Stability in non-supersymmetric open strings

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S. Abel, E. Dudas and D. Lewis [arXiv:1811.xxxxx].

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Introduction

■ Important properties of String Theory (dualities, branes,...) have been discovered in presence of exact supersymmetry in flat space.

Susy guaranties stability of the backgrounds from weak to strong coupling.

■ For Phenomonology and Cosmology, susy must be broken :

- **If "hard" breaking,** the susy breaking scale and effective potential are

$$M = M_{\rm s} \qquad \Longrightarrow \qquad \mathcal{V}_{\rm quantum} \sim M_{\rm s}^d$$

- **If susy spontaneously broken, at tree level, in flat space** e.g. by a **stringy Scherk-Schwarz mechanism,** [Kounnas, Porrati,'88] [Antoniadis, Dudas, Sagnotti, '98]

$$M = \frac{M_{\rm s}}{2R} \qquad \Longrightarrow \qquad \mathcal{V}_{\rm quantum} \sim M^d$$

■ **We study the moduli stability of backgrounds at the Quantum level, at weak coupling, in open strings compactified on a torus.**

■ **In a background where $M <$ all masses scales,** the 1-loop effective potential $\mathcal{V}$ is dominated by the light Kaluza-Klein states,

$$\mathcal{V} = \big(n_F - n_B\big)\,\xi\, M^d + \mathcal{O}\Big((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\Big)\,, \qquad \xi > 0$$

- M_0 is the string scale, or an Higgs-like scale.

- **n_F, n_B are the numbers of massless fermionic and bosonic degrees of freedom.**

■ Because we compactify on a torus ($\mathcal{N} = 4$ in 4D), **all moduli are Wilson lines (WL) :**

$$\mathcal{V} = \mathcal{V}\big|_{a=0} + M^d \sum_{\substack{\text{massless}\\ \text{spectrum}}} \sum_{r,I} Q_r a_r^I + \cdots$$

- a_r^I is the WL along the internal circle I of the r-th Cartan $U(1)$.
- Q_r is the charge of the massless spectrum (and Kaluza-Klein towers) running in the loop.
- combining states Q_r and $-Q_r \implies 0$: **No Tadpole.**

■ At quadratic order [Kounnas, H.P, '16] [Coudarchet, H.P., '18]

$$\mathcal{V} = \big(n_{\mathrm{F}} - n_{\mathrm{B}}\big)\, \xi\, M^d + M^d \Bigg(\sum_{\substack{\text{massless} \\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless} \\ \text{fermions}}} Q_r^2 \Bigg) \big(a_r^I\big)^2 + \cdots$$

$\Longrightarrow$ **The higher $\mathcal{V}$ is, the more unstable it is.**

■ We show that **tachyon free models with $\mathcal{V} \geq 0$ do exist at the quantum level.**

In 9 dimensions

■ Type I compactified on $S^1(R_9)$ with **Sherk-Schwarz** susy breaking

$$\mathcal{W} = \mathrm{diag}\big(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \dots, e^{2i\pi a_{16}}, e^{-2i\pi a_{16}}\big)$$

$$\text{momentum} \quad \frac{m_9}{R_9} \longrightarrow \frac{m_9 + \frac{F}{2} + a_r - a_s}{R_9}$$

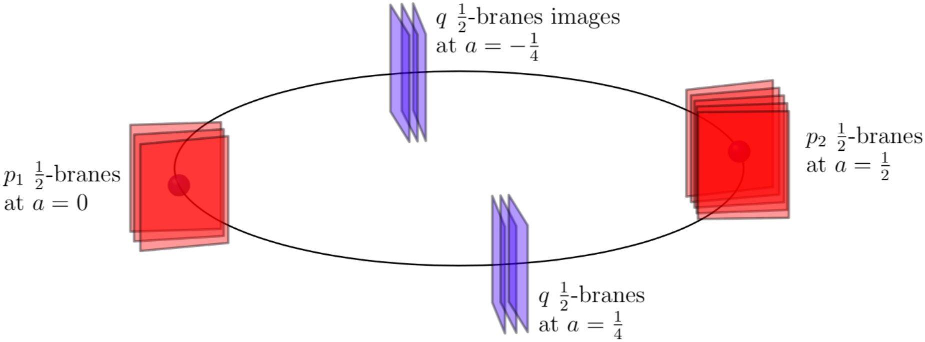
■ **T-duality $R_9 \to \tilde{R}_9 = \frac{1}{R_9}$ yields a geometric picture in Type I', where WLs become positions along $S^1(\tilde{R}_9)$:**

• There are 2 O8-orientifold planes at $\tilde{X}^9 = 0$ and $\tilde{X}^9 = \pi \tilde{R}_9$.

• The D9-branes become 32 D8 "half"-branes :
16 at $\tilde{X}^9 = 2\pi a_r \tilde{R}_9$ and 16 mirror $\frac{1}{2}$-branes at $\tilde{X}^9 = -2\pi a_r \tilde{R}_9$.

• Branes and mirrors branes can be coincident on an O8-plane,
$a_r = 0$ or $\frac{1}{2}$ $\implies$ ***SO(p), p* even**

• Elsewhere, a bunch of q $\frac{1}{2}$-branes and the mirror bunch $\Longrightarrow$ ***U(q)***



q $\frac{1}{2}$-branes images
at $a = -\frac{1}{4}$
p_1 $\frac{1}{2}$-branes
at $a = 0$
p_2 $\frac{1}{2}$-branes
at $a = \frac{1}{2}$
q $\frac{1}{2}$-branes
at $a = \frac{1}{4}$

■ We look for **stable brane configurations.**

• **A sufficient condition for $\mathcal{V}$ to be extremal** with respect to the a_r is that **no mass scale exist between 0 and M.**

This corresponds to $\boldsymbol{a = 0}$ **or** $\boldsymbol{\frac{1}{2}}$ **only.**

• Moreover, $m_9 + \frac{1}{2} + \frac{1}{2} - 0$ can vanish :
Super-Higgs and Higgs compensate $\Longrightarrow$ massless fermions.

This is necessary to have $n_\mathrm{F} - n_\mathrm{B} \geq 0$.

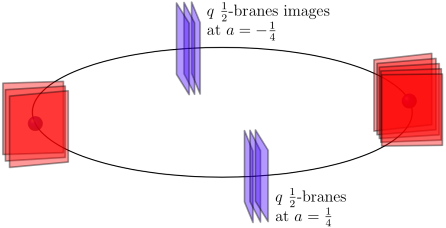
■ However, $\boldsymbol{a = \pm\frac{1}{4}}$ is also special :

• $m_9 + \frac{1}{2} + \frac{1}{4} - (-\frac{1}{4})$ can vanish $\Longrightarrow$ massless fermions.

• Moreover, Bosons $m_9 + 0 + \frac{1}{4} - 0$ and Fermions $m_9 + \frac{1}{2} + 0 - \frac{1}{4}$ have degenerate mass $M/2$. They cancel accidentally in

$$\mathcal{V} \propto \int \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}} \,\mathrm{Str}\, \frac{1+\Omega}{2}\, e^{-\pi\tau_2\mathcal{M}^2}$$

$$\simeq (n_\mathrm{F} - n_\mathrm{B})\, \xi\, M^d \qquad \text{which remains true}$$



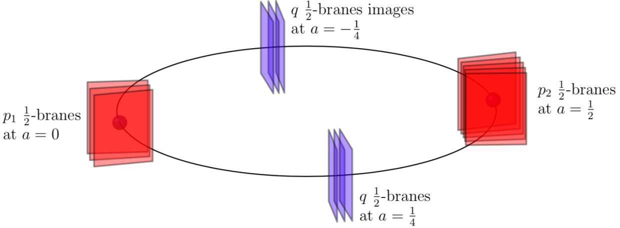
- $SO(p_1)\times SO(p_2)\times U(q) \quad \times U(1)^2$ for $G_{\mu 9}$, RR-2-form $C_{\mu 9}$

$$n_{\rm B}=8\left(8+\frac{p_1(p_1-1)}{2}+\frac{p_2(p_2-1)}{2}+q^2\right)$$

$$n_{\rm F}=8\left(p_1p_2+\frac{q(q-1)}{2}+\frac{q(q-1)}{2}\right)$$

- Bifundamental $(\boldsymbol{p_1},\boldsymbol{p_2})$ and antisymmetric $\oplus$ $\overline{\text{antisymmetric}}$

- **$n_{\rm F}-n_{\rm B}$ is minimal for $p_1=32$, $p_2=0$, $q=0$,** which **suggests that the $SO(32)$ brane configuration is stable.**



■ We have described the moduli space where p_1, p_2 are even.

• **The moduli space admits a second, disconnected part, where p_1, p_2 are odd $\Longrightarrow$ One $\frac{1}{2}$-brane is frozen at $a = 0$, and one frozen at $a = \frac{1}{2}$** [Schwarz,'99]

$$\mathcal{W} = \mathrm{diag}\left(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \ldots, e^{2i\pi a_{15}}, e^{-2i\pi a_{15}}, 1, -1\right)$$

• **$n_F - n_B$ is minimal for $p_1 = 31$, $p_2 = 1$, $q = 0$,** which **suggest that the $SO(31) \times SO(1)$ brane configuration is stable.** ($SO(1)$ is to remind the frozen brane ie $(p_1, 1)$ bifundamental fermion)

■ **To demonstrate these expectations, we compute the 1-loop potential**

$$\mathcal{V} = \frac{\Gamma(5)}{\pi^{14}}\, M^9 \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\mathcal{W})}{(2n_9+1)^{10}} + \mathcal{O}\Big((M_s M)^{\frac{9}{2}} e^{-\pi\frac{M_s}{M}}\Big)$$

It involves the torus + Klein bottle + annulus + Möbius amplitudes :

$$\mathcal{N}_{2n_9+1}(\mathcal{W}) = 4\Big(-16 - 0 - (\mathrm{tr}\,\mathcal{W}^{2n_9+1})^2 + \mathrm{tr}\,(\mathcal{W}^{2(2n_9+1)})\Big)$$

$$= -16\left(\sum_{\substack{r,s=1\\ r\neq s}}^{N} \cos\big(2\pi(2n_9+1)a_r\big)\cos\big(2\pi(2n_9+1)a_s\big) + N - 4\right)$$

(where $N = 16$ or 15)

■ **For $a_r = 0, \frac{1}{2}, \pm\frac{1}{4}$**

$$\mathcal{N}_{2n_9+1}(\mathcal{W}) = n_{\rm F} - n_{\rm B} \quad \Longrightarrow \quad \mathcal{V} = \big(n_{\rm F} - n_{\rm B}\big)\,\xi\, M^d + \cdots$$

- **Some of the WLs of $U(q)$ are always unstable** $\Longrightarrow$ $q=0$**.**

- For $p_1 \geq 2$, the WLs of $SO(p_1)$ have masses $\propto p_1 - 2 - p_2$.

  For $p_2 \geq 2$, those of $SO(p_2)$ have masses $\propto p_2 - 2 - p_1$.

  Both cannot be ≥ 0, $\Longrightarrow$ p_2 **must be 0 or 1.**

■ Conclusion in 9D :

$SO(32)$ and $SO(31)\times SO(1)$ are stable brane configurations

with M **running away**

NB : $0 - n_{\rm B} = -4032$ and $n_{\rm F} - n_{\rm B} = -3536$, which is higher because

- the dimension of $SO(31)$ is lower

- the frozen $\frac{1}{2}$-brane at $a = \frac{1}{2}$ induces a fermionic bifundam $(p_1, 1)$.

NB : In lower dim, we have more O-planes on which we can freeze more $\frac{1}{2}$-branes $\Longrightarrow n_{\rm F} - n_{\rm B} \geq 0$.

In d dimensions

■ **Type I on T^{10-d}** with metric G_{IJ} and **Scherk-Schwarz along X^9**

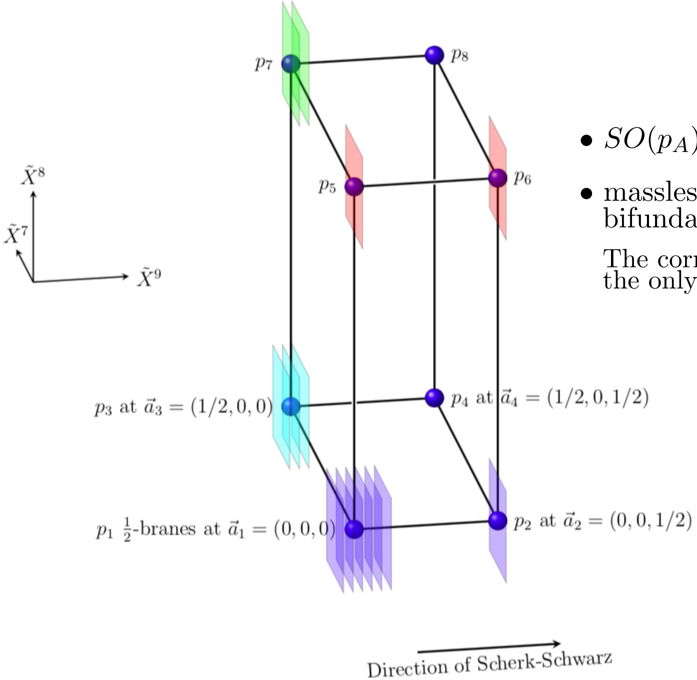
$$M = \frac{\sqrt{G^{99}}}{2} M_s$$

■ **Type I'** picture obtained by **T-dualizing T^{10-d} :**

• **2^{10-d} O$(d-1)$-planes located at the corners of a $(10-d)$-dimensional box.**

• **32 "half" $(d-1)$-branes.**

■ **$\mathcal{V}$ is extremal when the 32 $\frac{1}{2}$-branes are located on the O-planes.**



- $SO(p_A)$ at corner A
- massless fermionic bifundamental (p_{2A-1}, p_{2A})

  The corners $2A-1$, $2A$ are the only ones close

- **The WLs masses can be found from the potential, or**

$$\text{mass}^2 \propto \Big(\sum_{\substack{\text{massless}\\ \text{bosons}}} Q_r^2 - \sum_{\substack{\text{massless}\\ \text{fermions}}} Q_r^2 \Big)$$

$$\propto T_{\mathcal{R}_\mathrm{B}} - T_{\mathcal{R}_\mathrm{F}} \qquad \text{where} \qquad T_\mathcal{R}\delta_{ab} = \frac{1}{2}\,\mathrm{tr}\, T_a T_b, \quad (a,b=1\dots,\dim G)$$

$$\propto p_{2A-1} - 2 - p_{2A} \qquad \text{as in 9D}$$

- Stability $\implies$

$SO(p_{2A-1})$ with 0 or 1 frozen $\frac{1}{2}$-brane at corner $2A$

- **$n_\mathrm{F} - n_\mathrm{B}$ can be positive or negative.**
  - 23 models have $n_\mathrm{F} - n_\mathrm{B} = 0$, e.g. in $d \le 5$:

$$SO(4) \times \big[SO(1) \times SO(1)\big]^{14}$$

$$\big[SO(5) \times SO(1)\big] \times \big[SO(1) \times SO(1)\big]^{13} \;\; i.e. \;\; SO(5) + 1 \text{ fermionic fundam}$$

■ **The potential depends on G_{IJ} and**

$$a^I_\alpha = \langle a^I_\alpha \rangle + \varepsilon^I_\alpha, \qquad \langle a^I_\alpha \rangle \in \left\{0, \frac{1}{2}\right\}, \qquad \alpha = 1, \dots, 32, \quad I = d, \dots, 9$$

• $\mathcal{V}$ **does not depend on the Ramond-Ramond moduli C_{IJ}** because they are also WLs, but there are no perturbative states charged under the associated $U(1)$ gauge bosons $C_{\mu I}$.

• We take $G^{99} \ll |G_{ij}| \ll G_{99}$, $i, j = d, \dots, 8$, to not have mass scales $< M$

$$\mathcal{V} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\varepsilon, G)}{|2n_9+1|^{d+1}} + \mathcal{O}\left((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\right)$$

$$\mathcal{N}_{2n_9+1}(\varepsilon,G) = 4\left\{-16 - \sum_{(\alpha,\beta)\in L} (-1)^F \cos\left[2\pi(2n_9+1)\left(\varepsilon^9_\alpha - \varepsilon^9_\beta + \frac{G^{9i}}{G^{99}}(\varepsilon^i_\alpha - \varepsilon^i_\beta)\right)\right] \times \mathcal{H}_{\frac{d+1}{2}}\left(\pi|2n_9+1| \frac{(\varepsilon^i_\alpha - \varepsilon^i_\beta)\hat{G}^{ij}(\varepsilon^j_\alpha - \varepsilon^j_\beta)}{\sqrt{G^{99}}}\right) + \sum_\alpha \cos\left[4\pi(2n_9+1)\left(\varepsilon^9_\alpha + \frac{G^{9i}}{G^{99}}\varepsilon^i_\alpha\right)\right] \mathcal{H}_{\frac{d+1}{2}}\left(4\pi|2n_9+1| \frac{\varepsilon^i_\alpha \hat{G}^{ij} \varepsilon^j_\alpha}{\sqrt{G^{99}}}\right)\right\}$$

where $\hat{G}^{ij} = G^{ij} - \frac{G^{i9}}{G^{99}} G^{99} \frac{G^{9j}}{G^{99}}$ and $\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)} z^\nu K_\nu(2z)$

■ However,

$$\mathcal{V} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\varepsilon, G)}{|2n_9+1|^{d+1}} + \mathcal{O}\Big((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\Big)$$

becomes **for $\boldsymbol{\varepsilon^I_\alpha = 0}$,**

$$\mathcal{N}_{2n_9+1}(0, G) = n_\mathrm{F} - n_\mathrm{B} \quad \Longrightarrow \quad \mathcal{V} = \big(n_\mathrm{F} - n_\mathrm{B}\big)\, \xi\, M^d + \cdots$$

$\Longrightarrow$ **all components of $\boldsymbol{G_{IJ}}$ are flat directions !**

(Except $M = M_\mathrm{s}\sqrt{G^{99}}/2$ unless $n_\mathrm{F} - n_\mathrm{B} = 0$)

Conclusion

■ **In open string theory compactified on a torus,** we have found **at the quantum level but weak coupling,** backgrounds

- where **the open string moduli are massive.**

- **If $n_{\mathrm{F}} - n_{\mathrm{B}} < 0$ or > 0, all NS-NS closed string moduli G_{IJ} except M** and **Ramond-Ramond C_{IJ} are flat directions.**

- **If $n_{\mathrm{F}} - n_{\mathrm{B}} = 0$ vanishing, we have true vacua** (up to exponentially suppressed terms). See also [Abel, Dienes, Mavroudi,'15] [Kounnas, H.P.,'15] [Kachru, Kumar, Silverstein,'98] [Harvey,'98] [Shiu, Tye,'98] [Blumenhagen, Gorlich,'98] [Angelantonj, Antoniadis, Forger,'99] [Satoh, Sugawara, Wada,'15]