# Modular $S_4$ Models of Lepton Masses and Mixing

in collaboration with J. T. Penedo, S. T. Petcov and A. V. Titov
[arXiv: 1811.04933]

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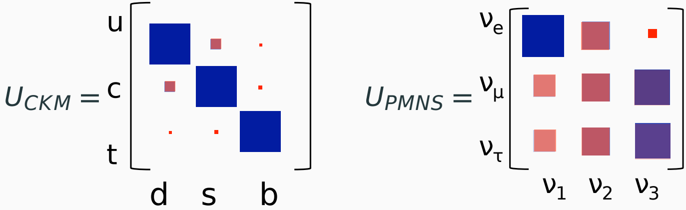
November 29, 2018

## Number of parameters

- SM forces: 3 couplings to rule them all
- EW breaking: 2
- Flavour: $\geq 20$: 10 (quarks) and $\geq 10$ (leptons)

## Mixing patterns




Adapted from [arXiv:1611.07770]

# Discrete symmetry to explain lepton flavour

☺ Successfully reproduces the mixing pattern

☹ Large number of flavons ⇒ loss of predictivity

☹ VEV alignment ⇒ elaborate potentials

Modular symmetry to the rescue! [arXiv:1706.08749]

Based on:

1. $\{S_3, A_4, S_4, A_5\} \subset \{\Gamma_N\}_{N=1}^{\infty}$ finite modular groups
2. Modular groups arise in string theory compactifications

# Outline of my talk

1. Modular symmetry: construction and systematic exploration

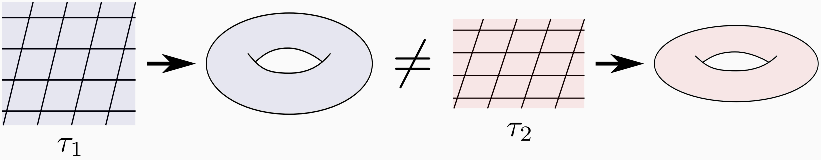
2. $\Gamma_4 \simeq S_4$ seesaw models phenomenology

3. The modulus potential and residual symmetries

# Modular symmetry: construction and systematic exploration

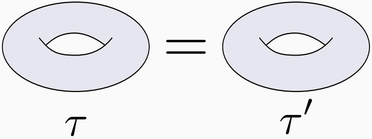
# Modular symmetry

Torus compactification:




Grid shape is encoded in the modulus $\tau \in \mathbb{C}$, $\mathrm{Im}\tau > 0$

Redundancy:




$$\tau' = \frac{a\tau + b}{c\tau + d}$$

$$\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \begin{matrix} a, b, c, d \in \mathbb{Z}, \\ ad - bc = 1 \end{matrix} \right\}$$

modular group

# Modular transformation of fields

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad \chi_i \to (c\tau + d)^{-k} \; \rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij} \chi_j$$

Weight $k$ prefactor
"charge"

Unitary representation of $\Gamma$
infinite, but we want finite!

Assumption: $\rho$ is "almost trivial"

- Infinite $\Gamma(N) \subset \Gamma$: $\rho[\,\Gamma(N)\,] = 1$
- $\rho$ is a representation of $\Gamma_N \equiv \Gamma/\Gamma(N)$ discrete finite group
- Analogy: $\mathbb{Z}_N \equiv \mathbb{Z}/N\mathbb{Z}$

| $N$ | $\Gamma_N$ |
|---|---|
| 2 | $S_3$ |
| 3 | $A_4$ |
| 4 | $S_4$ |
| 5 | $A_5$ |

$N$ is fixed $\qquad$ $k$ depends on $\chi$

# Field couplings

$$\underbrace{Y}_{\downarrow} \quad \underbrace{\chi_1 \chi_2 \cdots \chi_n}_{\downarrow}$$

$$(c\tau + d)^k \rho \quad (c\tau + d)^{-(k_1+k_2+\ldots+k_n)} \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n \quad \times \text{ itself}$$

$$k = k_1 + k_2 + .. + k_n \qquad \rho \otimes \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n \supset \mathbf{1}$$

$Y_i(\tau)$
modular forms
of weight $k$ and level $N$

form a linear space
of finite dimension

| $N$ \ $k$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| 2 ($S_3$) | 1 | 2 | 3 | 4 |
| 3 ($A_4$) | 1 | 3 | 5 | 7 |
| 4 ($S_4$) | 1 | 5 | 9 | 13 |
| 5 ($A_5$) | 1 | 11 | 21 | 31 |

# $\Gamma_4 \simeq S_4$ seesaw models phenomenology

# Systematic exploration

1. Choose the field content: seesaw type I

$$E^c L H_d \qquad N^c L H_u \qquad N^c N^c$$

2. Choose $N = 4$: $\Gamma_4 \simeq S_4$ with irreps $\mathbf{1}, \mathbf{1}', \mathbf{2}, \mathbf{3}, \mathbf{3}'$
3. Choose the field representations:

$$H_d,\, H_u,\, E^c_1,\, E^c_2,\, E^c_3 \sim \mathbf{1} \text{ or } \mathbf{1}' \qquad L,\, N^c \sim \mathbf{3} \text{ or } \mathbf{3}'$$

4. Choose the modular weights for each possible coupling. Start with the lowest weights ⇒ less parameters
5. Construct all possible singlets ⇒ Lagrangian terms
6. Search for viable regions in the parameter space

# Example: heavy neutrinos mass term

$$Y\,N^c N^c$$

$k = 0$

$$2\,\Lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$k = 2$

$$2\,\Lambda \begin{pmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{pmatrix}_{\mathbf{2}}$$

$k = 4$

$$2\,\Lambda \left[ Y_{\mathbf{1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{\Lambda'}{\Lambda} \begin{pmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{pmatrix}_{\mathbf{2}} + \frac{\Lambda''}{\Lambda} \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}_{\mathbf{3}} \right]$$

# Search for viable regions

- Parameters:

$$\tau,\ \alpha,\ \beta,\ \gamma,\ g,\ g'/g,\ \Lambda'/\Lambda, \ldots$$

- Scan to fit the current "knowns":

$$m_e,\ m_\mu,\ m_\tau,\ \theta_{12},\ \theta_{13},\ \theta_{23},\ \delta m^2,\ \left|\Delta m^2\right|$$

- Predict the "unknowns":

$$m_{\rm min}, \delta, \alpha_{21}, \alpha_{31}$$

- Correlations

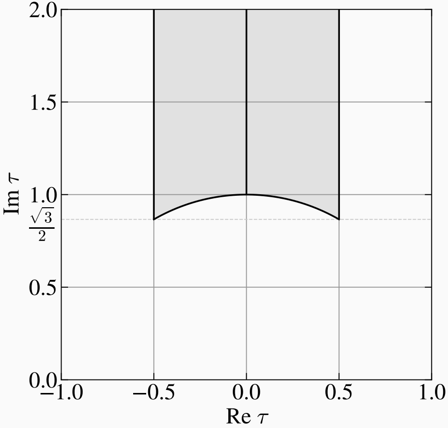
# The modulus scan range

"All moduli are equal"

$$\tau \longleftrightarrow \frac{a\tau + b}{c\tau + d}$$

The fundamental domain of Γ:

$$|\mathrm{Re}\tau| \le \frac{1}{2}, |\tau| \ge 1$$



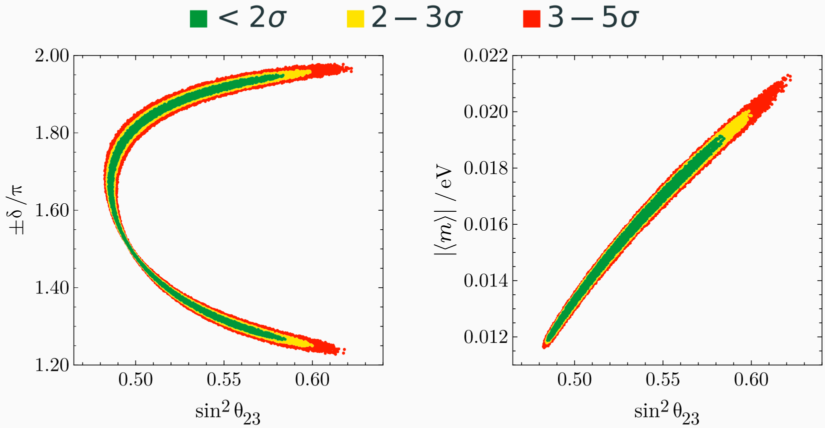
# Scan results

- Minimal viable models have 8 real parameters
- Viable models come in pairs with $\pm\delta, \pm\alpha_{21}, \pm\alpha_{31}$
- 5 pairs of viable models: 3 with NO, 2 with IO
- Mass predictions:

$$m_{\text{min}} \sim 0.003 - 0.024 \text{ eV}$$
$$\sum_i m_i \sim 0.077 - 0.12 \text{ eV}$$
$$|\langle m \rangle| \sim 0.006 - 0.045 \text{ eV}$$

- Phases and masses correlate with $\theta_{23}$

# Correlations



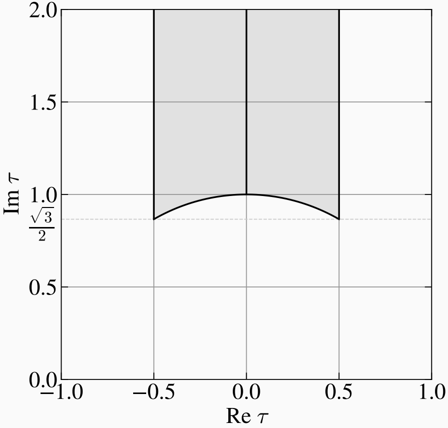
# The modulus potential and residual symmetries

# The modulus potential

Top-down conjecture

All extrema of $V(\tau)$ lie on the boundary of the fundamental domain and on the imaginary axis.

Cvetic, Font, Ibanez, Lust, Quevedo, Nucl. Phys. B361 (1991) 194



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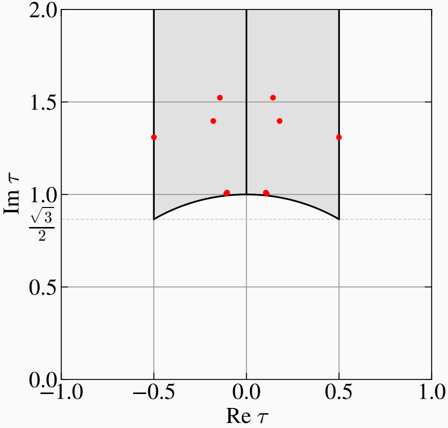
Bottom-up phenomenology



# Residual symmetries

$$\tau = i \qquad\qquad \tau = \exp\left(\frac{2\pi i}{3}\right)$$

$$\tau \to -\frac{1}{\tau} \quad \mathbb{Z}_2 \qquad\qquad \tau \to -\frac{1}{\tau+1} \quad \mathbb{Z}_3$$

Same symmetry for both mass matrices:

$$\rho_L^\dagger M_e M_e^\dagger \rho_L = M_e M_e^\dagger$$
$$\rho_L^\dagger M_\nu^\dagger M_\nu \rho_L = M_\nu^\dagger M_\nu$$

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PMNS matrix contains zeros

PMNS matrix contains even more zeros

# Different residual symmetries

## Assumption

1. $M_e$ is diagonal:
    - $\tau = \exp\left(\frac{2\pi i}{3}\right)$
    - flavon
2. $M_\nu$ invariant under $\mathbb{Z}_2$: $\tau = i$

Benchmark point found $\sim 1\sigma$, NO:

$$\sum_i m_i = 0.13 \text{ eV} \qquad |\langle m \rangle| = 0.023 \text{ eV}$$

$$\delta = 1.57\pi \qquad \alpha_{21} = 1.38\pi \qquad \alpha_{31} = 1.23\pi$$

# Conclusions

- Modular symmetry is a string-derived realisation of discrete flavour symmetry
- Predictive models can be explored in a systematic way
- Constrain masses, mixing angles and phases
- Viable $\tau$ correspond to the conjectured potential minima
- Possible residual symmetries are $\mathbb{Z}_2$ and $\mathbb{Z}_3$

# Outlook

- Try different field contents, representations, $N$
- Include the quark sector (different $\tau$?)
- Account for corrections: running, SUSY breaking, Kähler

Thank you!

# Neutrino and charged lepton Yukawa terms

$$Y\, N^c L$$

$k = 0$

$$g \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$k = 2$

$$g \left[ \begin{pmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{pmatrix}_{\mathbf{2}} + \frac{g'}{g} \begin{pmatrix} 0 & Y_3 & -Y_2 \\ -Y_3 & 0 & Y_1 \\ Y_2 & -Y_1 & 0 \end{pmatrix}_{\mathbf{3'}} \right]$$

$$Y\, E^c L$$

Weights (2, 4, 4) (minimal non-degenerate):

$$\alpha \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\mathbf{3},2} + \beta \begin{pmatrix} 0 & 0 & 0 \\ Y_1 & Y_3 & Y_2 \\ 0 & 0 & 0 \end{pmatrix}_{\mathbf{3},4} + \gamma \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Y_1 & Y_3 & Y_2 \end{pmatrix}_{\mathbf{3'},4}$$