Tr}\left[\frac{1}{\not p+i\epsilon}\frac{1}{\not p+\not q+i\epsilon}\right],\\
\Pi_{\rm P}(q^2) &= \int d^4x e^{ip\cdot x}\langle\Omega|T(\bar\psi(x)i\gamma_5\psi(x)\bar\psi(0)i\gamma_5\psi(0))|\Omega\rangle = -i\int\frac{d^4p}{(2\pi)^4}{\rm Tr}\left[i\gamma_5\frac{1}{\not p+i\epsilon}i\gamma_5\frac{1}{\not p+\not q+i\epsilon}\right].
\end{aligned} \tag{11}$$

$$\Pi_{\rm S}(q^2) = \Pi_{\rm P}(q^2) = -\frac{\Lambda^2}{4\pi^2} - \frac{q^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{-q^2}\right) - \frac{q^2}{8\pi^2}. \tag{12}$$

$$\begin{aligned}
T_{\rm S}(q^2) &= g + g\Pi_{\rm S}(q^2)g + g\Pi_S(q^2)g\Pi_S(q^2)g + ... = \frac{g}{1-g\Pi_{\rm S}(q^2)} = \frac{1}{g^{-1}-\Pi_{\rm S}(q^2)},\\
T_{\rm P}(q^2) &= g + g\Pi_{\rm P}(q^2)g + g\Pi_P(q^2)g\Pi_P(q^2)g + ... = \frac{g}{1-g\Pi_{\rm P}(q^2)} = \frac{1}{g^{-1}-\Pi_{\rm P}(q^2)}.
\end{aligned} \tag{13}$$

$$T_{\rm S}(q^2) = T_{\rm P}(q^2) = \frac{Z^{-1}}{(q^2+2M^2)},\ Z = \frac{1}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) \tag{14}$$

Tachyonic poles, but at cutoff independent masses. Normal vacuum is unstable. g^{-1} takes care of the quadratic divergence.

2.4 The Collective Goldstone and Higgs Modes when the Fermion is Massive

$$
\begin{aligned}
\Pi_{\rm S}(q^2, M) &= -i \int \frac{d^4p}{(2\pi)^4} {\rm Tr}\left[\frac{1}{\not p - M + i\epsilon}\frac{1}{\not p + \not q - M + i\epsilon}\right] \\
&= -\frac{\Lambda^2}{4\pi^2} + \frac{M^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) + \frac{(4M^2 - q^2)}{8\pi^2} + \frac{(4M^2 - q^2)}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) \\
&-\frac{1}{8\pi^2}\frac{(4M^2 - q^2)^{3/2}}{(-q^2)^{1/2}}\ln\left(\frac{(4M^2 - q^2)^{1/2} + (-q^2)^{1/2}}{(4M^2 - q^2)^{1/2} - (-q^2)^{1/2}}\right).
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\Pi_{\rm P}(q^2, M) &= -i \int \frac{d^4p}{(2\pi)^4} {\rm Tr}\left[i\gamma_5\frac{1}{\not p - M + i\epsilon} i\gamma_5 \frac{1}{\not p + \not q - M + i\epsilon}\right] \\
&= -\frac{\Lambda^2}{4\pi^2} + \frac{M^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) - \frac{q^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) + \frac{(4M^2 - q^2)}{8\pi^2} \\
&+\frac{(8M^4 - 8M^2q^2 + q^4)}{8\pi^2(-q^2)^{1/2}(4M^2 - q^2)^{1/2}}\ln\left(\frac{(4M^2 - q^2)^{1/2} + (-q^2)^{1/2}}{(4M^2 - q^2)^{1/2} - (-q^2)^{1/2}}\right).
\end{aligned} \tag{16}
$$

$$
T_{\rm S}(q^2) \;=\; \frac{R_{\rm S}^{-1}}{(q^2 - 4M^2)}, \qquad T_{\rm P}(q^2) = \frac{R_{\rm P}^{-1}}{q^2}, \tag{17}
$$

$$
R_{\rm S} \;=\; R_{\rm P} = \frac{1}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right). \tag{18}
$$

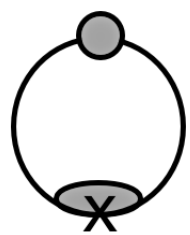
The scalar Higgs mass is finite and of order the dynamical fermion mass, and residue is determined.

3 SCALE INVARIANT QED COUPLED TO FOUR FERMI THEORY AT $\gamma_\theta(\alpha) = -1$

$$\begin{aligned}
\mathcal{L}_{\rm QED-FF} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi\gamma^\mu(i\partial_\mu - eA_\mu)\psi - \frac{g}{2}[\bar\psi\psi]^2 - \frac{g}{2}[\bar\psi i\gamma_5\psi]^2 \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi\gamma^\mu(i\partial_\mu - eA_\mu)\psi - m\bar\psi\psi + \frac{m^2}{2g} - \frac{g}{2}\left(\bar\psi\psi - \frac{m}{g}\right)^2 - \frac{g}{2}\left(\bar\psi i\gamma_5\psi\right)^2 \\
&= \mathcal{L}_{\rm QED-MF} + \mathcal{L}_{\rm QED-RI}.
\end{aligned} \tag{19}$$

$$\tilde S^{-1}(p) = \not p - m\left(\frac{-p^2 - i\epsilon}{\mu^2}\right)^{-1/2} + i\epsilon, \quad \tilde\Gamma_{\rm S}(p,p,0) = \left(\frac{-p^2 - i\epsilon}{\mu^2}\right)^{-1/2} \tag{20}$$

as renormalized at μ^2. With dimension of $(\bar\psi\psi)^2$ dropping from 6 to 4 when $\gamma_\theta = -1$, quadratic divergences become logarithmic, and four-fermion interaction becomes renormalizable to all orders in g (Mannheim 2017).

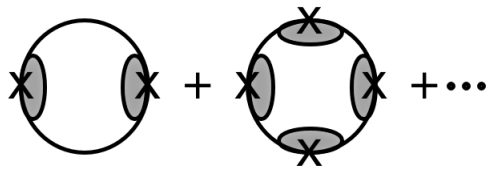


$$\langle\Omega_m|\bar\psi\psi|\Omega_m\rangle = -\frac{m\mu^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{m\mu}\right) = \frac{m}{g}. \tag{21}$$

$$-\frac{\mu^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{M\mu}\right) = \frac{1}{g}, \qquad M = \frac{\Lambda^2}{\mu}\exp\left(\frac{4\pi^2}{\mu^2 g}\right). \tag{22}$$

Gap equation gives $-g \sim 1/\ln\Lambda^2$. Thus g is negative, i.e. attractive, and becomes very small as $\Lambda \to \infty$, with BCS-type essential singularity in gap equation at $g = 0$. Hence dynamical symmetry breaking with weak coupling.

3.1 Vacuum Energy

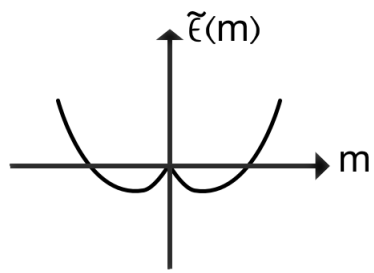


$$\epsilon(m) \;=\; \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\,\ln\left[1-\frac{m^2}{p^2+i\epsilon}\left(\frac{-p^2-i\epsilon}{\mu^2}\right)^{-1}\right] = -\frac{m^2\mu^2}{8\pi^2}\left[\ln\left(\frac{\Lambda^2}{m\mu}\right)+\frac{1}{2}\right] \tag{23}$$

and is only log divergent. Due to presence of $m^2/2g$ term we obtain the completely finite

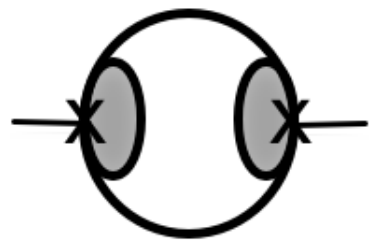
$$\tilde{\epsilon}(m) = \epsilon(m) - \frac{m^2}{2g} = \frac{m^2\mu^2}{16\pi^2}\left[\ln\left(\frac{m^2}{M^2}\right)-1\right], \tag{24}$$

which we recognize as a double-well potential, dynamically induced.



We thus see the power of dynamical symmetry breaking. It reduces divergences. Moreover, since $m^2/2g$ is a cosmological term, dynamical symmetry breaking has a control over the cosmological constant problem that an elementary Higgs field potential does not. When coupled to conformal gravity, the cosmological constant problem is completely solved.

3.2 The Collective Tachyon Modes when the Fermion is Massless



$$\Pi_{\rm S}(q^2, m=0) = \Pi_{\rm P}(q^2, m=0) = -\frac{\mu^2}{4\pi^2}\left[\ln\left(\frac{\Lambda^2}{(-q^2)}\right) - 3 + 4\,\ln 2\right]. \tag{25}$$

$$\begin{aligned} T_{\rm S}(q^2) &= \frac{g}{1-g\Pi_{\rm S}(q^2)} = \frac{1}{g^{-1}-\Pi_{\rm S}(q^2)}, \\ T_{\rm P}(q^2) &= \frac{g}{1-g\Pi_{\rm P}(q^2)} = \frac{1}{g^{-1}-\Pi_{\rm P}(q^2)}, \end{aligned} \tag{26}$$

$$q^2 = -M\mu e^{4\ln 2-3} = -0.797M\mu, \tag{27}$$

$$T_{\rm S}(q^2) = T_{\rm P}(q^2) = \frac{31.448M\mu}{(q^2+0.797M\mu)} \tag{28}$$

3.3 The Collective Goldstone Mode when the Fermion is Massive

$$\begin{aligned}\Pi_{\rm P}(q^2=0,m) &= -4i\mu^2\int\frac{d^4p}{(2\pi)^4}\frac{(p^2)(-p^2)-m^2\mu^2}{((p^2+i\epsilon)^2+m^2\mu^2)^2}.\\ &= 4i\mu^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+i\epsilon)^2+m^2\mu^2}=\frac{1}{g}.\end{aligned} \tag{29}$$

$$T_{\rm P}(q^2)=\frac{128\pi M}{7\mu q^2}=\frac{57.446M}{\mu q^2}. \tag{30}$$

3.4 The Collective Higgs Mode when the Fermion is Massive

$$q_0(\text{Higgs}) = (1.480-0.017i)(M\mu)^{1/2}, \qquad q^2(\text{Higgs}) = (2.189-0.051\text{i})\text{M}\mu. \tag{31}$$

$$q_0(\text{Higgs}) = (1.480-0.017i)M, \qquad q^2(\text{Higgs}) = (2.189-0.051\text{i})\text{M}^2, \tag{32}$$

Higgs mass is close to dynamical fermion mass, but above threshold, and thus has a width. In na double well elementary Higgs field theory Higgs mass is real. Width can be used to distinguish an elementary Higgs from a dynamical one.

4 WE GET LOCAL CONFORMAL SYMMETRY FOR FREE

To implement local Lorentz invariance for a Dirac spinor (i.e. $\psi \to \exp(\omega_{\mu\nu}(x)M^{\mu\nu})\psi$ where $\omega_{\mu\nu}(x)$ depends on x^μ), one introduces a set of vierbeins V^a_μ and a spin connection ω^{ab}_μ, where

$$-\omega^{ab}_\mu = V^b_\nu \partial_\mu V^{a\nu} + V^b_\lambda \Gamma^\lambda{}_{\nu\mu} V^{a\nu}, \qquad \Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\nu\mu}). \tag{33}$$

In a standard curved Riemannian space the massless Dirac action is given by

$$I_{\rm D} = \int d^4x (-g)^{1/2} i\bar{\psi}\gamma^a V^\mu_a (\partial_\mu + \Gamma_\mu)\psi, \qquad \Gamma_\mu = \frac{1}{8}(\gamma_a\gamma_b - \gamma_b\gamma_a)\omega^{ab}_\mu. \tag{34}$$

The Dirac action $I_{\rm D}$ possesses four local translation invariances and six local Lorentz invariances. However, $I_{\rm D}$ also possesses one local conformal invariance as well, since it is left invariant under

$$g_{\mu\nu}(x) \to e^{2\beta(x)} g_{\mu\nu}(x), \quad V^a_\mu(x) \to e^{\beta(x)} V^a_\mu(x), \quad \psi(x) \to e^{-3\beta(x)/2}\psi(x), \tag{35}$$

with arbitrary spacetime-dependent $\beta(x)$.

Local Lorentz invariance implies local conformal invariance, with $\Gamma_\mu = \Sigma_{bc}\omega^{bc}_\mu$ acting as a gauge field for local conformal transformations, just as A_μ is the gauge field for local gauge transformations. In $\psi \to e^{\beta + i\alpha}\psi$, gauging β is gravity, gauging α is Yang-Mills.

Thus unify fundamental forces with gravity through gauging real and imaginary parts of fermion phase. No longer need to unify them using a Lie group, So no need to evade Coleman-Mandula theorem, and thus no need for supersymmetry.

5 WE GET LOCAL CONFORMAL GRAVITY FOR FREE

$$I_W = -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} - 2\alpha_g \int d^4x(-g)^{1/2} \left[R_{\mu\kappa} R^{\mu\kappa} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2 \right]. \quad (36)$$

For Dirac action $I_{\rm D}$ path integration over the fermion fields is direct, and leads ('t Hooft) to

$$\int [D\psi][D\bar{\psi}] \exp(I_{\rm D}) = \exp(iI_{\rm EFF}),$$

$$I_{\rm EFF} = \int d^4x(-g)^{1/2} C \left[R_{\mu\nu} R^{\mu\nu} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2 \right] \quad (37)$$

with leading term with log divergent constant C. (Path integration is same as a one loop Feynman diagram.)

Conformal gravity is unavoidable. Standard Dirac action is ghost free, so conformal gravity must be ghost free also. Confirmed by Bender and Mannheim (2008) with conformal gravity being a non-Hermitian PT theory.

When we set

$$\epsilon(m) \; = \; i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} \, \ln \left[\not{p} - m + i\epsilon \right] - i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} \, \ln \left[\not{p} + i\epsilon \right] \quad (38)$$

we were only looking at energy difference. But gravity sees full

$$\epsilon(m) \; = \; i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} \, \ln \left[\not{p} - m + i\epsilon \right] \quad (39)$$

and it is quartically divergent. Cancelled by graviton loop quartic divergence.

5.1 Conformal Gravity and the Cosmological Constant Problem

$$\mathcal{L}_{\mathrm{EFF}} = \frac{M^4}{16\pi^2} - \frac{M^2}{512\pi} R^{\alpha}{}_{\alpha}. \tag{40}$$

$$T^{\mu\nu}_{\mathrm{M}} \;=\; i\hbar\bar{\psi}\gamma^{\mu}\partial^{\nu}\psi - \frac{M^2}{256\pi}\left(R^{\mu\nu} - \frac{g^{\mu\nu}}{2}R^{\alpha}{}_{\alpha}\right) - g^{\mu\nu}\frac{M^4}{16\pi^2}. \tag{41}$$

$$d_L = -\frac{c}{H_0}\frac{(1+z)^2}{q_0}\left(1 - \left[1 + q_0 - \frac{q_0}{(1+z)^2}\right]^{1/2}\right), \tag{42}$$

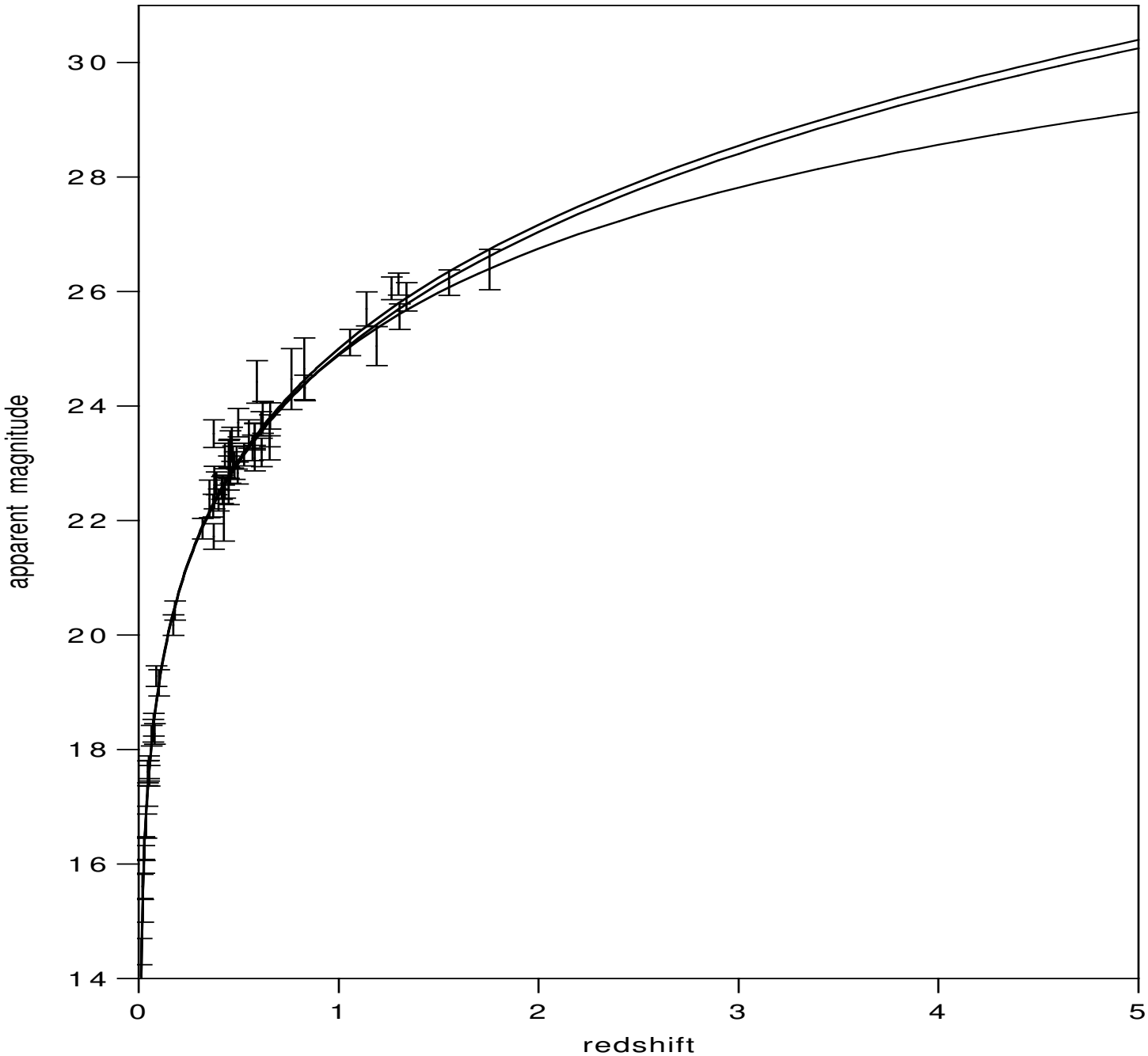


Figure 1: Hubble plot expectations for $q_0 = -0.37$ (highest curve) and $q_0 = 0$ (middle curve) conformal gravity and for $\Omega_M(t_0) = 0.3$, $\Omega_\Lambda(t_0) = 0.7$ standard gravity (lowest curve).

5.2 Conformal Gravity and the Dark Matter Problem

$$\nabla^4 B(r) = \frac{3}{4\alpha_g B(r)}(T^0{}_0 - T^r{}_r) = f(r). \tag{43}$$

$$\begin{aligned} B(r) &= -\frac{1}{6}\int_0^r dr' f(r')\left(3r'^2 r + \frac{r'^4}{r}\right) \\ &- \frac{1}{6}\int_r^\infty dr' f(r')(3r'^3 + r'r^2) + B_0(r), \end{aligned} \tag{44}$$

$$\begin{aligned} v_{\rm LOC}^2 &= \frac{N^*\beta^* c^2 R^2}{2R_0^3}\left[I_0\left(\frac{R}{2R_0}\right)K_0\left(\frac{R}{2R_0}\right)\right. \\ &- \left. I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right)\right] \\ &+ \frac{N^*\gamma^* c^2 R^2}{2R_0} I_1\left(\frac{R}{2R_0}\right)K_1\left(\frac{R}{2R_0}\right), \end{aligned} \tag{45}$$

$$v_{\rm TOT}^2 = v_{\rm LOC}^2 + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2 \to -\frac{N^*\beta^* c^2}{R} + \frac{N^*\gamma^* c^2 R}{2} + \frac{\gamma_0 c^2 R}{2} - \kappa c^2 R^2. \tag{46}$$

$$\begin{aligned} \beta^* &= 1.48\times 10^5 {\rm cm},\ \gamma^* = 5.42\times 10^{-41}{\rm cm}^{-1}, \\ \gamma_0 &= 3.06\times 10^{-30}{\rm cm}^{-1},\ \kappa = 9.54\times 10^{-54}{\rm cm}^{-2}, \end{aligned} \tag{47}$$

Fit 138 galaxies with VISIBLE N^* of each galaxy (i.e. galactic mass to light ratio M/L) as only variable, with β^*, γ^*, γ_0^* and κ all universal, and with NO DARK MATTER, and with 276 fewer free parameters than in dark matter calculations. Works since $(v^2/c^2R)_{\rm last} \sim 10^{-30}{\rm cm}^{-1}$ for every galaxy.

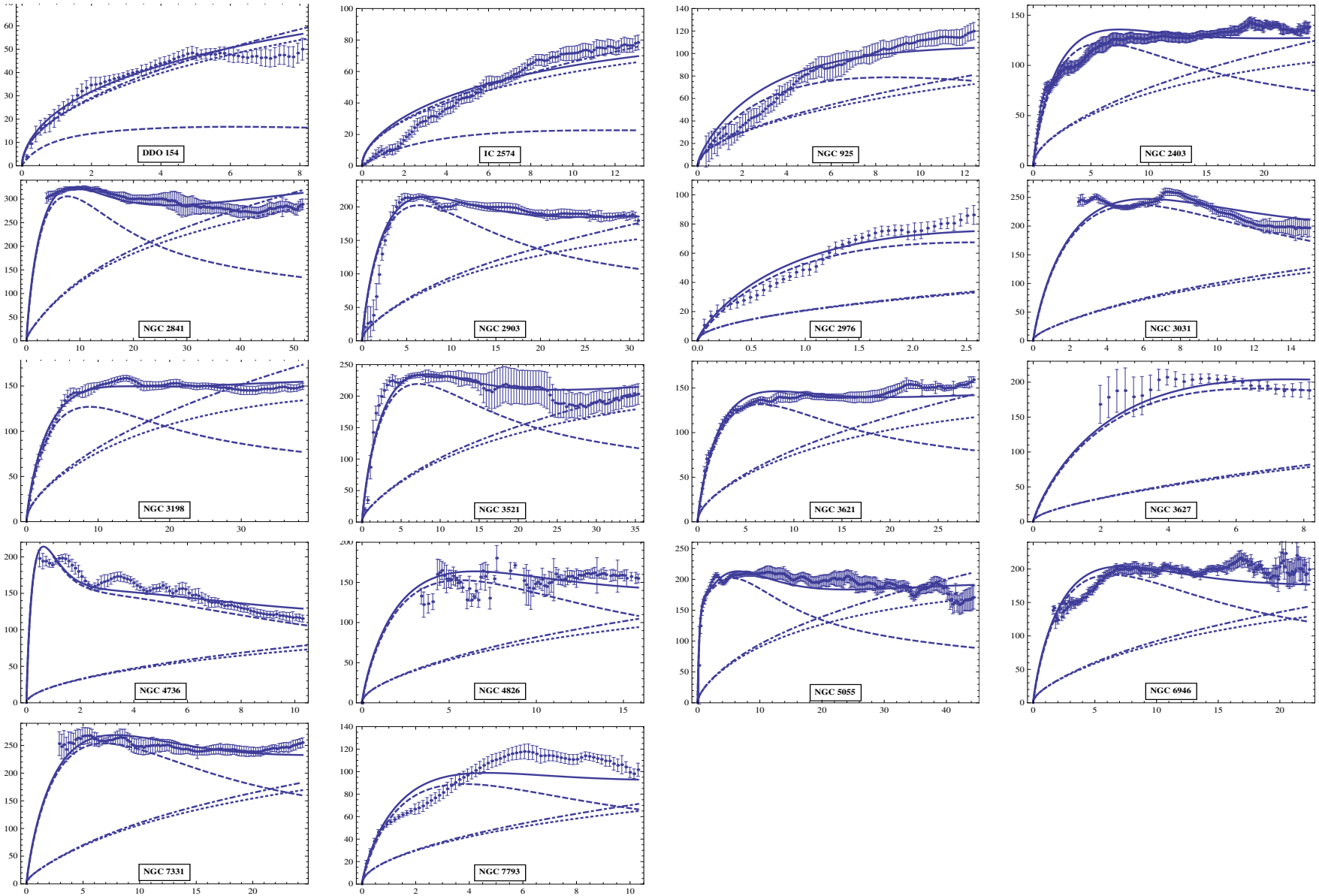


FIG. 1: Fitting to the rotational velocities (in km $\sec^{-1}$) of the THINGS 18 galaxy sample

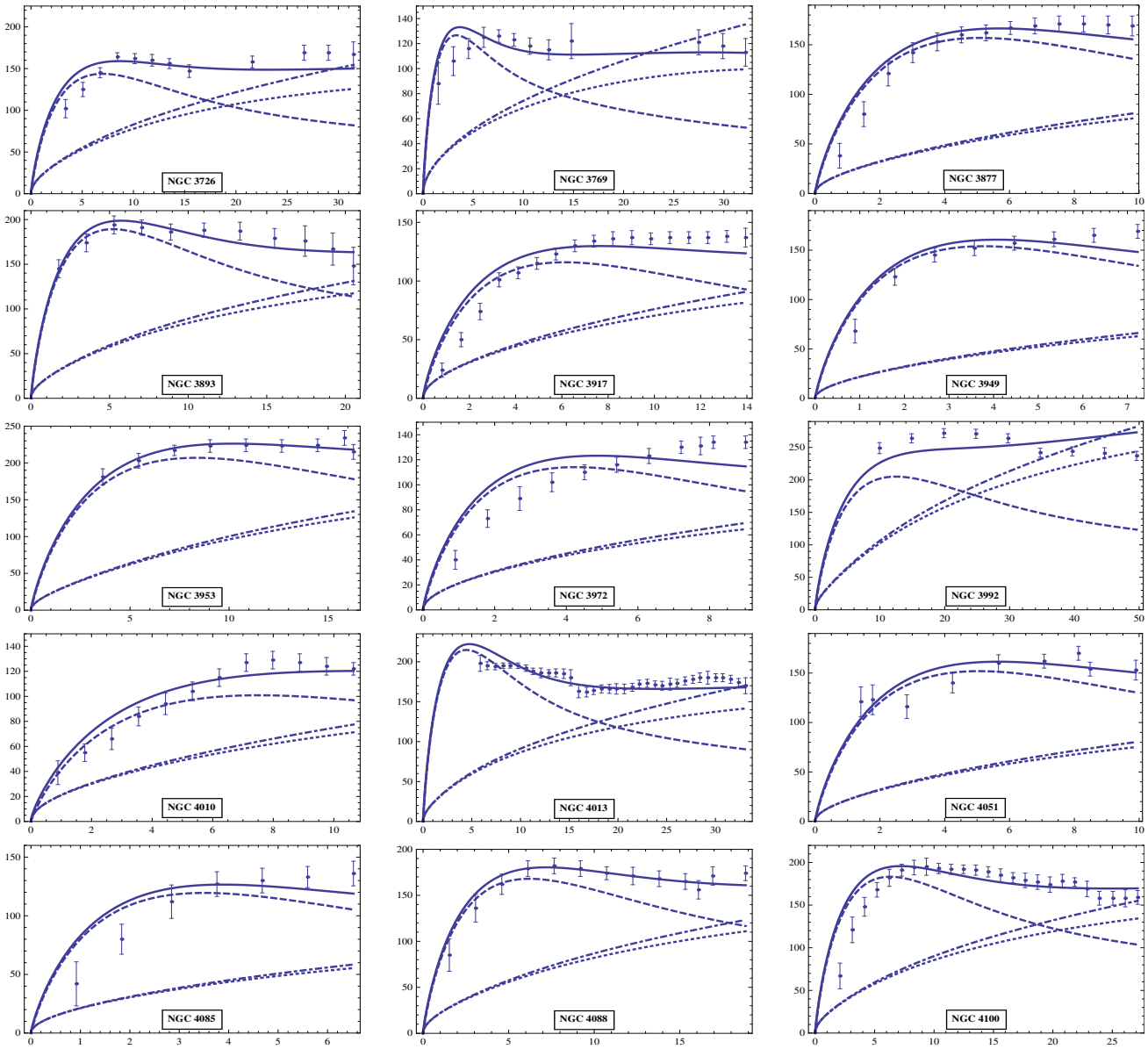


FIG. 2: Fitting to the rotational velocities of the Ursa Major 30 galaxy sample – Part 1

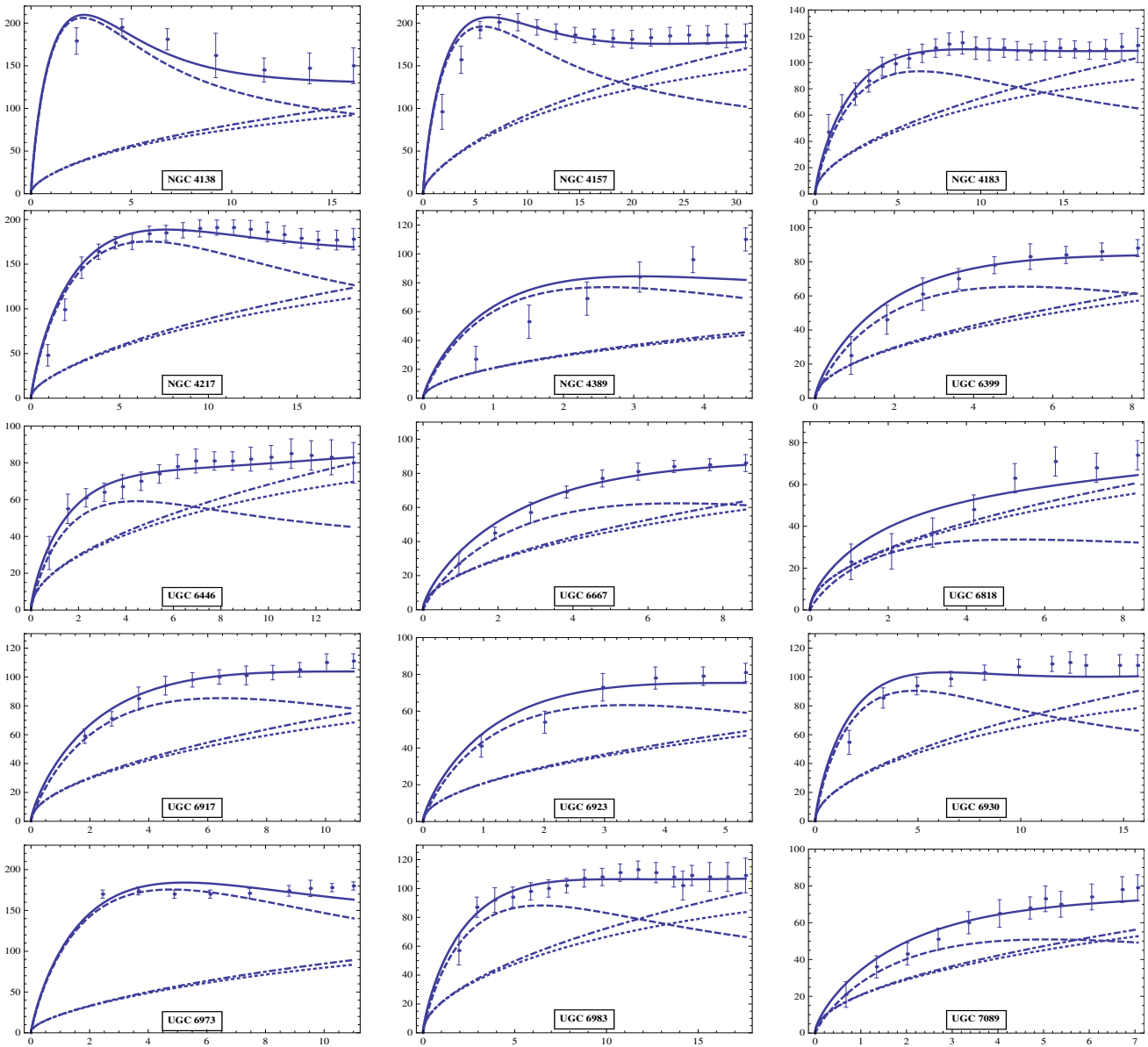


FIG. 3: Fitting to the rotational velocities of the Ursa Major 30 galaxy sample – Part 2

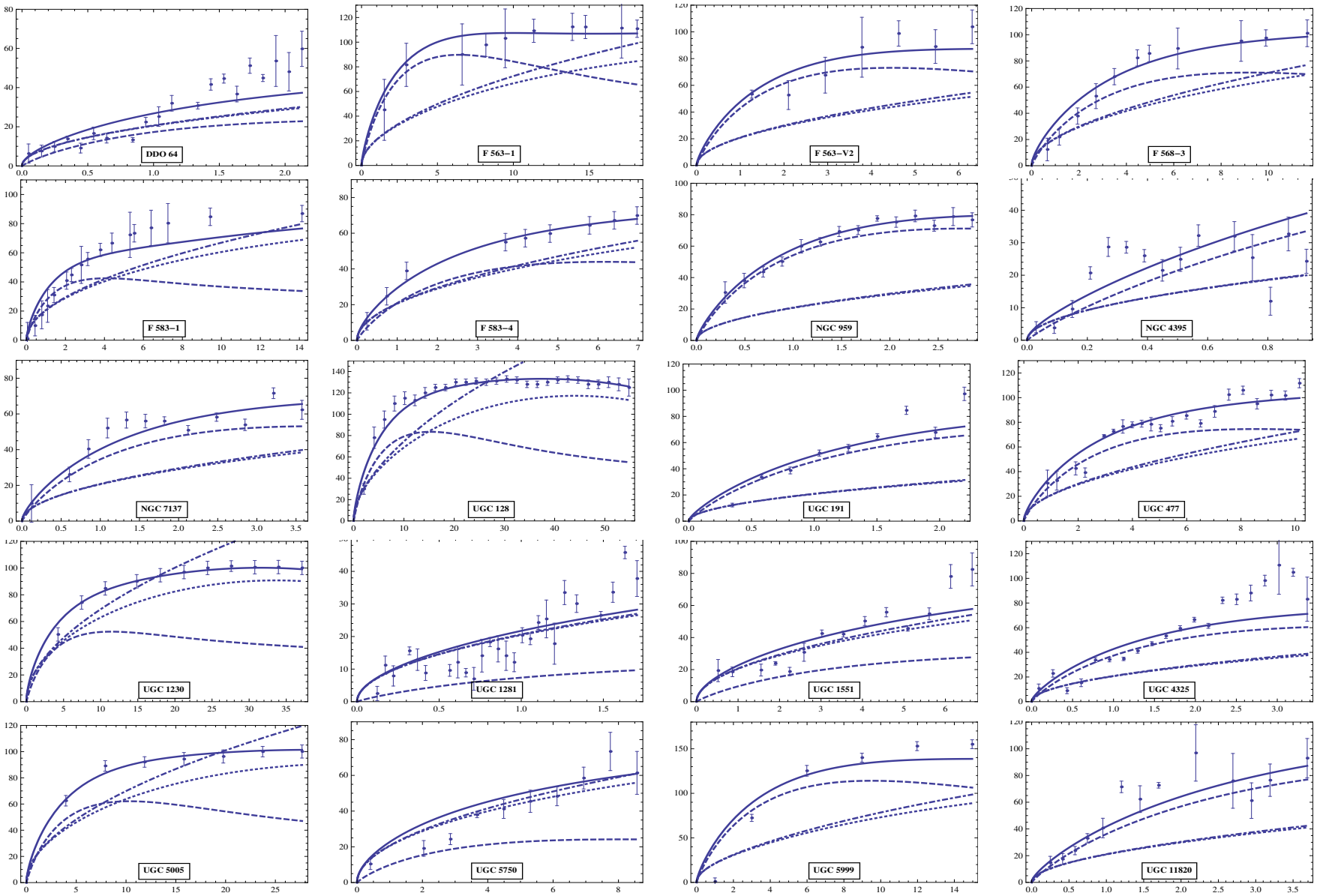


FIG. 4: Fitting to the rotational velocities of the LSB 20 galaxy sample

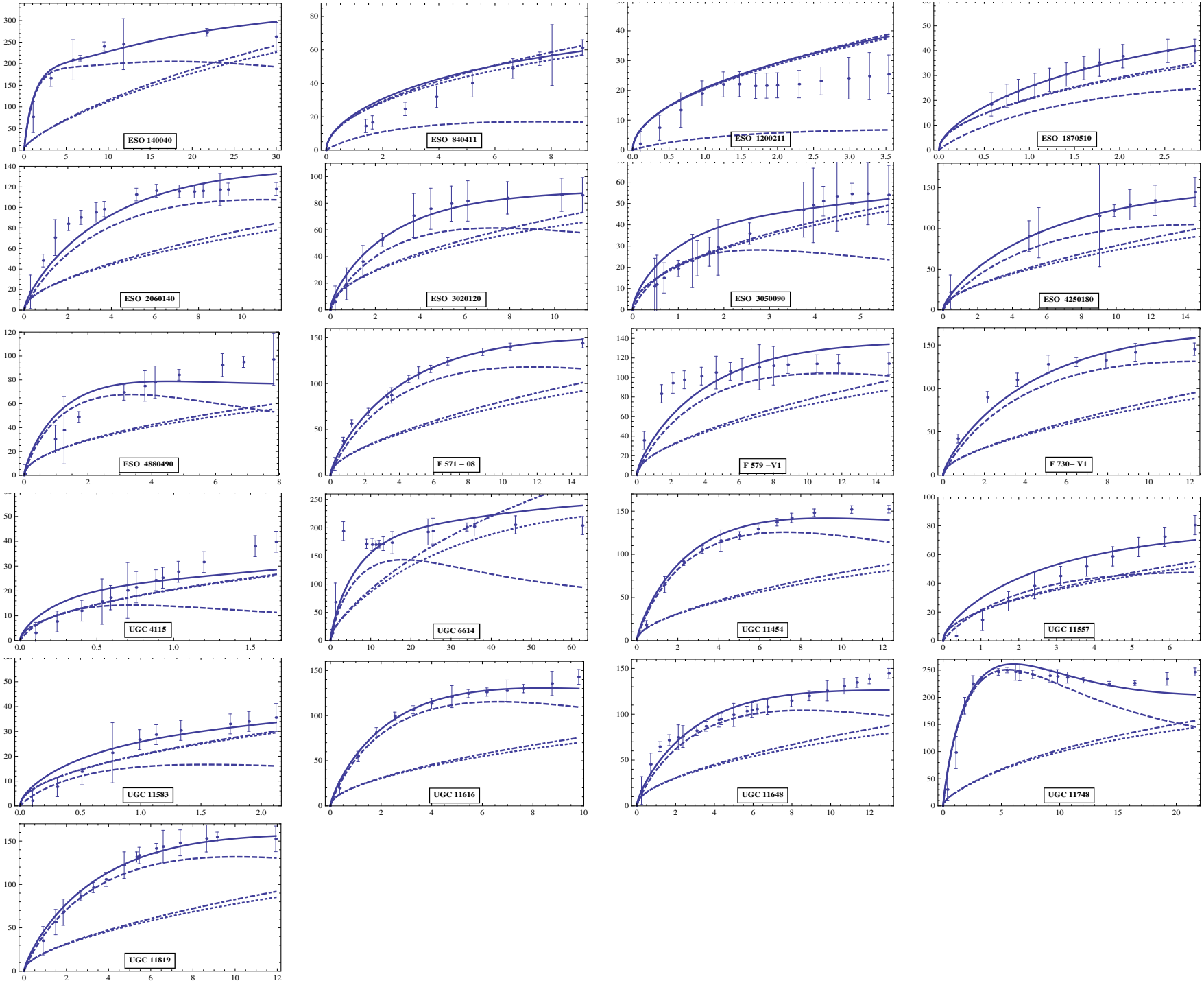


FIG. 5: Fitting to the rotational velocities of the LSB 21 galaxy sample

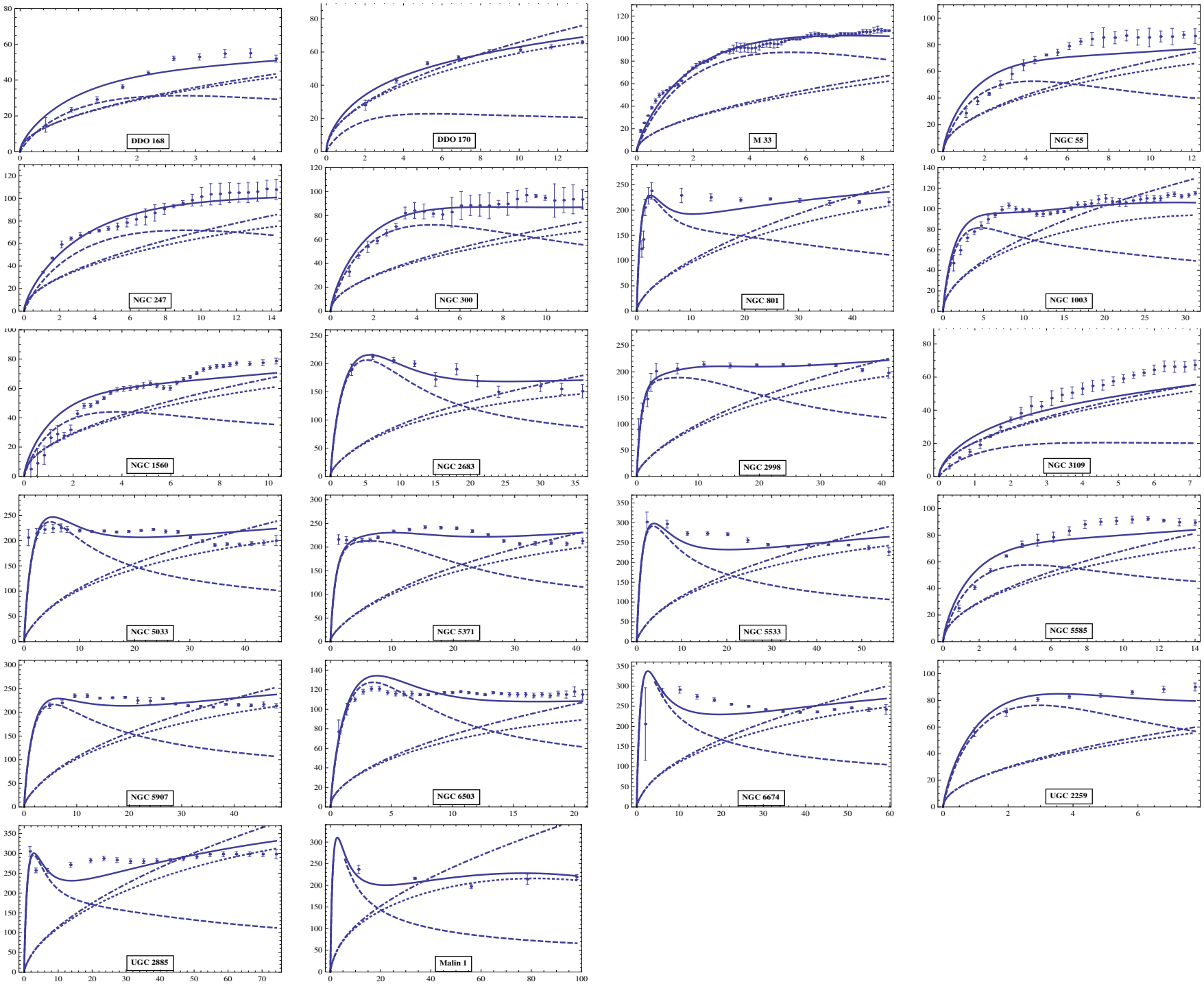


FIG. 6: Fitting to the rotational velocities of the Miscellaneous 22 galaxy sample

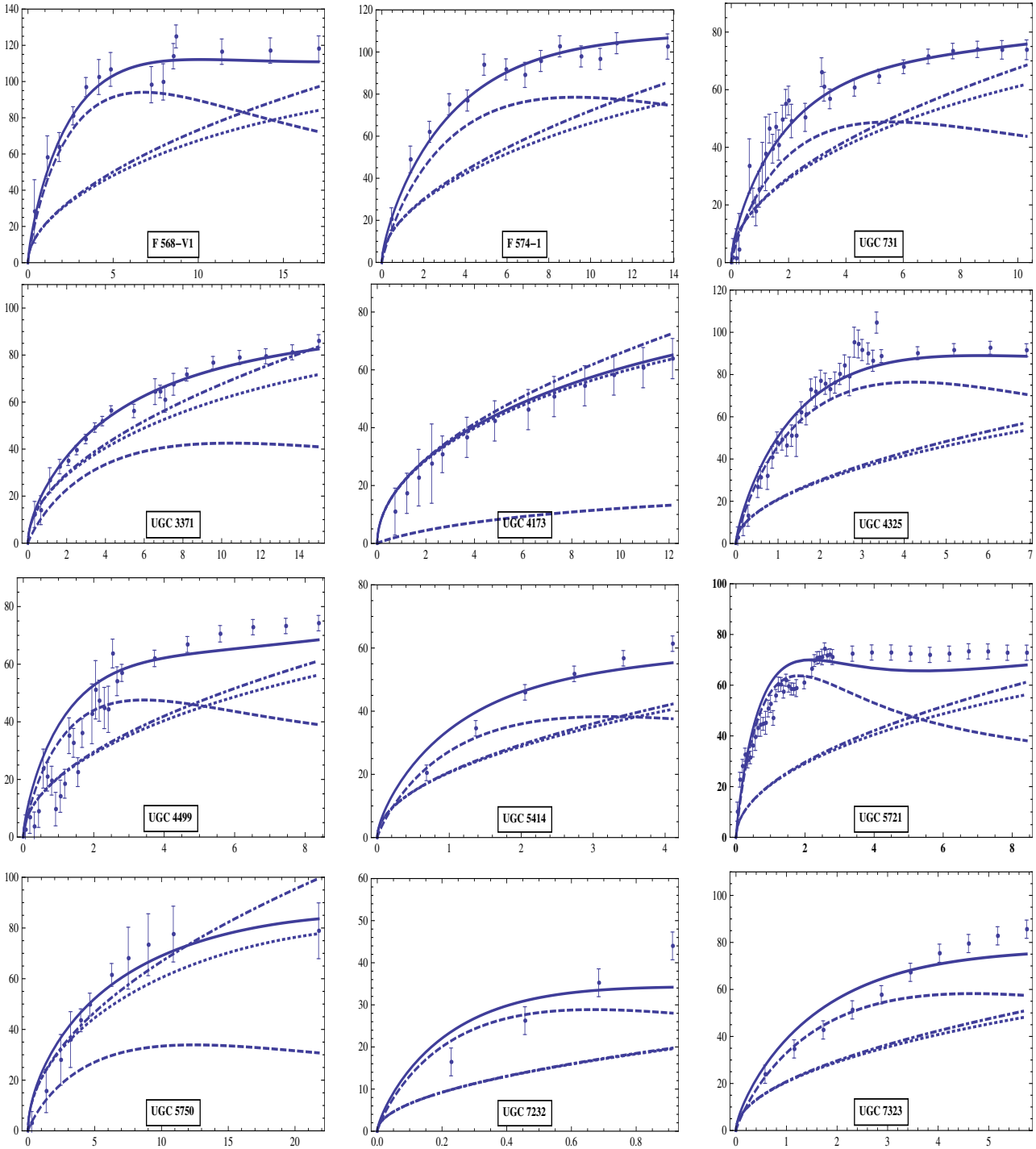


FIG. 7: Fitting to the rotational velocities (in km sec^{-1}) of the 24 dwarf galaxy sample – Part 1

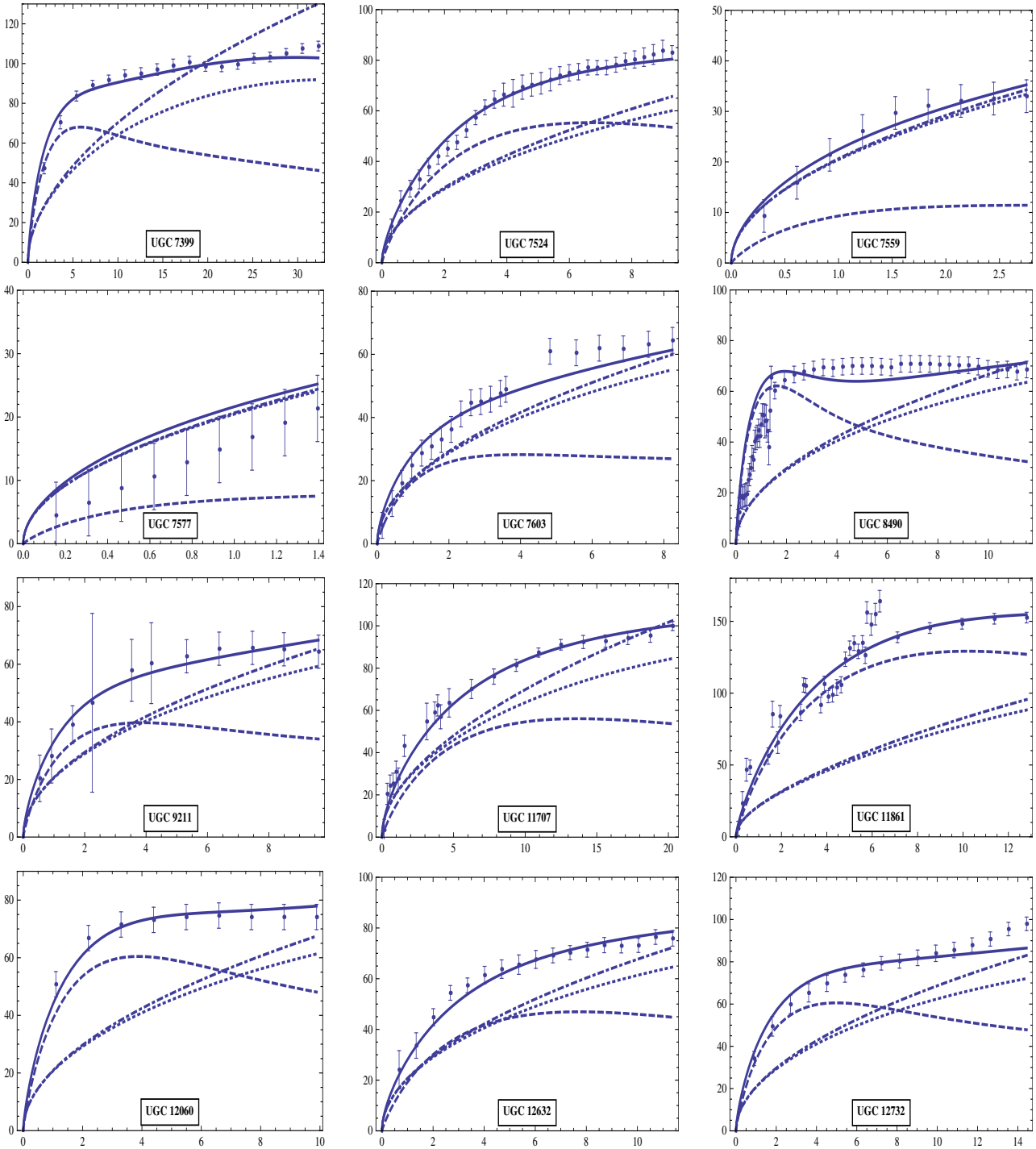


FIG. 8: Fitting to the rotational velocities of the 24 dwarf galaxy sample – Part 2

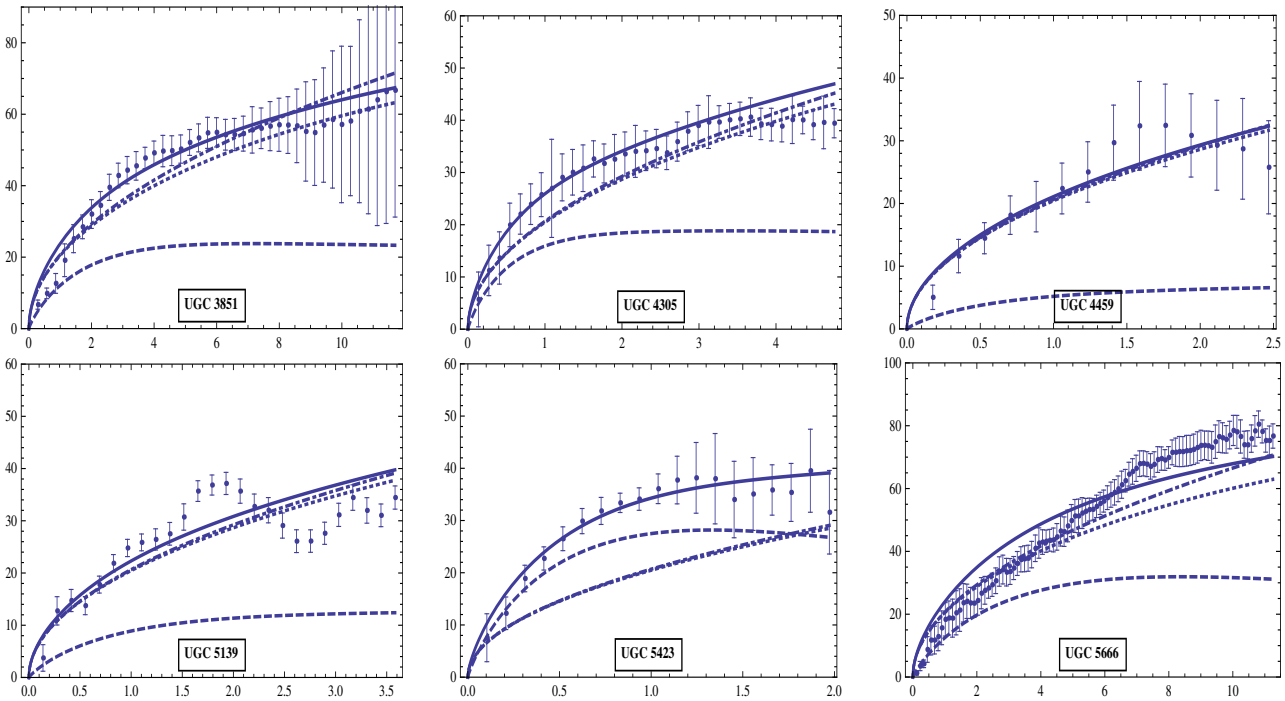


FIG. 9 Fitting to the rotational velocities of the 6 dwarf galaxy sample

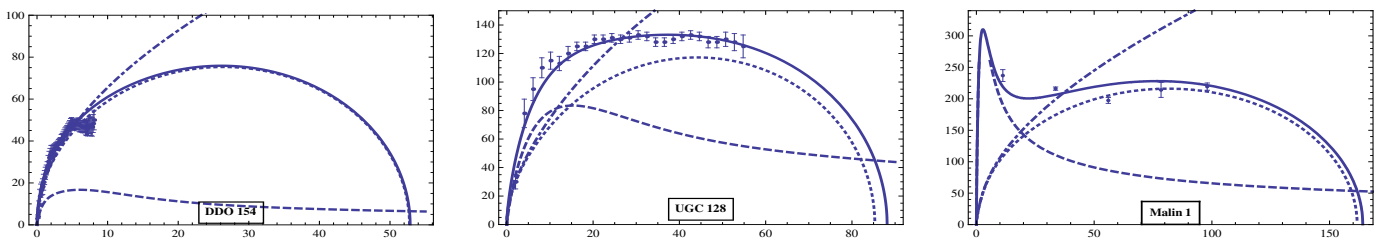


FIG. 10: Extended distance predictions for DDO 154, UGC 128, and Malin 1

6 SUPERSYMMETRY VERSUS CONFORMAL SYMMETRY – THE SCORECARD

(1) In flat space physics an interplay between bosons and fermions can render logarithmically divergent Feynman diagrams finite. **Conformal symmetry takes care of quartic and quadratic as well.**

(2) An interplay between bosons and fermions can cancel the perturbative quadratic divergence that an elementary Higgs scalar field would possess (the hierarchy problem). **Problem does not exist if Higgs is composite.**

(3) The existence of fermionic supersymmetry generators allows one to evade the Coleman-Mandula theorem that forbids the combining of spacetime and bosonic internal symmetry generators in a common Lie algebra. **No need to put spacetime in a Lie group**

(4) With the inclusion of supersymmetry one can potentially achieve a unification of the coupling constants of $SU(3) \times SU(2)_{\mathrm{L}} \times U(1)$ at a grand unified energy scale. **Not addressed by conformal symmetry, but would need something beyond standard model.**

(5) In the presence of gravity an interplay between bosons and fermions can cancel the quartic divergence in the vacuum energy. **Ditto conformal gravity since conformal gravity graviton is a boson. All contributions to vacuum energy under control and cosmological constant problem solved.**

(6) Cancellation of perturbative infinities can also be found in supergravity, the local version of supersymmetry. **Local conformal symmetry also cancels infinities, including those associated with mass generation.**

(7) With supersymmetry one can construct a consistent candidate quantum theory of gravity, string theory, which permits a possible unification of all of the fundamental forces and a metrication (geometrization) of them. **Conformal gravity is a consistent quantum gravity theory. Only requires four dimensions and no new particles. Admits of a metrication of the fundamental forces.**

(8) Finally, with supersymmetry one has a prime candidate for dark matter. **With conformal gravity, dark matter not needed, nor dark energy either.**

Properties of the THINGS 18 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\mathtt{cm}^{-1}$)	Data Sources v	L	R_0	HI
DDO 0154	LSB	4.2	0.007	0.8	8.1	0.03	0.003	0.45	1.12	[?]	[?]	[?]	[?]
IC 2574	LSB	4.5	0.345	4.2	13.1	0.19	0.098	0.28	1.69	[?]	[?]	[?]	[?]
NGC 0925	LSB	8.7	1.444	3.9	12.4	0.41	1.372	0.95	4.17	[?]	[?]	[?]	[?]
NGC 2403	HSB	4.3	1.647	2.7	23.9	0.46	2.370	1.44	2.89	[?]	[?]	[?]	[?]
NGC 2841	HSB	14.1	4.742	3.5	51.6	0.86	19.552	4.12	5.83	[?]	[?]	[?]	[?]
NGC 2903	HSB	9.4	4.088	3.0	30.9	0.49	7.155	1.75	3.75	[?]	[?]	[?]	[?]
NGC 2976	LSB	3.6	0.201	1.2	2.6	0.01	0.322	1.60	10.43	[?]	[?]	[?]	[?]
NGC 3031	HSB	3.7	3.187	2.6	15.0	0.38	8.662	2.72	9.31	[?]	[?]	[?]	[?]
NGC 3198	HSB	14.1	3.241	4.0	38.6	1.06	3.644	1.12	2.09	[?]	[?]	[?]	[?]
NGC 3521	HSB	12.2	4.769	3.3	35.3	1.03	9.245	1.94	4.21	[?]	[?]	[?]	[?]
NGC 3621	HSB	7.4	2.048	2.9	28.7	0.89	2.891	1.41	3.18	[?]	[?]	[?]	[?]
NGC 3627	HSB	10.2	3.700	3.1	8.2	0.10	6.622	1.79	15.64	[?]	[?]	[?]	[?]
NGC 4736	HSB	5.0	1.460	2.1	10.3	0.05	1.630	1.60	4.66	[?]	[?]	[?]	[?]
NGC 4826	HSB	5.4	1.441	2.6	15.8	0.03	3.640	2.53	5.46	[?]	[?]	[?]	[?]
NGC 5055	HSB	9.2	3.622	2.9	44.4	0.76	6.035	1.87	2.36	[?]	[?]	[?]	[?]
NGC 6946	HSB	6.9	3.732	2.9	22.4	0.57	6.272	1.68	6.39	[?]	[?]	[?]	[?]
NGC 7331	HSB	14.2	6.773	3.2	24.4	0.85	12.086	1.78	9.61	[?]	[?]	[?]	[?]
NGC 7793	HSB	5.2	0.910	1.7	10.3	0.16	0.793	0.87	3.61	[?]	[?]	[?]	[?]

Properties of the Ursa Major 30 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\mathtt{cm}^{-1}$)	Data Sources v	L	R_0	HI
NGC 3726	HSB	17.4	3.340	3.2	31.5	0.60	3.82	1.15	3.19	[?]	[?]	[?]	[?]
NGC 3769	HSB	15.5	0.684	1.5	32.2	0.41	1.36	1.99	1.43	[?]	[?]	[?]	[?]
NGC 3877	HSB	15.5	1.948	2.4	9.8	0.11	3.44	1.76	10.51	[?]	[?]	[?]	[?]
NGC 3893	HSB	18.1	2.928	2.4	20.5	0.59	5.00	1.71	3.85	[?]	[?]	[?]	[?]
NGC 3917	LSB	16.9	1.334	2.8	13.9	0.17	2.23	1.67	4.85	[?]	[?]	[?]	[?]
NGC 3949	HSB	18.4	2.327	1.7	7.2	0.35	2.37	1.02	14.23	[?]	[?]	[?]	[?]
NGC 3953	HSB	18.7	4.236	3.9	16.3	0.31	9.79	2.31	10.20	[?]	[?]	[?]	[?]
NGC 3972	HSB	18.6	0.978	2.0	9.0	0.13	1.49	1.53	7.18	[?]	[?]	[?]	[?]
NGC 3992	HSB	25.6	8.456	5.7	49.6	1.94	13.94	1.65	4.08	[?]	[?]	[?]	[?]
NGC 4010	LSB	18.4	0.883	3.4	10.6	0.29	2.03	2.30	5.03	[?]	[?]	[?]	[?]
NGC 4013	HSB	18.6	2.088	2.1	33.1	0.32	5.58	2.67	3.14	[?]	[?]	[?]	[?]
NGC 4051	HSB	14.6	2.281	2.3	9.9	0.18	3.17	1.39	8.52	[?]	[?]	[?]	[?]
NGC 4085	HSB	19.0	1.212	1.6	6.5	0.15	1.34	1.11	10.21	[?]	[?]	[?]	[?]
NGC 4088	HSB	15.8	2.957	2.8	18.8	0.64	4.67	1.58	5.79	[?]	[?]	[?]	[?]
NGC 4100	HSB	21.4	3.388	2.9	27.1	0.44	5.74	1.69	3.35	[?]	[?]	[?]	[?]
NGC 4138	LSB	15.6	0.827	1.2	16.1	0.11	2.97	3.59	5.04	[?]	[?]	[?]	[?]
NGC 4157	HSB	18.7	2.901	2.6	30.9	0.88	5.83	2.01	3.99	[?]	[?]	[?]	[?]
NGC 4183	HSB	16.7	1.042	2.9	19.5	0.30	1.43	1.38	2.36	[?]	[?]	[?]	[?]
NGC 4217	HSB	19.6	3.031	3.1	18.2	0.30	5.53	1.83	6.28	[?]	[?]	[?]	[?]
NGC 4389	HSB	15.5	0.610	1.2	4.6	0.04	0.42	0.68	9.49	[?]	[?]	[?]	[?]
UGC 6399	LSB	18.7	0.291	2.4	8.1	0.07	0.59	2.04	3.42	[?]	[?]	[?]	[?]
UGC 6446	LSB	15.9	0.263	1.9	13.6	0.24	0.36	1.36	1.70	[?]	[?]	[?]	[?]
UGC 6667	LSB	19.8	0.422	3.1	8.6	0.10	0.71	1.67	3.09	[?]	[?]	[?]	[?]
UGC 6818	LSB	21.7	0.352	2.1	8.4	0.16	0.11	0.33	2.35	[?]	[?]	[?]	[?]
UGC 6917	LSB	18.9	0.563	2.9	10.9	0.22	1.24	2.20	4.05	[?]	[?]	[?]	[?]
UGC 6923	LSB	18.0	0.297	1.5	5.3	0.08	0.35	1.18	4.43	[?]	[?]	[?]	[?]
UGC 6930	LSB	17.0	0.601	2.2	15.7	0.29	1.02	1.69	2.68	[?]	[?]	[?]	[?]
UGC 6973	HSB	25.3	1.647	2.2	11.0	0.35	3.99	2.42	10.58	[?]	[?]	[?]	[?]
UGC 6983	LSB	20.2	0.577	2.9	17.6	0.37	1.28	2.22	2.43	[?]	[?]	[?]	[?]
UGC 7089	LSB	13.9	0.352	2.3	7.1	0.07	0.35	0.98	3.18	[?]	[?]	[?]	[?]

Properties of the LSB 20 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\mathtt{cm}^{-1}$)	Data Sources v	L	R_0	HI
DDO 0064	LSB	6.8	0.015	1.3	2.1	0.02	0.04	2.87	6.05	[?]	[?]	[?]	[?]
F563-1	LSB	46.8	0.140	2.9	18.2	0.29	1.35	9.65	2.44	[?]	[?]	[?]	[?]
F563-V2	LSB	57.8	0.266	2.0	6.3	0.20	0.60	2.26	6.15	[?]	[?]	[?]	[?]
F568-3	LSB	80.0	0.351	4.2	11.6	0.30	1.20	3.43	3.16	[?]	[?]	[?]	[?]
F583-1	LSB	32.4	0.064	1.6	14.1	0.18	0.15	2.32	1.92	[?]	[?]	[?]	[?]
F583-4	LSB	50.8	0.096	2.8	7.0	0.06	0.31	3.25	2.52	[?]	[?]	[?]	[?]
NGC 0959	LSB	13.5	0.333	1.3	2.9	0.05	0.37	1.11	7.43	[?]	[?]	[?]	[?]
NGC 4395	LSB	4.1	0.374	2.7	0.9	0.13	0.83	2.21	2.29	[?]	[?]	[?]	[?]
NGC 7137	LSB	25.0	0.959	1.7	3.6	0.10	0.27	0.28	3.91	[?]	[?]	ES	[?]
UGC 0128	LSB	64.6	0.597	6.9	54.8	0.73	2.75	4.60	1.03	[?]	[?]	[?]	[?]
UGC 0191	LSB	15.9	0.129	1.7	2.2	0.26	0.49	3.81	15.48	[?]	[?]	[?]	[?]
UGC 0477	LSB	35.8	0.871	3.5	10.2	1.02	1.00	1.14	4.42	[?]	[?]	ES	[?]
UGC 1230	LSB	54.1	0.366	4.7	37.1	0.65	0.67	1.82	0.97	[?]	[?]	[?]	[?]
UGC 1281	LSB	5.1	0.017	1.6	1.7	0.03	0.01	0.53	3.02	[?]	[?]	[?]	[?]
UGC 1551	LSB	35.6	0.780	4.2	6.6	0.44	0.16	0.20	3.69	[?]	[?]	[?]	[?]
UGC 4325	LSB	11.9	0.373	1.9	3.4	0.10	0.40	1.08	7.39	[?]	[?]	[?]	[?]
UGC 5005	LSB	51.4	0.200	4.6	27.7	0.28	1.02	5.11	1.30	[?]	[?]	[?]	[?]
UGC 5750	LSB	56.1	0.472	3.3	8.6	0.10	0.10	0.21	1.58	[?]	[?]	[?]	[?]
UGC 5999	LSB	44.9	0.170	4.4	15.0	0.18	3.36	19.81	5.79	[?]	[?]	[?]	[?]
UGC 11820	LSB	17.1	0.169	3.6	3.7	0.40	1.68	9.95	8.44	[?]	[?]	[?]	[?]

Properties of the LSB 21 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\mathtt{cm}^{-1}$)	Data Sources v	L	R_0	HI
ESO 0140040	LSB	217.8	7.169	10.1	30.0		20.70	3.38	8.29	[?]	[?]	[?]	NA
ESO 0840411	LSB	82.4	0.287	3.5	9.1		0.06	0.21	1.49	[?]	[?]	ES	NA
ESO 1200211	LSB	15.2	0.028	2.0	3.5		0.01	0.20	0.66	[?]	[?]	ES	NA
ESO 1870510	LSB	16.8	0.054	2.1	2.8		0.09	1.62	2.02	[?]	[?]	[?]	NA
ESO 2060140	LSB	59.6	0.735	5.1	11.6		3.51	4.78	4.34	[?]	[?]	[?]	NA
ESO 3020120	LSB	70.9	0.717	3.4	11.2		0.77	1.07	2.37	[?]	[?]	ES	NA
ESO 3050090	LSB	13.2	0.186	1.3	5.6		0.06	0.32	1.87	[?]	[?]	ES	NA
ESO 4250180	LSB	88.3	2.600	7.3	14.6		4.79	1.84	5.17	[?]	[?]	[?]	NA
ESO 4880490	LSB	28.7	0.139	1.6	7.8		0.43	3.07	4.34	[?]	[?]	ES	NA
F571-8	LSB	50.3	0.191	5.4	14.6	0.16	4.48	23.49	5.10	[?]	[?]	[?]	[?]
F579-V1	LSB	86.9	0.557	5.2	14.7	0.21	3.33	5.98	3.18	[?]	[?]	[?]	[?]
F730-V1	LSB	148.3	0.756	5.8	12.2		5.95	7.87	6.22	[?]	[?]	[?]	NA
UGC 04115	LSB	5.5	0.004	0.3	1.7		0.01	0.97	3.42	[?]	[?]	[?]	NA
UGC 06614	LSB	86.2	2.109	8.2	62.7	2.07	9.70	4.60	2.39	[?]	[?]	[?]	[?]
UGC 11454	LSB	93.9	0.456	3.4	12.3		3.15	6.90	6.79	[?]	[?]	[?]	NA
UGC 11557	LSB	23.7	1.806	3.0	6.7	0.25	0.37	0.20	3.49	[?]	[?]	[?]	[?]
UGC 11583	LSB	7.1	0.012	0.7	2.1		0.01	0.96	2.15	[?]	[?]	[?]	NA
UGC 11616	LSB	74.9	2.159	3.1	9.8		2.43	1.13	7.49	[?]	[?]	[?]	NA
UGC 11648	LSB	49.0	4.073	4.0	13.0		2.57	0.63	5.79	[?]	[?]	[?]	NA
UGC 11748	LSB	75.3	23.930	2.6	21.6		9.67	0.40	1.01	[?]	[?]	[?]	NA
UGC 11819	LSB	61.5	2.155	4.7	11.9		4.83	2.24	7.03	[?]	[?]	[?]	NA

Properties of the Miscellaneous 22 Galaxy Sample

Galaxy	Type	Distance (Mpc)	$L_{\rm B}$ ($10^{10}{\rm L}_\odot$)	R_0 (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^{10}M_\odot$)	$M_{\rm disk}$ ($10^{10}M_\odot$)	$(M/L)_{\rm stars}$ ($M_\odot/L_\odot$)	$(v^2/c^2R)_{\rm last}$ (10^{-30} cm^{-1})	Data Sources v	L	R_0	HI
DDO 0168	LSB	4.5	0.032	1.2	4.4	0.03	0.06	2.03	2.22	[?]	[?]	[?]	[?]
DDO 0170	LSB	16.6	0.023	1.9	13.3	0.09	0.05	1.97	1.18	[?]	[?]	[?]	[?]
M 0033	HSB	0.9	0.850	2.5	8.9	0.11	1.13	1.33	4.62	[?]	[?]	[?]	[?]
NGC 0055	LSB	1.9	0.588	1.9	12.2	0.13	0.30	0.50	2.22	[?]	[?]	[?]	[?]
NGC 0247	LSB	3.6	0.512	4.2	14.3	0.16	1.25	2.43	2.94	[?]	[?]	[?]	[?]
NGC 0300	LSB	2.0	0.271	2.1	11.7	0.08	0.65	2.41	2.69	[?]	[?]	[?]	[?]
NGC 0801	HSB	63.0	4.746	9.5	46.7	1.39	6.93	2.37	3.59	[?]	[?]	[?]	[?]
NGC 1003	LSB	11.8	1.480	1.9	31.2	0.63	0.66	0.45	1.53	[?]	[?]	[?]	[?]
NGC 1560	LSB	3.7	0.053	1.6	10.3	0.12	0.17	3.16	2.16	[?]	[?]	[?]	[?]
NGC 2683	HSB	10.2	1.882	2.4	36.0	0.15	6.03	3.20	2.28	[?]	[?]	[?]	[?]
NGC 2998	HSB	59.3	5.186	4.8	41.1	1.78	7.16	1.75	3.43	[?]	[?]	[?]	[?]
NGC 3109	LSB	1.5	0.064	1.3	7.1	0.06	0.02	0.35	2.29	[?]	[?]	[?]	[?]
NGC 5033	HSB	15.3	3.058	7.5	45.6	1.07	0.27	3.28	3.16	[?]	[?]	[?]	[?]
NGC 5371	HSB	35.3	7.593	4.4	41.0	0.89	8.52	1.44	3.98	[?]	[?]	[?]	[?]
NGC 5533	HSB	42.0	3.173	7.4	56.0	1.39	2.00	4.14	3.31	[?]	[?]	[?]	[?]
NGC 5585	HSB	9.0	0.333	2.0	14.0	0.28	0.36	1.09	2.06	[?]	[?]	[?]	[?]
NGC 5907	HSB	16.5	5.400	5.5	48.0	1.90	2.49	1.89	3.44	[?]	[?]	[?]	[?]
NGC 6503	HSB	5.5	0.417	1.6	20.7	0.14	1.53	3.66	2.30	[?]	[?]	[?]	[?]
NGC 6674	HSB	42.0	4.935	7.1	59.1	2.18	2.00	2.52	3.57	[?]	[?]	[?]	[?]
UGC 2259	LSB	10.0	0.110	1.4	7.8	0.04	0.47	4.23	3.76	[?]	[?]	[?]	[?]
UGC 2885	HSB	80.4	23.955	13.3	74.1	3.98	8.47	0.72	4.31	[?]	[?]	[?]	[?]
Malin 1	LSB	338.5	7.912	84.2	98.0	5.40	1.00	1.32	1.77	[?]	[?]	[?]	[?]

Properties of the 30 Dwarf Galaxy Sample

Galaxy	Distance (Mpc)	$L_{\rm B}$ ($10^9 {\rm L}_\odot^{\rm B}$)	i $^\circ$	$(R_0)_{\rm disk}$ (kpc)	$R_{\rm last}$ (kpc)	$M_{\rm HI}$ ($10^9 M_\odot$)	$M_{\rm disk}$ ($10^9 M_\odot$)	$(M/L_{\rm B})_{\rm disk}$ ($M_\odot/L_\odot^{\rm B}$)	$(v^2/c^2R)_{\rm last}$ ($10^{-30}\,{\rm cm}^{-1}$)
F568-V1	78.20	2.15	40	3.11	17.07	2.32	16.00	7.45	2.95
F574-1	94.10	3.42	65	4.20	13.69	3.31	14.90	4.35	2.77
UGC 731	11.80	0.69	57	2.43	10.30	1.61	3.21	4.63	1.91
UGC 3371	18.75	1.54	49	4.53	15.00	2.62	4.49	2.91	1.78
UGC 4173	16.70	0.33	–5	4.43	12.14	2.24	0.07	0.20	1.21
UGC 4325	11.87	1.71	41	1.92	6.91	1.04	6.51	3.82	4.37
UGC 4499	12.80	1.01	50	1.46	8.38	1.15	1.80	1.79	2.37
UGC 5414	9.40	0.49	55	1.40	4.10	0.57	1.13	2.29	3.31
UGC 5721	7.60	0.48	+10	0.76	8.41	0.57	1.90	3.96	2.27
UGC 5750	56.10	4.72	64	5.60	21.77	1.00	3.68	0.78	1.03
UGC 7232	3.14	0.08	59	0.30	0.91	0.06	0.14	1.76	7.64
UGC 7323	7.90	2.39	47	2.13	5.75	0.70	4.19	1.75	4.59
UGC 7399	24.66	4.61		2.32	32.30	6.38	5.42	1.18	1.32
UGC 7524	4.12	1.37	46	3.02	9.29	1.34	5.29	3.86	2.67
UGC 7559	4.20	0.04	61	0.87	2.75	0.12	0.05	1.32	1.43
UGC 7577	2.13	0.05	–15	0.51	1.39	0.04	0.01	0.20	1.18
UGC 7603	9.45	0.80	78	1.24	8.24	1.04	0.41	0.52	1.81
UGC 8490	5.28	0.95	+10	0.71	11.51	0.72	1.53	1.61	1.47
UGC 9211	14.70	0.33	44	1.54	9.62	1.43	1.23	3.69	1.55
UGC 11707	21.46	1.13	68	5.82	20.30	6.78	9.89	8.76	1.77
UGC 11861	19.55	9.44	50	4.69	12.80	4.33	45.84	4.86	6.55
UGC 12060	15.10	0.39	40	1.70	9.89	1.67	3.45	8.94	2.00
UGC 12632	9.20	0.86	46	3.43	11.38	1.55	4.26	4.97	1.82
UGC 12732	12.40	0.71	39	2.10	14.43	3.23	4.11	5.76	2.40
UGC 3851	4.85	2.33		2.07	11.70	1.32	0.47	0.20	1.37
UGC 4305	2.34	0.41	–5	0.68	4.75	0.31	0.08	0.20	1.18
UGC 4459	3.06	0.03	27	0.60	2.47	0.04	0.01	0.20	0.97
UGC 5139	4.69	0.20	14	0.96	3.58	0.21	0.05	0.24	1.19
UGC 5423	7.14	0.14	45	0.61	1.97	0.05	0.28	2.01	1.82
UGC 5666	3.85	2.53	56	3.56	11.25	1.37	1.96	0.77	1.88

7 The Nambu-Jona-Lasinio (NJL) Chiral Four-Fermion Model

7.1 Quick Review of the NJL Model as a Mean-Field Theory in Hartree-Fock Approximation

$$
\begin{aligned}
I_{\rm NJL} &= \int d^4x \left[i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{g}{2}[\bar{\psi}\psi]^2 - \frac{g}{2}[\bar{\psi}i\gamma_5\psi]^2 \right]. \\
&= \int d^4x \left[i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + \frac{m^2}{2g} \right] + \int d^4x \left[-\frac{g}{2}\left(\bar{\psi}\psi - \frac{m}{g}\right)^2 - \frac{g}{2}\left(\bar{\psi}i\gamma_5\psi\right)^2 \right] \\
I_{\rm NJL} &= I_{\rm MF} + I_{\rm RI}, \quad \text{i.e. introduce mass term with } m \text{ as a trial parameter}
\end{aligned} \tag{48}
$$

$$
\langle\Omega_{\rm m}| \left[\bar{\psi}\psi - \frac{m}{g}\right]^2 |\Omega_{\rm m}\rangle = \langle\Omega_{\rm m}| \left[\bar{\psi}\psi - \frac{m}{g}\right] |\Omega_{\rm m}\rangle^2 = 0, \tag{49}
$$

$$
\langle\Omega_{\rm m}|\bar{\psi}\psi|\Omega_{\rm m}\rangle = -i\int \frac{d^4k}{(2\pi)^4}{\rm Tr}\left[\frac{1}{\not{p} - m + i\epsilon}\right] = \frac{m}{g}, \tag{50}
$$

$$
-\frac{M\Lambda^2}{4\pi^2} + \frac{M^3}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) = \frac{M}{g}. \tag{51}
$$

$$
\tilde{\epsilon}(m) = i\int \frac{d^4p}{(2\pi)^4}{\rm Tr}\ \ln\left[\frac{\not{p} - m + i\epsilon}{\not{p} + i\epsilon}\right] - \frac{m^2}{2g} = \frac{m^4}{16\pi^2}\ln\left(\frac{\Lambda^2}{m^2}\right) - \frac{m^2M^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) + \frac{m^4}{32\pi^2} \tag{52}
$$

7.2 Higgs-Like Lagrangian

Vacuum to vacuum functional due to $m(x)\bar{\psi}\psi$: $\langle\Omega(t=-\infty)|\Omega(t=+\infty)\rangle = e^{iW(m(x))}$

$$W(m(x)) = \sum(1/n!)\int d^4x_1...d^4x_n G_0^n(x_1,...,x_n)m(x_1)...m(x_n),$$

$$W(m(x)) = \int d^4x[-\epsilon(m(x)) + (1/2)Z(m(x))\partial_\mu m(x)\partial^\mu m(x) + ...].$$

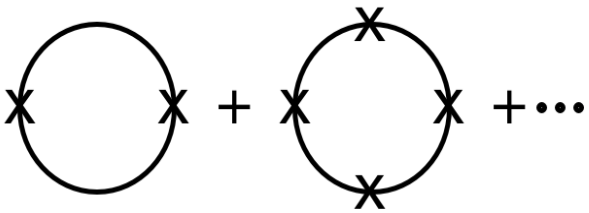


Figure 2: Vacuum energy density $\epsilon(m)$ via an infinite summation of massless graphs with zero-momentum point $m\bar{\psi}\psi$ insertions.

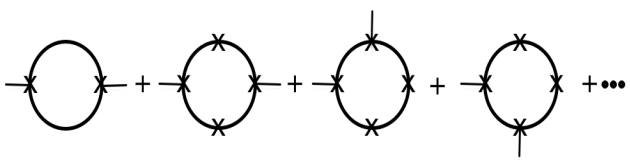


Figure 3: $\Pi_{\rm S}(q^2, m(x))$ developed as an infinite summation of massless graphs, each with two point $m\bar{\psi}\psi$ insertions carrying momentum q_μ (shown as external lines), with all other point $m\bar{\psi}\psi$ insertions carrying zero momentum.

Eguchi and Sugawara (1974), Mannheim (1976):

$$I_{\rm EFF} = \int \frac{d^4x}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right)\left[\frac{1}{2}\partial_\mu m(x)\partial^\mu m(x) + m^2(x)M^2 - \frac{1}{2}m^4(x)\right]. \tag{53}$$

Set $\phi = \langle\Omega_m|\bar{\psi}(1+\gamma^5)\psi|\Omega_m\rangle$. Couple to an axial gauge field via $\bar{\psi}g_A\gamma^\mu\gamma^5 A_{\mu 5}\psi$. Get effective Higgs:

$$I_{\rm EFF} = \int \frac{d^4x}{8\pi^2} \ln\left(\frac{\Lambda^2}{M^2}\right) \left[\frac{1}{2}|(\partial_\mu - 2ig_{\rm A}A_{\mu 5})\phi(x)|^2 + |\phi(x)|^2 M^2 - \frac{1}{2}|\phi(x)|^4 - \frac{g_{\rm A}^2}{6} F_{\mu\nu 5}F^{\mu\nu 5}\right]. \tag{54}$$

7.3 The Collective Tachyon Modes when the Fermion is Massless

$$\begin{aligned}\Pi_{\rm S}(x) &= \langle\Omega|T(\bar\psi(x)\psi(x)\bar\psi(0)\psi(0))|\Omega\rangle,\\ \Pi_{\rm P}(x) &= \langle\Omega|T(\bar\psi(x)i\gamma_5\psi(x)\bar\psi(0)i\gamma_5\psi(0))|\Omega\rangle\end{aligned} \tag{55}$$

$$\begin{aligned}\Pi_{\rm S}(q^2) &= -i\int\frac{d^4p}{(2\pi)^4}{\rm Tr}\left[\frac{1}{\not p+i\epsilon}\frac{1}{\not p+\not q+i\epsilon}\right],\\ \Pi_{\rm P}(q^2) &= -i\int\frac{d^4p}{(2\pi)^4}{\rm Tr}\left[i\gamma_5\frac{1}{\not p+i\epsilon}i\gamma_5\frac{1}{\not p+\not q+i\epsilon}\right],\end{aligned} \tag{56}$$

$$\Pi_{\rm S}(q^2)=\Pi_{\rm P}(q^2)=-\frac{\Lambda^2}{4\pi^2}-\frac{q^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{-q^2}\right)-\frac{q^2}{8\pi^2}. \tag{57}$$

$$\begin{aligned}T_{\rm S}(q^2) &= g+g\Pi_{\rm S}(q^2)g+g\Pi_S(q^2)g\Pi_S(q^2)g+...=\frac{g}{1-g\Pi_{\rm S}(q^2)}=\frac{1}{g^{-1}-\Pi_{\rm S}(q^2)},\\ T_{\rm P}(q^2) &= g+g\Pi_{\rm P}(q^2)g+g\Pi_P(q^2)g\Pi_P(q^2)g+...=\frac{g}{1-g\Pi_{\rm P}(q^2)}=\frac{1}{g^{-1}-\Pi_{\rm P}(q^2)}.\end{aligned} \tag{58}$$

$$T_{\rm S}(q^2)=T_{\rm P}(q^2)=\frac{Z^{-1}}{(q^2+2M^2)},\ Z=\frac{1}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) \tag{59}$$

7.4 The Collective Goldstone and Higgs Modes when the Fermion is Massive

$$
\begin{aligned}
\Pi_{\rm S}(q^2, M) &= -i\int \frac{d^4p}{(2\pi)^4}{\rm Tr}\left[\frac{1}{\not p - M + i\epsilon}\frac{1}{\not p + \not q - M + i\epsilon}\right] \\
&= -\frac{\Lambda^2}{4\pi^2} + \frac{M^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) + \frac{(4M^2 - q^2)}{8\pi^2} + \frac{(4M^2 - q^2)}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) \\
&-\frac{1}{8\pi^2}\frac{(4M^2 - q^2)^{3/2}}{(-q^2)^{1/2}}\ln\left(\frac{(4M^2 - q^2)^{1/2} + (-q^2)^{1/2}}{(4M^2 - q^2)^{1/2} - (-q^2)^{1/2}}\right). \qquad (60)
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{\rm P}(q^2, M) &= -i\int \frac{d^4p}{(2\pi)^4}{\rm Tr}\left[i\gamma_5\frac{1}{\not p - M + i\epsilon}i\gamma_5\frac{1}{\not p + \not q - M + i\epsilon}\right] \\
&= -\frac{\Lambda^2}{4\pi^2} + \frac{M^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) - \frac{q^2}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right) + \frac{(4M^2 - q^2)}{8\pi^2} \\
&+\frac{(8M^4 - 8M^2q^2 + q^4)}{8\pi^2(-q^2)^{1/2}(4M^2 - q^2)^{1/2}}\ln\left(\frac{(4M^2 - q^2)^{1/2} + (-q^2)^{1/2}}{(4M^2 - q^2)^{1/2} - (-q^2)^{1/2}}\right). \qquad (61)
\end{aligned}
$$

$$
T_{\rm S}(q^2) \;=\; \frac{R_{\rm S}^{-1}}{(q^2 - 4M^2)}, \qquad T_{\rm P}(q^2) = \frac{R_{\rm P}^{-1}}{q^2}, \qquad (62)
$$

$$
R_{\rm S} \;=\; R_{\rm P} = \frac{1}{8\pi^2}\ln\left(\frac{\Lambda^2}{M^2}\right). \qquad (63)
$$

7.5 Fixing the Wick Contour for the Vacuum Energy Density

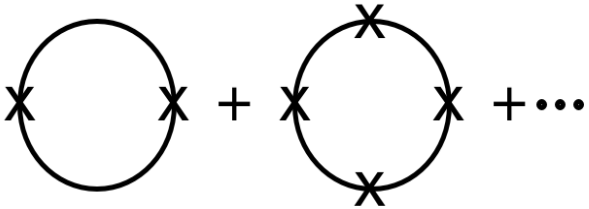


Figure 4: Vacuum energy density $\epsilon(m)$ via an infinite summation of massless graphs with zero-momentum point $m\bar{\psi}\psi$ insertions.

$$\epsilon(m) \;=\; \langle\Omega_m|H_m|\Omega_m\rangle - \langle\Omega_0|H_0|\Omega_0\rangle, \tag{64}$$

$$\epsilon(m) = \langle\Omega_m|H_m|\Omega_m\rangle - \langle\Omega_0|H_m|\Omega_0\rangle, \tag{65}$$

$$\epsilon(m) = \sum \frac{1}{n!} G_0^{(n)}(q_\mu = 0, m = 0) m^n \tag{66}$$

Massive theory contour fixed by massless theory graphs

$$\begin{aligned} G^{(2)}(q_\mu = 0, m = 0) &= -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\not{p} + i\epsilon}\frac{1}{\not{p} + i\epsilon}\right] \\ G^{(4)}(q_\mu = 0, m = 0) &= -i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\not{p} + i\epsilon}\frac{1}{\not{p} + i\epsilon}\frac{1}{\not{p} + i\epsilon}\frac{1}{\not{p} + i\epsilon}\right] \end{aligned} \tag{67}$$

$$\epsilon(m) \;=\; i \int \frac{d^4p}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(-1)}{2n} \mathrm{Tr}\left[(-i)^2 \left(\frac{i}{\not{p} + i\epsilon}\right)^2 m^2\right]^n = i \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\, \ln\left[\frac{\not{p} - m + i\epsilon}{\not{p} + i\epsilon}\right], \tag{68}$$

$$\int_{-\infty}^{\infty} dp_0 + \int_{\infty}^{i\infty} dp_0 + \int_{i\infty}^{-i\infty} dp_0 + \int_{-i\infty}^{-\infty} dp_0 = 0. \tag{69}$$

$$-i \int_{-\infty}^{\infty} dp_0 = -i \int_{-i\infty}^{i\infty} dp_0 = \int_{-\infty}^{\infty} dp_4 \tag{70}$$

8 Johnson-Baker-Willey Electrodynamics

8.1 Vanishing of the Bare Fermion Mass

Johnson, Baker, Willey (1964-1973)

$$m_0 = m\left(\Lambda^2/m^2\right)^{\gamma(\alpha)/2} \tag{71}$$

m_0 thus vanishes if $\gamma(\alpha) < 0$, and $\delta(m) = m - m_0$ is finite and m is non-zero. So looks like dynamical symmetry breaking.

$$\text{Adler, Bardeen (1971)}: \qquad \left[m\frac{\partial}{m} + \beta(\alpha)\frac{\partial}{\partial\alpha}\right]\tilde{S}^{-1}(p) = -m[1-\gamma_\theta(\alpha)]\tilde{\Gamma}_{\rm S}(p,p,0) \tag{72}$$

$$\text{asymptotically} \qquad \tilde{S}^{-1}(p) \;=\; \not{p} - m\left(\frac{-p^2-i\epsilon}{m^2}\right)^{\gamma_\theta(\alpha)/2} + i\epsilon, \qquad \tilde{\Gamma}_{\rm S}(p,p,0) = \left(\frac{-p^2-i\epsilon}{m^2}\right)^{\gamma_\theta(\alpha)/2}. \tag{73}$$

Scaling achievable if $\beta(\alpha) = 0$. However also achievable if keep photon propagator canonical and sum all photon exchange diagrams (Johnson, Baker and Willey 1964). And if keep planar graphs only then (Maskawa and Nakajima 1974, 1975) get scaling if $\alpha \leq \pi/3$, with $3 > D[\theta = \bar{\psi}\psi(\alpha < \pi/3)] = 3 + \gamma_\theta(\alpha) > 2$ and $D[\theta = \bar{\psi}\psi(\alpha = \pi/3)] = 3 + \gamma_\theta(\alpha = \pi/3) = 2$. But since $Z_\theta = (\Lambda^2/m^2)^{-\gamma_\theta(\alpha)/2}$, we see that $m_0(\bar{\psi}\psi)_0 = m(\bar{\psi}\psi) \neq 0$. Thus (Baker-Johnson evasion of the Goldstone theorem)) the theory corresponds to a non-chiral symmetric Abelian gluon model with a non-zero bare fermion mass term, and no Goldstone boson. If $\alpha > \pi/3$ get Goldstone boson (Maskawa and Nakajima).

Prevailing wisdom: no Goldstone boson if $\alpha \leq \pi/3$. Only get one if $\alpha > \pi/3$, i.e only if strong coupling. In this talk we challenge this wisdom and show that there is a Goldstone boson at $\alpha = \pi/3$ or at the α that obeys $\beta(\alpha) = 0$, which could be weak.

$$\{\gamma_5, \Sigma(p)\} \;=\; \int d^4k K(p,k,0)S(k)\{\gamma_5,\Sigma(k)\}S(k) + 2m_0\int d^4k K(p,k,0)S(k)\gamma_5 S(k). \tag{74}$$

$$\tilde{\Gamma}_{\rm P}(p,p+q,q) = Z_{\rm P} i\gamma^5 + \int d^4k \tilde{K}(p,k,q)\tilde{S}(k)\tilde{\Gamma}_{\rm P}(k,k+q,q)\tilde{S}(k+q). \tag{75}$$

$$-i\gamma_5\tilde{\Gamma}_{\rm P} = \frac{Z_{\rm P}}{1 - Z_{\rm P}\Pi} = \frac{1}{Z_{\rm P}^{-1} - \Pi}, \qquad T_{\rm P} = \frac{g}{1 - g\Pi} = \frac{1}{g^{-1} - \Pi}. \tag{76}$$

Two possibilities: (1) $m_0 = 0$ identically, $Z_{\rm P}$ non-zero is dynamical symmetry breaking. (2) $m_0 \to 0$ and $Z_{\rm P} = Z_{\rm S} \to 0$ as $\Lambda^2 \to \infty$ is Baker-Johnson evasion.

8.2 Non-Zero Vacuum Expectation Value for $\bar{\psi}\psi$ and the condition $\gamma_\theta(\alpha) = -1$

Even if m_0 vanishes in the limit of infinite cut-off, physical mass could still be zero since it satisfies a homogeneous equation. However (Mannheim 1974, 1975), if

$$\gamma_\theta(\alpha) = -1, \tag{77}$$

then $\tilde{\epsilon}(m) < \tilde{\epsilon}(m = 0)$, with the energy density of the vacuum having a double-well potential form and minimum at non-zero m:

$$\tilde{\epsilon}(m) = \frac{m^2\mu^2}{16\pi^2}\left[\ln\left(\frac{m^2}{M^2}\right) - 1\right]. \tag{78}$$

Compatibility of short-distance Wilson expansion with massive Johnson-Baker-Willey propagator $S^{-1}(p) \sim \not{p} - (-p^2)^{\gamma_\theta(\alpha)/2}$ also gives $\gamma_\theta(\alpha) = -1$ (Mannheim 1975). Specifically, in a scale invariant theory the Wilson expansion is of the form

$$T(\psi(x)\bar{\psi}(0)) \;=\; \langle\Omega_0|T(\psi(x)\bar{\psi}(0))|\Omega_0\rangle + (\mu^2x^2)^{\gamma_\theta(\alpha)/2} : \psi(0)\bar{\psi}(0) : \tag{79}$$

where the normal ordering is done with respect to the unbroken massless vacuum $|\Omega_0\rangle$. Now take matrix element in the spontaneously broken vacuum $|\Omega_m\rangle$, to obtain

$$\tilde{S}(p) = \frac{1}{\not{p}} + (-p^2)^{(-\gamma_\theta(\alpha)/2-2)}, \quad \tilde{S}^{-1}(p) = \not{p} - (-p^2)^{(-\gamma_\theta(\alpha)-2)/2}. \tag{80}$$

Compatibility with $S^{-1}(p) \sim \not{p} - (-p^2)^{\gamma_\theta(\alpha)/2}$ then gives $\gamma_\theta(\alpha) = -\gamma_\theta(\alpha) - 2$, i. e. $\gamma_\theta(\alpha) = -1$. Also shows that vacuum of Jonhnson-Baker-Willey electrodynamics is a broken vacuum even though no Goldstone boson. We expain this conundrum below.

Thus in this talk we explore symmetry breaking with $\gamma_\theta(\alpha) = -1$, i. e. with $D[\bar{\psi}\psi] = 2$, which makes the four-fermion interaction $[\bar{\psi}\psi]^2$ non-perturbatively renormalizable (Mannheim 1975).

8.3 Evasion of the Baker-Johnson Evasion of the Goldstone Theorem

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{g}{2}(\bar{\psi}\psi)^2 - \frac{g}{2}(\bar{\psi}i\gamma_5\psi)^2 = \mathcal{L}_{\rm MF} + \mathcal{L}_{\rm RI}, \tag{81}$$

$$\begin{aligned} \mathcal{L}_{\rm MF} &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + \frac{m^2}{2g} \\ \mathcal{L}_{\rm RI} &= -\frac{g}{2}\left(\bar{\psi}\psi - \frac{m}{g}\right)^2 - \frac{g}{2}(\bar{\psi}i\gamma_5\psi)^2. \end{aligned} \tag{82}$$

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - m\bar{\psi}\psi + \frac{m^2}{2g}, \tag{83}$$

We thus interpret Johnson-Baker-Willey or Maskawa-Nakajima as a mean-field theory, and as such there is no Goldstone boson, since the mean-field never has a Goldstone boson. Rather it is generated by the residual interaction. So to see how things work we adapt the Nambu-Jona-Lasinio chiral four-fermion model to the Johnson-Baker-Willey case. However we note that the induced m^2/g term will serve as a counter-term for the vacuum energy density.

9 Johnson-Baker-Willey Electrodynamics Coupled to a Four-Fermion Interaction

9.1 Vacuum Energy Density for Arbitrary $\gamma_\theta(\alpha)$

$$\begin{aligned}
\mathcal{L}_{\mathrm{QED-FF}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - \frac{g}{2}[\bar{\psi}\psi]^2 - \frac{g}{2}[\bar{\psi}i\gamma_5\psi]^2 \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - m\bar{\psi}\psi + \frac{m^2}{2g} - \frac{g}{2}\left(\bar{\psi}\psi - \frac{m}{g}\right)^2 - \frac{g}{2}\left(\bar{\psi}i\gamma_5\psi\right)^2 \\
&= \mathcal{L}_{\mathrm{QED-MF}} + \mathcal{L}_{\mathrm{QED-RI}}. \qquad (84)
\end{aligned}$$

$$\tilde{S}^{-1}(p, m=0) = \not{p} + i\epsilon, \qquad \tilde{\Gamma}_{\mathrm{S}}(p,p,0) = \left(\frac{-p^2 - i\epsilon}{\mu^2}\right)^{\gamma_\theta(\alpha)/2} \qquad (85)$$

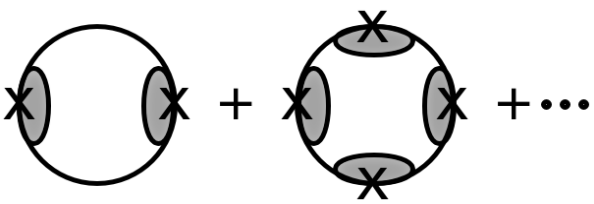


Figure 5: Vacuum energy density $\epsilon(m)$ via an infinite summation of massless graphs with zero-momentum dressed $m\bar{\psi}\psi$ insertions.

$$\langle\Omega_0|T(\psi(x):\bar{\psi}(z)\psi(z):\bar{\psi}(y))|\Omega_0\rangle = \frac{\mu^{-\gamma_\theta}(\not{y} - \not{z})(\not{z} - \not{x})}{[(y-z)^2(z-x)^2]^{(1+d_\theta)/2}[(x-y)^2]^{(3-d_\theta)/2}}, \qquad (86)$$

$$
\begin{aligned}
\epsilon(m) &= i\int \frac{d^4p}{(2\pi)^4}\sum_{n=1}^{\infty}\frac{(-1)}{2n}\mathrm{Tr}\left[(-i)^2\left(\frac{-p^2-i\epsilon}{\mu^2}\right)^{\gamma_\theta(\alpha)}\left(\frac{i}{\not p+i\epsilon}\right)^2 m^2\right]^n \\
&= \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\ \ln\left[1-\frac{m^2}{p^2+i\epsilon}\left(\frac{-p^2-i\epsilon}{\mu^2}\right)^{\gamma_\theta(\alpha)}\right],
\end{aligned}
\tag{87}
$$

$$
\tilde{S}_\mu^{-1}(p) = \not p - m\left(\frac{-p^2-i\epsilon}{\mu^2}\right)^{\gamma_\theta(\alpha)/2} + i\epsilon, \tag{88}
$$

$$
\epsilon(m) = i\int \frac{d^4p}{(2\pi)^4}\left[\mathrm{Tr}\ \ln(\tilde{S}_\mu^{-1}(p)) - \mathrm{Tr}\ \ln(\not p + i\epsilon)\right]. \tag{89}
$$

$$
\left[m\frac{\partial}{m} + \beta(\alpha)\frac{\partial}{\partial\alpha} + \gamma_\theta(\alpha)\right]\tilde{\Gamma}_{\mathrm{S}}(p,p,0) = m(1-\gamma_\theta(\alpha)]\tilde{\Gamma}_{\mathrm{SS}}(p,p,0), \tag{90}
$$

$\tilde{S}_\mu^{-1}(p)$ thus develops non-leading terms, so it cannot be the exact propagator of Johnson-Baker-Willey Electrodynamics. However, it is the exact mean-field propagator. It will be modified by the residual interaction.

9.2 Vacuum Energy Density for $\gamma_\theta(\alpha) = -1$

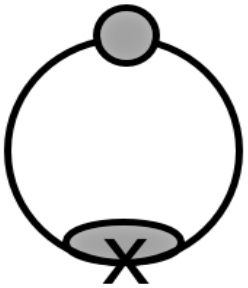


Figure 6: The $\gamma_\theta(\alpha) = -1$ tadpole graph for $\langle\Omega_m|\bar{\psi}\psi|\Omega_m\rangle$ with a zero-momentum dressed $m\bar{\psi}\psi$ insertion and a dressed $\tilde{S}_\mu(p)$ propagator.

$$\epsilon(m) = -\frac{m^2\mu^2}{8\pi^2}\left[\ln\left(\frac{\Lambda^2}{m\mu}\right) + \frac{1}{2}\right]. \tag{91}$$

$$\begin{aligned}\langle\Omega_m|\bar{\psi}\psi|\Omega_m\rangle &= \epsilon'(m) = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\tilde{\Gamma}_{\mathrm{S}}(p,p,0)\tilde{S}_\mu(p)] \\ &= 4i\int\frac{d^4p}{(2\pi)^4}\frac{m\mu^2}{(p^2+i\epsilon)^2+m^2\mu^2} = -\frac{m\mu^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{m\mu}\right) = \frac{m}{g}.\end{aligned} \tag{92}$$

$$-\frac{\mu^2}{4\pi^2}\ln\left(\frac{\Lambda^2}{M\mu}\right) = \frac{1}{g}, \qquad M = \frac{\Lambda^2}{\mu}\exp\left(\frac{4\pi^2}{\mu^2 g}\right). \tag{93}$$

$$\tilde{\epsilon}(m) = \epsilon(m) - \frac{m^2}{2g} = \frac{m^2\mu^2}{16\pi^2}\left[\ln\left(\frac{m^2}{M^2}\right) - 1\right], \tag{94}$$

Gap equation gives $-g \sim 1/\ln\Lambda^2$. Thus g is negative, i.e. attractive, and becomes very small as $\Lambda \to \infty$, with BCS-type essential singularity in gap equation at $g = 0$. Hence dynamical symmetry breaking with weak coupling.

9.3 Higgs-Like Lagrangian

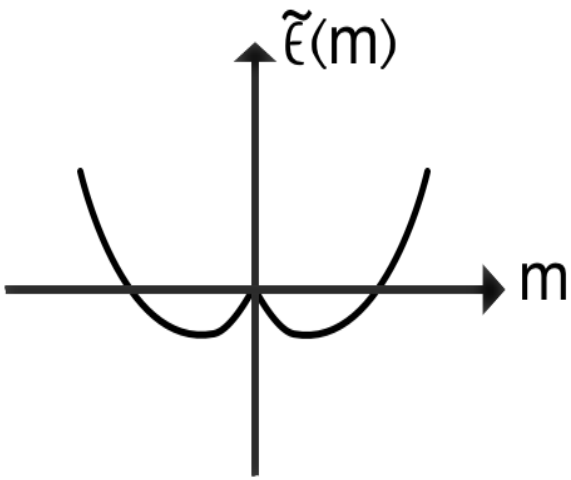


Figure 7: Dynamically generated double-well potential for the renormalized vacuum energy density when $\gamma_\theta(\alpha) = -1$.

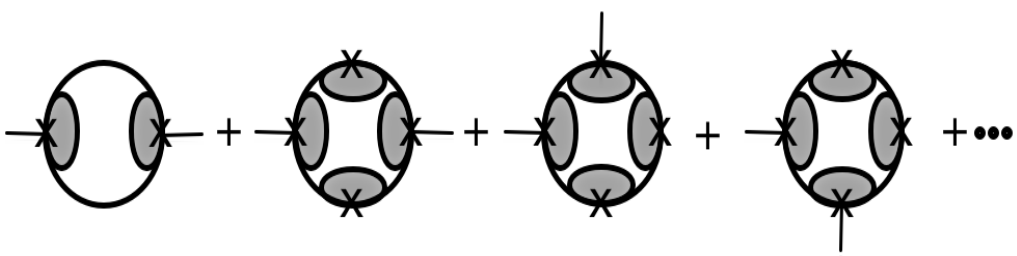


Figure 8: $\Pi_{\rm S}(q^2, m(x))$ developed as an infinite summation of massless graphs, each with two dressed $m\bar{\psi}\psi$ insertions carrying momentum q_μ (shown as external lines), with all other dressed $m\bar{\psi}\psi$ insertions carrying zero momentum.

$$\langle\Omega_0|T(:\bar{\psi}(x)\psi(x)::\bar{\psi}(y)\psi(y):)|\Omega_0\rangle = \frac{\mu^{-2\gamma_\theta}\mathrm{Tr}[(\not{x}-\not{y})(\not{y}-\not{x})]}{[(x-y)^2]^{(d_\theta+1)/2}[(y-x)^2]^{(d_\theta+1)/2}}. \tag{95}$$

$$\Pi_{\mathrm{S}}(q^2, m=0) \;=\; -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[[p^2(p+q)^2]^{\gamma_\theta(\alpha)/4}\frac{1}{\not{p}}[p^2(p+q)^2]^{\gamma_\theta(\alpha)/4}\frac{1}{\not{p}+\not{q}}\right]. \tag{96}$$

$$\Pi_{\mathrm{S}}(q^2, m=0) = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\tilde{\Gamma}_{\mathrm{S}}(p+q,p,-q)\;\tilde{S}_\mu(p,m=0)\tilde{\Gamma}_{\mathrm{S}}(p,p+q,q)\tilde{S}_\mu(p+q,m=0)\right], \tag{97}$$

$$\tilde{\Gamma}_{\mathrm{S}}(p,p+q,q) = \left[\frac{(-p^2)}{\mu^2}\frac{(-(p+q)^2)}{\mu^2}\right]^{\gamma_\theta(\alpha)/4}. \tag{98}$$

$$\Pi_{\mathrm{S}}(q^2, m) = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\tilde{\Gamma}_{\mathrm{S}}(p+q,p,-q)\tilde{S}_\mu(p)\tilde{\Gamma}_{\mathrm{S}}(p,p+q,q)\tilde{S}_\mu(p+q)\right], \tag{99}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{EFF}} &= -\tilde{\epsilon}(m(x)) - \frac{1}{2}m(x)[\Pi_{\mathrm{S}}(-\partial_\mu\partial^\mu, m(x)) - \Pi_{\mathrm{S}}(0, m(x))]m(x) + ... \\ &= -\frac{m^2(x)\mu^2}{16\pi^2}\left[\ln\left(\frac{m^2(x)}{M^2}\right) - 1\right] + \frac{3\mu}{256\pi m(x)}\partial_\mu m(x)\partial^\mu m(x) + ... \quad \text{Mannheim (1978)}\end{aligned} \tag{100}$$

9.4 The Collective Tachyon Modes when the Fermion is Massless

$$\Pi_{\rm P}(q^2, m=0) \;=\; -i\int \frac{d^4p}{(2\pi)^4}{\rm Tr}\left[[p^2(p+q)^2]^{\gamma_\theta(\alpha)/4} i\gamma_5 \frac{1}{\not p}[p^2(p+q)^2]^{\gamma_\theta(\alpha)/4} i\gamma_5 \frac{1}{\not p+\not q}\right] \tag{101}$$

$$\Pi_{\rm S}(q^2, m=0) = \Pi_{\rm P}(q^2, m=0) = -\frac{\mu^2}{4\pi^2}\left[\ln\left(\frac{\Lambda^2}{(-q^2)}\right) - 3 + 4\,\ln 2\right]. \tag{102}$$

$$\begin{aligned} T_{\rm S}(q^2) \;&=\; \frac{g}{1-g\Pi_{\rm S}(q^2)} = \frac{1}{g^{-1}-\Pi_{\rm S}(q^2)}, \\ T_{\rm P}(q^2) \;&=\; \frac{g}{1-g\Pi_{\rm P}(q^2)} = \frac{1}{g^{-1}-\Pi_{\rm P}(q^2)}, \end{aligned} \tag{103}$$

$$q^2 = -M\mu e^{4\ln 2-3} = -0.797M\mu, \tag{104}$$

$$T_{\rm S}(q^2) = T_{\rm P}(q^2) = \frac{31.448M\mu}{(q^2+0.797M\mu)} \tag{105}$$

9.5 The Collective Goldstone Mode when the Fermion is Massive

$$\Pi_{\rm P}(q^2,m) = -i\int \frac{d^4p}{(2\pi)^4}{\rm Tr}\left[\tilde{\Gamma}_{\rm S}(p+q,p,-q)\times i\gamma_5\tilde{S}_\mu(p)\tilde{\Gamma}_{\rm S}(p,p+q,q)i\gamma_5\tilde{S}_\mu(p+q)\right] \tag{106}$$

$$\begin{aligned}\Pi_{\rm S}(q^2,m) &= -4i\mu^2\int\frac{d^4p}{(2\pi)^4}\frac{N(q,p)+m^2\mu^2}{D(q,p,m)},\\ \Pi_{\rm P}(q^2,m) &= -4i\mu^2\int\frac{d^4p}{(2\pi)^4}\frac{N(q,p)-m^2\mu^2}{D(q,p,m)},\end{aligned} \tag{107}$$

$$\begin{aligned}N(q,p) &= (p^2+i\epsilon-q^2/4)\times(-(p-q/2)^2-i\epsilon)^{1/2}(-(p+q/2)^2-i\epsilon)^{1/2},\\ D(q,p,m) &= (((p-q/2)^2+i\epsilon)^2+m^2\mu^2)\times(((p+q/2)^2+i\epsilon)^2+m^2\mu^2).\end{aligned} \tag{108}$$

$$\begin{aligned}\Pi_{\rm P}(q^2=0,m) &= -4i\mu^2\int\frac{d^4p}{(2\pi)^4}\frac{(p^2)(-p^2)-m^2\mu^2}{((p^2+i\epsilon)^2+m^2\mu^2)^2}.\\ &= 4i\mu^2\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2+i\epsilon)^2+m^2\mu^2}=\frac{1}{g}.\end{aligned} \tag{109}$$

$$T_{\rm P}(q^2) = \frac{128\pi M}{7\mu q^2} = \frac{57.446M}{\mu q^2}. \tag{110}$$

9.6 The Collective Higgs Mode when the Fermion is Massive – the Needed Contour

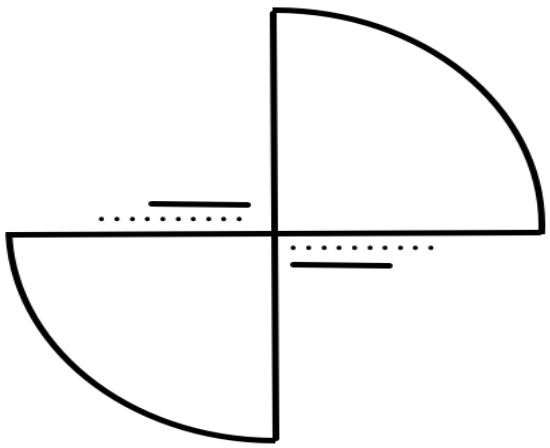


Figure 9: The standard Wick contour in the complex p_0 plane. The branch cuts are shown as lines and the poles as dots.

$$\Pi_{\rm S}(q^2, m=0) = -4i\mu^2 \int \frac{d^4p}{(2\pi)^4} \frac{N(q,p)}{D(q,p,m=0)}, \tag{111}$$

Set timelike $q_\mu = (q_0, 0, 0, 0)$. Branch points in $N(q,p)$ at:

$$\begin{aligned} p_0 &= q_0/2 + p - i\epsilon, \qquad & p_0 &= -q_0/2 + p - i\epsilon \\ p_0 &= q_0/2 - p + i\epsilon, \qquad & p_0 &= -q_0/2 - p + i\epsilon. \end{aligned} \tag{112}$$

They migrate into upper right and lower left hand planes in complex p_0 plane.

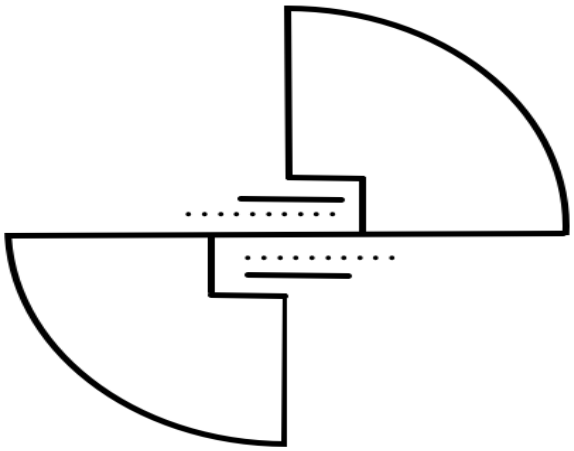


Figure 10: The migrated Wick contour in the complex p_0 plane. The branch cuts are shown as lines and the poles as dots.

$$-i\int_{-\infty}^{\infty} dp_0 = \int_{-\infty}^{\infty} dp_4 + I_{\rm cut}. \tag{113}$$

$$I_{\rm cut} = -\frac{4i\mu^2}{\pi^3}\int_0^{q_0/2} dpp^2 \int_0^{q_0/2-p} dp_0 \frac{N(q_0,p,p_0)}{D(q_0,p,p_0,m)}. \tag{114}$$

$$I_{\rm Wick} = \frac{\mu^2}{\pi^3}\int_0^{\infty} dpp^2 \int_{\infty}^{\infty} dp_4 \frac{N(q_0,p,p_4)+m^2\mu^2}{D(q_0,p,p_4,m)}. \tag{115}$$

9.7 The Collective Higgs Mode when the Fermion is Massive – Results

$$q_0(\text{Higgs}) \;=\; (1.480-0.017i)(M\mu)^{1/2}, \qquad q^2(\text{Higgs}) = (2.189-0.051\text{i})\text{M}\mu. \tag{116}$$

$$q_0(\text{Higgs}) \;=\; (1.480-0.017i)M, \qquad q^2(\text{Higgs}) = (2.189-0.051\text{i})\text{M}^2, \tag{117}$$

$$Z(\bar{\eta},\eta) = \int D[\bar{\psi},\psi]\exp\left[i\int d^4x\left(\bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - \frac{g}{2}(\bar{\psi}\psi)^2 + \bar{\eta}\psi + \bar{\psi}\eta\right)\right] \quad (118)$$

$$\begin{aligned} Z(\bar{\eta},\eta) &= \int D[\bar{\psi},\psi,\sigma] \\ &\times \exp\left[i\int d^4x\left(\bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - \frac{g}{2}(\bar{\psi}\psi)^2 + \frac{g}{2}\left(\frac{\sigma}{g} - \bar{\psi}\psi\right)^2 + \bar{\eta}\psi + \bar{\psi}\eta\right)\right] \quad (119) \end{aligned}$$

$$Z(\bar{\eta},\eta) = \int D[\bar{\psi},\psi,\sigma]\exp\left[i\int d^4x\left(\bar{\psi}\gamma^\mu(i\partial_\mu - eA_\mu)\psi - \sigma\bar{\psi}\psi + \frac{\sigma^2}{2g} + \bar{\eta}\psi + \bar{\psi}\eta\right)\right] \quad (120)$$

$$\begin{aligned} Z(0,0) &= \int D[\sigma]\exp\left[i\int d^4x\left(-\tilde{\epsilon}(\sigma) + Z(\sigma)\partial_\mu\sigma\partial^\mu\sigma/2 + ...\right)\right] \\ &= \int D[\sigma]\exp\left[\int d^4x\left(-\frac{\sigma^2(x)\mu^2}{16\pi^2}\left[\ln\left(\frac{\sigma^2(x)}{M^2}\right) - 1\right] + \frac{3\mu}{256\pi\sigma(x)}\partial_\mu\sigma(x)\partial^\mu\sigma(x) + ..\right.\right. \end{aligned}$$

Elementary and dynamical scalar Feynman diagrams are identical when scalar is off-shell. Since there is no $\int d^4x J(x)\sigma(x)$ source term for $\sigma(x)$, on-shell there are differences, such as the Higgs width. newpage

10 Conformal Symmetry Challenges Supersymmetry

10.1 Cancellation of Infinities

Supersymmetry cancellations are perturbative.
Conformal invariance cancellations are non-perturbative.
In QED key infinities are Z_3 and $m_0 = m - \delta m$, with Z_1 and Z_2 being gauge dependent.
If $\beta(\alpha) = 0$ then Z_3 is finite.
And if $\gamma_\theta(\alpha) < 0$ then δm is finite.

$$m_0 \;=\; m\left[1 + \frac{\gamma_\theta(\alpha)}{2}\ln\left(\frac{\Lambda^2}{m^2}\right) + \frac{\gamma_\theta^2(\alpha)}{8}\ln^2\left(\frac{\Lambda^2}{m^2}\right) + ...\right] = m\left(\frac{\Lambda^2}{m^2}\right)^{\gamma_\theta(\alpha)/2} \to 0. \quad (122)$$

If $\gamma_\theta = -1$ then $\tilde{\epsilon} = \epsilon - m^2/2g$ is finite, as are the scalar and pseudoscalar $T_S(q^2)$, $T_P(q^2)$ channels in the fermion-antifermion scattering amplitude.

10.2 Supersymmetry Treatment of the Vacuum Energy Density

$$T_{\rm M}^{\mu\nu} = i\hbar\bar{\psi}\gamma^\mu\partial^\nu\psi \tag{123}$$

$$\langle\Omega_0|T_{\rm M}^{\mu\nu}|\Omega_0\rangle = -\frac{2\hbar}{(2\pi)^3}\int_{-\infty}^{\infty} d^3k\frac{k^\mu k^\nu}{\omega_k}. \tag{124}$$

$$\langle\Omega_0|T_{\rm M}^{\mu\nu}|\Omega_0\rangle \;=\; (\rho_{\rm M}+p_{\rm M})U^\mu U^\nu + p\eta^{\mu\nu}, \tag{125}$$

$$\rho_{\rm M} = \langle\Omega_0|T_{\rm M}^{00}|\Omega_0\rangle = -\frac{2\hbar}{(2\pi)^3}\int_{-\infty}^{\infty} d^3k\omega_k, \tag{126}$$

$$p_{\rm M} \;=\; \langle\Omega_0|T_{\rm M}^{11}|\Omega_0\rangle = \langle\Omega_0|T_{\rm M}^{22}|\Omega_0\rangle = \langle\Omega_0|T_{\rm M}^{33}|\Omega_0\rangle = -\frac{2\hbar}{3(2\pi)^3}\int_{-\infty}^{\infty} d^3k\frac{k^2}{\omega_k}. \tag{127}$$

$$\eta_{\mu\nu}\langle\Omega_0|T_{\rm M}^{\mu\nu}|\Omega_0\rangle \;=\; 3p_{\rm M} - \rho_{\rm M} = 0 \tag{128}$$

$$\rho_{\rm M} = -\frac{\hbar K^4}{4\pi^2}, \qquad p_{\rm M} = -\frac{\hbar K^4}{12\pi^2}. \tag{129}$$

$$\begin{aligned}
\rho_{\rm M} &= -\frac{\hbar K^4}{4\pi^2} - \frac{m^2K^2}{4\pi^2\hbar} + \frac{m^4}{16\pi^2\hbar^3}\ln\left(\frac{4\hbar^2K^2}{m^2}\right) - \frac{m^4}{32\pi^2\hbar^3}, \\
p_{\rm M} &= -\frac{\hbar K^4}{12\pi^2} + \frac{m^2K^2}{12\pi^2\hbar} - \frac{m^4}{16\pi^2\hbar^3}\ln\left(\frac{4\hbar^2K^2}{m^2}\right) + \frac{7m^4}{96\pi^2\hbar^3},
\end{aligned} \tag{130}$$

Supersymmetry (i. e. add in boson loop with opposite overall sign) cancels the quartic divergence, but not the quadratic one unless superpartners are degenerate in mass, which they are known not to be.

10.3 Conformal Gravity Treatment of the Vacuum Energy Density

Gravity couples to energy density and not to energy density difference. Hence in the presence of gravity one cannot normal order away any infinities such as zero point infinities. Must cancel them by counterterms (provided they can naturally be induced) or by gravity itself, provided it is consistent quantum-mechanically as it has zero-point energy density too.

$$I_{\rm W} \quad = -\alpha_g \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa} \equiv -2\alpha_g \int d^4x(-g)^{1/2} \left[R_{\mu\kappa} R^{\mu\kappa} - \frac{1}{3}(R^\alpha{}_\alpha)^2 \right], \tag{131}$$

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{2} g^{\mu\nu} (R^\alpha{}_\alpha)^{;\beta}{}_{;\beta} + R^{\mu\nu;\beta}{}_{;\beta} - R^{\mu\beta;\nu}{}_{;\beta} - R^{\nu\beta;\mu}{}_{;\beta} - 2R^{\mu\beta} R^\nu{}_\beta + \frac{1}{2} g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - \frac{2}{3} g^{\mu\nu} (R^\alpha{}_\alpha)^{;\beta}{}_{;\beta} \\ &+ \frac{2}{3} (R^\alpha{}_\alpha)^{;\mu;\nu} + \frac{2}{3} R^\alpha{}_\alpha R^{\mu\nu} - \frac{1}{6} g^{\mu\nu} (R^\alpha{}_\alpha)^2, \end{aligned} \tag{132}$$

$$-4\alpha_g W^{\mu\nu} + T^{\mu\nu}_{\rm M} = 0, \qquad T^{\mu\nu}_{\rm UNIV} = T^{\mu\nu}_{\rm GRAV} + T^{\mu\nu}_{\rm M} = 0, \tag{133}$$

$$\langle \Omega_0 | T^{\mu\nu}_{\rm GRAV} | \Omega_0 \rangle = \frac{2\hbar}{(2\pi)^3} \int_{-\infty}^{\infty} d^3k \frac{Z(k) k^\mu k^\nu}{\omega_k}, \tag{134}$$

Conformal gravity cancellation when fermion $m = 0$ if graviton $Z(k) = 1$.

Comparison with standard gravity.

$$I_{\rm EH} = -\frac{1}{16\pi G} \int d^4x(-g)^{1/2} R^\alpha{}_\alpha, \qquad I_\Lambda = -\int d^4x(-g)^{1/2} \Lambda \tag{135}$$

$$-\frac{1}{8\pi G} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R^\alpha{}_\alpha \right) = T^{\mu\nu}_{\rm M}. \qquad \text{quantum} \;=\; \text{quantum? or classical} \;=\; \text{classical?} \tag{136}$$

$$-\frac{1}{8\pi G} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R^\alpha{}_\alpha \right)_{\rm CL} = \langle \Omega | T^{\mu\nu}_{\rm M} | \Omega \rangle. \qquad \text{classical} \;=\; \text{matrix element of quantum} \tag{137}$$

$$-\frac{1}{8\pi G}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R^{\alpha}{}_{\alpha}\right)_{\rm CL}=\langle\Omega|T^{\mu\nu}_{\rm M}|\Omega\rangle_{\rm FIN}. \qquad \text{classical} \;=\; \text{finite matrix element of quantum} \tag{138}$$

$$\langle\Omega_M|T^{00}_{\rm GRAV}|\Omega_M\rangle-\frac{\hbar K^4}{4\pi^2}-\frac{M^4}{16\pi^2\hbar^3}=0. \tag{139}$$

$Z(k)$ readjusts if fermion is massive – gravity is quantized by its source and not quantized independently.

$$kZ(k)=(k^2+iM^2/\hbar^2)^{1/2}-\frac{iM^2}{4\hbar^2(k^2+iM^2/\hbar^2)^{1/2}}+(k^2-iM^2/\hbar^2)^{1/2}+\frac{iM^2}{4\hbar^2(k^2-iM^2/\hbar^2)^{1/2}}. \tag{140}$$

$$(T^{\mu\nu}_{\rm GRAV})_{\rm DIV}+(T^{\mu\nu}_{\rm M})_{\rm DIV}=0, \qquad (T^{\mu\nu}_{\rm GRAV})_{\rm FIN}+(T^{\mu\nu}_{\rm M})_{\rm FIN}=0. \tag{141}$$

$$\Lambda^{\lambda}{}_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\nu\mu}), \tag{142}$$

$$-\tilde{\omega}^{ab}_{\mu} = -\omega^{ab}_{\mu} + V^{b}_{\lambda}\delta\Gamma^{\lambda}{}_{\nu\mu}V^{a\nu}, \tag{143}$$

$$-\omega^{ab}_{\mu} = V^{b}_{\nu}\partial_\mu V^{a\nu} + V^{b}_{\lambda}\Lambda^{\lambda}{}_{\mu\nu}V^{a\nu} = \omega^{ba}_{\mu}. \tag{144}$$

$$I_{\mathrm{D}} \;=\; \frac{1}{2}\int d^4x(-g)^{1/2} i\bar{\psi}\gamma^a V^{\mu}_{a}(\partial_\mu + \Sigma_{bc}\tilde{\omega}^{bc}_{\mu})\psi + \mathrm{H.\ c.}, \tag{145}$$

$$\delta\Gamma^{\lambda}{}_{\mu\nu} \;=\; -\frac{2i}{3} g^{\lambda\alpha}\left(g_{\nu\alpha}A_\mu + g_{\mu\alpha}A_\nu - g_{\nu\mu}A_\alpha\right) + \frac{1}{2} g^{\lambda\alpha}(Q_{\mu\nu\alpha} + Q_{\nu\mu\alpha} - Q_{\alpha\nu\mu}). \tag{146}$$

$$I_{\mathrm{D}} \;=\; \int d^4x(-g)^{1/2} i\bar{\psi}\gamma^a V^{\mu}_{a}(\partial_\mu + \Sigma_{bc}\omega^{bc}_{\mu} - iA_\mu - i\gamma^5 S_\mu)\psi, \tag{147}$$

$$S^{\mu} \;=\; \frac{1}{8}(-g)^{-1/2}\epsilon^{\mu\alpha\beta\gamma}Q_{\alpha\beta\gamma}. \tag{148}$$

$$I_{\mathrm{D}} \;=\; \int d^4x(-g)^{1/2} i\bar{\psi}\gamma^a V^{\mu}_{a}(\partial_\mu + \Sigma_{bc}\omega^{bc}_{\mu} - ig_V T^i A^i_\mu - ig_A\gamma^5 T^i S^i_\mu)\psi. \tag{149}$$

$$I_{\rm EFF} \;=\; \int d^4x(-g)^{1/2}C\left[\frac{1}{20}\left[R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right]+\frac{1}{3}G^i_{\mu\nu}G_i^{\mu\nu}+\frac{1}{3}S^i_{\mu\nu}S_i^{\mu\nu}\right], \quad (150)$$

$$I_{\rm W}+I_{\rm YM}=\int d^4x(-g)^{1/2}\left[-2\alpha_g\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right)-\frac{1}{4}G^i_{\mu\nu}G_i^{\mu\nu}-\frac{1}{4}S^i_{\mu\nu}S_i^{\mu\nu}\right] \quad (151)$$

$$I_{\rm FF} \;=\; -\int d^4x(-g)^{1/2}\frac{g_{\rm FF}}{2}\left[\bar\psi T^i\psi\bar\psi T^i\psi+\bar\psi i\gamma_5 T^i\psi\bar\psi i\gamma_5 T^i\psi\right], \quad (152)$$

$$I_{\rm UNIV}=I_{\rm D}+I_{\rm W}+I_{\rm YM}+I_{\rm FF}. \quad (153)$$

$$I_{\rm MF} \;=\; \int d^4x(-g)^{1/2}C\left[-M^4(x)+\frac{1}{6}M^2(x)R^{\alpha}{}_{\alpha}-(\partial_\mu+iA_\mu)M(x)(\partial^\mu-iA^\mu)M(x)\right]. \quad (154)$$

Potential theory of everything – string theory with extra dimensions and supersymmetry is a theory of more than everything.

A The Collective Higgs Mode when the Fermion is Massive – the Calculation

A.1 The Basic Equations

$$
\begin{aligned}
I_{\rm cut} &= -\frac{4i\mu^2}{\pi^3}\int_0^1 d\sigma\sigma^2\int_0^{1-\sigma} d\lambda\frac{N_{\rm cut}}{D_{\rm cut}},\\
N_{\rm cut} &= -(\lambda^2-\sigma^2-1)[(\lambda^2-\sigma^2+1)^2-4\sigma^2]^{1/2}q_0^8,\\
D_{\rm cut} &= 256m^4\mu^4+32m^2\mu^2[(\lambda^2-\sigma^2+1)^2+4\sigma^2]q_0^4+[(\lambda^2-\sigma^2+1)^2-4\sigma^2]^2q_0^8.
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
N(q,p) &= -(p_4^2+p^2+q_0^2/4)\times[(p^2+p_4^2-q_0^2/4)^2+p_4^2q_0^2]^{1/2},\\
D(q,p) &= [(p^2+p_4^2-q_0^2/4)^2+p_4^2q_0^2-m^2\mu^2]^2+4m^2\mu^2(p^2+p_4^2-q_0^2/4)^2.
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
I_{\rm Wick} &= \frac{2\mu^2}{\pi^3}\int_0^\infty drr^3\int_0^1 dz(1-z^2)^{1/2}\times\left[\frac{N(q,r,z)+m^2\mu^2}{D(q,r,z)}\right],\\
N(q,r,z) &= -(r^2+q_0^2/4)[(r^2-q_0^2/4)^2+r^2z^2q_0^2]^{1/2},\\
D(q,r,z) &= [(r^2-q_0^2/4)^2+r^2z^2q_0^2-m^2\mu^2]^2+4m^2\mu^2(r^2-q_0^2/4)^2.
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
\alpha &= \frac{(r^2-q_0^2/4)^2-m^2\mu^2}{r^2q_0^2},\\
\beta &= \frac{(r^2-q_0^2/4)^2+m^2\mu^2}{r^2q_0^2},
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
N(q,r,z) &= -(r^2+q_0^2/4)rq_0(z^2+\alpha/2+\beta/2)^{1/2},\\
D(q,r,z) &= r^4q_0^4(z^4+2\alpha z^2+\beta^2).
\end{aligned}
\tag{A5}
$$

$z = y/(1+y^2)^{1/2}$

$$\begin{aligned} I_2 &= \int_0^1 dz \frac{(1-z^2)^{1/2}}{z^4+2\alpha z^2+\beta^2} \\ &= \int_0^\infty dy \frac{1}{y^4(1+2\alpha+\beta^2)+2y^2(\alpha+\beta^2)+\beta^2} \\ &= \frac{\pi}{4\beta(\alpha^2-\beta^2)^{1/2}}[(\alpha+\beta^2+(\alpha^2-\beta^2)^{1/2})^{1/2}-(\alpha+\beta^2-(\alpha^2-\beta^2)^{1/2})^{1/2}], \end{aligned} \tag{A6}$$

$$\begin{aligned} I_1 &= \int_0^1 dz \frac{(1-z^2)^{1/2}(z^2+\alpha/2+\beta/2)^{1/2}}{z^4+2\alpha z^2+\beta^2} \\ &= \int_0^\infty dy \frac{1}{2^{1/2}(1+y^2)^{1/2}} \times \frac{(2y^2+(\alpha+\beta)(1+y^2))^{1/2}}{y^4(1+2\alpha+\beta^2)+2y^2(\alpha+\beta^2)+\beta^2} \\ &= -\frac{i}{8^{1/2}\beta^2(\alpha+\beta)(\alpha-\beta)^{1/2}} \times [(\alpha+\beta)(\alpha^2-\beta^2)^{1/2}(F_- + F_+) + (\alpha^2+\alpha\beta-2\beta^2)(F_- - F_+)], \end{aligned} \tag{A7}$$

$$F_\pm = \int_0^\phi d\theta \frac{1}{(1-j_\pm \sin^2\theta)(1-k\sin^2\theta)^{1/2}}, \tag{A8}$$

$$\begin{aligned} j_\pm &= \frac{1+2\alpha+\beta^2}{\alpha+\beta^2 \pm (\alpha^2-\beta^2)^{1/2}}, \\ \phi = i\ \mathrm{arcsinh}(\infty), \quad k &= \frac{2+\alpha+\beta}{\alpha+\beta}. \end{aligned} \tag{A9}$$

$\sin\theta = i\tan\nu$

$$\begin{aligned} F_\pm &= i\int_0^{\pi/2} \frac{d\nu}{\cos\nu}\frac{1}{(1+j_\pm\tan^2\nu)(1+k\tan^2\nu)^{1/2}} \\ &= -\frac{i}{j_\pm-1}K(1-k) + \frac{ij_\pm}{j_\pm-1}E(1-j_\pm,1-k), \end{aligned} \tag{A10}$$

$$\begin{aligned} K(1-k) &= \int_0^{\pi/2} d\nu \frac{1}{(1-(1-k)\sin^2\nu)^{1/2}} \\ E(1-j_\pm,1-k) &= \int_0^{\pi/2} d\nu \times \frac{1}{(1-(1-j_\pm)\sin^2\nu)(1-(1-k)\sin^2\nu)^{1/2}}. \end{aligned} \tag{A11}$$

$$\Pi_{\rm S}(q^2,m) \;=\; \frac{2\mu^2}{\pi^3}\int_0^\infty dr r^3\Big[-\frac{(r^2+q_0^2/4)I_1}{r^3q_0^3} + \frac{m^2\mu^2 I_2}{r^4q_0^4}\Big]. \tag{A12}$$

$$\hat{\Pi}_{\rm S}(q^2,M) = \Pi_{\rm S}(q^2,M) - g^{-1}. \tag{A13}$$

A.2 The Discontinuity

$r = q_{\rm R}/2 + \Gamma e^{i\theta}$

$$
\begin{aligned}
I_1 \quad &\to \int_0^\infty dy \frac{q_{\rm R}^8 y}{(1+y^2)^{1/2}(y^2(q_{\rm R}^4 - 4M^2\mu^2) - 4M^2\mu^2)^2} \\
&= \Bigg(\frac{q_{\rm R}^4 (1+y^2)^{1/2}}{2(4M^2\mu^2 + (4M^2\mu^2 - q_{\rm R}^4)y^2)} \\
&+ \frac{q_{\rm R}^2}{4(q_{\rm R}^4 - 4M^2\mu^2)^{1/2}} \times \ln\left(\frac{q_{\rm R}^2 + (q_{\rm R}^4 - 4M^2\mu^2)^{1/2}(1+y^2)^{1/2}}{q_{\rm R}^2 - (q_{\rm R}^4 - 4M^2\mu^2)^{1/2}(1+y^2)^{1/2}} \right) \Bigg) \Bigg|_0^\infty \\
&= \frac{i\pi q_{\rm R}^2}{4(q_{\rm R}^4 - 4M^2\mu^2)^{1/2}} - \frac{q_{\rm R}^4}{8M^2\mu^2} - \frac{q_{\rm R}^2}{4(q_{\rm R}^4 - 4M^2\mu^2)^{1/2}} \\
&\times \ln\left(\frac{q_{\rm R}^2 + (q_{\rm R}^4 - 4M^2\mu^2)^{1/2}}{q_{\rm R}^2 - (q_{\rm R}^4 - 4M^2\mu^2)^{1/2}} \right).
\end{aligned} \tag{A14}
$$

$$
\begin{aligned}
I_2 \to \quad &\int_0^\infty dy \frac{q_{\rm R}^8}{(y^2(q_{\rm R}^4 - 4M^2\mu^2) - 4M^2\mu^2)^2} \\
&= \Bigg(\frac{q_{\rm R}^8 y}{8M^2\mu^2(4M^2\mu^2 + (4M^2\mu^2 - q_{\rm R}^4)y^2)} \\
&+ \frac{q_{\rm R}^8}{32M^3\mu^3(q_{\rm R}^4 - 4M^2\mu^2)^{1/2}} \times \ln\left(\frac{2M\mu + (q_{\rm R}^4 - 4M^2\mu^2)^{1/2} y}{2M\mu - (q_{\rm R}^4 - 4M^2\mu^2)^{1/2} y} \right) \Bigg) \Bigg|_0^\infty \\
&= \frac{i\pi q_{\rm R}^8}{32M^3\mu^3(q_{\rm R}^4 - 4M^2\mu^2)^{1/2}}.
\end{aligned} \tag{A15}
$$

$$\begin{aligned}\mathrm{Re}[I_{\mathrm{COM}}] &= \frac{q_{\mathrm{R}}^3\Gamma}{4\pi^3 M^2} + \frac{\mu^2 q_{\mathrm{R}}\Gamma}{2\pi^3(q_{\mathrm{R}}^4 - 4M^2\mu^2)^{1/2}} \\ &\times \ln\left(\frac{q_{\mathrm{R}}^2 + (q_{\mathrm{R}}^4 - 4M^2\mu^2)^{1/2}}{q_{\mathrm{R}}^2 - (q_{\mathrm{R}}^4 - 4M^2\mu^2)^{1/2}}\right),\end{aligned} \tag{A16}$$

$$\mathrm{Im}[I_{\mathrm{COM}}] = \frac{i\mu\Gamma q_{\mathrm{R}}(q_{\mathrm{R}}^2 - 2M\mu)^{1/2}}{4\pi^2 M(q_{\mathrm{R}}^2 + 2M\mu)^{1/2}}. \tag{A17}$$

A.3 Numerical Results

$$\hat{I}_{\mathrm{GGAP}} = \frac{2\mu^2 q_{\mathrm{R}}^3\Gamma}{\pi^2(q_{\mathrm{R}}^4 + 16M^2\mu^2)}. \tag{A18}$$

$$\begin{aligned}&\hat{\Pi}_{\mathrm{S}}(q^2, M) = \hat{I}_{\mathrm{WICK}}(\mathrm{LOW}) + \hat{I}_{\mathrm{WICK}}(\mathrm{HIGH}) \\ &+\hat{I}_{\mathrm{GGAP}} + \mathrm{Re}[I_{\mathrm{COM}}] + \mathrm{Im}[I_{\mathrm{COM}}] + I_{\mathrm{CUT}}.\end{aligned} \tag{A19}$$

$$\begin{aligned}q_{\mathrm{R}} &= 1.480(M\mu)^{1/2}, \quad \Gamma = 0.017i(M\mu)^{1/2} \\ q^2 &= (2.189 - 0.051i)M\mu.\end{aligned} \tag{A20}$$

$$\begin{aligned}\hat{\Pi}_{\mathrm{S}}(q^2, M) &= (q^2 - (q_{\mathrm{R}} - i\Gamma)^2) \\ &\times (-0.021662 + 0.000484i),\end{aligned} \tag{A21}$$

$$T_{\mathrm{S}}(q^2) = \frac{46.141 + 1.030i}{q^2 - 2.2189M\mu + 0.051iM\mu}. \tag{A22}$$

B WE GET LOCAL CONFORMAL GRAVITY FOR FREE

The Weyl tensor $C_{\lambda\mu\nu\kappa}$

$$C_{\lambda\mu\nu\kappa} = R_{\lambda\mu\nu\kappa} - \frac{1}{2}(g_{\lambda\nu}R_{\mu\kappa} - g_{\lambda\kappa}R_{\mu\nu} - g_{\mu\nu}R_{\lambda\kappa} + g_{\mu\kappa}R_{\lambda\nu}) + \frac{1}{6}R^{\alpha}{}_{\alpha}(g_{\lambda\nu}g_{\mu\kappa} - g_{\lambda\kappa}g_{\mu\nu}) \quad (23)$$

obeys $g^{\lambda\nu}C_{\lambda\mu\nu\kappa} = 0$, and has the remarkable property that under a local conformal transformation $g_{\mu\nu} \to e^{2\beta(x)}g_{\mu\nu}(x)$, all derivatives of $\beta(x)$ drop out identically and $C^{\lambda}{}_{\mu\nu\kappa} \to C^{\lambda}{}_{\mu\nu\kappa}$. Analog to Maxwell tensor $F_{\mu\nu} \to F_{\mu\nu}$ under a gauge transformation and to $g^{\mu\nu}F_{\mu\nu} = 0$.

In terms of a dimensionless coupling constant α_g the action I_{W}

$$I_W = -\alpha_g \int d^4x(-g)^{1/2}C_{\lambda\mu\nu\kappa}C^{\lambda\mu\nu\kappa} = -\alpha_g \int d^4x(-g)^{1/2}\left[R_{\lambda\mu\nu\kappa}R^{\lambda\mu\nu\kappa} - 2R_{\mu\kappa}R^{\mu\kappa} + \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right] \quad (24)$$

is the unique locally conformal invariant action for the gravitational sector (compare $F_{\mu\nu}F^{\mu\nu}$).

With $(-g)^{1/2}\left[R_{\lambda\mu\nu\kappa}R^{\lambda\mu\nu\kappa} - 4R_{\mu\kappa}R^{\mu\kappa} + (R^{\alpha}{}_{\alpha})^2\right]$ being a total divergence (Gauss-Bonnet theorem), the Weyl action can be written more compactly just as in (26):

$$I_W = -2\alpha_g \int d^4x(-g)^{1/2}\left[R_{\mu\kappa}R^{\mu\kappa} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2\right]. \quad (25)$$

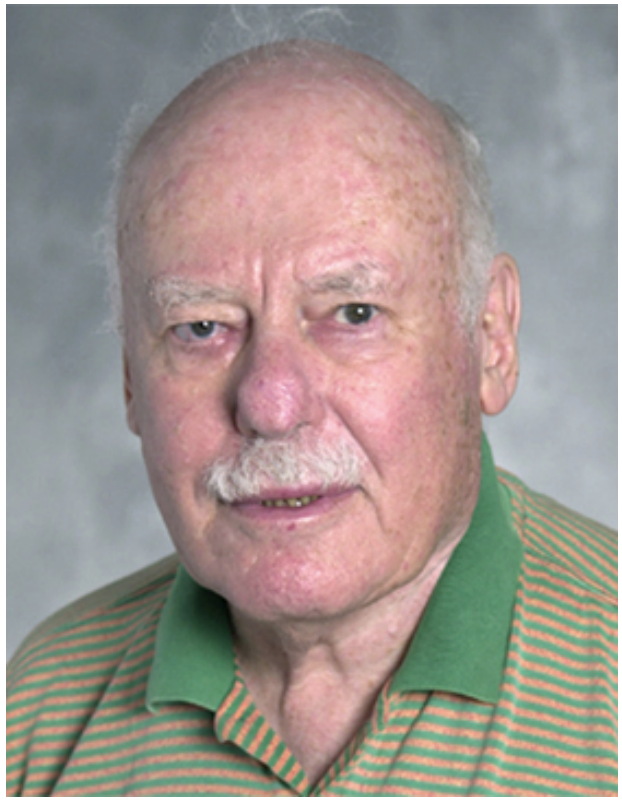
THE TAKEAWAY

For Dirac action I_{D} path integration over the fermion fields is direct, and leads ('t Hooft) to

$$\int [D\psi][D\bar{\psi}] \exp(I_{\mathrm{D}}) = \exp(iI_{\mathrm{EFF}}),$$

$$I_{\mathrm{EFF}} = \int d^4x(-g)^{1/2} C \left[R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}(R^{\alpha}{}_{\alpha})^2 \right] \tag{26}$$

with leading term with log divergent constant C. (Path integration is same as a one loop Feynman diagram.)



Harry J. Lipkin 1921 - 2015
My teacher, my colleague, and my friend. This talk is dedicated to his memory.

From: "The Work of Behram Kursunoglu" by Philip D. Mannheim, arXiv:gr-qc/0405035, May 2004.
Talk presented at the 2003 Coral Gables conference in honor and appreciation of the work of Professor Behram Kursunoglu, general relativist extraordinaire and founder of the Coral Gables series of conferences, whose untimely death occurred shortly before the 2003 conference.

"The very first of the Coral Gables Conferences took place in 1964. This was an auspicious year for me personally as it was my first year in graduate school at the Weizmann Institute, and even now I can still recall the excitement of that

period when Professors Yuval Ne'eman, Harry Lipkin and (frequent Weizmann visitor and Coral Gables Conference Series co-founder) Sydney Meshkov would report on the Coral Gables conferences."