Lepton masses and mixing in 2HDM

S. Zajac[1] B. Dziewit[2] J. Holeczek[2] P. Chaber[2] M. Richter[2]
M. Zralek[2]

[1] Faculty of Mathematics and Natural Studies
Cardinal Stefan Wyszyński University

[2] Institute of Physics
University of Silesia

6th Symposium on Prospects
in the Physics of Discrete Symmetries

Outline

1. Motivation
   - Masses and flavour symmetry problem
   - Discrete flavour symmetry in SM
2. 2HDM with a flavour symmetry
   - Dirac case
   - Majorana case
3. Numerical results
   - Dirac case
   - Majorana case

Outline

Masses And Flavour In Particle Physics

Standard Model of particles cannot be considered as a complete theory.

- Values of Mass - obtain just from experiments.
- Many others problems: gravity, dark matter etc.

Higgs particle is a partial solutions to the problem.

$$\textit{Mass} \rightarrow \textit{Yukawa couplings}$$

How to get Yukawa couplings ?

Solutions

Before 2012 $\theta_{13} = 0$

One of the solutions:

- Flavour symmetry on the leptonic part of Yukawa Lagrangian
- TriBiMaximal (TBM) (A_4 symmetry group) mixing fully explained parameters of U_{PMNS}

On 8 March 2012, Daya Bay $\theta_{13} \neq 0$ (5.2σ)

$What's\ now?$

Outline

Mass in neutrino sector of SM

Conventionally Standard Model is theory with one Higgs doublet and messles neutrinos.

Add Masses with:

- New three Dirac right handed fields, or...
- Majorana mass from left handed fields.

Mass in neutrino sector of SM

Conventionally Standard Model is theory with one Higgs doublet and messles neutrinos.

Add Masses with:

- New three Dirac right handed fields, or...
- Majorana mass from left handed fields.

Discrete symmetry in SM

Discrete symmetry for Yukawa couplings provides the relations for three-dim mass matrices

$$A_L^{i\dagger}\left(M_l M_l^\dagger\right)A_L^i = \left(M_l M_l^\dagger\right)$$

$$A_L^{i\dagger}\left(M_\nu M_\nu^\dagger\right)A_L^i = \left(M_\nu M_\nu^\dagger\right)$$

where $A_L^i = A_L(g_i)$, for $i = 1, 2, \ldots, N$ are 3-dim representation matrices for the left-handed lepton doublets for some N-order falvour symmetry group $\mathcal{G}$
The Schur's lemma implies that $M_l M_l^\dagger$ and $M_\nu M_\nu^\dagger$ are proportional to *Id* matrices, so You get trivial lepton mixing matrix.

Discrete symmetry in SM

What You can do ?

- Break the family symmetry group by scalar singlet called flavon.
- Add more Higgs multiplets

Outline

2HDM type III

We consider Two–Higgs–Doublet–Model of type III with Yukawa Lagrangian

$$\mathcal{L}_Y = -\sum_{i=1,2}\sum_{\alpha,\beta=e,\mu,\tau}\left((h_i^{(l)})_{\alpha,\beta}\left[\bar{L}_{\alpha L}\tilde{\Phi}_i l_{\beta R}\right] + (h_i^{(\nu)})_{\alpha,\beta}\left[\bar{L}_{\alpha L}\Phi_i \nu_{\beta R}\right]\right)$$

Where:

$$L_{\alpha L} = \begin{pmatrix}\nu_{\alpha L}\\ l_{\alpha L}\end{pmatrix},\ \Phi_i = \begin{pmatrix}\phi_i^0\\ \phi_i^-\end{pmatrix},\ i = 1,2$$

are gauge doublets and $l_{\beta R}, \nu_{\beta R}$ are singlets.
$h_i^{(l)}$, and $h_i^{(\nu)}$ are 3-dim Yukawa matrices.

2HDM type III

After spontaneous gauge symmetry breaking we get nonzero VEV's $v_i = |v_i| e^{i\varphi_i}$.
Mass matrices read as follows:

$$M^l = -\frac{1}{\sqrt{2}} \left(v_1^* h_1^{(l)} + v_2^* h_2^{(l)} \right)$$

$$M^\nu = \frac{1}{\sqrt{2}} \left(v_1 h_1^{(\nu)} + v_2 h_2^{(\nu)} \right)$$

with VEV's restricted to:

$$\sqrt{|v_1|^2 + |v_2|^2} = \left(\sqrt{2} G_F \right)^{-1/2} \sim 246 GeV$$

Family symmetry in 2HDM type III

For some discrete flavour group $\mathcal{G}$ Family Symmetry means that:

- After fields transformations Lagrangian does not change
- We need 3-dim representation for:

$$
\begin{aligned}
L_{\alpha L} \to L'_{\alpha L} &= (A_L)_{\alpha,\chi} L_{\chi L}, \\
l_{\beta R} \to l'_{\beta R} &= (A_l^R)_{\beta,\delta} l_{\delta R}, \\
\nu_{\beta R} \to \nu'_{\beta R} &= (A_\nu^R)_{\beta,\delta} \nu_{\delta R}
\end{aligned}
$$

- And 2-dim representation for:

$$
\Phi_i \to \Phi'_i = (A_\Phi)_{ik} \Phi_k
$$

All transformation matrices are unitary.

$$\mathcal{L}(L_{\alpha L}, l_{\beta R}, \nu_{\beta R}, \Phi_i) = \mathcal{L}(L'_{\alpha L}, l'_{\beta R}, \nu'_{\beta R}, \Phi'_i)$$

In the Higgs potential two possibilities:

- Coefficients in the potential remain the same

$$V(\Psi'_1, \Psi'_2) = V(\Psi_1, \Psi_2)$$

- Form of the Higgs potential does not change, but

$$v'_i = (A_\Psi)_{ik} v_k$$

Symmetry conditions can be written as an eigenequation problem for direct product of unitary group representations to the eigenvalue 1

For any group elements (so also for generators) we have:

$$((A_\Phi)^\dagger \otimes (A_L)^\dagger \otimes (A_l^R)^T)_{k,\alpha,\delta;i,\beta,\gamma}(h_i^l)_{\beta,\gamma} = (h_k^l)_{\alpha,\delta}$$

$$((A_\Phi)^T \otimes (A_L)^\dagger \otimes (A_l^R)^T)_{k,\alpha,\delta;i,\beta,\gamma}(h_i^\nu)_{\beta,\gamma} = (h_k^\nu)_{\alpha,\delta}$$

The invariance equations for the mass matrices are not trivial, so we avoid the consequences of Schur's Lemma.

$$(A_L)M^{l(\nu)}(A_l^R)^\dagger = \frac{1}{\sqrt{2}}\sum_{i,k=1}^{2} h_i^{l(\nu)}(A_\Phi)_{ik} v_k \neq M^{l(\nu)}$$

We can obtain non-trivial mass matrices without additional flavon fields.

Outline

For Majorana neutrinos the Yukawa term must to be changed.
We can take (non-renormalizable Weinberg term):

$$\mathcal{L}_Y^{\nu} = -\frac{g}{M}\sum_{i,k=1}^{2}\sum_{\alpha,\beta=e,\mu,\tau} h_{\alpha,\beta}^{(i,k)}\left(\bar{L}_{\alpha L}\Phi_i\right)\left(\Phi_k L_{\beta R}^c\right)$$

where $L_{\beta R}^c = C\bar{L}_{\beta L}^T$ are charge conjugated lepton doublet fields. After symmetry breaking neutrino mass matrix has form:

$$M_{\alpha,\beta}^{\nu} = \frac{g}{M}\sum_{i,k=1}^{2} v_i v_k h_{\alpha,\beta}^{(i,k)}$$

Relation for Yukawa couplings:

$$\left((A_\Phi)^T \otimes (A_\Phi)^T \otimes (A_L)^\dagger \otimes (A_L)^\dagger\right)_{k,m,\chi,\eta;i,j,\alpha,\beta}\left(h_{\alpha,\beta}^{i,j}\right) = \left(h_{\chi,\eta}^{k,m}\right)$$

Candidates for the flavour group

The flavour group $\mathcal{G}$ in 2HDM cannot be arbitrary.

- The group must posses at least one 2-dim and one 3-dim irreducible (faithful) representation (10862 groups).
- Subgroup of $U(3)$ (reduce to 413 groups).

We use `GAP` system for computational discrete algebra (`Small Groups Library` and `REPSN` packages)

$[o, i]$	Structure description	2D	3D	$U(2)$	$U(3)$	L	DN	MN	L + DN	L + MN
[24, 3]	SL(2,3)	3	1	2	$1+2$			3		
[24, 12]	S4	1	2		3	4	4	2	8	4
[48, 28]	C2.S4=SL(2,3).C2	3	2	2	$1+2$	4	4	6	8	4
[48, 29]	GL(2,3)	3	2	2	$1+2$	4	4	6	8	4
[48, 30]	A4:C4	2	4		3	16	16	8	32	16
[48, 32]	C2xSL(2,3)	6	2		$1+2$			12		
[48, 33]	SL(2,3):C2	6	2	2	$1+2$					
[48, 48]	C2xS4	2	4		3	16	16	8	32	16
[54, 8]	((C3xC3):C3):C2	4	4		3	32	32		64	
[72, 3]	Q8:C9	9	3	2	$1+2$			9		
[72, 25]	C3xSL(2,3)	9	3	2	$1+2$			9		
[72, 42]	C3xS4	3	6		3	36	36	6	72	12
[96, 64]	((C4xC4):C3):C2	1	6		3	12	12	2	24	4
[96, 65]	A4:C8	4	8		3	64	64	16	128	32
[96, 66]	SL(2,3):C4	6	4		$1+2$	16	16	24	32	16
[96, 67]	SL(2,3):C4	6	4	2	$1+2$	16	16	8	32	16
[96, 69]	C4xSL(2,3)	12	4		$1+2$			24		
[96, 74]	((C8xC2):C2):C3	12	4	2	$1+2$					
[96, 186]	C4xS4	4	8		3	64	64	16	128	32
[96, 188]	C2x(C2.S4=SL(2,3).C2)	6	4		$1+2$	16	16	24	32	16
[96, 189]	C2xGL(2,3)	6	4		$1+2$	16	16	24	32	16
[96, 192]	(C2.S4=SL(2,3).C2):C2	6	4	2	$1+2$	16	16	8	32	16
[96, 200]	C2x(SL(2,3):C2)	12	4		$1+2$					

Outline

Dirac neutrinos

There exist 267 groups that gave 748672 different combinations of 2 and 3 dim irred. representations that gave 1-dim degeneration subspace for all generators.
All possible solutions for Yukawa matrices can be expressed in 7 base forms. For example:.

$$h_1^{(7)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad h_2^{(7)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \tag{1}$$

- For Dirac neutrinos:

$$\{h_1^{(\nu)}, h_2^{(\nu)}\} = \{h_1^{(i)}, e^{i\varphi} h_2^{(i)}\}$$

- For charged leptons:

$$\{h_1^{(l)}, h_2^{(l)}\} = \{h_2^{(i)}, e^{-i(\delta_l+\varphi)} h_1^{(i)}\}$$

where φ is a phase distinctive for a group and $\delta_l = 0, \pi$

For lepton masses and mixing matrix we construct $M^l M^{l\dagger}$ $M^\nu M^{\nu\dagger}$ (for all possible Yukawa matrices) as:

$$M_x M_x^\dagger = |c_x|^2 \begin{pmatrix} 1+\kappa^2 & \kappa e^{-i(\eta_x+2k\pi/3)} & \kappa e^{i(\eta_x-2k\pi/3)} \\ \kappa e^{i(\eta_x+2k\pi/3)} & 1+\kappa^2 & \kappa e^{-i\eta_x} \\ \kappa e^{-i(\eta_x-2k\pi/3)} & \kappa e^{i\eta_x} & 1+\kappa^2 \end{pmatrix} \quad (2)$$

with $k=-1,0,+1$ and $\kappa = |v_2|/|v_1|$.

The only difference lies in the phase η_x.

For Dirac neutrino : $\eta_\nu = \varphi + \varphi_2 - \varphi_1$

For charged leptons: $\eta_l = \delta_l + \varphi + \varphi_2 - \varphi_1$

where $\varphi_i (i=1,2)$ are phases of the VEVs v_i

After diagonalization: $U^{\dagger}\left(M_x M_x^{\dagger}\right)U = diag\left(m_{x1}^2, m_{x2}^2, m_{x3}^2\right)$, we obtain $(\omega = e^{\frac{2}{3}\pi i})$:

- masses :

$$
\begin{aligned}
m_{x1}^2 &= |c_x|^2\left(1+\kappa^2+2\kappa cos\left(\eta_x\right)\right),\\
m_{x2}^2 &= |c_x|^2\left(1+\kappa^2+2\kappa sin\left(\eta_x-\frac{\pi}{6}\right)\right),\\
m_{x3}^2 &= |c_x|^2\left(1+\kappa^2-2\kappa sin\left(\eta_x+\frac{\pi}{6}\right)\right),
\end{aligned}
$$

- diagonaliztion matrix

$$
U = \frac{1}{\sqrt{3}}\begin{pmatrix} e^{-\frac{2}{3}\pi ik} & \omega e^{-\frac{2}{3}\pi ik} & \omega^2 e^{-\frac{2}{3}\pi ik} \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix}
$$

Dirac case summary

- U matrix does not depend on the phase η_x, so it is identical for charged leptons and for the neutrino. Therefore, it is not possible to reconstruct the correct mixing matrix.
- In the case $\delta_l = 0$ neutrinos and lepton masses are proportional and the mixing matrix is 3×3 identity matrix.
- For groups and for representations where $\delta_l = \pi$, masses of charged leptons and neutrinos are not proportional, but in this case we cannot reconstruct masses of the electron muon and tau

The SM extended by one additional doublet of Higgs particles (2HDM) does not posses a discrete family symmetry that can explain the masses of charged leptons, masses of neutrions, and PMNS matrix.

Outline

1. Motivation
   - Masses and flavour symmetry problem
   - Discrete flavour symmetry in SM
2. 2HDM with a flavour symmetry
   - Dirac case
   - Majorana case
3. Numerical results
   - Dirac case
   - Majorana case

There exist 195 groups that gave in total 20888 solutions that gave 2-dim subspace for all generators.
Yukawa matrices for Majorana neutrinos are given by:

$$h^{(i,k)} = x\, p_{i,k} + y\, r_{i,k}$$

where x,y are complex number and $\vec{p}$, $\vec{r}$ are 36-dim vectors.

- Majorana

$$h^{(1,1)} = xh_2^{(7)}, \qquad h^{(1,2)} = yI_3,$$
$$h^{(2,1)} = ye^{i\delta} I_3, \qquad h^{(2,2)} = xe^{i(\delta+2\varphi)} h_1^{(7)}$$

- Charged leptons

$$\{h_1^{(l)}, h_2^{(l)}\} = \{h_2^{(7)}, e^{-i(\delta_l+\varphi)} h_1^{(7)}\}$$

Majorana mass matrix has more complicated form:

$$M^{\nu} = \frac{g}{2M}\left(x|v_1|^2 e^{2i\varphi_1} h_2^{(7)} + y|v_1 v_2| e^{i(\varphi_1+\varphi_2)} I_3 \left(1 + e^{i\delta}\right) + x|v_2|^2 e^{i(\delta+2(\varphi_2+\varphi))} h_1^{(7)}\right)$$

but masses for charged leptons are given by the same formula as in the Dirac case.

Symmetry condition for the Majorana neutrinos does not give any new flavour symmetry group with new 3-dim repr. A_L.

There is no good solutions for mass of charged leptons

Summary

- We try to find some discrete flavour symmetry to explain masses and mixing of leptons in SM and 2HDM (with full Lagrangian symmetry)
- We have investigate discrete groups (subgroups of U(3)) up to the order of 1025 (our choice)
- Models with Dirac and Majorana neutrino cases. Yukawa matrices (for charged leptons) are independent from the nature of neutrino.

We haven't find a symmetry that gave valid masses of charged leptons