Original works by: Peng Chen, Cai-Chang Li, Chang-Yuan Yao, Gui-Jun Ding, Felix González-Canales, José WF Valle

Works done in collaboration with: Peng Chen, Gui-Jun Ding, Rahul Srivastava, José WF Valle

Flavour mixing predictions from generalized CP symmetries

Salvador Centelles Chuliá – AHEP group IFIC (CSIC-Univ. Valencia)
Outline

• Introduction: Generalized CP Symmetries.

• Generalized $\mu\tau$ reflection in the neutrino sector.

• Generalized $e\mu$, $e\tau$ and $\mu\tau$ reflections in the charged lepton sector.

• Revamped TBM from generalized CP symmetries.
Generalized CP Symmetries

- Definition:

$$\psi \rightarrow iX_\psi \psi^c, \quad \psi \in \{\nu_L, \nu_R, l_L, l_R\}$$

- If this (remnant) symmetry is conserved in the Lagrangian (after SSB), it implies:

$$X_\psi^T m_\psi X_\psi = m_\psi^*, \quad \text{for Majorana fields}$$

$$X_\psi^\dagger M_\psi^2 X_\psi = M_\psi^{2*}, \quad \text{for Dirac fields}$$

Where $$M_\psi^2 \equiv m_\psi^\dagger m_\psi$$

1412.8352 Peng Chen, Cai-Chang Li, Gui-Jun Ding
1507.03419 Peng Chen, Chang-Yuan Yao, Gui-Jun Ding
Generalized CP symmetries

- After some algebra, we can find a constraint for the mixing matrix given the gCP symmetry:

\[ U^\dagger_\psi X_\psi U^*_\psi \equiv P = \begin{cases} \text{diag}(\pm 1, \pm 1, \pm 1), & \text{for Majorana fields}, \\ \text{diag}(e^{i\delta_e}, e^{i\delta_\mu}, e^{i\delta_\tau}), & \text{for Dirac fields}, \end{cases} \]

- We can Takagi decompose \( X_\psi \):

\[ X_\psi = \Sigma \cdot \Sigma^T \]

- Which leads to the 'master formula':

\[ U_\psi = \Sigma O_3^\dagger P^{-\frac{1}{2}} \]
Generalized $\mu \tau$ reflection

- Assumptions:
  Neutrinos are Majorana fields.
  The charged lepton mass matrix is diagonal.

We impose a generalized $\mu \tau$ reflection in the neutrino sector:

$$X_{\mu \tau} = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\beta} \cos \Theta & i e^{\frac{1}{2} i (\beta + \gamma)} \sin \Theta \\
0 & i e^{\frac{1}{2} i (\beta + \gamma)} \sin \Theta & e^{i \gamma} \cos \Theta
\end{pmatrix}$$
Generalized $\mu\tau$ reflection

- Imposing generalized $\mu\tau$ reflection leads to:

$$\sin^2 \theta_{13} = \sin^2 \theta_2, \quad \sin^2 \theta_{12} = \sin^2 \theta_3, \quad \sin^2 \theta_{23} = \frac{1}{2} (1 - \cos \Theta \cos 2\theta_1)$$

$$J_{CP} = \frac{1}{4} \sin \Theta \sin \theta_2 \sin 2\theta_3 \cos^2 \theta_2, \quad \sin \delta_{CP} = \frac{\sin \Theta \text{sign} \left[ \sin \theta_2 \sin 2\theta_3 \right]}{\sqrt{1 - \cos^2 \Theta \cos^2 2\theta_1}},$$

$$\tan \delta_{CP} = \tan \Theta \csc 2\theta_1, \quad \phi_{12} = \frac{k_2-k_1}{2} \pi, \quad \phi_{13} = \frac{k_3-k_1}{2} \pi, \quad \delta_{CP} = \frac{k_3-k_2}{2} \pi - \phi_{23}$$
Generalized $\mu \tau$ reflection

- Imposing generalized $\mu \tau$ reflection leads to:
Generalized $\mu \tau$ reflection

- Imposing generalized $\mu \tau$ reflection leads to:
Charged lepton sector: generalized $e\mu$ reflection

- Same spirit, now in the charged lepton sector.
- The analogous situation to $\mu\tau$ in the neutrino sector is $e\mu$ in the charged lepton sector.

$$X_{e\mu} = \begin{pmatrix} e^{i\alpha} \cos \Theta & ie^{\frac{1}{2}i(\alpha+\beta)} \sin \Theta & 0 \\ ie^{\frac{1}{2}i(\alpha+\beta)} \sin \Theta & e^{i\beta} \cos \Theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Charged lepton sector: generalized eμ reflection

- Analogue predictions for mixing angles changing $23 \leftrightarrow 12$:

$$\sin^2 \theta_{13} = \sin^2 \theta_2, \quad \sin^2 \theta_{12} = \frac{1}{2} (1 - \cos 2\theta_3 \cos \Theta), \quad \sin^2 \theta_{23} = \sin^2 \theta_1$$

- Different predictions for the Majorana phases:

$$\tan \delta_{CP} = -\csc 2\theta_3 \tan \Theta,$$
$$\sin 2\phi_{12} = -\frac{\sin 4\theta_3 \sin \Theta}{\cos^2 2\theta_3 \cos^2 \Theta - 1},$$
$$\sin 2\phi_{13} = -\frac{\sin 2\theta_3 \sin \Theta}{\cos 2\theta_3 \cos \Theta + 1}.$$
Charged lepton sector: generalized eμ reflection

- Analogue predictions for mixing angles changing $23 \leftrightarrow 12$: 

![Graphs showing mixing angles and CP phases](image)
Charged lepton sector: generalized $e\mu$ reflection

- Very different predictions for the phases:

![Graph showing correlations among CP violation phases $\delta_{CP}$, $\phi_{12}$, and $\phi_{13}$ for the generalized $e - \mu$ reflection in the charged lepton sector, taking $\alpha = \beta = 0$. The light blue and green areas correspond to $\phi_{12} - \delta_{CP}$ and $\phi_{13} - \delta_{CP}$ respectively.](image)

Figure 3. Correlations among the CP violation phases $\delta_{CP}$, $\phi_{12}$ and $\phi_{13}$ for the generalized $e - \mu$ reflection in the charged lepton sector, taking $\alpha = \beta = 0$. The light blue and green areas correspond to $\phi_{12} - \delta_{CP}$ and $\phi_{13} - \delta_{CP}$ respectively.
Charged lepton sector: generalized $e\mu$ reflection

- Neutrinoless double beta decay predictions:
Reflections in the charged lepton sector

- We can also impose generalized $e\tau$ and $\mu\tau$ generalized reflections.
- Results are ‘messier’ but some nice predictions can be obtained. See paper for details 1802.04275.
GCP as guiding posts: generalized TBM mixing

• Given the gCP symmetry you obtain a restricted mixing. But given the mixing you can extract the compatible CP symmetries.

• We can do this exercise with the famous TBM mixing matrix.

• Starting with TBM you can do perturbations to the mass matrix that satisfy just partial symmetry of the full TBM matrix.

• Many possibilities, see future paper for details 1812.xxxxx.
Realistic TBM neutrino mixing

- Of those possibilities the golden boy is the “realistic tribimaximal neutrino mixing”.
- Derived from generalized CP symmetries principles this matrix takes the form:

\[
U = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{e^{-i\rho} \cos \theta}{\sqrt{3}} & -\frac{ie^{-i\rho} \sin \theta}{\sqrt{3}} \\
-\frac{e^{i\rho}}{\sqrt{6}} & \frac{\cos \theta}{\sqrt{3}} & -\frac{i e^{-i\sigma} \sin \theta}{\sqrt{2}} \\
\frac{e^{i(\rho+\sigma)}}{\sqrt{6}} & \frac{e^{i\sigma} \cos \theta}{\sqrt{3}} & -\frac{i \sin \theta}{\sqrt{2}} \\
\end{pmatrix}
\]
Realistic TBM neutrino mixing

- This matrix has nice correlations between physical observables.
- Several interesting limits (real TBM, $\mu\tau$ reflection, maximal atmospheric mixing...)
- The most general case leads to:

\[
\begin{align*}
\sin^2 \theta_{12} &= \frac{\cos^2 \theta}{\cos^2 \theta + 2}, \\
\sin^2 \theta_{23} &= \frac{1}{2} + \frac{\sqrt{6} \sin 2\theta \sin \sigma}{2 \cos^2 \theta + 4}, \\
\sin^2 \theta_{13} &= \frac{\sin^2 \theta}{3}, \\
\tan \delta_{CP} &= \frac{(\cos^2 \theta + 2) \cot \sigma}{(5 \cos^2 \theta - 2)}, \\
\phi_{12} &= \rho, \\
\phi_{13} &= \rho + \frac{\pi}{2},
\end{align*}
\]
Realistic TBM neutrino mixing
Conclusions

- Generalized CP symmetries are a powerful method to obtain testable correlations between observables.
- Given a CP symmetry you can restrict the lepton mixing matrix.
- Given a mixing pattern (for example TBM) you can extract the compatible CP symmetries and then do a perturbation that satisfies only partial symmetry.
Thank you for your attention

- References:
  - 1412.8352 Lepton Flavor Mixing and CP Symmetry Peng Chen, Cai-Chang Li, Gui-Jun Ding, PhysRevD.91.033003
  - 1507.03419 Neutrino Mixing from CP Symmetry Peng Chen, Chang-Yuan Yao, Gui-Jun Ding PhysRevD.92.073002
  - 1812.xxxxx Peng Chen, SCC, Gui-Jun Ding, Rahul Srivastava, J.W.F. Valle Coming soon, stay tuned