DISCRETE18

Lepton masses and mixing from modular S_4 symmetry

in collaboration with S.T. Petcov [arXiv:1806.03203],
A.V. Titov and P.P. Novichkov [arXiv: 1811.04933]

DF Departamento de Física, Técnico Lisboa
CFTP Centro de Física Teórica de Partículas

João Penedo (CFTP, Lisbon)

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FCT Fundação para a Ciência e a Tecnologia
INFN
europa.eu European Union
QREN Quadro de Referência Estratégico Nacional Portugal 2007.2013
COMPETE Programa Operacional Factores de Competitividade

Plan

- Neutrino oscillation data
- Symmetry and flavour
- The modular symmetry framework
- The case of S_4
- Model building and phenomenology

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3ν flavour paradigm

Recall/see e.g. talks by L. Ludhova, M. Zito, M. Tórtola, W. Wang, ...

Consistent description and remarkable progress in the last two decades

Standard parameterisation

$$c_{ij} \equiv \cos\theta_{ij}, \; s_{ij} \equiv \sin\theta_{ij}$$

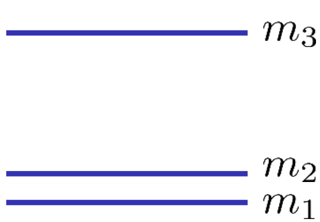
$$U_{\text{PMNS}} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha_{21}/2} & \\ & & e^{i\alpha_{31}/2} \end{pmatrix}$$

Mass ordering

$$\frac{\Delta m^2_\odot}{|\Delta m^2_A|} \sim \frac{1}{30}$$

Normal ordering (NO)

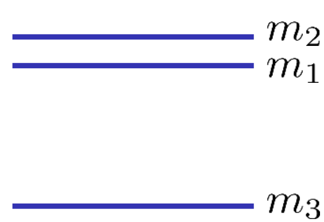
$m_1 < m_2 < m_3$



vs.

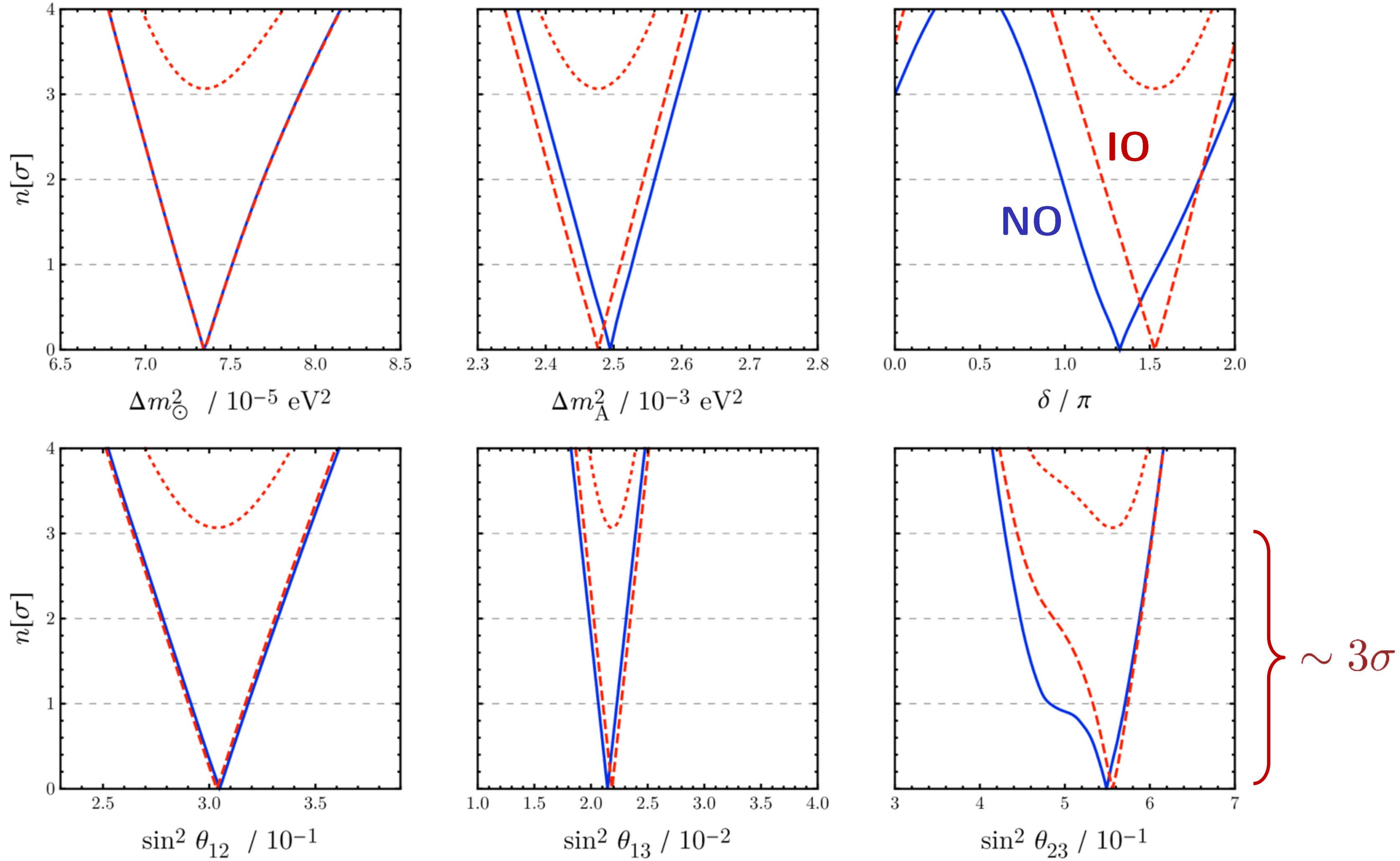
Inverted ordering (IO)

$m_3 < m_1 < m_2$



3ν flavour paradigm (cont.)

Capozzi et al, 1804.09678,
see also 1811.05487 recently

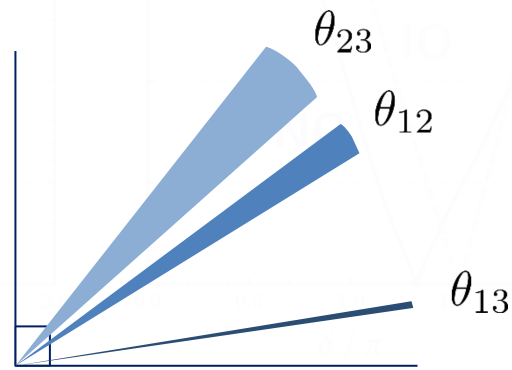


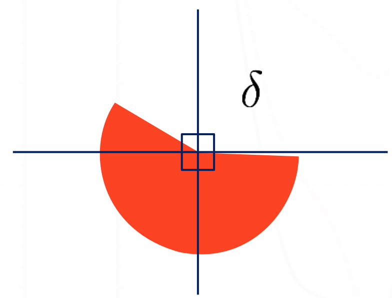
3ν flavour paradigm (cont.)

Capozzi et al, 1804.09678,
see also 1811.05487 recently

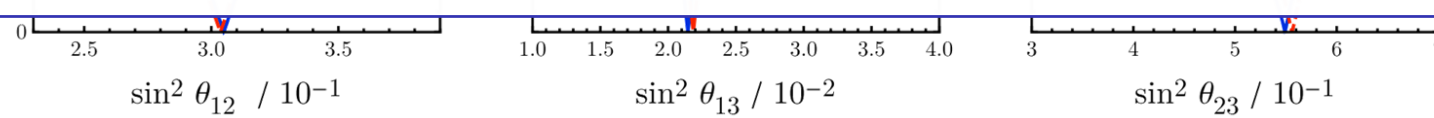
For a spectrum with NO:

Parameter	Best-fit value
$\Delta m^2_\odot$	7.34×10^{-5} eV^2
$\lvert\Delta m^2_{\mathrm{A}}\rvert$	2.49×10^{-3} eV^2
$\sin^2\theta_{12}$	0.304
$\sin^2\theta_{13}$	0.0214
$\sin^2\theta_{23}$	0.551
δ	$1.32\,\pi$





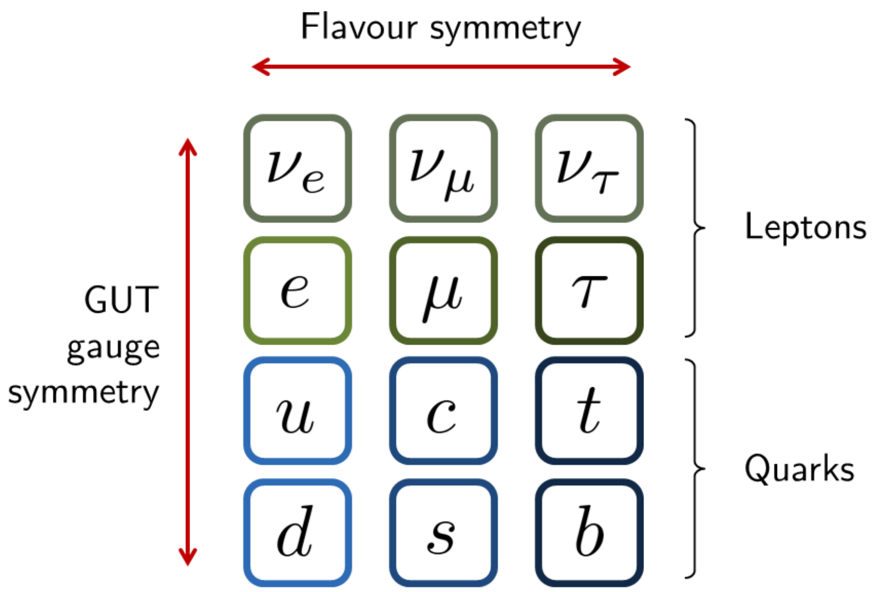
$\sim 3\sigma$



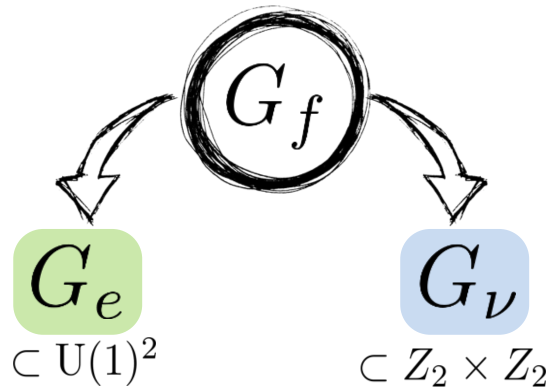
Symmetry and flavour

For the lepton sector, at low energy and in some flavour basis:

$$\mathcal{L}_\ell = -\left(M_e\right)_{ij} \overline{\ell_{iL}}\, \ell_{jR} - \frac{1}{2} \left(M_\nu\right)_{ij} \overline{\nu^C_{iR}}\, \nu_{jL} + \text{h.c.}$$

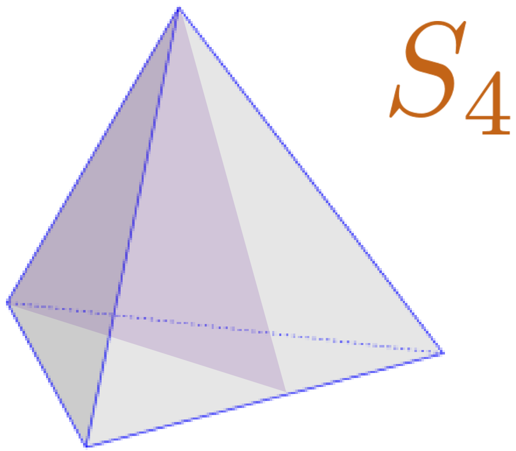


Non-Abelian discrete flavour symmetries



residual symmetries
constrain mixing and Dirac phase

S_4 as the flavour group



rotations + **reflection**
⇔ **permutation of vertices**

Group presentations

3 generators, *S*, *T*, and *U*, obeying:

$$S^2 = T^3 = U^2 = (ST)^3$$
$$= (SU)^2 = (TU)^2 = (STU)^4 = 1$$

vs.

2 generators, *S* and *T*, obeying:

$$S^2 = (ST)^3 = T^4 = 1$$

For reviews on the discrete sym. approach to flavour, see e.g.: Altarelli, Feruglio, 1002.0211; Ishimori et al, 1003.3552; King et al, 1402.4271; Petcov, 1711.10806.

Problems with the usual approach

- many scalar multiplets to arrange breaking
- "barroque"/complicated potentials
- need care with higher-dim. operators
- may need large groups, or large corrections

Predictability is at risk

Framework
(Modular-invariant SUSY actions)

Modular-invariant SUSY actions

Ferrara et al, '89

Interest renewed by Feruglio, 1706.08749

a new model builing avenue

$$\mathcal{S} = \int d^4x\, d^2\theta\, d^2\overline{\theta}\; K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) + \int d^4x\, d^2\theta\; W(\chi_i; \tau) + \text{h.c.}$$

- small number of parameters
- small number of symmetry breaking spurions
- control over higher-dim. operators
- leading-order predictions only potentially modified by corrections from Kähler, SUSY breaking

Modular transformations: the modulus

τ is a dimensionless spurion, $\langle\tau\rangle$ only source of modular sym. breaking

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\text{with} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2,\mathbb{Z})$$

$$a, b, c, d \in \mathbb{Z} \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1$$

<u>Modular group</u>

$$S^2 = (ST)^3 = 1$$

$$S: \tau \to -1/\tau\,, \qquad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$T: \tau \to \tau + 1\,, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Bottom-up approach

We will scan $\langle\tau\rangle$; for top-down, see e.g. Kobayashi et al, 1804.06644

Modular transformations: the superfields

$$\begin{cases} \tau \to \dfrac{a\tau + b}{c\tau + d} \\ \chi_i \to (c\tau + d)^{-k_i}\, \rho_i(\gamma)\, \chi_i \end{cases} \quad \text{with } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

"weight"

infinite subgroup

$$\overline{\Gamma}(N) \equiv \left\{ \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| \, \gamma \in PSL(2,\mathbb{Z}) \wedge \gamma = 1 \,(\mathrm{mod}\ N) \right\}$$

quotient

$$\Gamma_N$$

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$$S^2 = (ST)^3 = T^N = 1$$

Modular transformations: the superfields

$$\Gamma_2 \simeq S_3$$

Kobayashi, Tanaka, Tatsuishi, 1803.10391

$$\Gamma_3 \simeq A_4$$

Feruglio, 1706.08749
Feruglio, Criado, 1807.01125
Kobayashi et al., 1808.03012

$$\Gamma_4 \simeq S_4$$

JP, Petcov, 1806.11040
Novichkov, JP, Petcov, Titov, 1811.04933

...

quotient

$$\Gamma_N$$

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$$S^2 = (ST)^3 = T^N = 1$$

The superpotential

$$W(\chi_i;\tau) = \sum_n \sum_{\{i_1,\,\ldots,\,i_n\}} \left(Y_{\{i_1,\,\ldots,\,i_n\}}(\tau)\ \chi_{i_1}\cdots\chi_{i_n}\right)_{\mathbf{1}}$$

$$\begin{cases} \tau \to \gamma\tau = \dfrac{a\tau+b}{c\tau+d} \\ \chi_i \to (c\tau+d)^{-k_i}\rho_i(\gamma)\,\chi_i \\ Y(\tau) \to Y(\gamma\tau) = (c\tau+d)^{2k_Y}\rho_Y(\gamma)\,Y(\tau) \end{cases} \quad \text{with } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

weights

$Y(\tau)$ are **modular forms** obeying $\begin{cases} 2k_Y - k_{i_1} - \ldots - k_{i_n} = 0 \\ \rho_Y \otimes \rho_{i_1} \otimes \ldots \otimes \rho_{i_1} \supset \mathbf{1} \end{cases}$

How to build them?

Generators of S_4 modular forms

The Dedekind eta function

Useful to build the sought-out modular forms

$$\eta(\tau) \equiv q^{1/24} \prod_{k=1}^{\infty} (1 - q^k), \quad \text{with } q = e^{2\pi i \tau}$$

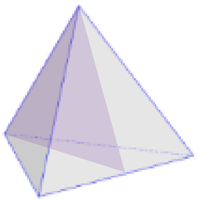
why this function?

$$S: \; \tau \to -1/\tau \qquad \qquad \eta(-1/\tau) = \sqrt{-i\tau}\, \eta(\tau)$$

$$\Longrightarrow$$

$$T: \; \tau \to \tau + 1 \qquad \qquad \eta(\tau + 1) = e^{i\pi/12}\, \eta(\tau)$$

Action of S_4 generators

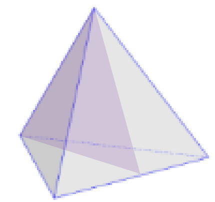


$$S^2 = (ST)^3 = T^4 = 1$$

Set of 'seed' functions

$$\{\eta_i\} = \left\{\eta\left(\tau + \frac{1}{2}\right), \eta\left(4\tau\right), \eta\left(\frac{\tau}{4}\right), \eta\left(\frac{\tau+1}{4}\right), \eta\left(\frac{\tau+2}{4}\right), \eta\left(\frac{\tau+3}{4}\right)\right\}$$

Action of S_4 generators



$$S^2 = (ST)^3 = T^4 = 1$$

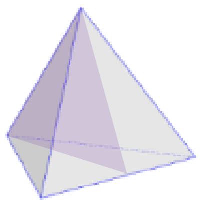
Set of 'seed' functions

$S:$

$$\{\eta_i\} = \left\{\eta\left(\tau + \frac{1}{2}\right), \eta\left(4\tau\right), \eta\left(\frac{\tau}{4}\right), \eta\left(\frac{\tau+1}{4}\right), \eta\left(\frac{\tau+2}{4}\right), \eta\left(\frac{\tau+3}{4}\right)\right\}$$

up to multiplicative factors

Action of S_4 generators



$$S^2 = (ST)^3 = T^4 = 1$$

Set of 'seed' functions

S :

$$\{\eta_i\} = \left\{ \eta\left(\tau + \frac{1}{2}\right), \eta(4\tau), \eta\left(\frac{\tau}{4}\right), \eta\left(\frac{\tau+1}{4}\right), \eta\left(\frac{\tau+2}{4}\right), \eta\left(\frac{\tau+3}{4}\right) \right\}$$

T :

up to multiplicative factors

Building lowest-weight forms

$$Y(a_1,\dots,a_6|\tau) \equiv \frac{d}{d\tau}\left(\sum_{i=1}^{6} a_i \log \eta_i(\tau)\right)$$

$$\sum_i a_i = 0$$

$$S:\ Y(a_1,\dots,a_6|\tau) \to Y(a_1,a_2,a_3,a_4,a_5,a_6|-1/\tau) = \tau^2\, Y(a_5,a_3,a_2,a_6,a_1,a_4|\tau)$$

$$T:\ Y(a_1,\dots,a_6|\tau) \to Y(a_1,a_2,a_3,a_4,a_5,a_6|\tau+1) = Y(a_1,a_2,a_6,a_3,a_4,a_5|\tau)$$

$$Y(\tau) \to (c\tau+d)^{2k_Y}\,\rho_Y(\gamma)\,Y(\tau) \Longrightarrow$$ **Modular forms of weight 2**

Building lowest-weight forms: multiplets

$$Y(a_1, \ldots, a_6|\tau) \equiv \frac{d}{d\tau}\left(\sum_{i=1}^{6} a_i \log \eta_i(\tau)\right)$$

$$\sum_i a_i = 0$$

Lowest weight forms arrange into:

$$Y_{\mathbf{2}}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$ **doublet 2**

$$Y_{\mathbf{3'}}(\tau) = \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}$$ **triplet 3'**

$$Y_1(\tau) \equiv Y(1, 1, \omega, \omega^2, \omega, \omega^2|\tau)$$
$$Y_2(\tau) \equiv Y(1, 1, \omega^2, \omega, \omega^2, \omega|\tau)$$
$$Y_3(\tau) \equiv Y(1, -1, -1, -1, 1, 1|\tau)$$
$$Y_4(\tau) \equiv Y(1, -1, -\omega^2, -\omega, \omega^2, \omega|\tau)$$
$$Y_5(\tau) \equiv Y(1, -1, -\omega, -\omega^2, \omega, \omega^2|\tau)$$

Correct dimension (5)

Products generate higher weight forms

Modular S_4 phenomenology

What are the consequences of having a spontaneously broken modular symmetry?

Guidelines for model building

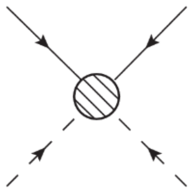
Using minimality as a guiding principle...

- **No flavons** are introduced,
- Higgs multiplets transform trivially,
- Lepton doublets transform as an S_4 **triplet**,
- Lepton singlets transform as S_4 **singlets**, and
- Lowest possible weights are chosen such that all charged-leptons are massive

Lepton masses and mixing from the Weinberg operator

JP, Petcov, 1806.03203

$$W = \sum_i \alpha_i \left[E_i^c\, L\, H_d\, f_i(Y)\right]_{\mathbf{1}} + \frac{g}{\Lambda}\left[L\, H_u\, L\, H_u\, f_W(Y)\right]_{\mathbf{1}}$$



- Study models systematically by increasing weight of L
- Minimal working model ($k_L = 2$) has 10 real parameters, predicting 12 observables
- Correlations between observables are expected

Lepton masses and mixing from the Weinberg operator: a benchmark

	H_u	H_d	L	E_1^c	E_2^c	E_3^c
ρ_i	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$ $\mathbf{3}'$	$\mathbf{1}'$ $\mathbf{1}$	$\mathbf{1}$ $\mathbf{1}'$	$\mathbf{1}'$ $\mathbf{1}$
k_i	0	0	2	0	2	2

NO spectrum

$$\frac{m_e}{m_\mu} \simeq 0.0048\,, \quad \sin^2\theta_{12} \simeq 0.292\,, \quad \delta \simeq 1.64\pi\,,$$

$$\frac{m_\mu}{m_\tau} \simeq 0.0560\,, \quad \sin^2\theta_{13} \simeq 0.021\,, \quad \alpha_{21} \simeq 0.10\pi\,,$$

$$r \simeq 0.0298\,, \quad \sin^2\theta_{23} \simeq 0.493\,, \quad \alpha_{31} \simeq 1.10\pi\,.$$

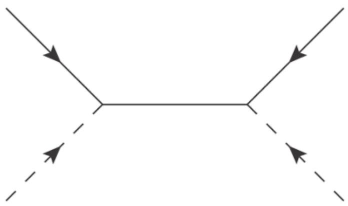
A distinctive feature of this framework is the prediction of the **Dirac and Majorana CPV phases**

$$|\langle m \rangle| \simeq 0.042 \text{ eV}$$

Lepton masses and mixing from Seesaw type I

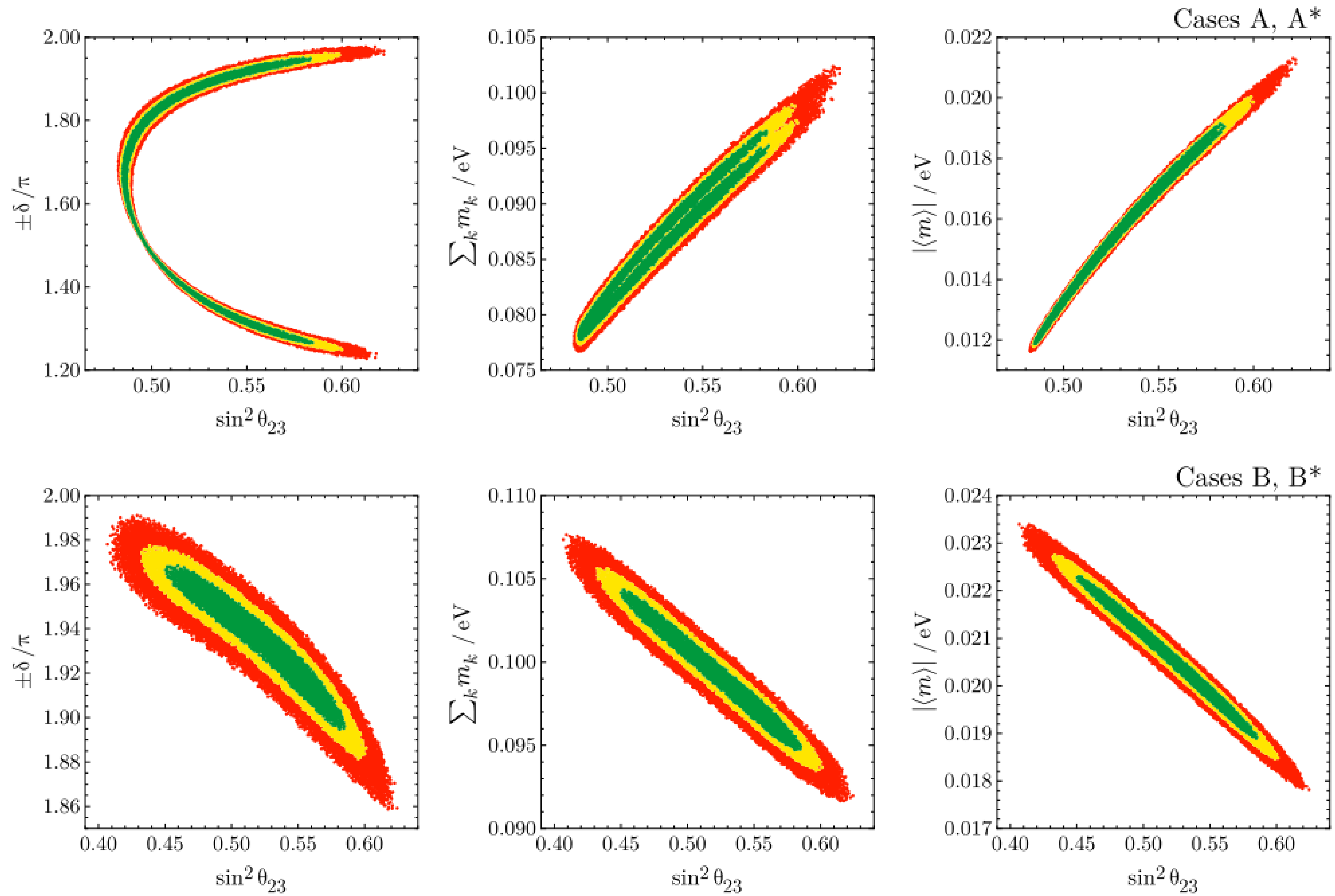
Novichkov, JP, Petcov, Titov, 1811.04933

$$W = \sum_i \alpha_i \left[E_i^c\, L\, f_i(Y)\right]_{\mathbf{1}} H_d + g\left[N^c\, L\, f_N(Y)\right]_{\mathbf{1}} H_u + \Lambda\left[N^c\, N^c\, f_M(Y)\right]_{\mathbf{1}}$$

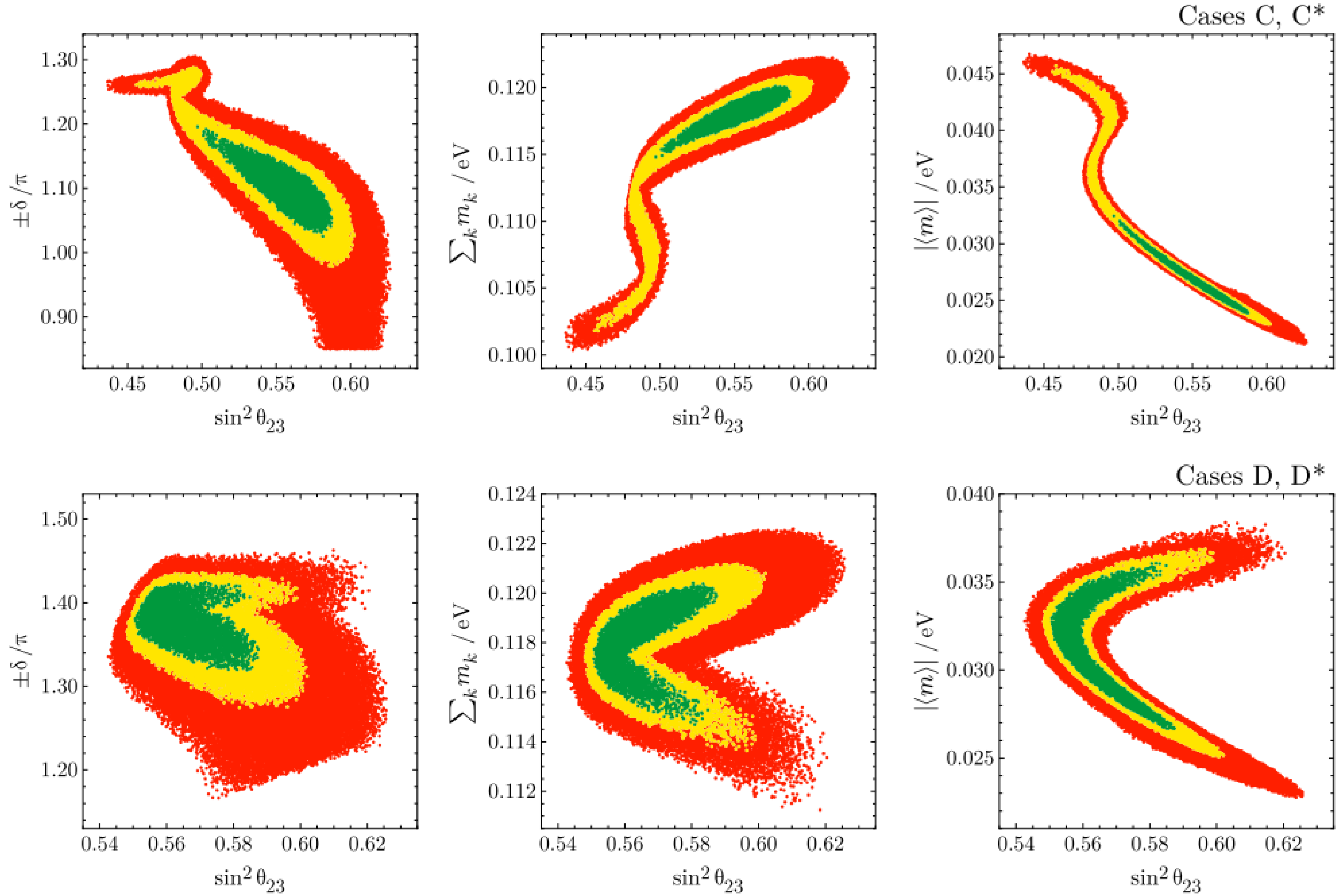


- UV completion, more predictive
- Minimal working models have 8 parameters (vs. 12 observables)
- Parameter space fully scanned and correlations studied in detail
- In working models heavy singlets can be integrated out

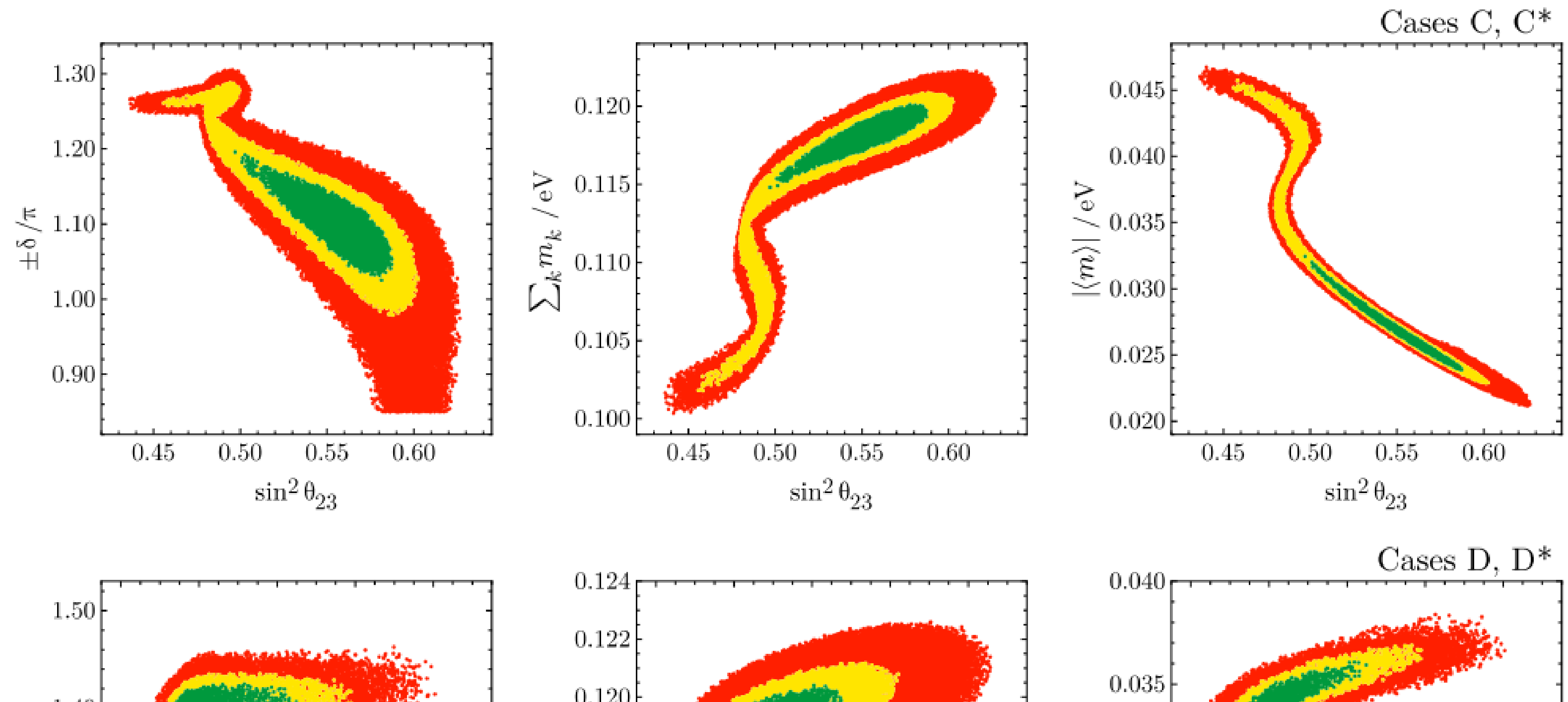
Modular Seesaw correlations



Modular Seesaw correlations



Modular Seesaw correlations



Promising: more model building needed to reduce number of parameters (and perhaps produce additional phenomenology)

Summary and Conclusions (1/2)

- **Modular symmetry** may strongly constrain masses and mixing.
- Fields carrying a non-trivial modular weight transform with a scale factor in addition to the usual unitary rotation.
- To build invariants one needs **modular forms**.

Summary and Conclusions (2/2)

- We have shown how lepton mixing and Dirac and Majorana CPV phases can be predicted in **minimal models**.
- The existence of successful benchmarks warrants further exploration of such an approach.

STAY TUNED...

<u>Thu 29/11 (the day after tomorrow)</u>

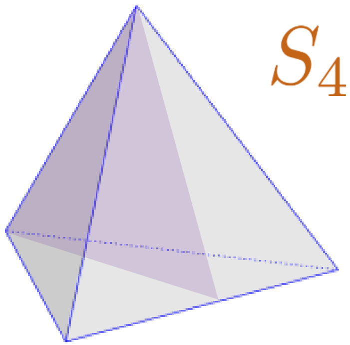
at 15:15

P. Novichkov: “S_4 Modular Symmetry, Seesaw Mechanism and Lepton Masses and Mixing”

Thank you / Danke schön

Backup slides

S_4 in the 2-generator presentation



$$S^2 = (ST)^3 = T^4 = 1$$

$$\mathbf{1}: \quad \rho(S) = 1, \quad \rho(T) = 1$$

$$\mathbf{1}': \quad \rho(S) = -1, \quad \rho(T) = -1$$

$$\mathbf{2}: \quad \rho(S) = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix}, \quad \rho(T) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathbf{3}: \quad \rho(S) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix}, \quad \rho(T) = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}$$

$$\mathbf{3}': \quad \rho(S) = -\frac{1}{3}\begin{pmatrix} -1 & 2\omega^2 & 2\omega \\ 2\omega & 2 & -\omega^2 \\ 2\omega^2 & -\omega & 2 \end{pmatrix}, \quad \rho(T) = -\frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}$$

see, e.g., Bazzocchi, Merlo, Morisi, 0901.2086

$$\omega = e^{2\pi i/3}$$

Kähler potential

The minimal choice; a definition of the setup, at this point:

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) = -h\,\Lambda_0^2\,\log(-i(\tau - \overline{\tau})) + \sum_i \frac{|\chi_i|^2}{(-i(\tau - \overline{\tau}))^{k_i}}$$

$$\mathcal{L} \supset \sum_i \frac{\partial_\mu \overline{\chi}_i\, \partial^\mu \chi_i}{(2\,\mathrm{Im}\langle\tau\rangle)^{k_i}}$$

Under a modular transformation, **invariant up to a Kähler transformation:**

$$K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) \to K(\chi_i, \overline{\chi}_i; \tau, \overline{\tau}) + f(\chi_i; \tau) + f(\overline{\chi}_i; \overline{\tau})$$

There may exist potentially dangerous
corrections to the Kähler, to be studied

Generators of modular forms: *q*-expansions

$$Y_{\mathbf{2}}(\tau) \equiv \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}$$ **doublet 2**

$$Y_{\mathbf{3'}}(\tau) \equiv \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}$$ **triplet 3'**

$$-\frac{8i}{3\pi}\,Y_1(\tau) \;=\; 1 - 24y - 72y^2 + 288y^3 + 216y^4 + \ldots\,,$$

$$-\frac{8i}{3\pi}\,Y_2(\tau) \;=\; 1 + 24y - 72y^2 - 288y^3 + 216y^4 + \ldots\,,$$

$$\frac{4i}{\pi}\,Y_3(\tau) \;=\; 1 - 8z + 64z^3 + 32z^4 + 192z^5 - 512z^7 + 384z^8 + \ldots\,,$$

$$\frac{2i}{\pi}\,[Y_4(\tau) + Y_5(\tau)] \;=\; 1 + 4z - 32z^3 + 32z^4 - 96z^5 + 256z^7 + 384z^8 + \ldots\,,$$

$$\frac{i}{\pi}\,[Y_4(\tau) - Y_5(\tau)] \;=\; 2\sqrt{3}\,z\,\left(1 + 8z^2 - 24z^4 - 64z^6 + \ldots\right)\,,$$

where $y \equiv i\sqrt{q/3}$, $z \equiv e^{i\pi/4}(q/4)^{1/4}$, and $q = e^{2\pi i\,\tau}$

Building higher-weight forms

At weight 4

$$Y^{(4)}_{\mathbf{1}} = Y_1 Y_2 \sim \mathbf{1}$$

$$Y^{(4)}_{\mathbf{2}} = (Y_2^2,\, Y_1^2)^T \sim \mathbf{2}$$

$$Y^{(4)}_{\mathbf{3}} = (Y_1 Y_4 - Y_2 Y_5,\, Y_1 Y_5 - Y_2 Y_3,\, Y_1 Y_3 - Y_2 Y_4)^T \sim \mathbf{3}$$

$$Y^{(4)}_{\mathbf{3'}} = (Y_1 Y_4 + Y_2 Y_5,\, Y_1 Y_5 + Y_2 Y_3,\, Y_1 Y_3 + Y_2 Y_4)^T \sim \mathbf{3'}$$

1' at weight 6

Constraints

guarantee correct dimensionality

$$\frac{1}{3}\left(Y_3^2 + 2Y_4Y_5\right) = Y_1Y_2\,, \qquad -\frac{1}{\sqrt{3}}\left(Y_3^2 - Y_4Y_5\right) = Y_1Y_4 - Y_2Y_5\,,$$
$$\frac{1}{3}\left(Y_4^2 + 2Y_3Y_5\right) = Y_2^2\,, \qquad -\frac{1}{\sqrt{3}}\left(Y_5^2 - Y_3Y_4\right) = Y_1Y_5 - Y_2Y_3\,,$$
$$\frac{1}{3}\left(Y_5^2 + 2Y_3Y_4\right) = Y_1^2\,, \qquad -\frac{1}{\sqrt{3}}\left(Y_4^2 - Y_3Y_5\right) = Y_1Y_3 - Y_2Y_4\,.$$

Mass matrices in Weinberg operator case

$k_L = 1$ ✗

$$M_e^\dagger = v_d \begin{pmatrix} \alpha Y_3 & \alpha Y_5 & \alpha Y_4 \\ \beta (Y_1 Y_4 - Y_2 Y_5) & \beta (Y_1 Y_3 - Y_2 Y_4) & \beta (Y_1 Y_5 - Y_2 Y_3) \\ \gamma (Y_1 Y_4 + Y_2 Y_5) & \gamma (Y_1 Y_3 + Y_2 Y_4) & \gamma (Y_1 Y_5 + Y_2 Y_3) \end{pmatrix}$$

$$M_\nu = \frac{2 g_1 v_u^2}{\Lambda} \begin{pmatrix} 0 & Y_2 & Y_1 \\ Y_2 & Y_1 & 0 \\ Y_1 & 0 & Y_2 \end{pmatrix}$$

$$Y_i \equiv Y_i(\tau)$$

Mass matrices in Weinberg operator case

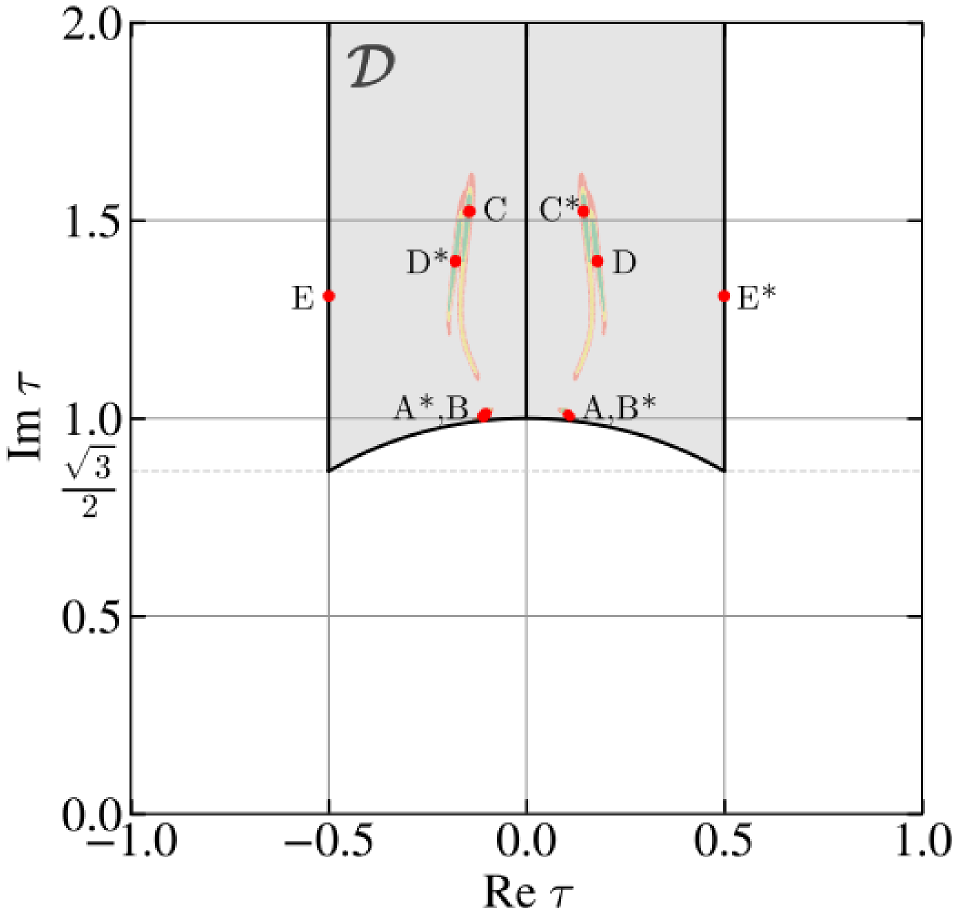
$k_L = 2$ ✓

$$M_e^\dagger = v_d \begin{pmatrix} \alpha\, Y_3 & \alpha\, Y_5 & \alpha\, Y_4 \\ \beta\,(Y_1 Y_4 - Y_2 Y_5) & \beta\,(Y_1 Y_3 - Y_2 Y_4) & \beta\,(Y_1 Y_5 - Y_2 Y_3) \\ \gamma\,(Y_1 Y_4 + Y_2 Y_5) & \gamma\,(Y_1 Y_3 + Y_2 Y_4) & \gamma\,(Y_1 Y_5 + Y_2 Y_3) \end{pmatrix}$$

$$M_\nu = \frac{2 g' v_u^2}{\Lambda} \left[\begin{pmatrix} (g/g') Y_1 Y_2 & Y_2^2 & Y_1^2 \\ Y_2^2 & Y_1^2 & (g/g') Y_1 Y_2 \\ Y_1^2 & (g/g') Y_1 Y_2 & Y_2^2 \end{pmatrix} + \frac{1}{2} \frac{g''}{g'} \begin{pmatrix} 2(Y_1 Y_4 - Y_2 Y_5) & -(Y_1 Y_3 - Y_2 Y_4) & -(Y_1 Y_5 - Y_2 Y_3) \\ -(Y_1 Y_3 - Y_2 Y_4) & 2(Y_1 Y_5 - Y_2 Y_3) & -(Y_1 Y_4 - Y_2 Y_5) \\ -(Y_1 Y_5 - Y_2 Y_3) & -(Y_1 Y_4 - Y_2 Y_5) & 2(Y_1 Y_3 - Y_2 Y_4) \end{pmatrix} \right]$$

$$Y_i \equiv Y_i(\tau)$$

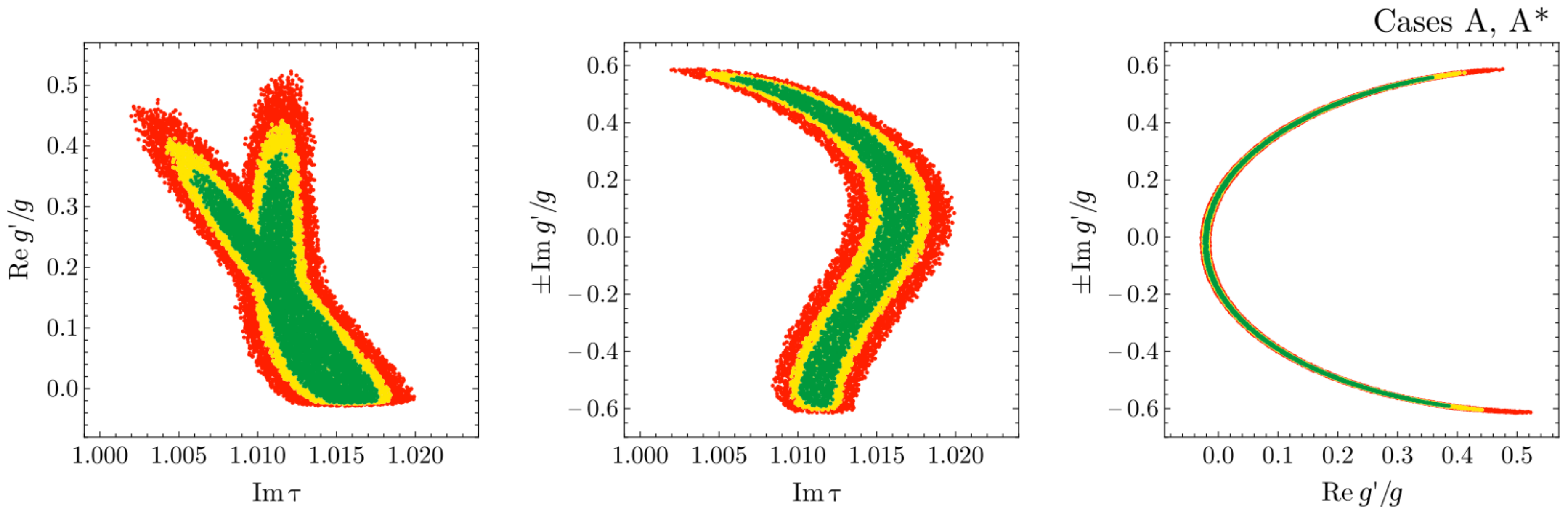
Viable seesaw regions



Novichkov, JP, Petcov, Titov, 1811.04933

Conjecture: Cvetic, Font, Ibanez, Lust, Quevedo, Nucl. Phys. B361 (1991) 194

Correlations between parameters



see Novichkov, JP, Petcov, Titov, 1811.04933