Classification of left-right symmetric heterotic string vacua

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Overview

Introduction

- String theory ought to reproduce Standard Model at low energies.
- Huge number of vacua in four dimensions
- Seek to classify vacua and identify general features of (quasi-)realistic vacua.
- Phenomenological requirements:

1. $\mathcal{N} = 1$ SUSY
2. $SO(10)$ GUT with 3 generations in **16** rep
3. Higgs particles
4. top quark mass coupling
5. Generation mass hierarchy, Seesaw mechanism, proton stability
6. ...

Aims

- 4 key aims of talk:

1. To introduce free fermionic SO(10) models.
2. To review the results of previous classifications: Pati-Salam, Flipped SU(5), Standard-like and Left-Right Symmetric.
3. To describe fertility conditions in Left-Right symmetric case.
4. Future outlook.

- $D = 4 \implies$ introduction of free fermions on worldsheet, traditionally written:

$$\{\underbrace{\psi^\mu}_{\text{S'partners of } X^\mu}, \underbrace{\chi^i}_{\text{S'partners to six compactified dimensions}}, \underbrace{y^i, w^i \,\|\, \bar{y}^i, \bar{w}^i}_{\text{"compactified" directions}}, \underbrace{\bar{\psi}^{1,2,3,4,5}, \bar{\eta}^{1,2,3}}_{\text{observable G. G.}}, \underbrace{\bar{\phi}^{1,2,3,4,5,6,7,8}}_{\text{rank 8 Hidden G. G.}}\} \quad (1)$$

$i = 1, ..., 6$

- Partition function modular invariance sufficiently encoded at one-loop, i.e. a torus:

$$f_i \to -e^{i\pi\alpha(f_i)} f_i, \qquad \alpha(f) \in (-1, +1] \quad (2)$$

$$Z = \sum_{\text{All Spin structures}} C\begin{pmatrix} v_i \\ v_j \end{pmatrix} Z\begin{pmatrix} v_i \\ v_j \end{pmatrix} \quad (3)$$

Free Fermion Construction

- Model defined through:

1. Basis vectors:

$$v_i = \{\alpha(f_1), \alpha(f_2), ..., \alpha(f_N)\}, \tag{4}$$

2. GGSO phases:

$$C\begin{pmatrix} v_i \\ v_j \end{pmatrix} = \pm 1 \text{ or } \pm i, \quad i > j \tag{5}$$

$2^{\frac{N(N-1)}{2} - \#\text{constraints}}$: 'ABK rules'.

References:

I. Antoniadis and C. Bachas, Nuclear Physics B, 298(3):586 - 612, 1988. I. Antoniadis and C. Bachas, and C. Kounnas, Nuclear Physics B, 289(0):87 - 108, 1987.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ $SO(10)$ heterotic models

- Basis vectors:

  $\mathbf{1} = \{\text{ALL}\}\}$ None transform

  $\mathbf{S} = \{\psi^\mu, \chi^{1,\dots,6}\}\}$ SUSY generator

  $\mathbf{e_i} = \{y^i, w^i | \bar{y}^i, \bar{w}^i\}, \quad i = 1, \dots 6\}$ Internal symmetric shifts

  $\mathbf{b_1} = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, , \bar{\psi}^{1,..,5}\}$
  $\mathbf{b_2} = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,..,5}\}$ } $\mathbb{Z}_2$ twists

  $\mathbf{z_1} = \{\bar{\phi}^{1234}\}$
  $\mathbf{z_2} = \{\bar{\phi}^{5678}\}$ } $SO(8) \times SO(8)$ hidden

- $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$ gauge group.

Observable Sectors (twisted)

- $\mathbf{16}$'s / $\overline{\mathbf{16}}$'s of $SO(10)$ (48 possible sectors):

$$\begin{aligned} B^{(1)}_{pqrs} &= \mathbf{S} + \mathbf{b_1} + p\mathbf{e_3} + q\mathbf{e_4} + r\mathbf{e_5} + s\mathbf{e_6} \\ B^{(2)}_{pqrs} &= \mathbf{S} + \mathbf{b_2} + p\mathbf{e_1} + q\mathbf{e_2} + r\mathbf{e_5} + s\mathbf{e_6} \\ B^{(3)}_{pqrs} &= \mathbf{S} + \mathbf{b_3} + p\mathbf{e_1} + q\mathbf{e_2} + r\mathbf{e_3} + s\mathbf{e_4} \end{aligned} \tag{6}$$

$p, q, r, s \in \{0, 1\}$.

- Vectorial $\mathbf{10}$'s:

$$V^{(I=1,2,3)}_{pqrs} = B^{(I)}_{pqrs} + \mathbf{x} \tag{7}$$

$$\mathbf{x} = \mathbf{1} + \mathbf{S} + \sum_{i=1}^{6} \mathbf{e_i} + \sum_{k=1}^{2} \mathbf{z_k} = \{\bar{\eta}^{123}, \bar{\psi}^{1,\dots,5}\} \tag{8}$$

$$\mathbf{b_3} = \mathbf{b_1} + \mathbf{b_2} + \mathbf{x} \tag{9}$$

Projectors

- Example projector ($B^{(1)}$):

$$\underbrace{\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix}}_{\Delta^1} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \underbrace{\begin{pmatrix} (e_1|b_1) \\ (e_2|b_1) \\ (z_1|b_1) \\ (z_2|b_1) \end{pmatrix}}_{Y^1}$$

where $C\begin{pmatrix} v_i \\ v_j \end{pmatrix} = e^{i\pi(v_i|v_j)}$. Similarly for $B^{(2)}$, $B^{(3)}$ and $V^{(I)}$.

- rank$([\Delta^I, Y^I])$'s related to: # $(\mathbf{16} + \overline{\mathbf{16}})$ and # $\mathbf{10}$'s

Key Finding: Spinor-Vector Duality

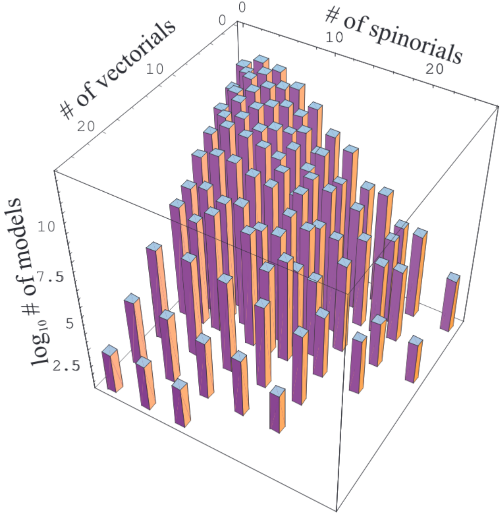
- $S \leftrightarrow V$ duality holds for each orbifold plane separately.
- self-dual case: $\#S = \#V$ often free of abelian anomalies.
- E_6 embedding $\implies$ always self-dual:

$$\begin{aligned} \mathbf{27} &\rightarrow \mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1}, \\ \overline{\mathbf{27}} &\rightarrow \overline{\mathbf{16}} \oplus \mathbf{10} \oplus \mathbf{1} \end{aligned} \tag{10}$$

Reference:

See e.g. A. Faraggi, C. Kounnas and J. Rizos, Phys. Lett. B648 (2007) arXiv:hep-th/0611251

Spinor-Vector Duality



Reference:

Fig. 1 from J. Rizos (2013), *Towards classi cation of SO(10) heterotic string vacua*.

SO(10) Breaking vectors

- α vector breaks $SO(10)$
- Classification methodology has been applied for $SO(10)$ subgroups:
  1. $\alpha(\bar{\psi}^{1,\dots,5}) = \{11100\} \implies SO(6)\times SO(4)$ (PS).
  2. $\alpha(\bar{\psi}^{1,\dots,5}) = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \implies SU(5)\times U(1)$ (FSU5).
  3. $\begin{cases}\alpha(\bar{\psi}^{1,\dots,5}) = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\}, \\ \beta(\bar{\psi}^{1,\dots,5}) = \{11100\}\end{cases} \implies SU(3)\times SU(2)\times U(1)^2$ (SLM)
  4. $\alpha(\bar{\psi}^{1,\dots,5}) = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\} \implies SU(3)\times SU(2)^2\times U(1)$ (LRS)
- Classify through set of numbers, in particular:
  1. Observables (e.g. n_g, n_H)
  2. Exotics (fractionally charged states)

Highlights of Pati-Salam Classification

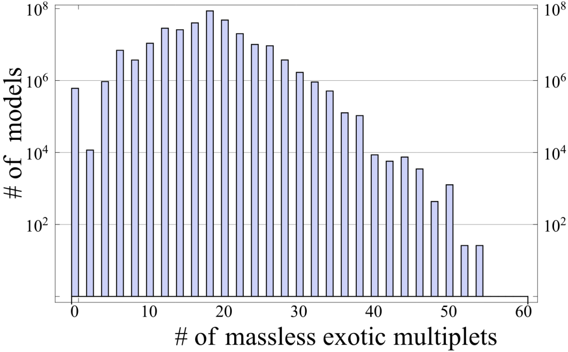
	constraint	# of models in sample	probability	estimated # of models in class
	None	100000000000	1	2.25×10^{15}
(a)	+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
(b)	+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
(c)	+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
(d)	+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
(e)	+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
(f)	+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}
(g)	+ Minimal PS Higgs	31804	3.18×10^{-7}	7.16×10^{8}

- $\sim 1 : 10^6$ are 3 generation, exophobic (!) with SM breaking Higgs.

Reference:

Table 2 from B. Assel, C. Christodoulides, A.E. Faraggi, C. Kounnas and J. Rizos arXiv:1007.2268
J. Pati and A. Salam, *Lepton number as fourth colour* (1974)

Highlights of Pati-Salam Classification



Number of 3 generation models versus number of exotics (random sample of 10^{11} vacua)

Reference:

Fig 3 from B. Assel, et al. (2010) arXiv:1007.2268

Classification of Flipped SU(5)

	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	1000000000000	1	1.76×10^{13}
(1)	+ No Enhancements	762269298719	7.62×10^{-1}	1.34×10^{13}
(2)	+ Anomaly Free Flipped $SU(5)$	139544182312	1.40×10^{-1}	2.45×10^{12}
(3)	+ 3 Generations	738045321	7.38×10^{-4}	1.30×10^{10}
(4a)	+ SM Light Higgs	706396035	7.06×10^{-4}	1.24×10^{10}
(4b)	+ Flipped $SU(5)$ Heavy Higgs	46470138	4.65×10^{-5}	8.18×10^{8}
(5)	+ SM Light Higgs + & Heavy Higgs	43624911	4.36×10^{-5}	7.67×10^{8}
(6a)	+ Minimal Flipped $SU(5)$ Heavy Higgs	42310396	4.23×10^{-5}	7.44×10^{8}
(6b)	+ Minimal SM Light Higgs	25333216	2.53×10^{-5}	4.46×10^{8}
(7)	+ Minimal Flipped $SU(5)$ Heavy Higgs + & Minimal SM Light Higgs	24636896	2.46×10^{-5}	4.33×10^{8}
(8)	+ Minimal Exotic States	1218684	1.22×10^{-6}	2.14×10^{7}

- No exophobic vacua with an odd number of generations(!) (in random sample 10^{12})

References:

Table 3 from A.E. Faraggi, J. Rizos and H. Sonmez, (2014) arXiv:1403.4107

I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos (1987)

Highlights of Flipped SU(5) Classification

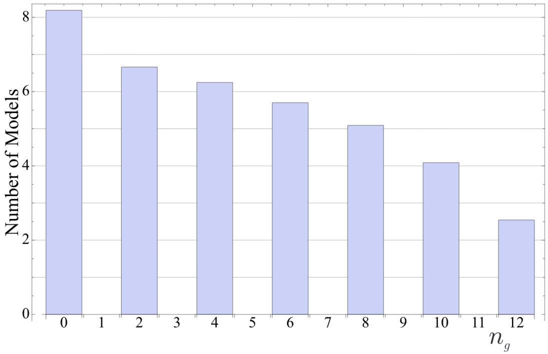


Figure 2: *Logarithm of the number of exophobic models against the number of generations (n_g) in a random sample of 10^{12} flipped $SU(5)$ configurations.*

Reference:

Fig 2 from A.E. Faraggi, J. Rizos and H. Sonmez, (2014) arXiv:1403.4107

Highlights of Standard-like Classification

- Complications:

  1. Combinations of PS and FSU(5) sectors $\implies$ exotics with standard charges w.r.t SM but fractional charge w.r.t. extra $U(1)$ $\implies$ dark matter possibilities (see e.g. L. Delle Rose, et al (2017) arXiv:1704.02579).

  2. Uses PS and FSU(5) $SO(10)$ breaking vectors $\implies$ space of vacua to explore far larger

  $\implies$ change methodology: **Fertility conditions**!

- Efficient way to find realistic models without full scan

Reference:

A. E. Faraggi, J. Rizos, H. Sonmez (2017) arXiv:1709.08229

Highlights Left-Right Symmetric Classification

	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	1000000000	1	7.38×10^{19}
(1)	+ No Enhancements	708830165	7.09×10^{-1}	5.23×10^{19}
(2)	+ Complete Families	70241057	7.02×10^{-2}	5.18×10^{18}
(3)	+ No Chiral Exotics	43660665	4.37×10^{-2}	3.30×10^{18}
(4)	+ Three Generations	1486	1.49×10^{-6}	1.10×10^{14}
(5)	+ SM Light Higgs + & Heavy Higgs	1	1.00×10^{-9}	7.38×10^{10}
(6)	+ Minimal Heavy Higgs & Minimal SM Light Higgs	0	0	N/A
(7)	+ Top Quark Mass Coupling	0	0	N/A

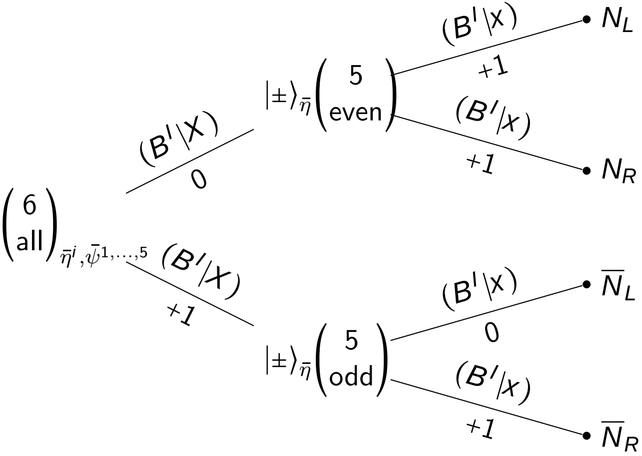
- Proliferation of exotic states
- 2α also breaks $SO(10)$
- Good models rare (as in SLM) $\Longrightarrow$ fertility conditions

Reference:

A. E. Faraggi, G. Harries, J. Rizos (2018) arXiv:1806.04434

LRS Spinorial Condition

- Classify with 'chirality' X, and **x** projection both **at** $SO(10)$ **level**:



- Demand: $\mathbf{N_L - \overline{N}_L \geq 6}$, $\mathbf{N_R - \overline{N}_R \geq 6}$ and $\mathbf{\overline{N}_R \geq 1}$

LRS Vectorial Condition

- Recall:

$$V^I = B^{(I)} + \mathbf{x} \tag{11}$$

Can show:

$$C\begin{pmatrix} V^I \\ x \end{pmatrix} = \begin{cases} +1 & \Longrightarrow \quad \text{bidoublet Higgs } (\mathbf{1}, \mathbf{2}, \mathbf{2}) \text{ survives} \\ \\ -1 & \Longrightarrow \quad \text{triplets survive} \end{cases} \tag{12}$$

- $\Longrightarrow$ at $SO(10)$ level can demand:

$$\mathbf{N}_{\text{doublets}} > \mathbf{1} \tag{13}$$

- Once $SO(10)$ fertile cores found, do full classification in α phases (in progress)

Current Work and outlook

- Complete fertility analysis for LRS
- Apply deep learning techniques to classification procedure.
- "Burgeoning enterprise" (Yang-Hui He, 2018): e.g. recent conferences: Data Science Strings (Northeastern); String Data (Munich); Machine Learning CY (Sanya)
- Genetic algorithms (see e.g. S. Abel and J. Rizos (2014) arXiv:1404.7359)
- Neural Nets (see e.g. for orbifolds: Andreas Mütter et al (2018) arXiv:1811.05993 and G. Harries PhD Thesis).

Thanks for listening! ☺