Tension between a vanishing cosmological constant and non-supersymmetric heterotic orbifolds

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Motivation

The main motivation for this work arises from the following:

- It is a experimental fact that the universe is equipped with a tiny but non-vanishing cosmological constant
- String theory comes with the promise of being a unified quantum theory of all interactions
- In string theory the cosmological constant is a calculable quantity

Is it possible to obtain a small but non-vanishing cosmological constant within string theory?

Motivation for a vanishing cosmological constant

The cosmological constant Λ is very very tiny: 10^{-120} smaller than its natural scale $\Lambda \sim m_P^4$

This may be taken as an indication that the cosmological constant should vanish perturbatively to all orders and only arises due to non-perturbative effects

For this to be feasible at least the cosmological constant should vanish at the one-loop level

Motivation for heterotic strings

All string theories are presumably related to each other by dualities, so in principle any question can be investigated within various string theoretical contexts

In heterotic strings both gravity, gauge interactions and chiral particle spectra arise at the same stage in the construction

Perturbative heterotic string theory is a well-studied subject

Have lead to many candidates of MSSM-like models

e.g. Faraggi,Nanopoulos,Yuan'90; Braun,He,Ovrut,Pantev'05; Bouchard,Donagi'05; Buchmuller,Hamaguchi,Lebedev,Ratz'05; Lebedev,Nilles,Ramos-Sanchez,Ratz,Vaudrevange'06

Motivation for non-supersymmetry strings

Given that

- so far there has been no experimental evidence for supersymmetry
- and a positive cosmological constant seems inconsistent with supersymmetry

this question should be investigated in the context of non-supersymmetric string theory

One-loop vanishing cosmological constants have been obtained for non-supersymmetric asymmetric orientifold constructions

Karchru,Silverstein'99; Blumenhagen,Gorlich'99; Satoh,Sugawara,Wada'15

There are also non-supersymmetric heterotic strings,

Dixon,Harvey'86; Alvarez-Gaume,Ginsparg,Moore,Vafa'86

on which Standard Model-like theories can be obtained

Faraggi and M. Tsulaia'07; Blaszczyk,SGN,Loukas,Ramos-Sanchez'14

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Motivation for orbifolds

Since string theories (except the bosonic one) tend to live in 10D, a compactification of 6D is necessary

As there are an infinite (over-countable) number of 6D manifolds, the close to 29 million 6D toroidal orbifolds Opgenorth,Plesken,Schulz'98 provide a large but trackable testing ground

Strings on orbifolds can be exactly quantized Dixon,Harvey,Vafa,Witten'85

Overview

Ingredients of toroidal orbifolds

A toroidal orbifold T^6/G is build as follows:

- A 6D lattice spanned by six basis vectors e_i:

$$\Gamma = \{e\,m | m \in \mathbb{Z}^6\}$$

- A torus $T^6 = \mathbb{R}^6/\Gamma$ defined by the periodicities:

$$X \sim X + e\,m\,, \qquad m \in \mathbb{Z}^6\,,$$

  where the basis vectors e_i are combined to the vielbein $e = (e_i)$
- A finite orbifold group G generated by

$$\begin{array}{lll} \text{twists:} & X \sim D_{\mathbf{v}}(\theta)\, X & \theta \neq \mathbb{1} \\ \text{roto-translations:} & X \sim D_{\mathbf{v}}(\theta)\, X + e\,q\,, & q \in \mathbb{Q}^6 \end{array}$$

Space group **S** description of orbifolds

The space group **S** combines the lattice and the orbifold group elements, e.g.:

$$(\mathbb{1}; e\, m) \in \mathbf{S} \quad m \in \mathbb{Z}^6$$
$$(\theta; e\, q) \in \mathbf{S} \quad \theta \neq \mathbb{1}\ ,\ q \in \mathbb{Q}^6$$

The finite point group **P** is a projection of the space group **S**:

$$\mathbf{S} \to \mathbf{P}: \quad (\theta, e\, q) \mapsto \theta$$

Twist action on vectors and spinors

The action $D_{\mathbf{v}}(\theta)$ associated to a given space group element $g = (\theta, e\,q)$ can be diagonalized in a complex coordinate basis as

$$D_{\mathbf{v}}(\theta) = \begin{pmatrix} e^{2\pi i\, v_g^1} & 0 & 0 \\ 0 & e^{2\pi i\, v_g^2} & 0 \\ 0 & 0 & e^{2\pi i\, v_g^3} \end{pmatrix}$$

in terms of a local twist vector $v_g = (0, v_g^1, v_g^2, v_g^3)$

Its action on eight-component 6D internal spinors is given by

$$D_{\mathbf{s}}(\theta) = e^{2\pi i\, v_g^1\, \frac{\sigma_3}{2}} \otimes e^{2\pi i\, v_g^2\, \frac{\sigma_3}{2}} \otimes e^{2\pi i\, v_g^3\, \frac{\sigma_3}{2}}$$

Double cover of Spin(6) over SO(6)

$$D_{\mathbf{v}}(\theta) = \begin{pmatrix} e^{2\pi i\, v_g^1} & 0 & 0 \\ 0 & e^{2\pi i\, v_g^2} & 0 \\ 0 & 0 & e^{2\pi i\, v_g^3} \end{pmatrix}$$

$$D_{\mathbf{s}}(\theta) = e^{2\pi i\, v_g^1\, \frac{\sigma_3}{2}} \otimes e^{2\pi i\, v_g^2\, \frac{\sigma_3}{2}} \otimes e^{2\pi i\, v_g^3\, \frac{\sigma_3}{2}}$$

The $\mathrm{Spin}(6) = \mathrm{SU}(4)$ is the double cover of $\mathrm{SO}(6)$:

- Both $D_{\mathbf{s}}(\theta)$ and $-D_{\mathbf{s}}(\theta)$ are associated to $D_{\mathbf{v}}(\theta)$
- $D_{\mathbf{v}}(\theta)$ are inert under $v_g^a \mapsto v_g^a + 1$, while $D_{\mathbf{s}}(\theta)$ changes sign
- $-\mathbb{1} \in \mathrm{Spin}(6)$ breaks all supersymmetries

It is often possible to make a choice per space group element $g = (\theta, e\, q)$ such that $D_{\mathbf{s}}(\theta)$ admits some Killing spinors

Existence of killing spinors

A space group element $g = (\theta; e\, q)$ admits a Killing spinor $\Psi_{\text{inv.}}$, if

$$D_{\mathbf{s}}(\theta)\, \Psi_{\text{inv.}} = \Psi_{\text{inv.}}$$

has non-trivial solutions $\Psi_{\text{inv.}} \neq 0$

The possible eigenvalues of $D_{\mathbf{s}}(\theta)$ are $\exp(\pm 2\pi i\, \widetilde{v}_g^a)$, $a = 0, 1, 2, 3$, where

$$\widetilde{v}_g = \frac{1}{2} \begin{pmatrix} v_g^1 + v_g^2 + v_g^3 \\ -v_g^1 + v_g^2 + v_g^3 \\ v_g^1 - v_g^2 + v_g^3 \\ v_g^1 + v_g^2 - v_g^3 \end{pmatrix}$$

Hence, for a space group element $g = (\theta; e\, q)$ to admit at least one Killing spinor, at least one of the entries of $\widetilde{v}_g$ needs to vanish modulo integers

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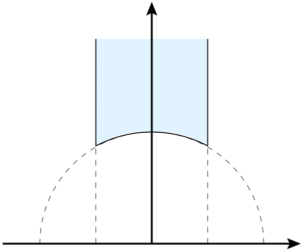
One-loop cosmological constant

In heterotic string theory the one-loop cosmological constant is computed via

$$\Lambda \sim \int_{\mathcal{F}} \frac{\mathrm{d}^2\tau}{\tau_2^2}\, \mathcal{Z}_{\text{full}}(\tau, \bar{\tau})$$



Given that the integral over the fundamental domain $\mathcal{F}$ can be very complicated, we asked:

Can we construct non-supersymmetric heterotic orbifolds which have a vanishing one-loop partition function?

Decomposition of the full partition function

The full partition function consists of

$$\mathcal{Z}_{\text{full}} = \mathcal{Z}_{\text{4D Mink.}} \, \mathcal{Z}_{\text{6D int.}} \qquad \mathcal{Z}_{\text{4D Mink.}} = \frac{1}{\tau_2} \left| \frac{1}{\eta^2} \right|^2 \neq 0$$

On orbifolds the internal part Dixon,Harvey,Vafa,Witten'85

$$\mathcal{Z}_{\text{6D int.}} = \frac{1}{|\mathbf{P}|} \sum_{[g,h]=0} \mathcal{Z}_X \begin{bmatrix} g \\ h \end{bmatrix} \overline{\mathcal{Z}_\psi \begin{bmatrix} g \\ h \end{bmatrix}} \, \mathcal{Z}_Y \begin{bmatrix} g \\ h \end{bmatrix}$$

is associated with

- the 6D internal coordinate fields X
- their worldsheet superpartners, the right-moving fermions ψ
- the 16D left-moving gauge degrees of freedom Y

and the sum over all commuting space group element $g, h \in \mathbf{S}$

Vanishing right-moving fermionic partition functions

Contrary the other partition functions, the right-moving fermionic partition function Antoniadis,Bachas,Kounnas'87, Kawai,Lewellen,Tye'87

$$\mathcal{Z}_\psi\begin{bmatrix} g \\ h \end{bmatrix}(\tau) = \frac{1}{2}\, e^{-\pi i\, v_g^T(v_h - e_4)} \sum_{s,s'=0}^{1} (-)^{s's}\, e^{-2\pi i\, \frac{s'}{2} e_4^T v_g}\, \frac{\theta_4\begin{bmatrix} \frac{1-s}{2} e_4 - v_g \\ \frac{1-s'}{2} e_4 - v_h \end{bmatrix}}{\eta^4}\,,$$

with $e_4 = (1,1,1,1)$ and $\theta_4\begin{bmatrix} \alpha \\ \alpha' \end{bmatrix} = \sum_{n\in\mathbb{Z}^4} e^{2\pi i\left\{\frac{\tau}{2}(n+\alpha)^2 + (n+\alpha)^T\alpha'\right\}}\,,$

may vanish under certain circumstances

Orbifolds with vanishing partition functions

Using Riemann identities one can shows that:

$$\mathcal{Z}_\psi\begin{bmatrix} g \\ h \end{bmatrix} = 0 \quad \Leftrightarrow \quad g, h \in \mathbf{S} \text{ share at least one Killing spinor}$$

Hence, all supersymmetric orbifolds have vanish partition functions

And so does any non-supersymmetric toroidal orbifold for which

i. a Killing spinor exists *locally* in every commuting (g, h)-sector

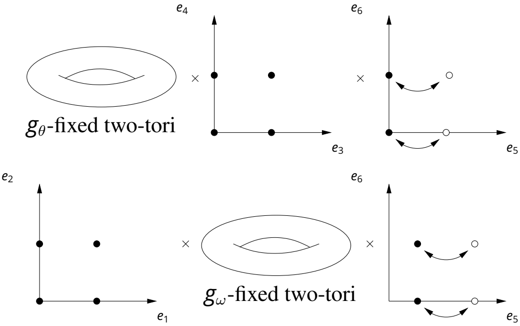
ii. but none *globally*

Do such orbifolds exist?

- One would say yes, since there are orbifold examples with different local and global supersymmetry breakings

(Non-)local supersymmetry breaking on orbifolds

The space group **S** the DW(0–2) $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold is generated by the elements: $g_\theta = (\theta, 0)$, $g_\omega = (\omega, \frac{1}{2}\, e_5)$, $g_i = (1, e_i)$ Donagi,Wendland'08



- These non-intersecting two-tori preserve different $\mathcal{N} = 2$, but combined only $\mathcal{N} = 1$ supersymmetry
- Less supersymmetry is preserved globally than locally

Classification of toroidal orbifolds

Toroidal orbifolds in 6D have been classified: Opgenorth,Plesken,Schulz'98

- **7,103 $\mathbb{Q}$-classes:**

  Inequivalent point groups **P**

- **85,308 $\mathbb{Z}$-classes:**

  Inequivalent lattices Γ on which these point groups can act

- **28,927,915 affine-classes:**

  Inequivalent space groups **S** (encoding roto-translations) that act on these lattices (labeled uniquely by CARAT-indices)

Among these there are **520** toroidal orbifolds that preserve $\mathcal{N} = 1$ supersymmetry or more Fischer,Ratz,Torrado,Vaudrevange'12

Local but no global Killing spinors

This classification can be used to identify toroidal orbifolds that admit Killing spinors in all sectors locally but none globally:

# $\mathbb{Q}$-classes	Restriction
7,103	All inequivalent geometrical point groups **P** $\subset$ O(6)
1,616	Orientable geometrical point groups **P** $\subset \mathrm{SO}(6)$
106	No element from **P** rotates in a two-dimensional plane only
63	Each element $\theta \in$ **P** admits a choice with some local Killing spinors
60	Geometrical point group compatible with some global Killing spinors

This leaves orbifolds with 3 candidate $\mathbb{Q}$-classes

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Candidate orbifold geometries

Some properties of the three candidate $\mathbb{Q}$-classes are:

CARAT-index	Point group	Generator relations	Order	Local twist vectors
3375	$\text{Dic}_3 = \mathbb{Z}_3 \rtimes \mathbb{Z}_4$	$\theta_1^4 = \theta_2^3 = \mathbb{1}$, $\theta_2\,\theta_1\,\theta_2 = \theta_1$	12	$\big(\frac{1}{4}, \frac{1}{4}, -\frac{1}{2}\big)$, $\big(\frac{1}{3}, -\frac{1}{3}, 0\big)$
5751	Q_8	$\theta_1^4 = \mathbb{1},\ \theta_1^2 = \theta_2^2$, $\theta_1\,\theta_2\,\theta_1 = \theta_2$	8	$\big(\frac{1}{4}, \frac{1}{4}, -\frac{1}{2}\big)$ $\big(\frac{1}{4}, -\frac{1}{4}, 0\big)$
6737	$\text{SL}(2,3)$	$\theta_1^3 = \theta_2^4 = \mathbb{1}$, $(\theta_2\,\theta_1)^2 = \theta_1^2\,\theta_2$	24	$\big(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\big)$, $\big(\frac{1}{4}, -\frac{1}{4}, 0\big)$

The local twist vectors, obtained in two different bases, separately indeed preserve some amount of supersymmetry

Candidate orbifold geometries

For these candidate geometries all possible embedding of the point group into spinor space were explicitly constructed:

In all cases either target-space supersymmetry is preserved *globally* or there is at least one element that does not preserve any Killing spinor

Hence, there does not exist any non-supersymmetric orbifold for which all point group elements separately preserve some Killing spinors

No locally but not globally supersymmetry orbifolds

Two proofs of the nonexistence of locally but not globally supersymmetric orbifolds:

1. Construct and analyze all spin representations associated to all point groups **P**
2. Make use of representation theory of finite groups

Abstract point group

The 7,103 $\mathbb{Q}$-classes of 6D orbifolds provided by CARAT correspond to only 1,594 different abstract point groups:

- For a given abstract group there can exist several inequivalent realizations as integral 6×6-matrices

The following two representations of an abstract point group **P** are relevant:

Geometrical point group		Abstract point group	
Name	Matrix repr.	Repr.	Character
Spinor	$D_{\mathbf{s}}(\theta)$	$\mathbf{4}$	$\chi_{\mathbf{4}}$
Vector	$D_{\mathbf{v}}(\theta)$	$\mathbf{6} = [\mathbf{4}]_2$	$\chi_{\mathbf{6}}$

Almost arbitrary four-dimensional representations

Now, consider an arbitrary **4**-representations of **P** (in particular, neither **4** nor **6** need to be irreducible) but:

1. The **4** needs to lie inside $\mathrm{SU}(4) = \mathrm{Spin}(6)$:

   On all conjugacy classes of **P**: $\chi_{[\mathbf{4}]_4} = 1$

2. The **6** should be isomorphic to a $\mathbb{Q}$-class:

   There such be an integral 6×6-matrix representation $\widehat{D}_{\mathbf{v}}$ within the CARAT $\mathbb{Q}$-classes such that $\chi_{\mathbf{6}} = \chi_{\mathbf{v}} = \mathrm{Tr}\widehat{D}_{\mathbf{v}}$

Abstract counting of Killing spinors

G-invariant Killing spinors satisfy: $D_{\mathbf{4}}(\theta)\,\Psi_{\text{inv.}} = \Psi_{\text{inv.}}\ , \quad \forall \theta \in \mathbf{G} \subset \mathbf{P}$

Consequently, the projector on the **G**-invariant subspace reads:

$$\mathcal{P}^{\mathbf{G}} = \frac{1}{|\mathbf{G}|} \sum_{\theta' \in \mathbf{G}} D_{\mathbf{4}}(\theta')$$

Hence, the number of **G**-invariant Killing spinors is counted by:

$$\mathcal{N}^{\mathbf{G}} = \text{Tr}\big(\mathcal{P}^{\mathbf{G}}\big) = \frac{1}{|\mathbf{G}|} \sum_{\theta' \in \mathbf{G}} \text{Tr}\,\big(D_{\mathbf{4}}(\theta')\big) = \frac{1}{|\mathbf{G}|} \sum_{\theta' \in \mathbf{G}} \chi_{\mathbf{4}}(\theta') = \langle \chi_{\mathbf{4}}, \chi_{\mathbf{1}} \rangle_{\mathbf{G}} = n_{\mathbf{1}}^{\mathbf{G}}$$

where **1** denotes the trivial singlet representation with $\chi_{\mathbf{1}} = 1$

$\Rightarrow$ The number of **G**-invariant Killing spinors equals the number of trivial singlet $n_{\mathbf{1}}^{\mathbf{G}}$ in the branching of a **4**-representation of **P** into irrepresentations of **G**

Abstract counting of Killing spinors

The number of *local* Killing spinors preserved by θ is given:

$$\mathcal{N}^{\langle\theta\rangle} = n_{\mathbf{1}}^{\langle\theta\rangle}$$

since any element $\theta \in \mathbf{P}$ of order N_θ generates a $\langle\theta\rangle \cong \mathbb{Z}_{N_\theta} \subset \mathbf{P}$

The number of *global* Killing spinors is given by:

$$\mathcal{N} = n_{\mathbf{1}}^{\mathbf{P}}$$

e.g. how many trivial singlets the **4**-representation contains

$\Rightarrow$ We look for point groups **P** such that for all $\theta \in \mathbf{P}$: $n_{\mathbf{1}}^{\langle\theta\rangle} > 0$, while $n_{\mathbf{1}}^{\mathbf{P}} = 0$

Nonexistence proof by finite group theory

For each of the 1,594 different abstract groups **P** we considered all faithful (but in general reducible) **4**-representations and required that they

- do not contain a trivial singlet representation to avoid global Killing spinors
- yet satisfy the two conditions mentioned above

By constructing all $\mathbb{Z}_N \subset \mathbf{P}$ subgroups we showed, that for each remaining **4**, there is at least one cyclic subgroup, for which the **4** does not contain the trivial $\mathbb{Z}_N$-singlet representation

$\Rightarrow$ **For all non-supersymmetric 6D toroidal orbifolds there is always a sector without any local Killing spinor**

Conclusion

There are no non-supersymmetric toroidal orbifolds that preserve some amount of supersymmetry in all sectors locally

This was proven by

1. explicit construction of all spin representations for all 7,103 $\mathbb{Q}$-classes
2. exploiting representation theory of finite group applied to all 1,594 abstract point groups

This results shows that it is also in string theory very challenging to obtain a very small yet non-zero cosmological constant

Epilog: Is this result surprising?

The leading contribution in the one-loop partition function of any heterotic string theory goes like

$$\mathcal{Z} \sim \frac{1}{q}$$

This corresponds to non-level-matched tachyons with right- and left-moving-masses $(0, -1)$ associated with the so-called Dienes'90, Abel,Dienes,Mavroudi'16

- proto-gravitons: $|p_R\rangle_R \otimes |0\rangle_L$, $p_R \in \mathbf{V}_4$
- proto-gravitinos: $|p_R\rangle_R \otimes |0\rangle_L$, $p_R \in \mathbf{S}_4$

which form gravitons and gravitinos when hit by the α^μ_{-1}, resp.

In supersymmetric theories their contributions cancel, but in non-supersymmetric theories they do not

Epilog: A group theoretical conjecture

Conjecture:

There does not exist any finite group **H** that has a four-dimensional representation $D_{\mathbf{4}}$ with the following three properties:

1. $D_{\mathbf{4}}$ has a trivial determinant, i.e. $\det(D_{\mathbf{4}}(\theta)) = 1$ for all $\theta \in \mathbf{H}$
2. $D_{\mathbf{4}}$ does not contain the trivial singlet representation of **H**
3. but the branchings of $D_{\mathbf{4}}$ to all $\mathbb{Z}_N \subset \mathbf{H}$ subgroups always contain the trivial $\mathbb{Z}_N$-singlet representation

We have checked this against the following finite group lists:

- all 1,594 different finite groups which originate from the 7,103 $\mathbb{Q}$-classes of CARAT;
- all finite groups of order up to 500 from the SmallGroups Library of GAP, amounting to $\mathcal{O}(100,000)$ finite groups