Froggatt-Nielsen mechanism in a model with 331-gauge group

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K. Huitu and N. Koivunen, arXiv:1706.09463 [hep-ph], **PRD**,
K. Huitu and N. Koivunen, arXiv:1812.xxxx [hep-ph]

Discrete, Vienna, 27.11.2018

The Flavour Problem

Three fermion families. Why?

$\Rightarrow$ 331 -models have the answer!

The origin of fermion mass hierarchy?

$$\begin{cases} y_e = 2.94 \times 10^{-6} \\ y_\mu = 6.07 \times 10^{-4} \\ y_\tau = 1.02 \times 10^{-2} \end{cases} \quad \begin{cases} y_u = 1,32 \times 10^{-5} \\ y_c = 7.33 \times 10^{-3} \\ y_t = 1 \end{cases} \quad \begin{cases} y_d = 2.76 \times 10^{-5} \\ y_s = 5.46 \times 10^{-4} \\ y_b = 2.4 \times 10^{-2} \end{cases}$$

$\Rightarrow$ Froggatt-Nielsen mechanism is the answer!

Froggatt-Nielsen mechanism

- Introduce new gauge singlet scalar, ϕ, called the flavon, and global $U(1)_{FN}$ symmetry.
- SM fermions, the Higgs and Flavon are charged under the $U(1)_{FN}$. Charges are such that SM Yukawa couplings are forbidden.
- Effective operator is allowed:

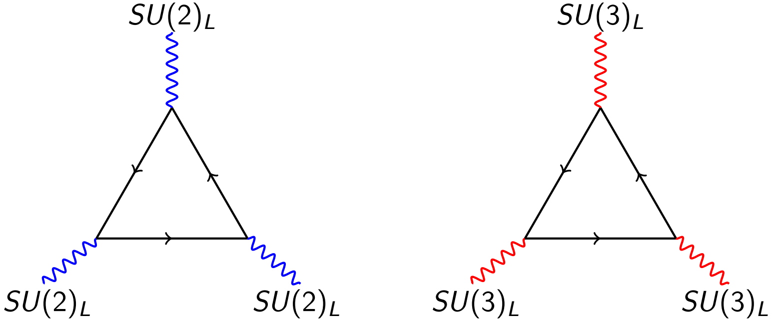
$$c_{ij}\left(\frac{\phi}{\Lambda}\right)^{n_{ij}}\bar{f}_{L,i}H\ f_{R,j}+h.c.\to\underbrace{c_{ij}\left(\frac{v_\phi}{\sqrt{2}\Lambda}\right)^{n_{ij}}}_{y^f_{ij}}\bar{f}_{L,i}Hf_{R,j}+\ldots$$

- Yukawa hierarchy determined by the charge assignment

$$n_{ij}=-\frac{1}{q(\phi)}[q(\bar{f}_{L,i})+q(f_{R,i})+q(H)]$$

The 331 – gauge extension

$$\text{Standard model} = SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$331 - \text{model} = SU(3)_C \times SU(3)_L \times U(1)_X$$



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331-model

- Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets. Possible only with 3 generations!
- One quark family must be in different representation than the others (due to anomaly cancellation)
  $\Rightarrow$ flavour changing neutral current at tree-level!
- Freedom in electric charge definition: $Q = T_3 + \beta T_8 + X$. Two types of models: $\beta = \pm\sqrt{3}$ and $\beta = \pm\frac{1}{\sqrt{3}}$.

The Model

$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X$

Fermion representations:

$$\text{Triplets:} \quad L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ N_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$Antitriplets: Q_{L,2} = \begin{pmatrix} s \\ c \\ D_1 \end{pmatrix}_L \sim (3, 3^*, 0), Q_{L,3} = \begin{pmatrix} b \\ t \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

$N_{L,i}$ = new neutrino-like fields, U = new up-type quark, D_1, D_2 = new down-type quarks.

- 6 triplets and 6 antitriplets ⇒ Gauge anomalies cancel!
- **Generic feature of 331-models**: Quark generations in different representations ⇒ Flavour Changing Neutral Currents!

Particle Content

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1,3,\frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \; \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1,3,-\frac{1}{3})$$

The most general vacuum is:

$$\langle\eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v' \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \\ v_2 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}$$

The symmetry breaking pattern is:

$$SU(3)_L \times U(1)_X \overset{u,\,v_2}{\rightarrow} SU(2)_L \times U(1)_Y \overset{v_1,\,v'}{\rightarrow} \times U(1)_{em}, \tag{1}$$

where $v_2, u \gtrsim 1\mathrm{TeV}$ and $v_1, v' = \mathcal{O}(100\mathrm{GeV})$

Minimal scalar sector admits Froggatt-Nielsen mechanism!

$$\rho = \begin{pmatrix} \rho^0 + \langle \rho^0 \rangle \\ \rho^- \\ \rho'^0 + \langle \rho'^0 \rangle \end{pmatrix}, \ \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 + \langle \chi'^0 \rangle \end{pmatrix} \sim (1, 3, -\frac{1}{3}).$$

- The combination $\rho^\dagger \chi$ is a gauge singlet.
- It can carry a non-zero $U(1)$ charge.
- $\rho^\dagger \chi$ has non-zero VEV:

$$\langle \rho^\dagger \chi \rangle = \frac{v_2 u}{2} \neq 0$$

The combination $\rho^\dagger \chi$ can play the role of the flavon in the Froggatt-Nielsen mechanism!

331 Froggatt-Nielsen mechanism

Froggatt-Nielsen mechanism works the usual way:

$$(c_s^f)_{ij}\left(\frac{\rho^\dagger\chi}{\Lambda^2}\right)^{(n_f^s)_{ij}}\bar{\psi}_{L,i}^f S f_{R,j} \to \underbrace{(c_s^f)_{ij}\overbrace{\left(\frac{v_2 u}{2\Lambda^2}\right)}^{\epsilon}{}^{(n_f^s)_{ij}}}_{(y_s^f)_{ij}}\bar{\psi}_{L,i}^f(S+\langle S\rangle)f_{R,j}+\ldots$$

The minimal scalar content of the 331-models with $\beta = \pm\frac{1}{\sqrt{3}}$ is enough for Froggatt-Nielsen mechanism. No new scalars required!
Models with $\beta = \pm\frac{1}{\sqrt{3}}$ thus explain both the number of fermion families and their hierarchy simultaneously!

Up-type quark mass matrix

$$m_u = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 y^u_{11} & v_1 y^u_{12} & v_1 y^u_{13} & v_1 y^u_{14} \\ -v' y^u_{21} & -v' y^u_{22} & -v' y^u_{23} & -v' y^u_{24} \\ -v' y^u_{31} & -v' y^u_{32} & -v' y^u_{33} & -v' y^u_{34} \\ v_2 y^u_{11} + u y^u_{11} & v_2 y^u_{12} + u y^u_{12} & v_2 y^u_{13} + u y^u_{13} & v_2 y^u_{14} + u y^u_{14} \end{pmatrix}$$

Two sources of hierarchy:

- Usual Froggatt-Nielsen hierarchy:

$$y_{ij} = c_{ij} \left(\frac{v_2 u}{2\Lambda^2} \right)^{n_{ij}} = c_{ij} \epsilon^{n_{ij}}, \quad c_{ij} = \mathcal{O}(1) \quad \text{and} \quad \epsilon < 1$$

- VEVs of two scales: $v_2, u \gtrsim 1\text{TeV}$ and $v_1, v' = \mathcal{O}(100\text{GeV})$

Higgs mediated FCNCs at tree-level

Up-type quark Yukawa couplings:

$$\mathcal{L}_{up} = \sum_{\gamma=1}^{4}(y^u_\rho)_{1\gamma}\,\bar{Q}'_{L,1}\rho u'_{R,\gamma} + \sum_{\gamma=1}^{4}(y^u_\chi)_{1\gamma}\,\bar{Q}'_{L,1}\chi u'_{R,\gamma} + \sum_{\alpha=2}^{3}\sum_{\gamma=1}^{4}(y^u_{\eta^*})_{\alpha\gamma}\,\bar{Q}'_{L,\alpha}\eta^* u'_{R,\gamma}$$

Down-type quark Yukawa couplings:

$$\mathcal{L}_{down} = \sum_{\gamma=1}^{5}(y^d_\eta)_{1\gamma}\,\bar{Q}'_{L,1}\eta d'_{R,\gamma} + \sum_{\alpha=2}^{3}\sum_{\gamma=1}^{5}(y^d_{\rho^*})_{\alpha\gamma}\,\bar{Q}'_{L,\alpha}\rho^* d'_{R,\gamma} + \sum_{\alpha=2}^{3}\sum_{\gamma=1}^{5}(y^d_{\chi^*})_{\alpha\gamma}\,\bar{Q}'_{L,\alpha}\chi^* d'_{R,\gamma}.$$

Borth up- and down-type quarks couple to three different scalar triplets.
$\Rightarrow$ FCNCs!

Up-type quark couplings to Higgs

Higgs is the linear combination of real parts of the neutral fields η^0, ρ^0, ρ'^0, χ^0 and χ'^0.

$$\mathcal{L}_{Yukawa} = \bar{u}'_L \frac{\Gamma'^u_h}{\sqrt{2}} u'_R \; h = \bar{u}_L \; \underbrace{(U^u_L)\frac{\Gamma'^u_h}{\sqrt{2}}(U^{d\dagger}_R)}_{\Gamma^u_h/\sqrt{2}} \; u_R \; h$$

The physical Up-type Higgs-Yukawa coupling can be written as:

$$\begin{aligned}(\Gamma^u_h)_{ij} \quad = \quad & \sqrt{2}\frac{m_j}{v_{SM}}\Big[\delta_{ij} + \alpha_1 (U^u_L)_{i1}(U^{u\dagger}_L)_{1j} - (U^u_L)_{i4}(U^{u\dagger}_L)_{4j} \\ & + \alpha_2 (U^u_L)_{i1}(U^{u\dagger}_L)_{4j} + \alpha_3 (U^u_L)_{i4}(U^{u\dagger}_L)_{1j}\Big],\end{aligned}$$

where

$$\alpha_i = \frac{SU(2)_L\text{-breaking scale}}{SU(3)_L\text{-breaking scale}} \ll 1 \Rightarrow \quad \text{Suppression!}$$

Additional suppression from U_L^u

$$(\Gamma_h^u)_{ij} = \sqrt{2}\frac{m_j}{v_{SM}}\Big[\delta_{ij} + \alpha_1 (U_L^u)_{i1}(U_L^{u\dagger})_{1j} - (U_L^u)_{i4}(U_L^{u\dagger})_{4j} + \alpha_2 (U_L^u)_{i1}(U_L^{u\dagger})_{4j} + \alpha_3 (U_L^u)_{i4}(U_L^{u\dagger})_{1j}\Big]$$

Example: Assume hierarchy:

$$m^u_{i,j} \leq m^u_{i+1,j}.$$

FN charges: $q(Q^c_{L,1}) = 3,\ q(Q^c_{L,2}) = 2,\ q(Q^c_{L,3}) = 0,$
expansion parameter: $\epsilon = 0.23$.

$$U_L^u \sim \begin{pmatrix} 1 & \epsilon^1 & \epsilon^3 & \epsilon^{3+x} \\ \epsilon^1 & 1 & \epsilon^2 & \epsilon^{2+x} \\ \epsilon^3 & \epsilon^2 & 1 & \epsilon^x \\ \epsilon^{3+x} & \epsilon^{2+x} & \epsilon^x & 1 \end{pmatrix} \approx \mathbb{1},$$

where $x = 2 + (\log\epsilon)^{-1}\log(v'/v_2) \geq 0$. $\Rightarrow$ More Suppression!

The Γ_h^u texture becomes:

$$\Gamma_h^u \sim \begin{pmatrix} y_u & y_c[\alpha\epsilon^1] & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^1] & y_c & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^x] & y_c[\alpha\epsilon^x] & y_t & 1 \\ y_u[\alpha] & y_c[\alpha] & y_t[\epsilon^x] & \epsilon^x \end{pmatrix}, \quad \text{where} \quad x \geq 0,$$

$$\alpha = \frac{SU(2)_L\text{-breaking scale}}{SU(3)_L\text{-breaking scale}} \ll 1, \quad \epsilon = 0.23$$

- The off-diagonal elements over the diagonal are largests. They are suppressed by α and ϵ.
- The suppression is enough when $SU(3)_L$-breaking scale $= v_2, u \gtrsim 4\text{TeV}$

Summary

- Froggatt-Nielsen mechanism naturally suppresses Higgs mediated FCNCs. No tricks required!
- The minimal 331-scalar sector is sufficient in housing Froggatt-Nielsen mechanism. No additional scalars required!
- 331-models with $\beta = \pm\frac{1}{\sqrt{3}}$ can explain the number of fermion generations and their mass hierarchy simultaneously! We are the first to point this out in the literature.

$\beta = (\pm)\frac{1}{\sqrt{3}}$

M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

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Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1,3,\frac{2}{3}),\quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix},\ \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1,3,-\frac{1}{3})$$

$\beta = (\pm)\sqrt{3}$

F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).
P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).

$$\textbf{Leptons:}\quad L_{L,i} = \begin{pmatrix} \nu_{L,i} \\ e_{L,i} \\ (e_{R,i})^c \end{pmatrix} \sim (1,3,0), \quad \mathcal{L}_m = \epsilon_{\alpha\beta\gamma} G_{ij} \overline{L}^{\alpha}_{L,i} (L^{\beta}_{L,j})^c \eta^{\gamma *}$$

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^+ \end{pmatrix} \sim (1,3,0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1,3,1),$$

$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1,3,-1), \quad S = \begin{pmatrix} \sigma_1^0 & h_2^- & h_1^+ \\ h_2^- & H_1^{--} & \sigma_2^0 \\ h_1^+ & \sigma_2^0 & H_2^{++} \end{pmatrix} \sim (1,6,0).$$

Quark Flavour Constraints

The most stringent bounds on flavour changing couplings comes from neutral meson mixing: $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$ and $D^0 - \bar{D}^0$.

Process	coupling	Current bound
$K^0 - \bar{K}^0$	$\sqrt{\mathrm{Re}({\Gamma_{ds}^{d}}^2)}$	$([-6.3, 6.1] \times 10^{-2}) \times y_s$
$B^0 - \bar{B}^0$	$\lvert\Gamma_{db}^{d}\rvert$	$< (8.8 \times 10^{-3}) \times y_b$
$B_s^0 - \bar{B}_s^0$	$\lvert\Gamma_{sb}^{d}\rvert$	$< (7.8 \times 10^{-2}) \times y_b$
$D^0 - \bar{D}^0$ (Tree-level)	$\lvert\Gamma_{uc}^{u}\rvert$	$< (1.4 \times 10^{-2}) \times y_c$
$D^0 - \bar{D}^0$ (Loop-level)	$\lvert\Gamma_{ut}^{u}\Gamma_{ct}^{u}\rvert$	$< (1.5 \times 10^{-2}) \times y_t^2$

Neutrino masses

$$\mathcal{L}_{\text{Neutrino mass}} = (\overline{\nu_L}\ \overline{\nu'}_L\ \overline{(N_R)^c}) \begin{pmatrix} 0 & (m^D)^T & \frac{1}{2}m^N \\ m^D & 0 & \frac{1}{2}(m')^N \\ \frac{1}{2}(m^N)^T & \frac{1}{2}(m'^N)^T & \frac{1}{2}M \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ (\nu'_L)^c \\ N_R \end{pmatrix} + h.c.$$

$m^D, m^N \sim 100\text{GeV}$, $(m')^N, M \sim$GUT

$\Rightarrow$ sub-eV neutrinos

Linear seesaw-mechanism!

Gauge bosons

$$D_\mu = \partial_\mu - ig_3 \sum_{a=1}^{8} T_a W_{a\mu} - ig_x X B_\mu,$$

$$\sum_{a=1}^{8} T_a W_{a\mu} =$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & W'^{+}_{\mu} & X'^{0}_{\mu} \\ W'^{-}_{\mu} & -\frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & V'^{-}_{\mu} \\ X'^{0*}_{\mu} & V'^{+}_{\mu} & -\frac{2}{\sqrt{6}} W_{8\mu} \end{pmatrix},$$

The neutral mass eigenstates are: photon, Z_μ, Z'_μ, X^0_μ and X^{0*}_μ. The charged mass eigenstates are $W^\pm_\mu$ and $V^\pm_\mu$.