Stability in non-supersymmetric open strings

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S. Abel, E. Dudas and D. Lewis [arXiv:1811.xxxxx].
Introduction

- Important properties of String Theory (dualities, branes,...) have been discovered in presence of exact supersymmetry in flat space.

Susy guaranties stability of the backgrounds from weak to strong coupling.

- For Phenomonology and Cosmology, susy must be broken:
  
  - If “hard” breaking, the susy breaking scale and effective potential are
    \[ M = M_s \quad \rightarrow \quad V_{\text{quantum}} \sim M_s^d \]
  
  - If susy spontaneously broken, at tree level, in flat space, e.g. by a stringy Scherk-Schwarz mechanism, [Kounnas, Porrati,’88] [Antoniadis, Dudas, Sagnotti, ’98]
    \[ M = \frac{M_s}{2R} \quad \rightarrow \quad V_{\text{quantum}} \sim M^d \]

- We study the moduli stability of backgrounds at the Quantum level, at weak coupling, in open strings compactified on a torus.
In a background where $M < \text{all masses scales}$, the 1-loop effective potential $\mathcal{V}$ is dominated by the light Kaluza-Klein states,

$$\mathcal{V} = (n_F - n_B) \xi M^d + \mathcal{O}\left((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\right), \quad \xi > 0$$

• $M_0$ is the string scale, or an Higgs-like scale.

• $n_F, n_B$ are the numbers of massless fermionic and bosonic degrees of freedom.

Because we compactify on a torus ($\mathcal{N} = 4$ in 4D), all moduli are Wilson lines (WL):

$$\mathcal{V} = \mathcal{V}\big|_{a=0} + M^d \sum_{\text{massless spectrum}} \sum_{r,I} Q_r a^I_r + \cdots$$

• $a^I_r$ is the WL along the internal circle $I$ of the $r$-th Cartan $U(1)$.

• $Q_r$ is the charge of the massless spectrum (and Kaluza-Klein towers) running in the loop.

• combining states $Q_r$ and $-Q_r \implies 0$ : No Tadpole.
At quadratic order [Kounnas, H.P, ’16] [Coudarchet, H.P., ’18]

\[ \mathcal{V} = (n_F - n_B) \xi M^d + M^d \left( \sum_{\text{massless bosons}} Q_r^2 - \sum_{\text{massless fermions}} Q_r^2 \right) (a_I^r)^2 + \cdots \]

⇒ The higher \( \mathcal{V} \) is, the more unstable it is.

We show that tachyon free models with \( \mathcal{V} \geq 0 \) do exist at the quantum level.
In 9 dimensions

- Type I compactified on $S^1(R_9)$ with **Sherk-Schwarz** susy breaking

\[ \mathcal{W} = \text{diag}(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \ldots, e^{2i\pi a_{16}}, e^{-2i\pi a_{16}}) \]

momentum

\[
\frac{m_9}{R_9} \rightarrow \frac{m_9 + \frac{F}{2} + a_r - a_s}{R_9}
\]

- **T-duality** $R_9 \rightarrow \tilde{R}_9 = \frac{1}{R_9}$ yields a geometric picture in Type I’, where WLs become positions along $S^1(\tilde{R}_9)$:

  - There are 2 O8-orientifold planes at $\tilde{X}^9 = 0$ and $\tilde{X}^9 = \pi \tilde{R}_9$.
  - The D9-branes become 32 D8 “half”-branes:
    16 at $\tilde{X}^9 = 2\pi a_r \tilde{R}_9$ and 16 mirror $\frac{1}{2}$-branes at $\tilde{X}^9 = -2\pi a_r \tilde{R}_9$.
  - Branes and mirrors branes can be coincident on an O8-plane, $a_r = 0$ or $\frac{1}{2}$ \(\implies SO(p), \ p \text{ even} \)
  - Elsewhere, a bunch of $q$ $\frac{1}{2}$-branes and the mirror bunch \(\implies U(q)\)
$p_1 \frac{1}{2}$-branes at $a = 0$

$q \frac{1}{2}$-branes images at $a = -\frac{1}{4}$

$q \frac{1}{2}$-branes at $a = \frac{1}{4}$

$p_2 \frac{1}{2}$-branes at $a = \frac{1}{2}$
We look for stable brane configurations.

- A sufficient condition for $\mathcal{V}$ to be extremal with respect to the $a_r$ is that no mass scale exist between 0 and $M$.

This corresponds to $a = 0$ or $\frac{1}{2}$ only.

- Moreover, $m_9 + \frac{1}{2} + \frac{1}{2} - 0$ can vanish: Super-Higgs and Higgs compensate $\Rightarrow$ massless fermions.

This is necessary to have $n_F - n_B \geq 0$.

- However, $a = \pm \frac{1}{4}$ is also special:

  - $m_9 + \frac{1}{2} + \frac{1}{4} - (-\frac{1}{4})$ can vanish $\Rightarrow$ massless fermions.

  - Moreover, Bosons $m_9 + 0 + \frac{1}{4} - 0$ and Fermions $m_9 + \frac{1}{2} + 0 - \frac{1}{4}$ have degenerate mass $M/2$. They cancel accidentally in

$$
\mathcal{V} \propto \int \frac{d\tau_2}{\tau_2^{1+\frac{d}{2}}} \text{Str} \frac{1+\Omega}{2} e^{-\pi \tau_2 M^2}
\approx (n_F - n_B) \xi M^d \quad \text{which remains true}
$$
• $SO(p_1) \times SO(p_2) \times U(q) \times U(1)^2$ for $G_{\mu 9}$, RR-2-form $C_{\mu 9}$

$$n_B = 8 \left( 8 + \frac{p_1(p_1 - 1)}{2} + \frac{p_2(p_2 - 1)}{2} + q^2 \right)$$

$$n_F = 8 \left( p_1 p_2 + \frac{q(q - 1)}{2} + \frac{q(q - 1)}{2} \right)$$

• Bifundamental $(p_1, p_2)$ and antisymmetric $\oplus$ antisymmetric

• $n_F - n_B$ is minimal for $p_1 = 32$, $p_2 = 0$, $q = 0$, which suggests that the $SO(32)$ brane configuration is stable.
We have described the moduli space where \( p_1, p_2 \) are even.

- The moduli space admits a second, disconnected part, where \( p_1, p_2 \) are odd \( \Rightarrow \) One \( \frac{1}{2} \)-brane is frozen at \( a = 0 \), and one frozen at \( a = \frac{1}{2} \) \([\text{Schwarz,'99}]\)

\[
\mathcal{W} = \text{diag}(e^{2i\pi a_1}, e^{-2i\pi a_1}, e^{2i\pi a_2}, e^{-2i\pi a_2}, \ldots, e^{2i\pi a_{15}}, e^{-2i\pi a_{15}}, 1, -1)
\]

- \( n_F - n_B \) is minimal for \( p_1 = 31, p_2 = 1, q = 0 \), which suggest that the \( SO(31) \times SO(1) \) brane configuration is stable. (\( SO(1) \) is to remind the frozen brane ie \( (p_1, 1) \) bifundamental fermion)
To demonstrate these expectations, we compute the 1-loop potential

\[ \mathcal{V} = \frac{\Gamma(5)}{\pi^{14}} M^9 \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\mathcal{W})}{(2n_9 + 1)^{10}} + \mathcal{O}\left((M_s M)^{9/2} e^{-\pi \frac{M_s}{M}}\right) \]

It involves the torus + Klein bottle + annulus + Möbius amplitudes:

\[ \mathcal{N}_{2n_9+1}(\mathcal{W}) = 4 \left(-16 - 0 - (\text{tr}\ \mathcal{W}^{2n_9+1})^2 + \text{tr}\ (\mathcal{W}^{2(2n_9+1)})\right) \]

\[ = -16 \left( \sum_{r,s=1}^{N} \cos(2\pi(2n_9 + 1)a_r) \cos(2\pi(2n_9 + 1)a_s) + N - 4 \right) \]

(\text{where } N = 16 \text{ or } 15)

For \( a_r = 0, \frac{1}{2}, \pm \frac{1}{4} \)

\[ \mathcal{N}_{2n_9+1}(\mathcal{W}) = n_F - n_B \quad \implies \quad \mathcal{V} = (n_F - n_B) \xi M^d + \cdots \]
• Some of the WLs of $U(q)$ are always unstable $\implies q = 0$.

• For $p_1 \geq 2$, the WLs of $SO(p_1)$ have masses $\propto p_1 - 2 - p_2$.
  For $p_2 \geq 2$, those of $SO(p_2)$ have masses $\propto p_2 - 2 - p_1$.
  Both cannot be $\geq 0$, $\implies p_2$ must be 0 or 1.

■ Conclusion in 9D:
$SO(32)$ and $SO(31) \times SO(1)$ are stable brane configurations
with $M$ running away

NB: $0 - n_B = -4032$ and $n_F - n_B = -3536$, which is higher because

• the dimension of $SO(31)$ is lower
• the frozen $\frac{1}{2}$-brane at $a = \frac{1}{2}$ induces a fermionic bifundam $(p_1, 1)$.

NB: In lower dim, we have more O-planes on which we can freeze more $\frac{1}{2}$-branes $\implies n_F - n_B \geq 0.$
In $d$ dimensions

- **Type I on** $T^{10-d}$ **with metric** $G_{IJ}$ **and Scherk-Schwarz along** $X^9$

\[ M = \frac{\sqrt{G^{99}}}{2} M_s \]

- **Type I’** picture obtained by **T-dualizing** $T^{10-d}$ :

  - $2^{10-d}$ **O($d-1$)-planes** located at the corners of a $(10-d)$-dimensional box.

  - 32 “half” $(d-1)$-branes.

- **$\mathcal{V}$ is extremal** when the 32 $\frac{1}{2}$-branes are located on the O-planes.
- $SO(p_A)$ at corner $A$
- massless fermionic bifundamental $(p_{2A-1}, p_{2A})$

The corners $2A - 1, 2A$ are the only ones close.
The WLs masses can be found from the potential, or

\[ \text{mass}^2 \propto \left( \sum_{\text{massless bosons}} Q_r^2 - \sum_{\text{massless fermions}} Q_r^2 \right) \]

\[ \propto T_{\mathcal{R}_B} - T_{\mathcal{R}_F} \]

where \( T_{\mathcal{R}} \delta_{ab} = \frac{1}{2} \text{tr} \ T_a T_b \), \( (a,b=1...,\text{dim } G) \)

\[ \propto p_{2A-1} - 2 - p_{2A} \]

as in 9D

Stability \[ \implies \]

\( SO(p_{2A-1}) \) with 0 or 1 frozen \( \frac{1}{2} \)-brane at corner \( 2A \)

\( n_F - n_B \) can be positive or negative.

- 23 models have \( n_F - n_B = 0 \), e.g. in \( d \leq 5 \) :

\( SO(4) \times [SO(1) \times SO(1)]^{14} \)

\( [SO(5) \times SO(1)] \times [SO(1) \times SO(1)]^{13} \) i.e. \( SO(5) + 1 \) fermionic fundam
The potential depends on $G_{IJ}$ and

$$a^I_\alpha = \langle a^I_\alpha \rangle + \varepsilon^I_\alpha, \quad \langle a^I_\alpha \rangle \in \left\{ 0, \frac{1}{2} \right\}, \quad \alpha = 1, \ldots, 32, \quad I = d, \ldots, 9$$

- $\mathcal{V}$ does not depend on the Ramond-Ramond moduli $C_{IJ}$ because they are also WLs, but there are no perturbative states charged under the associated $U(1)$ gauge bosons $C_{\mu I}$.

- We take $G^{99} \ll |G_{ij}| \ll G_{99}, \quad i, j = d, \ldots, 8$, to not have mass scales $< M$

$$\mathcal{V} = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\varepsilon, G)}{|2n_9 + 1|^{d+1}} + \mathcal{O}\left(\left(M_0 M\right)^{\frac{d}{2}} e^{-M_0/M}\right)$$

$$\mathcal{N}_{2n_9+1}(\varepsilon, G) = 4 \left\{ -16 - \sum_{(\alpha, \beta) \in L} (-1)^F \cos \left[ 2\pi(2n_9+1) \left( \varepsilon^9_\alpha - \varepsilon^9_\beta + \frac{G^{9i}}{G^{99}} (\varepsilon^i_\alpha - \varepsilon^i_\beta) \right) \right] \right. \right.$$  

$$\left. \times \mathcal{H}_{d+1} \left( \pi |2n_9 + 1| \frac{(\varepsilon^i_\alpha - \varepsilon^i_\beta) \hat{G}^{ij} (\varepsilon^j_\alpha - \varepsilon^j_\beta)}{\sqrt{G^{99}}} \right) \right.$$  

$$\left. + \sum_\alpha \cos \left[ 4\pi(2n_9+1) \left( \varepsilon^9_\alpha + \frac{G^{9i}}{G^{99}} \varepsilon^i_\alpha \right) \right] \mathcal{H}_{d+1} \left( 4\pi |2n_9 + 1| \frac{\varepsilon^i_\alpha \hat{G}^{ij} \varepsilon^j_\alpha}{\sqrt{G^{99}}} \right) \right\}$$

where $\hat{G}^{ij} = G^{ij} - \frac{G^{9i}}{G^{99}} G^{99} \frac{G^{9j}}{G^{99}}$ and $\mathcal{H}_\nu(z) = \frac{2}{\Gamma(\nu)} z^\nu K_\nu(2z)$
However,

\[ V = \frac{\Gamma\left(\frac{d+1}{2}\right)}{\pi^{\frac{3d+1}{2}}} M^d \sum_{n_9} \frac{\mathcal{N}_{2n_9+1}(\varepsilon, G)}{|2n_9 + 1|^{d+1}} + \mathcal{O}\left((M_0 M)^{\frac{d}{2}} e^{-M_0/M}\right) \]

becomes for \( \varepsilon^I_\alpha = 0 \),

\[ \mathcal{N}_{2n_9+1}(0, G) = n_F - n_B \quad \Rightarrow \quad V = (n_F - n_B) \xi M^d + \cdots \]

\( \Rightarrow \) all components of \( G_{IJ} \) are flat directions!

(Except \( M = M_s \sqrt{G^{\frac{99}{2}}} \) unless \( n_F - n_B = 0 \))
In open string theory compactified on a torus, we have found backgrounds at the quantum level but weak coupling, backgrounds where the open string moduli are massive.

- If $n_F - n_B < 0$ or $> 0$, all NS-NS closed string moduli $G_{IJ}$ except $M$ and Ramond-Ramond $C_{IJ}$ are flat directions.

- If $n_F - n_B = 0$ vanishing, we have true vacua (up to exponentially suppressed terms). See also [Abel, Dienes, Mavroudi,’15] [Kounnas, H.P.,’15] [Kachru, Kumar, Silverstein,’98] [Harvey,’98] [Shiu, Tye,’98] [Blumenhagen, Gorlich,’98] [Angelantonj, Antoniadis, Forger,’99] [Satoh, Sugawara, Wada,’15]