Master Majorana neutrino mass parametrization

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There are MANY Majorana neutrino mass models...

Tree-level

Radiative: 1-loop, 2-loop, 3-loop, ...

High scale

Low scale

Dimension-5: Weinberg operator

Higher dimensions: dim-7, dim-9, ...

The world of ~~beers~~ ν mass models

This talk

Can one unify all models in a *master* neutrino mass formula?

Can one find a *master* solution to this formula?

Outline

Master formula

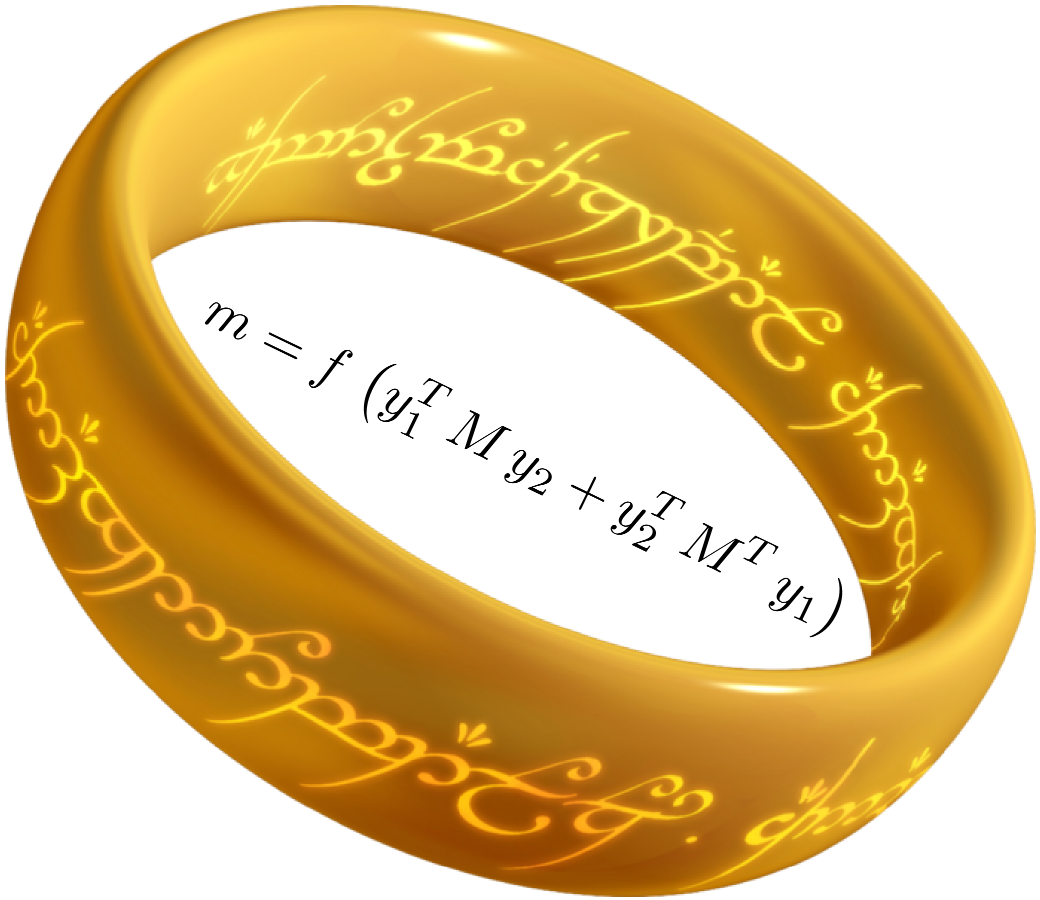
One formula to rule them all

Master parametrization

The most general solution to the master formula

An application

An illustrative example



One formula to rule them all

Master formula

The master formula

$$m = f \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)$$

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m : symmetric 3×3 Majorana neutrino mass matrix

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m : symmetric 3×3 Majorana neutrino mass matrix

f : global factor [numerical factors, model parameters, mass ratios, ...]

The master formula

$$m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1 \right)$$

m : symmetric 3×3 Majorana neutrino mass matrix

f : global factor [numerical factors, model parameters, mass ratios, ...]

y_1 : $n_1 \times 3$ Yukawa matrix

y_2 : $n_2 \times 3$ Yukawa matrix [$n_1 \geq n_2$]

The master formula

$$m = f \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)$$

m : symmetric 3×3 Majorana neutrino mass matrix

f : global factor [numerical factors, model parameters, mass ratios, ...]

y_1 : $n_1 \times 3$ Yukawa matrix

y_2 : $n_2 \times 3$ Yukawa matrix [$n_1 \geq n_2$]

M : $n_1 \times n_2$ matrix [dimensions of mass]

Some examples

To convince you that the **master formula** in indeed valid for all Majorana neutrino mass models

Type-I seesaw

everyone's model

$$m = f \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)$$

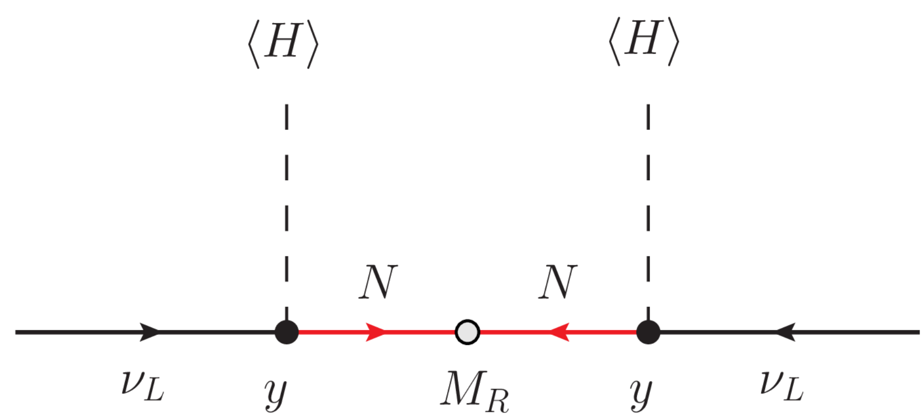
$$f = 1$$

$$n_1 = n_2 = 3$$

$$y_1 = y_2 = y/\sqrt{2}$$

$$M = \frac{v^2}{2} M_R^{-1}$$

$$\Rightarrow \quad m = \frac{v^2}{2} y^T \, M_R^{-1} \, y$$



[Minkowski, 1977]

Inverse seesaw

$$m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1 \right)$$

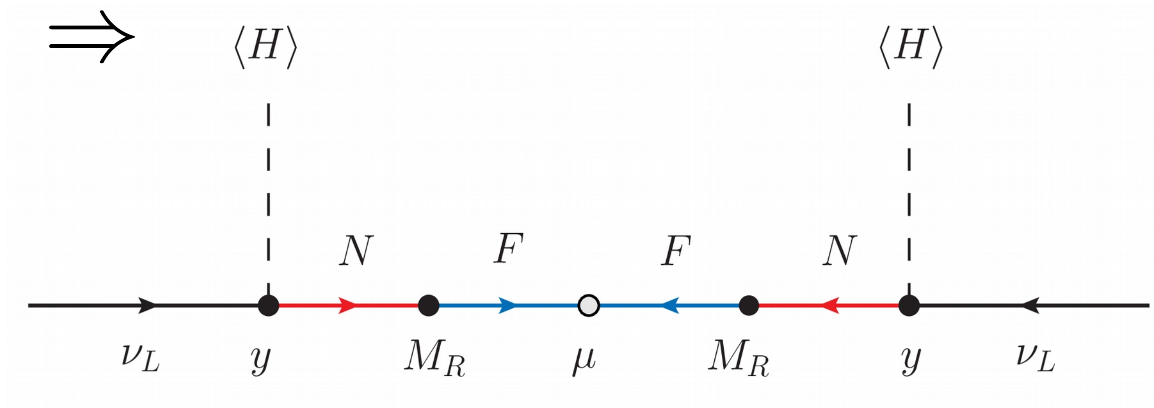
$$f = 1$$

$$n_1 = n_2 = 3$$

$$y_1 = y_2 = y/\sqrt{2}$$

$$M = \frac{v^2}{2} (M_R^T)^{-1} \mu\, M_R^{-1}$$

$$\Rightarrow \quad m = \frac{v^2}{2} y^T\, (M_R^T)^{-1} \mu\, M_R^{-1}\, y$$



[Mohapatra, Valle, 1986]

Scotogenic model

$$m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1\right)$$

$$m = \frac{\lambda_5 v^2}{32\pi^2}\, y^T\, M_R^{-1} f_{\rm loop}\, y$$

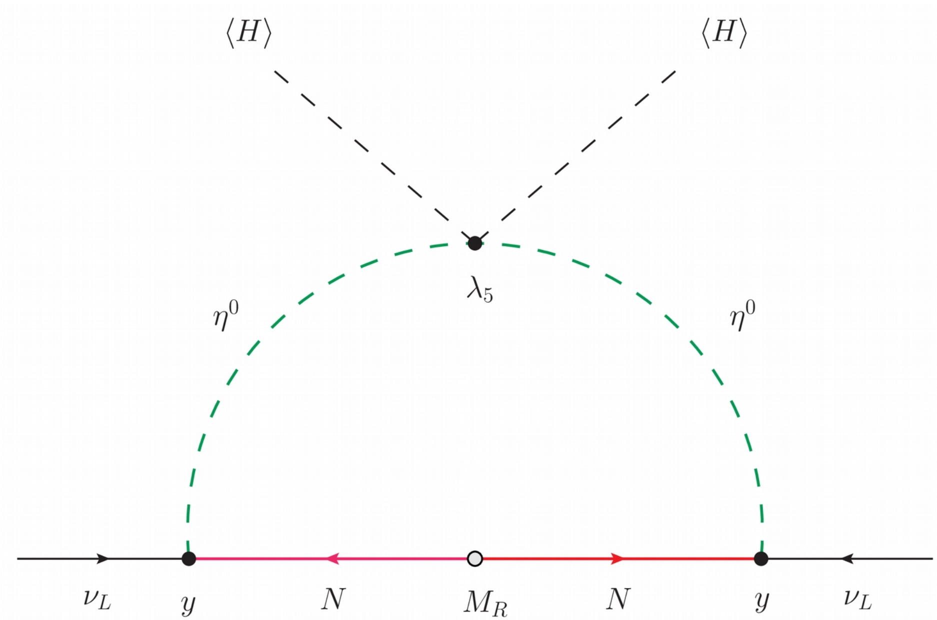
$$f = \frac{\lambda_5}{16\pi^2}$$

$$n_1 = n_2 = 3$$

$$y_1 = y_2 = y/\sqrt{2}$$

$$M = \frac{v^2}{2} M_R^{-1} f_{\rm loop}$$

$$\Rightarrow$$



[Ma, 2006]

BNT model

$$m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1\right)$$

$$f = \frac{\lambda_\Phi\, v^2}{2\, M_\Phi^2}$$

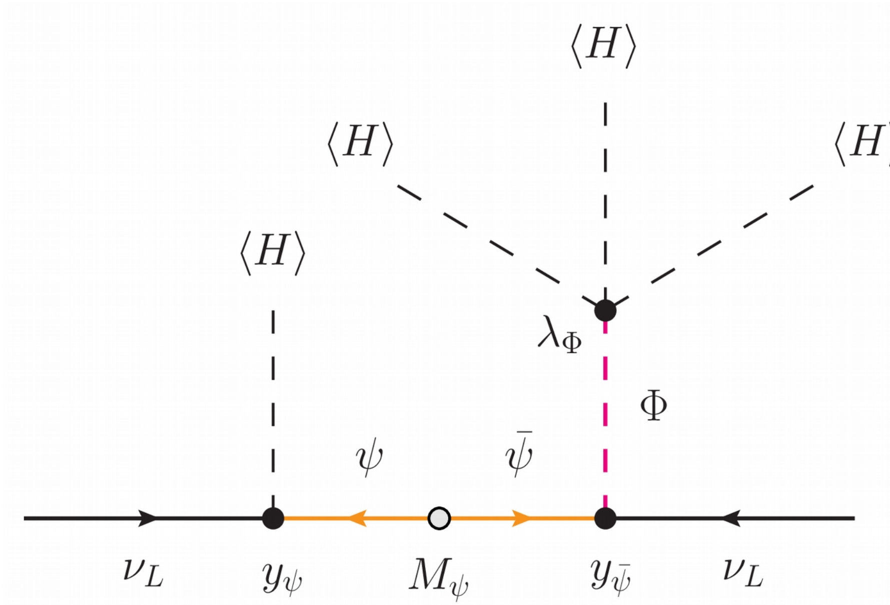
$$n_1 = n_2 = 3$$

$$y_1 = y_\psi \neq y_2 = y_{\bar\psi}$$

$$M = \frac{v^2}{2} M_\psi^{-1}$$

$$\Rightarrow$$

$$m = \frac{\lambda_\Phi\, v^4}{4\, M_\Phi^2}\ \left[y_\psi^T M_\psi^{-1} y_{\bar\psi} + y_{\bar\psi}^T (M_\psi^{-1})^T y_\psi\right]$$



[Babu, Nandi, Tavartkiladze, 2009]

Towards a master parametrization

$$m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1 \right)$$

Neutrino oscillation experiments

Model parameters

Goal:

To establish a <u>general</u>, <u>complete</u> and <u>programmable</u> parametrization of the y_1 and y_2 Yukawa matrices

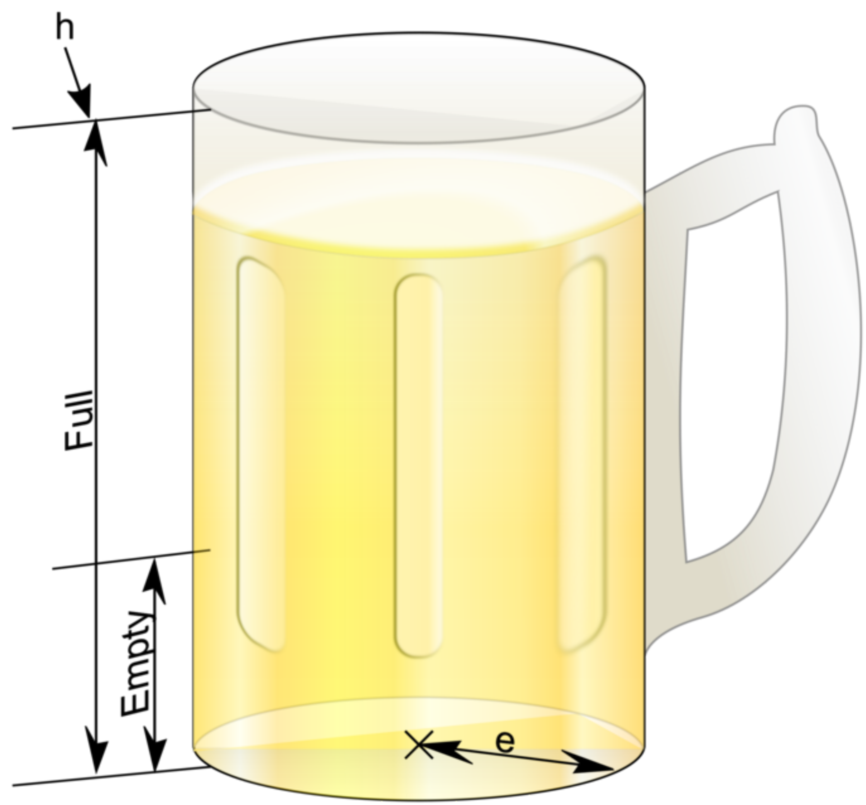
Particular case: Type-I seesaw

Casas-Ibarra parametrization

[Casas, Ibarra, 2001]

$\Longrightarrow$ **Master parametrization**

The master parametrization



$$V = G \cdot h = B \cdot e^2 \cdot r$$

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

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$$D_m = \text{diag}\,(m_1, m_2, m_3) = U^T\, m\, U$$

$r_m = \text{rank}(m)$ 2 or 3

$$\bar{D}_{\sqrt{m}} = \text{diag}\,(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$$

$$\bar{D}_{\sqrt{m}} = \text{diag}\,(\sqrt{m_1}, \sqrt{m_2}, \sqrt{v})$$

U : 3×3 unitary matrix

$$U^\dagger U = U U^\dagger = \mathbb{I}_3$$

Leptonic mixing matrix

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\; A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\; \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$M = V_1^T\, \widehat{\Sigma}\, V_2$$

Singular-value decomposition

$$\widehat{\Sigma} = \left(\begin{array}{cc} \Sigma & 0 \\ 0 & 0_{n_2-n} \\ \hline \multicolumn{2}{c}{0_{n_1-n_2}} \end{array} \right)$$

$\widehat{\Sigma}\ :\ n_1 \times n_2$

$V_1\ :\ n_1 \times n_1$

$V_2\ :\ n_2 \times n_2$

Unitary matrices

$\Sigma\ :\ n \times n$

$\Sigma = \text{diag}\,(\sigma_1, \sigma_2, \ldots, \sigma_n) \qquad (\sigma_i > 0)$

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}} V_1^\dagger \begin{pmatrix} \Sigma^{-1/2} W A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}} U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}} V_2^\dagger \begin{pmatrix} \Sigma^{-1/2} \widehat{W}^* \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}} U^\dagger$$

$$X_1 \, : \, (n_2 - n) \times 3$$

$$X_2 \, : \, (n_1 - n_2) \times 3$$

$$X_3 \, : \, (n_2 - n) \times 3$$

Arbitrary matrices

[Note: absent if $n_1 = n_2 = n$]

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$\widehat{W} = \begin{pmatrix} W & \bar{W} \end{pmatrix}$$

$$r = \text{rank}(W) \leq \min(n, 3)$$

$\widehat{W}$: $n \times n$ unitary matrix

$$\widehat{W}^\dagger \widehat{W} = \widehat{W} \widehat{W}^\dagger = \mathbb{I}_n$$

W : $n \times r$

$\bar{W}$: $n \times (n - r)$ → Absent if $n = r$

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

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$A \, : \, r \times 3$

$A = T\, C_1$

$T \, : \, r \times r$

Invertible upper triangular matrix

$(T)_{ii} \in \mathbb{R}, \, (T)_{ii} > 0$

$C_1 \, : \, r \times 3$

Numerical matrix whose form depends on r_m and r

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}} \, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2} \, W \, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}} \, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}} \, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2} \, \widehat{W}^* \, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}} \, U^\dagger$$

$$\widehat{B} = \begin{pmatrix} B \\ \bar{B} \end{pmatrix}$$

$\widehat{B} \, : \, n \times 3$

$B \, : \, r \times 3$

$$B = \left(T^T\right)^{-1} \left[C_1 \, C_2 + K \, C_1\right]$$

$\bar{B} \, : \, (n - r) \times 3$ → Absent if $n = r$

$C_2 \, : \, 3 \times 3$

Matrix whose form depends on r_m and r

$K \, : \, r \times r$ antisymmetric

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}} \, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2} \, W \, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}} \, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}} \, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2} \, \widehat{W}^* \, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}} \, U^\dagger$$

For $r_m = r = 3$:

$$C_1 = \mathbb{I}_3 \qquad C_2 = \mathbb{I}_3 + K_{12} \, \frac{T_{13}}{T_{11}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

[Different matrix forms for other values of (r_m, r). Ask me if you want to see them]

The master parametrization

$$y_1 = \frac{1}{\sqrt{2\,f}}\, V_1^\dagger \begin{pmatrix} \Sigma^{-1/2}\, W\, A \\ X_1 \\ X_2 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

$$y_2 = \frac{1}{\sqrt{2\,f}}\, V_2^\dagger \begin{pmatrix} \Sigma^{-1/2}\, \widehat{W}^*\, \widehat{B} \\ X_3 \end{pmatrix} \bar{D}_{\sqrt{m}}\, U^\dagger$$

Summary:

Experimental input:

ν-data $\left\{ \begin{array}{l} \bar{D}_{\sqrt{m}} \\ U \end{array} \right.$

Model input:

f

$M \left\{ \begin{array}{l} \Sigma \\ V_{1,2} \end{array} \right.$

Free parameters:

$\widehat{W}$ $\quad X_{1,2,3}$

$\left. \begin{array}{c} T \\ K \\ (C_2) \end{array} \right\} \begin{array}{c} A \\ \widehat{B} \end{array}$

The Casas-Ibarra limit

Particular case: Type-I seesaw

$$m = \frac{v^2}{2} y^T \, M_R^{-1} \, y$$

The Casas-Ibarra limit

Particular case: Type-I seesaw

$$m = \frac{v^2}{2} y^T \, M_R^{-1} \, y$$

$$\Downarrow$$

$$\left.\begin{aligned} f &= 1 \\ n_1 = n_2 &= n = 3 \\ r_m &= r = 3 \\ y_1 = y_2 &= y/\sqrt{2} \\ M &= \frac{v^2}{2} M_R^{-1} \end{aligned}\right\}$$

The Casas-Ibarra limit

Particular case: Type-I seesaw

$$m = \frac{v^2}{2} y^T \, M_R^{-1} \, y$$

$$\Downarrow$$

$$\left.\begin{array}{c} f = 1 \\ n_1 = n_2 = n = 3 \\ r_m = r = 3 \\ y_1 = y_2 = y/\sqrt{2} \\ M = \frac{v^2}{2} M_R^{-1} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} V_1 = V_2 = V \;\; (= \mathbb{I} \text{ in mass basis}) \\ X_{1,2,3}\text{ , } \bar{W} \text{ and } \bar{B} \text{ are absent} \\ W^T W A = B \Rightarrow \; B = \left(A^T\right)^{-1} \\ \qquad\qquad\quad \Rightarrow R = W\,A \end{array}\right.$$

orthogonal 3×3

$$R^T R = R R^T = \mathbb{I}$$

The Casas-Ibarra limit

Particular case: Type-I seesaw

$$m = \frac{v^2}{2} y^T \, M_R^{-1} \, y$$

$$\Downarrow$$

$$\left.\begin{array}{c} f = 1 \\ n_1 = n_2 = n = 3 \\ r_m = r = 3 \\ y_1 = y_2 = y/\sqrt{2} \\ M = \dfrac{v^2}{2} M_R^{-1} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} V_1 = V_2 = V \;\; (= \mathbb{I} \text{ in mass basis}) \\ X_{1,2,3} \text{ , } \bar{W} \text{ and } \bar{B} \text{ are absent} \\ W^T W A = B \; \Rightarrow \; B = \left(A^T\right)^{-1} \\ \qquad\qquad \Rightarrow R = W \, A \\ \qquad\qquad \text{orthogonal } 3 \times 3 \\ \qquad\qquad R^T R = R R^T = \mathbb{I} \end{array}\right.$$

Casas-Ibarra parametrization [Casas, Ibarra, 2001]

$$y = \Sigma^{-1/2} \, R \, D_{\sqrt{m}} \, U^\dagger$$

$$\Uparrow$$

An application

BNT model

	generations	$\mathrm{SU}(3)_c$	$\mathrm{SU}(2)_L$	$\mathrm{U}(1)_Y$
Φ	1	$\mathbf{1}$	$\mathbf{4}$	$3/2$
$\psi_{L,R}$	3	$\mathbf{1}$	$\mathbf{3}$	-1

$$\Phi = \begin{pmatrix} \Phi^{+++} \\ \Phi^{++} \\ \Phi^{+} \\ \Phi^{0} \end{pmatrix}$$

$$-\mathcal{L} \supset y_\psi \, \overline{L} \, H \, \psi_R + y_{\bar{\psi}} \, \overline{L^c} \, \Phi \, \psi_L + M_\psi \overline{\psi} \, \psi$$

An example of $y_1 \neq y_2$

$$\psi_{L,R} = \begin{pmatrix} \psi^{0} \\ \psi^{-} \\ \psi^{--} \end{pmatrix}_{L,R}$$

$$\mathcal{V} \supset \lambda_\Phi \, H^3 \, \Phi$$

$\Rightarrow$ $$\langle \Phi^0 \rangle = \frac{\lambda_\Phi v^3}{2\sqrt{2} M_\Phi^2}$$

$\Rightarrow$ $$\Delta L = 2$$

Lepton number violation

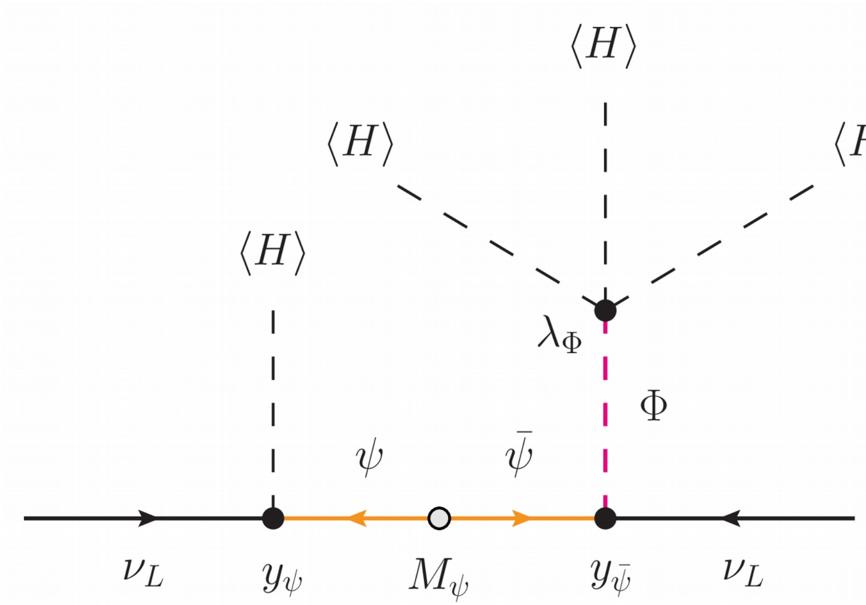
[Babu, Nandi, Tavartkiladze, 2009]

BNT model

$$m = f \left(y_1^T \, M \, y_2 + y_2^T \, M^T \, y_1 \right)$$

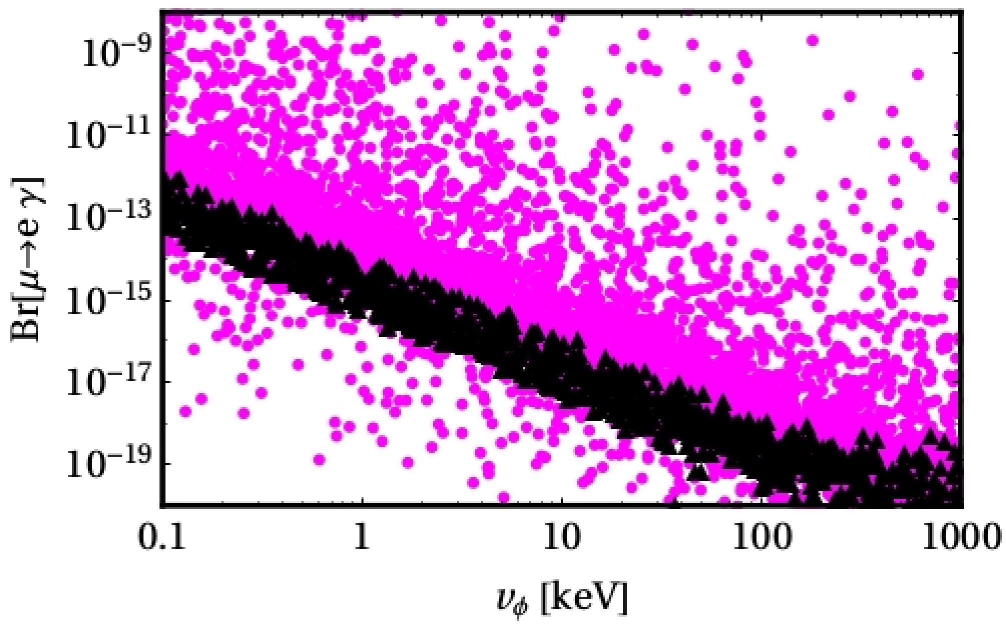
$$m = \frac{\lambda_\Phi \, v^4}{4 \, M_\Phi^2} \left[y_\psi^T M_\psi^{-1} y_{\bar{\psi}} + y_{\bar{\psi}}^T (M_\psi^{-1})^T y_\psi \right]$$

$$\left.\begin{aligned} f &= \frac{\lambda_\Phi \, v^2}{2 \, M_\Phi^2} \\ n_1 &= n_2 = 3 \\ y_1 &= y_\psi \neq y_2 = y_{\bar{\psi}} \\ M &= \frac{v^2}{2} M_\psi^{-1} \end{aligned}\right\} \Rightarrow$$



[Babu, Nandi, Tavartkiladze, 2009]

ν-data NH within 3σ $\quad M_\psi \in [0.5, 2]\,\mathrm{TeV}$

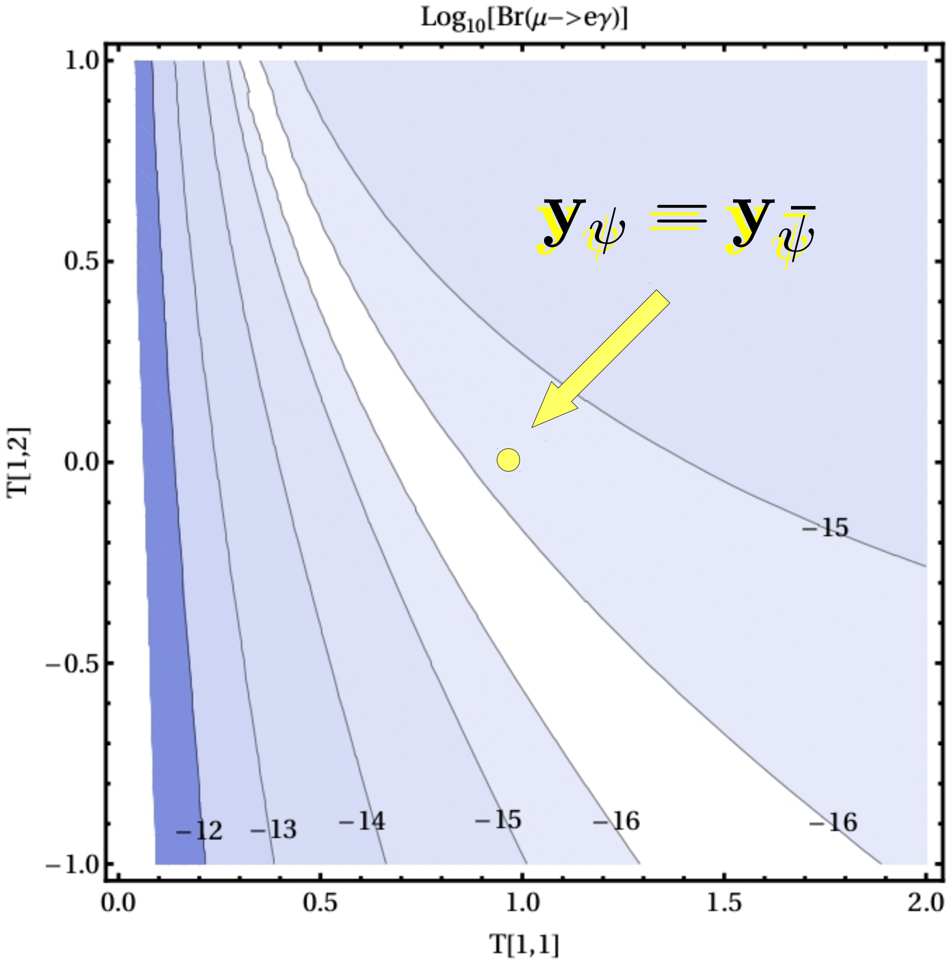


Black: Trivial scan $[W = T = \mathbb{I}\ \&\ K = 0]$

Purple: General scan

A large parameter space that can only be covered with the **master parametrization**

ν-data NH BFP $\quad v_\Phi = 10^{-5}\,\mathrm{GeV}$, $M_\psi = 0.5\,\mathrm{TeV}$



Model in **SARAH** and scans with **SSP** [Staub]
BR computed with **FlavorKit** [Porod, Staub, AV]

Final discussion

Final discussion

The master parametrization allows one to explore the parameter space of any Majorana neutrino mass model in a complete way

Potential limitation: models with Yukawas that have additional restrictions

Easy to program: parameter space exploration easier than ever

The master parametrization may also provide analytical insight on some scenarios

Cheers!

Apologies to those whose favorite beer was not mentioned

No, Heineken does not qualify as “beer”

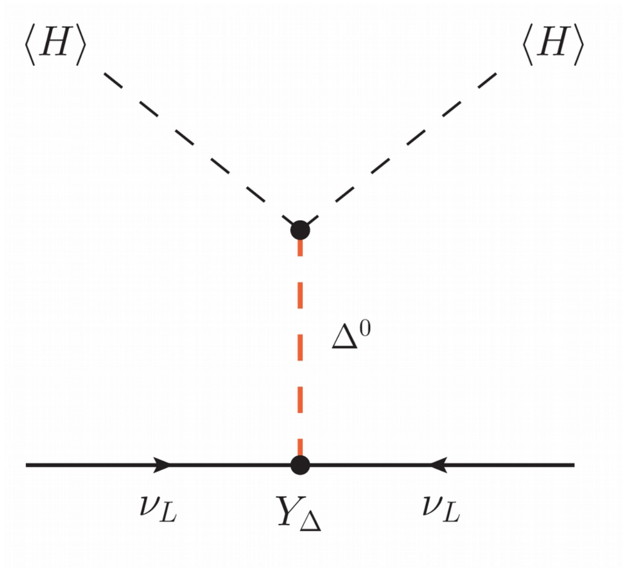
Backup slides

Type-II seesaw

$$m = f\ \left(y_1^T\, M\, y_2 + y_2^T\, M^T\, y_1\right)$$

$$\left.\begin{array}{c} f = 1 \\ n_1 = n_2 = 3 \\ y_1 = y_2 = \mathbb{I}/\sqrt{2} \\ M = Y_\Delta\, v_\Delta \end{array}\right\} \Rightarrow$$

$$m = Y_\Delta v_\Delta$$



[Schechter, Valle, 1982]

Cases (I)

$(r_m, r) = (3,3)$ $\quad C_1 = \mathbb{I}_3 \quad C_2 = \mathbb{I}_3 + K_{12} \frac{T_{13}}{T_{11}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

$(r_m, r) = (3,2)$

1st & 2nd columns of WA ...

l.i. $\quad C_1 = \begin{pmatrix} 1 & 0 & z_1 \\ 0 & 1 & z_2 \end{pmatrix} \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

with $1 + z_1^2 + z_2^2 = 0$

l.d. $\quad C_1 = \begin{pmatrix} 1 & \pm i & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$(r_m, r) = (2,3)$ $\quad C_1 = \mathbb{I}_3 \quad C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + K_{12} \frac{T_{13}}{T_{11}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

Cases (II)

$(r_m, r) = (2,2)$

1st & 2nd columns of WA ...

l.i. $C_1 = \begin{pmatrix} 1 & 0 & z_1 \\ 0 & 1 & z_2 \end{pmatrix}$ $C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

with $z_1^2 + z_2^2 = 0$

l.d. $C_1 = \begin{pmatrix} 1 & \pm i & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$(r_m, r) = (2,1)$ $C_1 = \begin{pmatrix} 1 & \pm i & 0 \end{pmatrix}$ $C_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Parameter counting

$$\#_{\text{free}} = \#_{y_1} + \#_{y_2} - \#_{\text{eqs}} - \#_{\text{extra}}$$

$$\#_{y_1} = 6\, n_1$$

$$\#_{y_2} = 6\, n_2$$

$$\#_{\text{eqs}} = \begin{cases} 12 & \text{for } r = 3 \text{ or } 2 \\ 10 & \text{for } r = 1 \end{cases}$$

$\#_{\text{extra}}$: any extra condition on $y_{1,2}$

$$\#_{\text{free}} = \#_{X_1} + \#_{X_2} + \#_{X_3} + \#_T + \#_W + \#_K + \#_{\bar{B}} + \#_{C_1}$$

$$\#_{X_1} = 6(n_2 - n)$$

$$\#_{X_2} = 6(n_1 - n_2)$$

$$\#_{X_3} = 6(n_2 - n)$$

$$\#_T = r^2$$

$$\#_K = r(r-1)$$

$$\#_{\bar{B}} = 6(n - r)$$

$$\#_W = r(2n - r)$$

$$\#_{C_1} = 0 \text{ or } 2$$

Parameter counting

Scenario	n_1	n_2	n	case	$\#_{\text{eqs}}$	$\#_{\text{extra}}$	$\#_{\text{free}}$	$\#_{X_1}$	$\#_{X_2}$	$\#_{X_3}$	$\#_T$	$\#_W$	$\#_K$	$\#_{\bar{B}}$	$\#_{C_1}$
1	3	3	3	$(3,3)$	12	0	24	-	-	-	9	9	6	-	-
2	4	3	2	$(3,3)$	12	0	42	6	6	6	9	9	6	-	-
3	3	3	3	$(3,2)_a$	12	2	22	-	-	-	4	8	2	6	2
4	2	2	2	$(3,2)_a$	12	0	12	-	-	-	4	4	2	-	2
5	3	3	3	$(3,2)_b$	12	4	20	-	-	-	4	8	2	6	-
6	2	2	2	$(2,2)_a$	12	0	12	-	-	-	4	4	2	-	2
7	2	2	2	$(2,2)_b$	12	2	10	-	-	-	4	4	2	-	-
8	2	2	2	$(2,1)$	10	4	10	-	-	-	1	3	-	6	-

(r_m, r)

Absent if $n_1 = n_2$

Absent if $n = r$

Absent if $n_2 = n$

The <u>general parameter counting</u> can be easily adapted <u>to any model</u>

A philosophical moment

Occam's razor:

The simplest explanation is the correct one

Occam's laser:

The most awesome explanation is the correct one

Occam's hammer:

My explanation is the correct one

All credit goes to
Alberto Aparici