Grand Unified Theories of Flavour

$SO(10) \times S_4$

Predicting Fermion Mass Matrices and Leptogenesis

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based on

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elusives Horizon 2020

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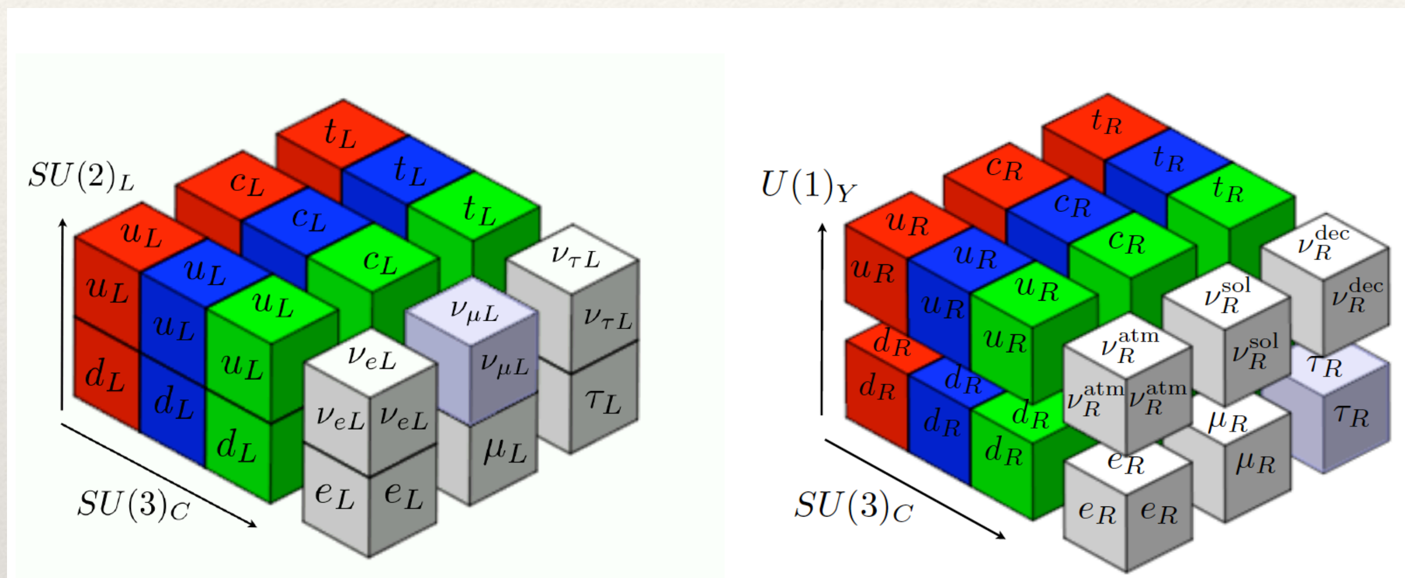
The Flavour Problem

The Standard Model

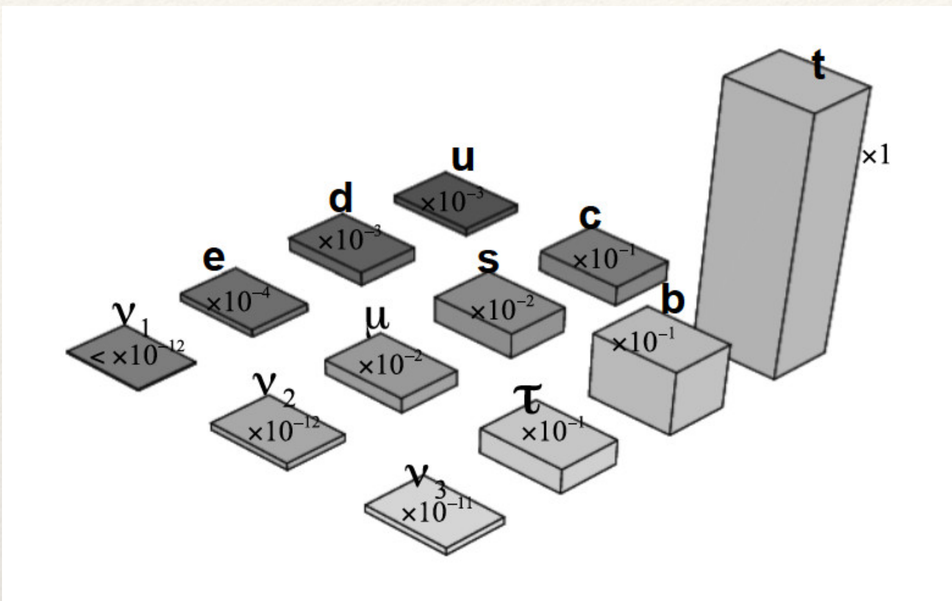


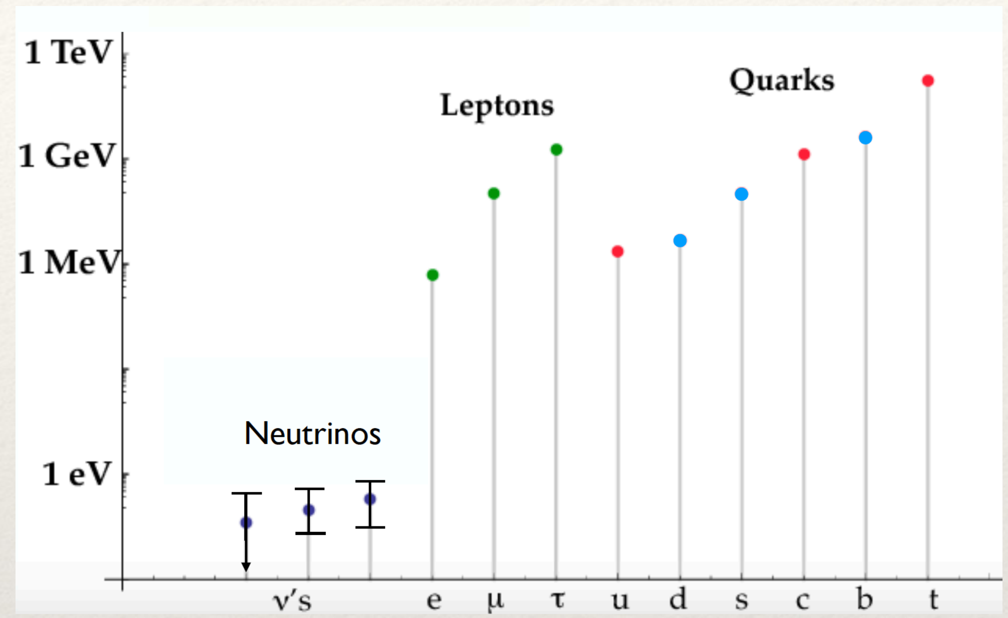
[King '17]

Why are there 3 families of each SM fermion field, in the same representation of the gauge group, differing only by their mass?

The Flavour Problem

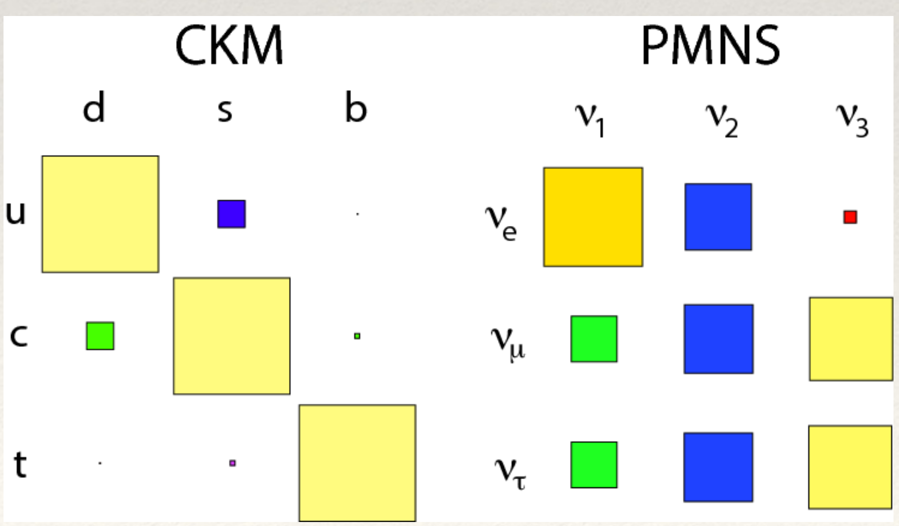
Why such large hierarchy among fermion masses?





[King '17]

Why is flavour mixing in the quark sector small compared to the lepton sector?



	θ_{12}	θ_{23}	θ_{13}	δ
Quarks	13° $\pm 0.1^\circ$	2.4° $\pm 0.1^\circ$	0.2° $\pm 0.05^\circ$	70° $\pm 5^\circ$
Leptons	34° $\pm 1^\circ$	45° $\pm 5^\circ$	8.5° $\pm 0.15^\circ$	-130^o $\pm 40^o$

[Stone '12]

Neutrino Masses

How are neutrino masses generated and why are they so small?

[Minkowski '77]
[Yanagida '79]
[Gell-Mann, Ramond, Slansky '79]
[Ramond '98]

Type I Seesaw mechanism

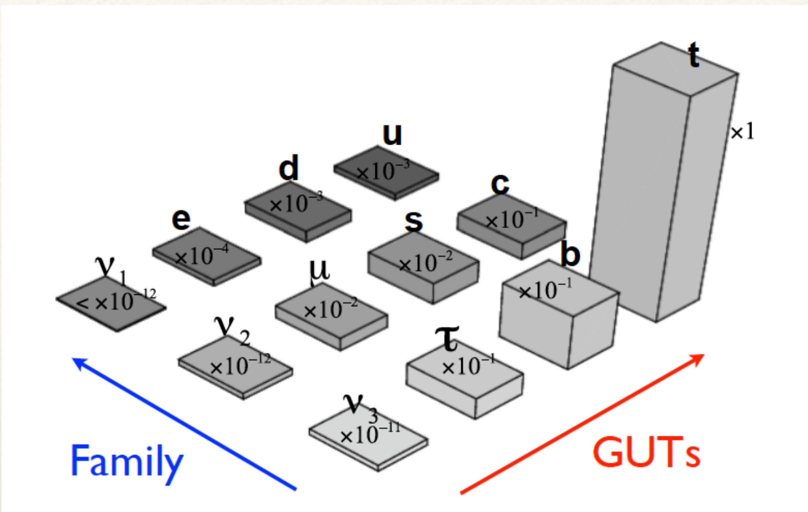
Include RH neutrinos which are singlets under the SM

$$\mathcal{L}_\nu = y_\nu L^\dagger H N + M_N N^c N + h.c.$$

Large Majorana RHN masses

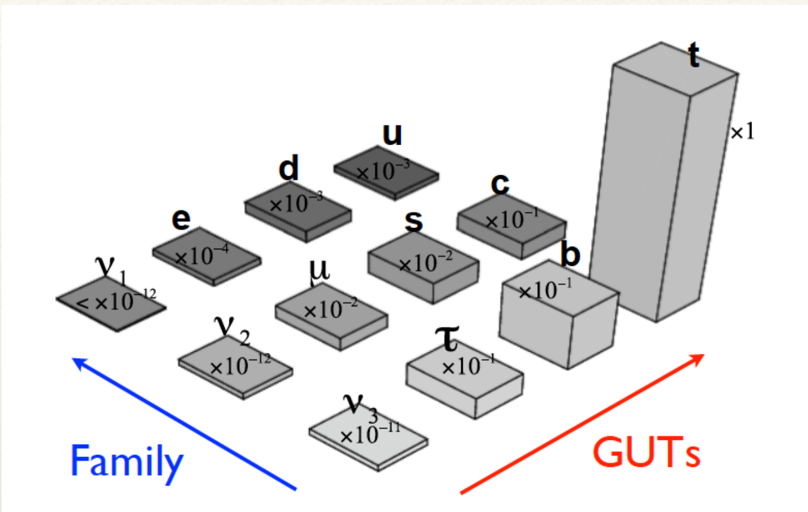
$$\boxed{m_\nu = v^2 y_\nu M_N^{-1} y_\nu^T}$$

SUSY FLAVOUR GUTs



- **SUSY:** gauge coupling unification, ameliorates the hierarchy problem.
- **Grand Unified Theory:** unifies fermions within each family and reproduces an universal mass matrix structure.
- **Family symmetry:** "Horizontal" unification of SM fermions.

SUSY FLAVOUR GUTs



- **SUSY:** gauge coupling unification, ameliorates the hierarchy problem.
- **Grand Unified Theory:** unifies fermions within each family and reproduces an universal mass matrix structure.
- **Family symmetry:** "Horizontal" unification of SM fermions.

$G_{\rm GUT}$ / $G_{\rm FAM}$	$SU(2)_L \times U(1)_Y$	$SU(5)$	PS	$SO(10)$
S_3	[29]			[142]
A_4	[30, 34, 51, 53, 64, 143–145]	[146–149]	[68, 150, 151]	
T'	[152]	[153]		
S_4	[31, 51, 53, 145, 155]	[156, 157]	[154]	[158]
A_5	[53, 159]	[160]		
T_7	[161, 162]			
$\Delta(27)$	[163]			[164]
$\Delta(96)$	[165, 166]	[167]		[168]
D_N	[169]			
Q_N	[170]			
other	[171]	[172]	[173]	

see review S.King 1701.04413

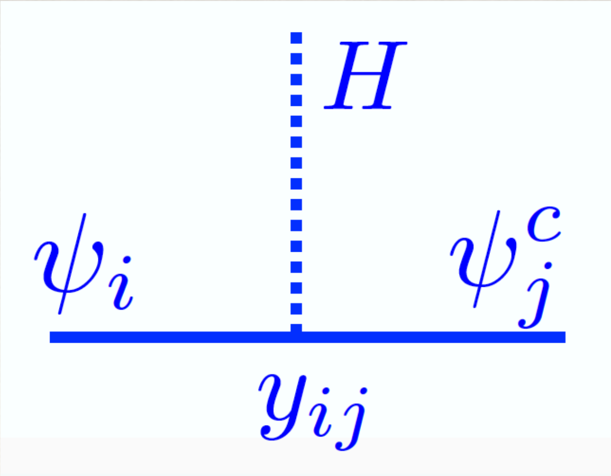
$S_4 \times SO(10) \times \mathbb{Z}_4^R \times \mathbb{Z}_4^3$

S_4 : symmetric group of permutations of 4 objects $\cong$ rigid rotation group of a cube

Global symmetry at high scale $\rightarrow$
broken spontaneously by the VEV of some scalar fields, called **flavons**

In the SM:
Yukawa couplings

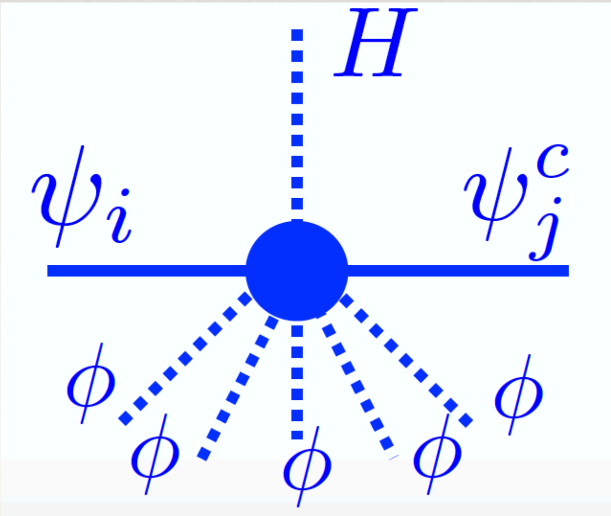
$$y_{ij} H \psi_i \psi_j^c$$



Why so small (except from top quark)?

Effective
Yukawa couplings

$$\left(\frac{\langle \phi_i \rangle}{\Lambda^{\psi}_{i,n}}\right)^n \left(\frac{\langle \phi_j \rangle}{\Lambda^{\psi^c}_{j,m}}\right)^m H \psi_i \psi_j^c$$



Small Yukawas due to powers of ratios $\frac{\langle \phi \rangle}{\Lambda}$

$S_4 \times SO(10) \times \mathbb{Z}_4^R \times \mathbb{Z}_4^3$

$SO(10)$: A complete family of quarks and leptons fits into a single $\mathbf{16}$ representation

It contains the righ-handed neutrino $\rightarrow$ non-zero neutrino masses

Yukawa couplings: Higgs in the $\mathbf{10}$ representation $\mathbf{10} \otimes \mathbf{16} \otimes \mathbf{16} \rightarrow \mathbf{1}$

RH Majorana Masses: from the non-renormalisable operators

$$\frac{\lambda_{ij}}{\Lambda} \bar{H} \bar{H} \psi_i \psi_j \rightarrow \frac{\lambda_{ij}}{\Lambda} \langle \bar{v}_H \rangle^2 \nu_i^c \nu_j^c \equiv M_R^{ij} \nu_i^c \nu_j^c$$

$\bar{H}$ are Higgs in the $\overline{\mathbf{16}}$ representation and break SO(10) down to SU(5)

The model

Field	Representation						
	S_4	$SO(10)$	$\mathbb{Z}_4^R$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	
ψ	$3'$	16	1	0	0	0	Quarks and leptons
H_{10}^u	1	10	0	0	0	0	Contain MSSM Higgs doublets
H_{10}^d	1	10	0	0	2	0	Break electroweak symmetry
$H_{\overline{16}}$	1	$\overline{16}$	0	0	0	0	Break $SO(10) \to SU(5)$ and
H_{16}	1	16	0	0	1	0	give RH neutrino masses
ϕ_1	$3'$	1	0	2	2	0	Flavons:
ϕ_2	$3'$	1	0	2	0	0	Break S_4 completely with
ϕ_3	$3'$	1	0	0	2	0	the CSD2 vacuum alignment

Additional fields:

- $5 \times H_{45}$: break SU(5) into the SM and introduce Clebsh-Gordan coefficients to distinguish between Yukawa couplings.
- Messenger superfileds χ : renormalizable masses.
- Alignment superfields X_i, Z_i : driving fields that coupled to the flavons give rise to the CSD2 flavon alignment.

The superpotential

Renormalisable at the GUT scale:

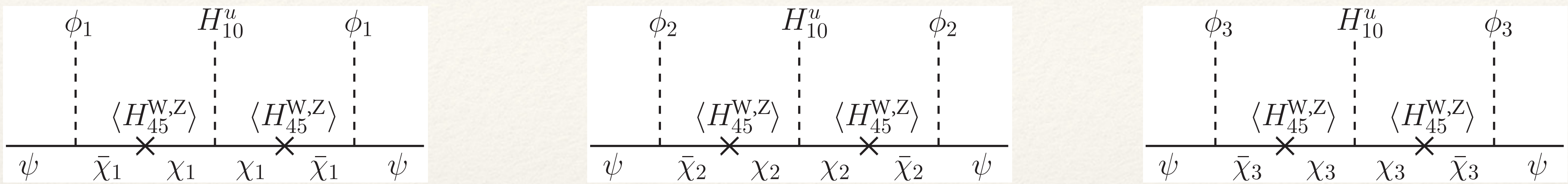
$$W_Y^{\rm GUT} = \sum_{a=1,2,3} \psi\phi_a\bar{\chi}_a + (H_{45}^W + H_{45}^Z)\chi_a\bar{\chi}_a + \chi_a\chi_a H_{10}^u$$
$$+ \sum_{b=2,3} \overline{\chi}_b\chi_b^d(H_{45}^X + H_{45}^Y) + \chi_b^d\chi_b^d H_{10}^d + \chi_1\chi_2 H_{10}^d$$

Planck-suppressed terms allowed by the symmetries:

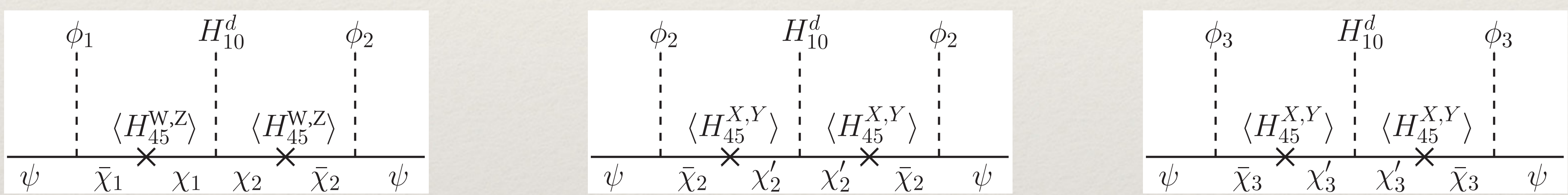
$$W_Y^{\rm Planck} = \chi_a\chi_a \frac{H_{\overline{16}}H_{\overline{16}}}{M_P} + \frac{(\psi\psi)_{3'}\,\phi_3 H_{10}^d}{M_P}$$

Diagrams

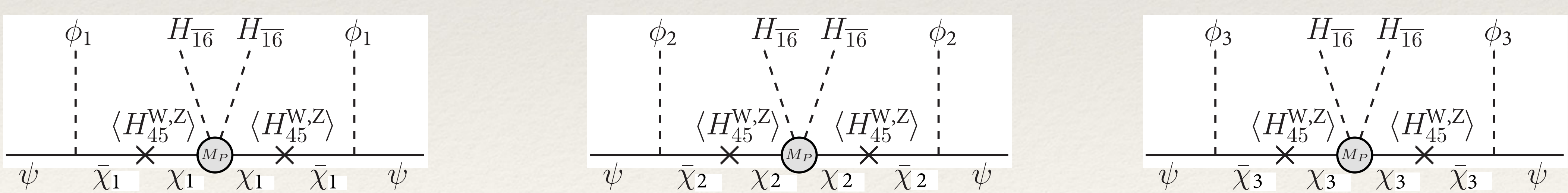
- Up-type quarks and Dirac neutrino Yukawa



- Down-type quarks and charged leptons



- Right-handed neutrinos



Vacuum alignment

The flavon superpotential fixes the symmetry breaking flavon VEVs

$$W_\phi \sim X_{3'}(\phi_{S,U})^2 + X_2(\phi_T)^2 + X_1(\phi_t)^2 + \tilde{X}_1\phi_T\phi_t + X_{1'}\phi_T\phi_3 + \tilde{X}_2\phi_t\phi_3$$
$$+ Z_{3'}(\phi_{S,U}\phi_T + \xi\phi_2) + \tilde{Z}_{3'}\xi\left(\frac{\phi_2\phi_3}{M_P} - \phi_1\right)$$

Supersymmetric F-terms equations lead to flavons alignment:

$$\langle\phi_1\rangle = v_1\begin{pmatrix}1\\0\\2\end{pmatrix}, \quad \langle\phi_2\rangle = v_2\begin{pmatrix}0\\1\\-1\end{pmatrix}, \quad \langle\phi_3\rangle = v_3\begin{pmatrix}0\\1\\0\end{pmatrix}$$

‘CSD2 vacuum alignment’

see also: [Antusch, King, Luhn, Spinrath ’11]
[Antusch, King, Spinrath ’13]
[Antusch, Hohl, Khosa, Music ‘18]

VEVs are driven to scales with the hierarchy: $v_1 \ll v_2 \ll v_3$

$$v_3 \simeq M_{GUT}, \quad v_2 \simeq 0.1 M_{GUT}, \quad v_1 \simeq 0.001 M_{GUT}$$

CP spontaneously broken by the complex VEVs of the flavons.

Yukawa matrices

Up-type quarks and neutrinos couple to one Higgs H_{10}^u, leading to Yukawa matrices $Y_{ij} \sim \langle\phi_i\rangle\langle\phi_j\rangle^T$ with an universal structure:

$$Y^{u,\nu} = y_1^{u,\nu} e^{i\eta} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + y_2^{u,\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + y_3^{u,\nu} e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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- Each matrix is **rank 1**
- **Natural** understanding of the hierarchical Yukawa couplings:

$$y_1^{u,\nu} \simeq v_1^2/M_{\rm GUT}^2 \simeq 10^{-6} \quad y_2^{u,\nu} \simeq v_2^2/M_{\rm GUT}^2 \simeq 10^{-2} \quad y_3^{u,\nu} \simeq v_3^2/M_{\rm GUT}^2 \simeq 1$$

- RH neutrino parameters are also estimated

$$M_1^R \simeq 10^7 {\rm GeV} \quad M_2^R \simeq 10^{11} {\rm GeV} \quad M_3^R \simeq 10^{13} {\rm GeV}$$

Yukawa matrices

Down-type quarks and **charged leptons** couple to a second Higgs H_{10}^d

with a new mixed term involving $Y_{12} \sim \langle\phi_1\rangle\langle\phi_2\rangle^T$

$$Y^{d,e} = y_{12}^{d,e} e^{i\frac{\eta}{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 0 \end{pmatrix} + y_2^{d,e} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + y_3^{d,e} e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y^P e^{i\gamma} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Yukawa matrices

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- This new term enforces a **texture zero** in the (1,1) element, giving the GST relation for the Cabbibo angle, i.e. $\vartheta_{12}^q \approx \sqrt{y_d/ys}$
- It also leads to a **milder hierarchy** in the down and charged lepton sectors.

$$y_{12}^{d,e} \simeq \cos\beta \frac{v_1 v_2}{M_{\text{GUT}}^2} \simeq 10^{-5} \quad y_2^{d,e} \simeq \cos\beta \frac{v_2^2}{M_{\text{GUT}}^2} \simeq 10^{-2} \quad y_3^{d,e} \simeq \cos\beta \frac{v_3^2}{M_{\text{GUT}}^2} \simeq 1$$

Seesaw Mechanism

The **light neutrino Majorana mass** matrix, after **seesaw**, will also have the CSD2 structure

$$m^\nu = \mu_1^\nu e^{i\eta} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \mu_2^\nu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mu_3^\nu e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where the parameters μ_i are given by

$$\mu_i = v_u^2 \frac{(y_i^\nu)^2}{M_i^R}$$

Flavons yield to **normal hierarchy.**

Numerical Fit

χ^2 test function to find the best fit

$$\chi^2 = \sum_n \left(\frac{P_n(x) - P_n^{\rm obs}}{\sigma_n} \right)^2$$

19 observables given by $\{\theta^q_{ij}, \delta^q, y_{u,c,t}, y_{d,s,b}, \theta^\ell_{ij}, \delta^l, y_{e,\mu,\tau}, \Delta m^2_{ij}\}$

- After seesaw, **15 effective parameters** $x = \{y^u_i, y^d_i, y^e_i, y^P, \mu_i, \eta', \gamma\}$

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- After seesaw, **15 effective parameters** $x = \{y^u_i, y^d_i, y^e_i, y^P, \mu_i, \eta', \gamma\}$
- Run up to the GUT scale:

[Antusch, Maurer '13]

  - Match the SM to the MSSM $\rightarrow$ supersymmetric radiative threshold corrections
  - At the GUT scale, the values depend only in two extra parameters $\overline{\eta}_b$ and $\tan\beta$
- Best fit found with a $\boldsymbol{\chi^2 \simeq 12}$ $(\tan\beta = 20, M_{\rm SUSY} = 1\ \text{TeV}\ \bar{\eta}_b = -0.9)$
  - Reduced $\chi^2_\nu = \chi^2/\nu \simeq 3$, where $\nu = n - n_i = 4$ d.o.f

Results

Leptons

Observable	Data		Model
	Central value	1σ range	Best fit
θ^ℓ_{12} /°	33.82	32.06 → 34.58	33.75
θ^ℓ_{13} /°	8.610	8.480 → 8.740	8.624
θ^ℓ_{23} /°	49.6	48.60 → 50.60	49.51
δ^ℓ /°	220.0	185.0 → 255.0	194.5
y_e $/10^{-5}$	6.023	5.987 → 6.059	6.024
y_μ $/10^{-2}$	1.272	1.264 → 1.280	1.272
y_τ	0.222	0.219 → 0.225	0.222
$\Delta m^2_{21}/(10^{-5}\,\mathrm{eV}^2)$	7.390	7.190 → 7.590	7.400
$\Delta m^2_{31}/(10^{-3}\,\mathrm{eV}^2)$	2.525	2.493 → 2.557	2.524
m_1 /meV			8.888
m_2 /meV			12.37
m_3 /meV			51.02
$\sum m_i$ /meV	< 230		72.28
α_{21} /°			2.161
α_{31} /°			227.5
$m_{\beta\beta}$ /meV	< 61-165		9.561

Fitted within 1_σ

Quarks

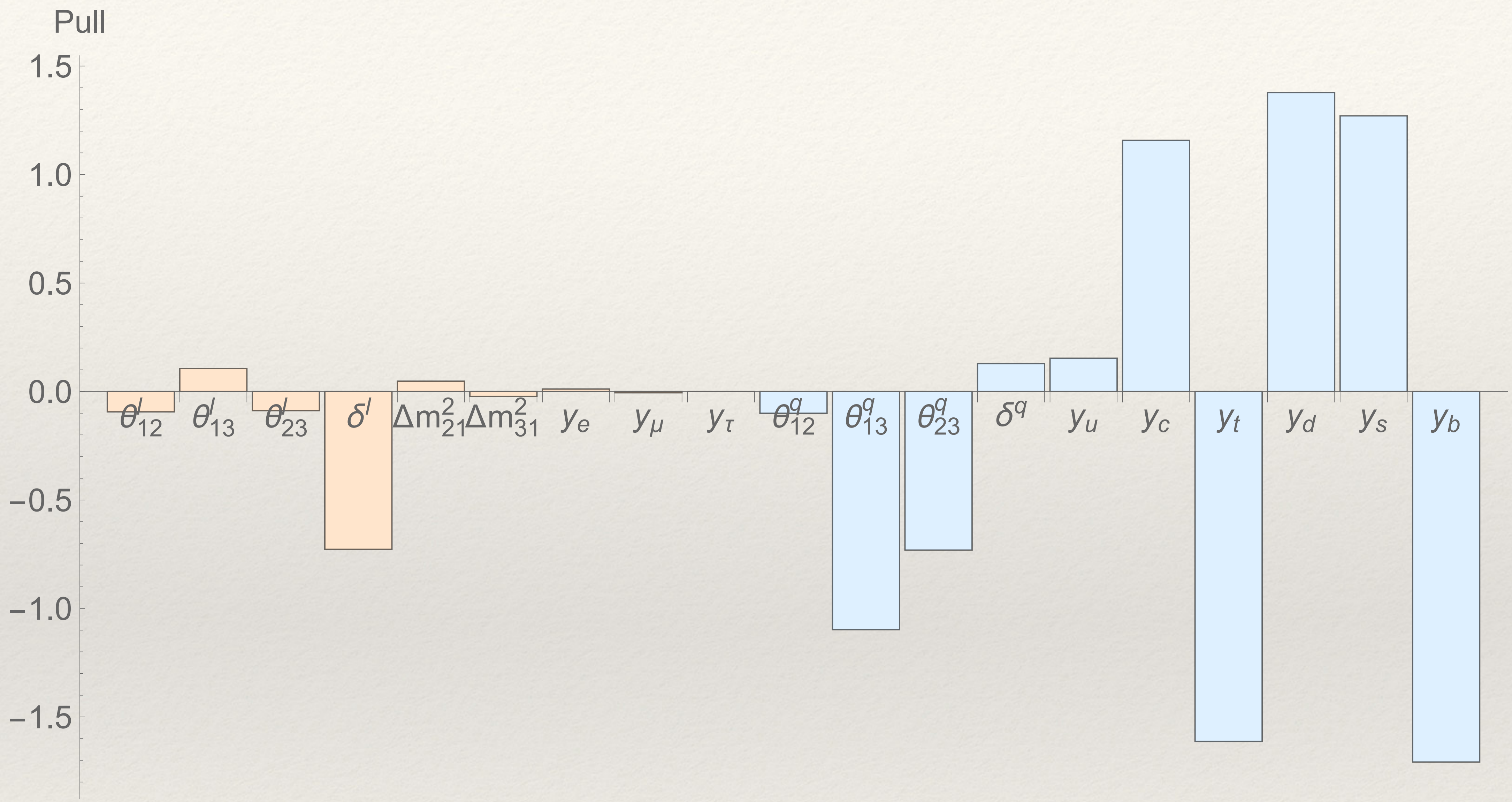
Observable	Data		Model
	Central value	1σ range	Best fit
θ^q_{12} /°	13.03	12.99 → 13.07	13.02
θ^q_{13} /°	0.016	0.016 → 0.017	0.016
θ^q_{23} /°	0.189	0.186 → 0.192	0.186
δ^q /°	69.22	66.12 → 72.31	69.61
y_u $/10^{-6}$	3.060	2.111 → 4.009	3.205
y_c $/10^{-3}$	1.497	1.444 → 1.549	1.558
y_t	0.666	0.637 → 0.694	0.620
y_d $/10^{-4}$	1.473	1.311 → 1.635	1.696
y_s $/10^{-3}$	2.918	2.760 → 3.075	3.118
y_b	2.363	2.268 → 2.457	2.201

Fitted within 2 σ

Most contribution to the total χ^2 comes from the quark sector

[Updated compared to the published version, to use nu-fit v4]

Results



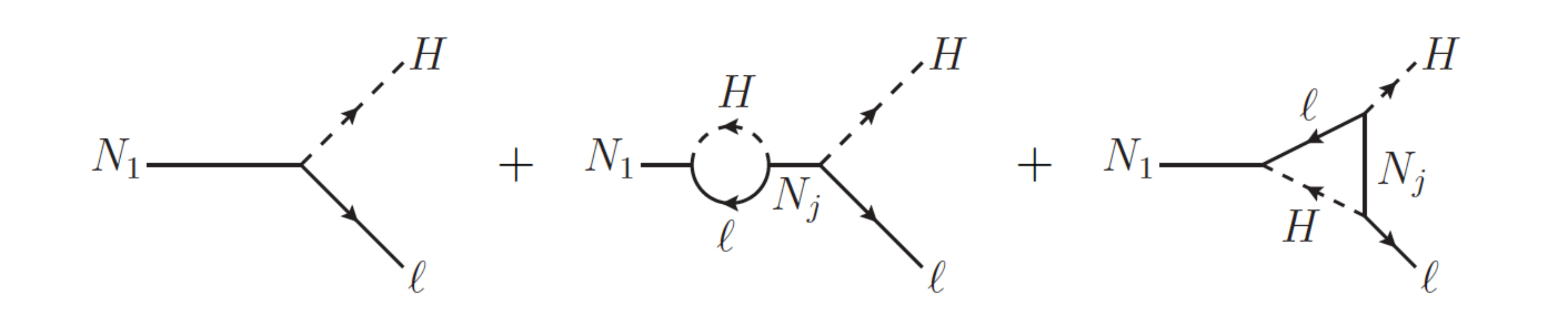
$$\text{Pull} = \left(\frac{P_n(x) - P_n^{\text{obs}}}{\sigma_n}\right)$$

Leptogenesis

- Baryon Asymmetry of the Universe (BAU)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1 \pm 0.1) \times 10^{-10}$$

can be generated through **CP breaking decays** of heavy RHNs into leptons, then converted into baryons through sphalerons [Fukugita, Yanagida '86]



- The RHN mass has to be of order 10^{10} GeV
- In our model, the expected natural value $M_2^R \sim 10^{11}$ GeV $\to$ **"N2 leptogenesis"**

Leptogenesis

- Solving the Boltzmann equation, the final *B*-*L* asymmetry is

$$N_{B-L}^{\rm f} \simeq \left[\frac{K_{2e}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2e} - \frac{K_{2e}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\,\kappa(K_{2\tau_2^\perp}/2)\right]\,e^{-\frac{3\pi}{8}\,K_{1e}} +$$
$$+\left[\frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2\mu} - \frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\,\kappa(K_{2\tau_2^\perp}/2)\right]\,e^{-\frac{3\pi}{8}\,K_{1\mu}} +$$
$$+\,\varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}$$

[Di Bari, Re Fiorentin '15]

$\varepsilon \to$ CP asymmetries $\quad \kappa(K_{2\alpha}) \to$ wash-out $\quad K_{i\alpha} \propto$ seesaw parameters

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$$+\,\varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}$$

[Di Bari, Re Fiorentin '15]

$$\varepsilon \to \text{ CP asymmetries} \quad \kappa(K_{2\alpha}) \to \text{ wash-out} \quad K_{i\alpha} \propto \text{ seesaw parameters}$$

- Finally, the baryon asymmetry is

$$\eta_B \simeq \sin\eta' \,\frac{3}{8\pi}\,\frac{\alpha_{sph}}{N_\gamma^{rec}}\;\kappa\left(\frac{\mu_2}{m_\star^{MSSM}}\right)\frac{\mu_3 M_2}{v^2}$$

Leptogenesis

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$$+ \varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}$$

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$\alpha_{sph} = 8/23$: fraction *B-L* asymmetry converted into baryon asymmetry

$N_\gamma^{rec} \simeq 78$: photon asymmetry at recombination

$m_\star^{\rm MSSM} \simeq 0.78 \times 10^{-3}{\rm eV}\sin^2\beta$: equilibrium neutrino mass

Leptogenesis

- Solving the Boltzmann equation, the final *B-L* asymmetry is

$$N_{B-L}^{\rm f} \simeq \left[\frac{K_{2e}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2e} - \frac{K_{2e}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\,\kappa(K_{2\tau_2^\perp}/2)\right]\,e^{-\frac{3\pi}{8}\,K_{1e}} +$$
$$+\left[\frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2\mu} - \frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\,\kappa(K_{2\tau_2^\perp}/2)\right]\,e^{-\frac{3\pi}{8}\,K_{1\mu}} +$$
$$+\,\varepsilon_{2\tau}\,\kappa(K_{2\tau})\,e^{-\frac{3\pi}{8}\,K_{1\tau}}$$

[Di Bari, Re Fiorentin '15]

$\varepsilon \to$ CP asymmetries $\quad \kappa(K_{2\alpha}) \to$ wash-out $\quad K_{i\alpha} \propto$ seesaw parameters

- Finally, the baryon asymmetry is

$$\eta_B \simeq \sin\eta' \,\frac{3}{8\pi}\,\frac{\alpha_{sph}}{N_\gamma^{rec}}\,\kappa\left(\frac{\mu_2}{m_\star^{MSSM}}\right)\frac{\mu_3 M_2}{v^2}$$

from the fit!

Leptogenesis

- Solving the Boltzmann equation, the final *B-L* asymmetry is

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$$+ \left[\frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2\mu} - \frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\kappa(K_{2\tau_2^\perp}/2)\right]\, e^{-\frac{3\pi}{8}\,K_{1\mu}} +$$
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only free parameter

Correct BAU when $M_2 \simeq 1.9\times 10^{11}\ \mathrm{GeV}$

natural value for the second RHN mass: the model naturally explains the origin of the BAU through **N_2 leptogenesis** without any need of tunning!

Conclusions

Matter fields unified into a single representation $(3', 16)$ of $S_4 \times SO(10)$

- **Minimal** field content and low-dimensional representations
- **Matter hierarchies** explained by the flavon VEVs when setting all the GUT scale parameters to be $\sim \mathcal{O}(1)$
- **Predictions**: neutrino mass masses, normal neutrino mass ordering, CP oscillation phase $\delta^l \sim 200^\circ$

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Not shown here but:

- Splits Higgs doublets and triplets via Dimopoulos-Wilczek mechanism.
- Generates a μ term of $\mathcal{O}(\mathrm{TeV}), W \in \mu h_u h_d$
- Proton decay satisfies experimental constraints.

Conclusions

Matter fields unified into a single representation $(3', 16)$ of $S_4 \times SO(10)$

- **Minimal** field content and low-dimensional representations
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Future work:

- Model with extra dimensions: vacuum alignments from boundary conditions?
- Flavour symmetry as modular symmetry?

Thank you!

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Back up slides

GUT problems?

- GUT symmetry breaking: gives rise to hierarchical VEVs ✓
- Flavour symmetry breaking: gives rise to CSD2 vacuum alignment ✓
- Har to obtain the correct masses: fit! ✓
- Doublet-triplet splitting ✓
- μ problem $W \in \mu h_u h_d$ ✓
- Proton decay ✓
- SUSY breaking: not discussed ▽

Doublet-triplet splitting

$$H_{10}^{u,d}, \quad H_{16,\overline{16}}: \quad \mathbf{10} \to \mathbf{2} + \mathbf{2} + \mathbf{3} + \bar{\mathbf{3}}$$

- Two of the doublets remain light: MSSM Higgs doublets.
- Extra pairs of heavy doublets: preserve gauge coupling unification.
- Heavy colour triplets: mediate proton decay.
- $\mu \sim 10^2 - 10^3$ GeV: to ensure Higgs VEV of order 174 GeV

Dimopoulos-Wilczek(DW) mechanism

$$\begin{aligned}\mathcal{W}_H = H_{45}^{B-L}&\left(H_{10}^u H_{10}^d + \zeta_2\zeta_2 + H_{\overline{16}}\chi_u + H_{16}\overline{\chi}_d\right)\\ &+ H_{\overline{16}}H_{10}^u\overline{\chi}_u + H_{16}H_{10}^d\chi_d + H_{16}H_{\overline{16}}\zeta_1 + \zeta\left(\zeta_1\zeta_2 + \overline{\chi}_u\chi_u + \overline{\chi}_d\chi_d\right)\\ &+ H_{45}^{B-L}\left(\frac{H_{\overline{16}}H_{\overline{16}}H_{10}^d}{M_P} + \frac{H_{16}H_{16}H_{10}^u}{M_P} + H_{10}^u H_{10}^d\frac{(H_{45}^{X,Y,W,Z})^4}{M_P^4}\right)\end{aligned}$$

Doublet-triplet splitting

After the fields get the VEVs, the mass matrices and eigenvalues are

$$M_T \sim \begin{array}{c} \\ \mathbf{3}_d(H_{10}^d) \\ \mathbf{3}_d(H_{10}^u) \\ \mathbf{3}_d(H_{16}) \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{3}_u(H_{10}^u) & \mathbf{3}_u(H_{10}^d) & \mathbf{3}_u(H_{\overline{16}}) \end{array} \\ \begin{pmatrix} \kappa_1 & 0 & \kappa_4 y \\ 0 & -\kappa_1 & \kappa_3 z \\ \kappa_5 y & \kappa_6 z & \kappa_2 z^2 \end{pmatrix} \end{array} M_{GUT}, \quad m_T \sim \begin{pmatrix} \kappa_1 \\ \kappa_1 \\ \kappa_2 z^2 \end{pmatrix} M_{GUT}$$

$$M_D \sim \begin{array}{c} \\ \mathbf{2}_d(H_{10}^d) \\ \mathbf{2}_d(H_{10}^u) \\ \mathbf{2}_d(H_{16}) \end{array} \begin{array}{c} \begin{array}{ccc} \mathbf{2}_u(H_{10}^u) & \mathbf{2}_u(H_{10}^d) & \mathbf{2}_u(H_{\overline{16}}) \end{array} \\ \begin{pmatrix} -\kappa_7 y^4 & 0 & \kappa_4 y \\ 0 & \kappa_7 y^4 & \kappa_3 z \\ \kappa_5 y & \kappa_6 z & \kappa_2 z^2 \end{pmatrix} \end{array} M_{GUT}, \quad m_D \sim \begin{pmatrix} -y^4 \\ \kappa_6 \kappa_3 z^2 \\ \kappa_2 z^2 \end{pmatrix} M_{GUT}$$

it requires $\kappa_1 \sim \kappa_2 z^2 \sim 1$ to get the triplets at the GUT scale and $\kappa_6 \kappa_3 z^2 \sim \kappa_2 z^2 \sim 1$ to get two doublet pairs at the GUT scale.

Furthermore, there is a μ term generated by $\mu \sim y^4 M_{GUT} \sim 1\ TeV$ which happens at the correct order.

Proton decay

- The proton lifetime is constrained to be $\tau_p > 10^{34}$ years.
- Proton decay mediated by extra gauge bosons of the GUT and by triplets accompanying the Higgs doublets.
- Constraints barely met when the triplets have a GUT scale mass.
- Additional fields may allow proton decay from effective terms

$$gQQQL\frac{\langle X\rangle}{M_P^2}$$

Such terms must obey the constraint $g\langle X\rangle < 3\times 10^9$ GeV.

- In our model, the largest contribution of this type comes from the term

$$\psi\psi\psi\psi\frac{\left\langle H_{45}^{B-L}(H_{45}^{X,Y})^2\right\rangle}{M_P^4} \quad\Rightarrow\quad \langle X\rangle = \frac{(M_{\mathrm{GUT}})^3}{M_P^2}\sim 10^{10}\ \mathrm{GeV}.$$

Constraints met when $g < 0.3$

Proton decay complies with experimental constraints but lies fairly close to detection!

Difference between Yukawa couplings

$$y_2^e = \lambda_2^d \frac{(\lambda_2^\phi)^2 |v_2|^2}{[\lambda_2^X \langle H_{45}^X \rangle + \lambda_2^Y \langle H_{45}^Y \rangle]_L [\lambda_2^X \langle H_{45}^X \rangle + \lambda_2^Y \langle H_{45}^Y \rangle]_{e^c}}$$

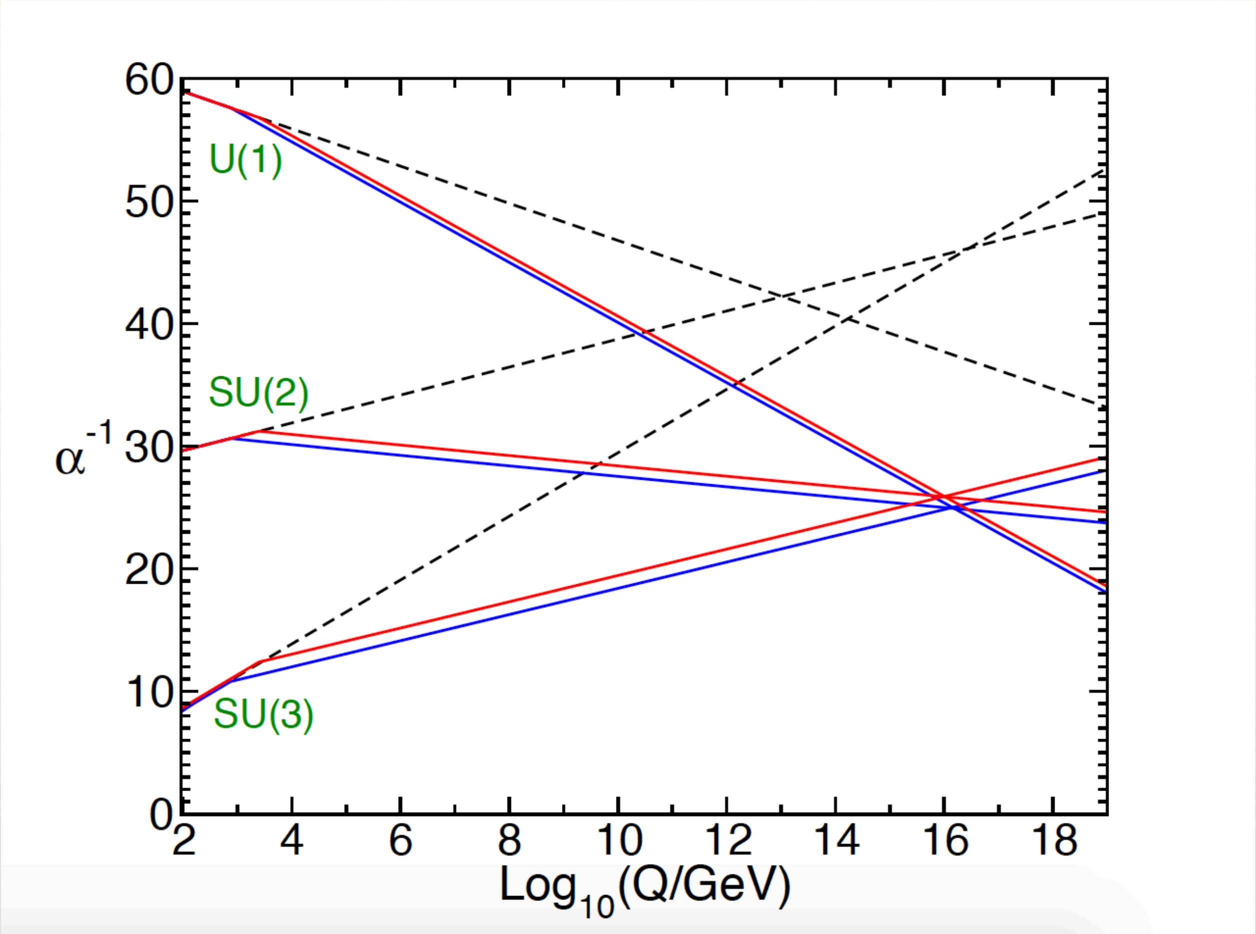
$$y_2^d = \lambda_2^d \frac{(\lambda_2^\phi)^2 |v_2|^2}{[\lambda_2^X \langle H_{45}^X \rangle + \lambda_2^Y \langle H_{45}^Y \rangle]_Q [\lambda_2^X \langle H_{45}^X \rangle + \lambda_2^Y \langle H_{45}^Y \rangle]_{d^c}}$$

- The $\langle H_{45}^{X,Y} \rangle$ obtain a VEV in an arbitrary SO(10) breaking direction. They need to be different from one another.
- Let us assume that $\langle H_{45}^{X,Y} \rangle$ is aligned in the $U(1)_{X,Y}$ direction respectively with an M_{GUT} magnitude. In this case the effective Yukawa couplings would be

$$y_2^e = \lambda_2^d \frac{(\lambda_2^\phi)^2 |v_2|^2}{[3\lambda_2^X - \lambda_2^Y/2][-\lambda_2^X + \lambda_2^Y] M_{GUT}^2} \quad y_2^d = \lambda_2^d \frac{(\lambda_2^\phi)^2 |v_2|^2}{[-\lambda_2^X + \lambda_2^Y/6][3\lambda_2^X + \lambda_2^Y/3] M_{GUT}^2}$$

where the coefficients multiplying each $\lambda^{X,Y}$ are the $U(1)_{X,Y}$ charges of the corresponding SM field.

Gauge coupling unification in the MSSM



[A supersymmetry Primer, Martin]

Two-loop renormalization group evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ in the SM (dashed lines) and the MSSM (solid lines).