Dispersion theoretic calculation of the H → Z+γ amplitude

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work done together with

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talk based on [1] - PRD 97 (2018) no.7, 073008 [arXiv:1711.07298]

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Motivation

- $H \rightarrow \gamma\gamma$ decay rate, W-loop contribution has become a subject of a controversy.
- Is a loop induced process, total amplitude is finite.
- Individual amplitudes UV divergent, thus most authors use dimensional regularisation of the loop integrals (DimReg)
- Direct computation within the unitary gauge is also possible.
- DimReg and unitary gauge results differ in general!
- $H \rightarrow \gamma\gamma$ automatically included in $H \rightarrow Z\gamma$ calculation
- Working with dispersion integral – no regularisation necessary
- SM has a broken SU(2) symmetry – massive vector bosons have three polarisations

We consider:

- H → Z+γ
- The W-loop contribution
- we use the dispersion method
- we compare to the commonly used DimReg

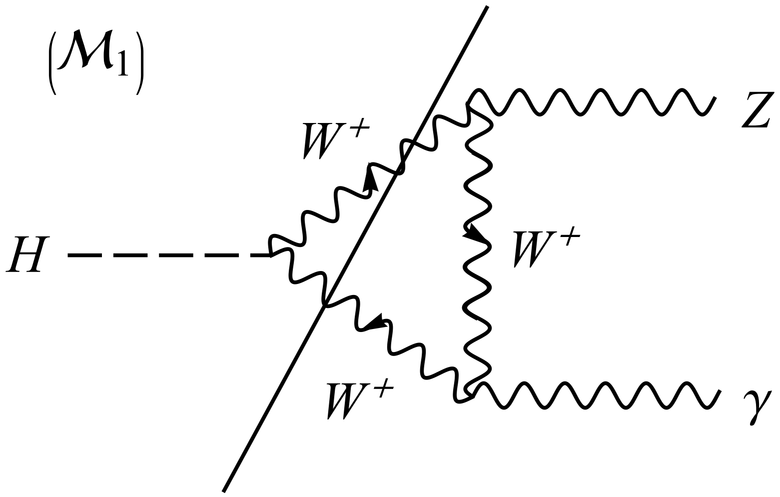
Our dispersion method in 2 steps:

1. We calculate Im(amplitude) – in the unitary gauge
2. We calculate the amplitude by using the dispersion integral

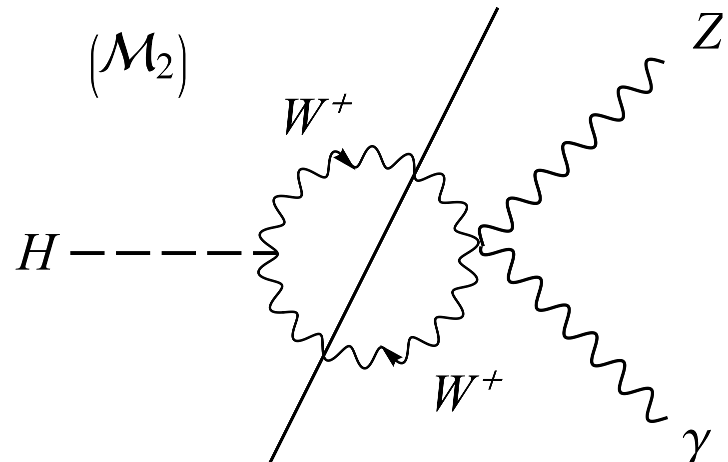
The Feynman diagrams

in the unitary gauge

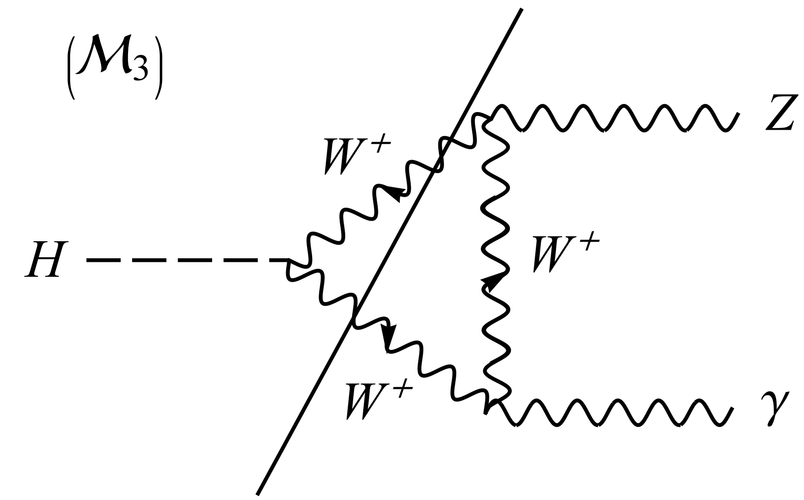
$(\mathcal{M}_1)$



$(\mathcal{M}_2)$

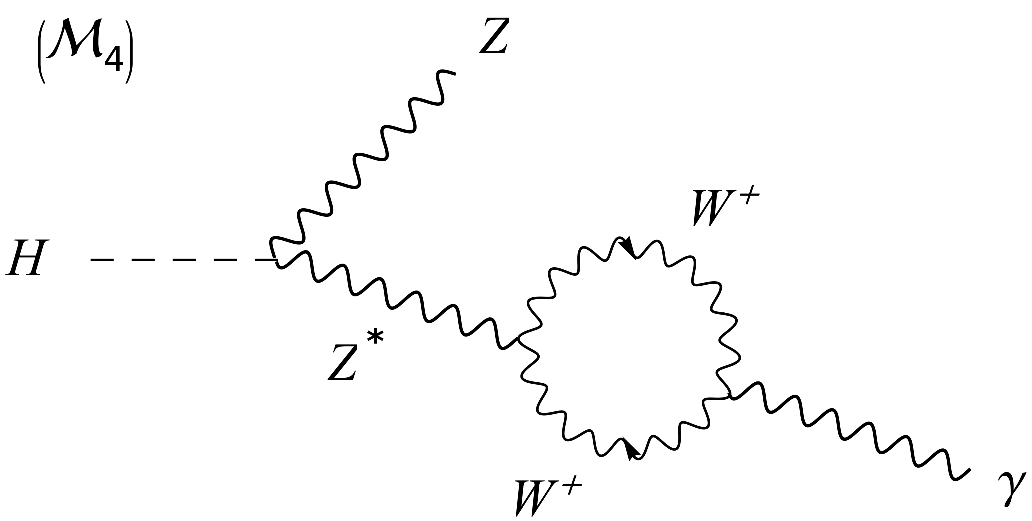


$(\mathcal{M}_3)$

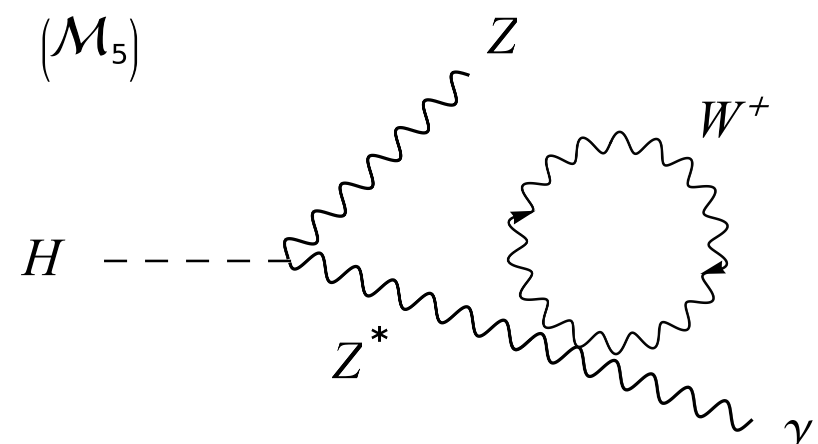


The inclined lines indicate the cuts.

$(\mathcal{M}_4)$



$(\mathcal{M}_5)$



These two selfenergy graphs do not contributed to the imaginary part of the amplitude.

The amplitude in the unitary gauge:

$$\mathcal{M} = \mathcal{M}_{\mu\nu}(k_1, k_2)\, \epsilon_1^{*\mu}\, \epsilon_2^{*\nu}$$

$$\mathcal{M}_{1\mu\nu} = \frac{-ieg^2 \cos\theta_W M}{(2\pi)^4} \int \mathrm{d}^4k\, \frac{V_{\mu\rho\beta}(-k_1, -P_2, P_1)\, V_{\nu\gamma\sigma}(-k_2, -P_3, P_2)}{D_1 D_2 D_3} \times$$
$$\times \left(g_\alpha^\beta - \frac{P_{1\alpha} P_1^\beta}{M^2} \right) \left(g^{\rho\sigma} - \frac{P_2^\rho P_2^\sigma}{M^2} \right) \left(g^{\alpha\gamma} - \frac{P_3^\alpha P_3^\gamma}{M^2} \right)$$

M is the W-mass

$$P_1 = k + \frac{p}{2}, \qquad P_2 = k - \frac{v}{2}, \qquad P_3 = k - \frac{p}{2},$$
$$D_i = P_i^2 - M^2 + i\epsilon, \qquad (i = 1, 2, 3),$$
$$\tilde{P}_2 = k + \frac{v}{2}, \qquad \tilde{D}_2 = \tilde{P}_2^2 - M^2 + i\epsilon$$
$$p = k_1 + k_2, \qquad v = k_1 - k_2.$$

$$\mathcal{M}_{3\mu\nu}(k \to -k) = \mathcal{M}_{1\mu\nu}$$

$$\mathcal{M}_{2\mu\nu} = \frac{ieg^2 \cos\theta_W M}{(2\pi)^4} \int \mathrm{d}^4k \frac{V_{\gamma\beta\mu\nu}}{D_1 D_3} \left(g_\alpha^\beta - \frac{P_{1\alpha} P_1^\beta}{M^2} \right) \left(g^{\alpha\gamma} - \frac{P_3^\alpha P_3^\gamma}{M^2} \right)$$

$$\mathcal{M}_{\mu\nu} = 2\mathcal{M}_{1\mu\nu} + \mathcal{M}_{2\mu\nu}$$

Absorptive part of the amplitude

Using Cutkosky rules - sets the momenta of the W's on-shell:

$$\frac{1}{p^2 - M^2 + i\epsilon} \longrightarrow (2\pi i)\, \theta(\pm p_0)\, \delta(p^2 - M^2)\,.$$

The invariant absorptive part$\mathcal{A}$ of the amplitude is defined by

$$\Im m\, \mathcal{M}_{\mu\nu} = \frac{eg^2 \cos\theta_W}{8\pi M}\, \mathcal{A}(\tau)\mathcal{P}_{\mu\nu} \qquad \text{with} \qquad \mathcal{A}(\tau)\mathcal{P}_{\mu\nu} = \frac{M^2}{\pi} \int \mathrm{d}^4k\, \mathcal{I}_{\mu\nu}\, \theta(P_{10})\theta(-P_{30})\delta(D_1)\delta(D_3),$$

The transverse factor is $\mathcal{P}_{\mu\nu} = k_{2\mu}k_{1\nu} - (k_1 \cdot k_2)\, g_{\mu\nu}$

and

$$\begin{aligned}
\mathcal{I}_{\mu\nu} = {} & \frac{8M_Z^2}{M^4 D_2}\, k^2 \left(k_\mu k_\nu + \frac{k_{2\mu}k_\nu}{2} - \frac{k_\mu k_{1\nu}}{2} - \frac{k_{2\mu}k_{1\nu}}{4}\right) + \frac{-2M_Z^2}{M^4}\, k^2 g_{\mu\nu} \\
& + \frac{8M_Z^2}{M^2 D_2} \left[-k_\mu k_\nu - \frac{k_{2\mu}k_\nu}{2} + \frac{k_\mu k_{1\nu}}{2} - \frac{k_{2\mu}k_{1\nu}}{8} + \frac{1}{4}\, g_{\mu\nu} k_1 \cdot k_2 - \frac{1}{8}\, g_{\mu\nu} k \cdot (k_1 - k_2)\right] + \frac{M_Z^2}{M^2}\, g_{\mu\nu} \\
& + \frac{2}{M^2 D_2} \left[4k_1 \cdot k_2\, k_\mu k_\nu + 2k^2 k_{2\mu}k_{1\nu} - 4k \cdot k_1\, k_{2\mu}k_\nu - 4k \cdot k_2\, k_\mu k_{1\nu}\right. \\
& \qquad\qquad \left. + g_{\mu\nu} \left(4k \cdot k_1\, k \cdot k_2 - 2k^2\, k_1 \cdot k_2\right)\right] \\
& + \frac{2}{D_2} \left[\left(-3k^2 + 3k \cdot k_1 - 3k \cdot k_2 - \frac{9}{2}\, k_1 \cdot k_2 + 3M^2 - \frac{3}{4}\, M_Z^2\right) g_{\mu\nu}\right. \\
& \qquad\qquad \left. + 12k_\mu k_\nu + 3k_{1\nu}k_{2\mu} - 6k_\mu k_{1\nu} + 6k_{2\mu}k_\nu\right].
\end{aligned}$$

$\tau = \frac{p^2}{4M^2}$, $a = \frac{M_Z^2}{4M^2}$, p - momentum of Higgs boson, $M = M_W$

Using the integrals given in Appendix B of [1] we get the non-zero result

$$\mathcal{A}(\tau) = \frac{a}{\tau - a}\left\{\left[1 + \frac{1}{\tau - a}\left(\frac{3}{2} - 2a\tau\right)\right]\beta - \left[1 - \frac{1}{2(\tau - a)}\left(2a - \frac{3}{2\tau}\right) - \frac{3}{2a}\left(1 - \frac{1}{2\tau}\right)\right]\ln\left(\frac{1+\beta}{1-\beta}\right)\right\}, \qquad \tau > 1\,.$$

$\beta = \sqrt{1 - \tau^{-1}}$

Real (dispersive) part of the amplitude

We define the full invariant amplitude $\mathcal{F}(\tau, a)$ by

$$\mathcal{M}_{\mu\nu} = -\frac{eg^2 \cos\theta_W}{8\pi M} \mathcal{F}(\tau, a) \mathcal{P}_{\mu\nu}$$

The invariant *unsubtracted* amplitude $\mathcal{F}_{un}(\tau, a)$ is defined by the convergent dispersion integral

$$\mathcal{F}_{un}(\tau, a) = \frac{1}{\pi} \int_1^\infty \frac{\mathcal{A}(y)}{y - \tau} \,\mathrm{d}y, \qquad \tau < 1.$$

$\mathcal{F}_{un}(\tau, a)$ defines the full amplitude $\mathcal{F}(\tau, a)$ up to an additive constant $C(a)$:

$$2\pi\, \mathcal{F}(\tau, a) = 2\pi\, \mathcal{F}_{un}(\tau, a) + \mathcal{C}(a)$$

We fix $\mathcal{C}(a)$ through the Goldstone Boson equivalence theorem (GBET), which fixes the behaviour of the amplitude at $\tau \to \infty$.

Using the integrals given in Appendix C of [1] we get the result for $\mathcal{F}_{un}(\tau, a)$:

$$2\pi\mathcal{F}_{un}(\tau, a) = \frac{3 - 4a^2}{\tau - a} + \left(6 - 4a - \frac{3 - 4a^2}{\tau - a}\right) F(\tau, a) - 2a \left(2 + \frac{3 - 4a\tau}{\tau - a}\right) G(\tau, a)\,,$$

F and G denote loop integrals and can be found in [1].

$\mathcal{F}_{un}(\tau, a)$ has the properties:
* is finite at threshold $\tau = a$
* it vanishes for $\tau \to \infty$ with fixed a
* for $a \to 0$ we get the corresponding amplitude for $H \to \gamma\gamma$

C(a) from GBET

We determine the subtraction constant $\mathcal{C}(a)$ through the charged ghost contribution adopting the Goldstone Boson Equivalence Theorem which implies that at $M_W \to 0$, i.e. at $\tau \to \infty$, the $SU(2) \times U(1)$ symmetry of the SM is restored and the longitudinal components of the physical $W^{\pm}$-bosons are replaced by the physical Goldstone bosons $\phi^{\pm}$. In the following $\mathcal{M}^{\phi}_{\mu\nu}$ denotes the amplitude of $H \to Z + \gamma$ in which the $W^{\pm}$ are replaced by their Goldstone bosons $\phi^{\pm}$. The GBET implies

$$\lim_{\tau\to\infty} \mathcal{M}_{\mu\nu}(\tau, a) = \lim_{\tau\to\infty} \mathcal{M}^{\phi}_{\mu\nu}(\tau, a)\,.$$

We calculate the charged ghost contribution in two different ways: through direct calculations and via the dispersion integral. Both calculations lead to the same result.

Again, applying Cutkosky rules to the amplitude we get

$$\Im m\, \mathcal{M}^{\phi}_{\mu\nu}(\tau, a) = -\frac{eg^2 \cos\theta_W}{8\pi M} \frac{M_H^2}{4M^2} \mathcal{A}^{\phi}(\tau, a)\, \mathcal{P}_{\mu\nu} \qquad \text{with} \quad \mathcal{A}^{\phi}(\tau, a) = (1-2a)\, \frac{2a\beta - \ln\frac{1+\beta}{1-\beta}}{2\,(\tau-a)^2}.$$

The dispersion integral is

$$\mathcal{F}^{\phi}(\tau, a) = \frac{1}{\pi}\int_1^{\infty} \frac{\mathcal{A}^{\phi}(y, a)}{y-\tau} \mathrm{d}y \quad \text{and we obtain} \quad \lim_{\tau\to\infty} \mathcal{F}^{\phi}(\tau, a) = \frac{2\,(1-2a)}{2\,\pi\,(\tau-a)}$$

$$\lim_{\tau\to\infty} 2\pi\mathcal{F}(\tau, a) = \lim_{\tau\to\infty} 2\pi\,[\tau\mathcal{F}^{\phi}(\tau, a)] = \mathcal{C}(a).$$

Thus we determine $C(a) = 2(1-2a)$.

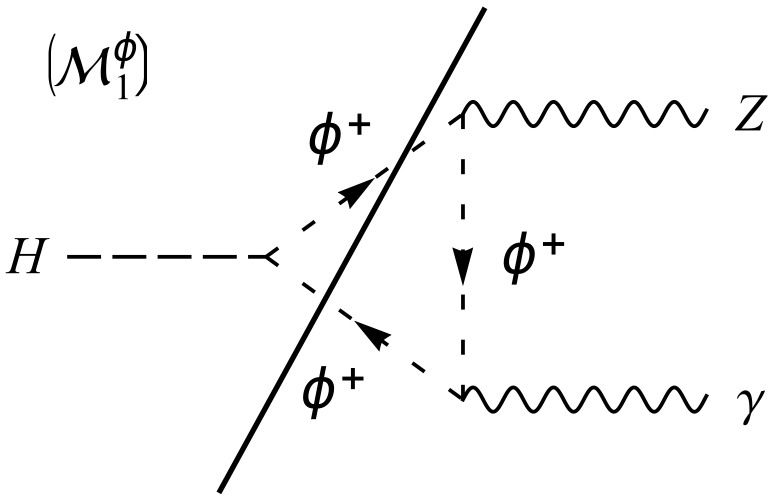
Important note:

M_H^2 in the coupling - large-τ behavior $\mathcal{F}^{\phi}(\tau, a) \sim \mathcal{O}(\tau^{-x})$ with $x \geq 1$.

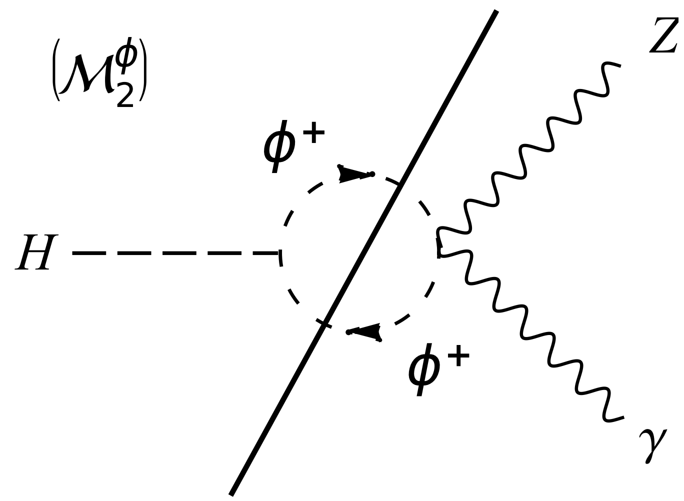
$\to$ $(1/\pi)\int_{ARC} \mathrm{d}y \mathcal{F}^{\phi}(y, a)/(y-\tau)$ over the infinite arc in the complex y-plane, is zero.

+ no physics reason as GBET for $\mathcal{F}(\tau, a)$

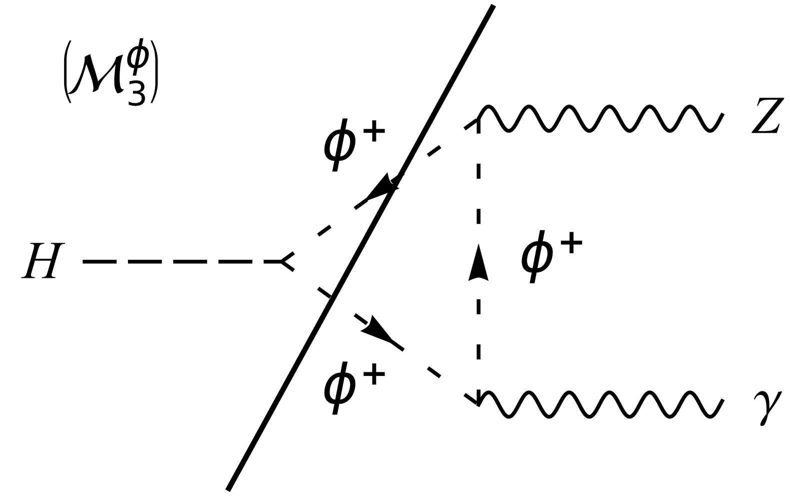
$(\mathcal{M}_1^{\phi})$



$(\mathcal{M}_2^{\phi})$

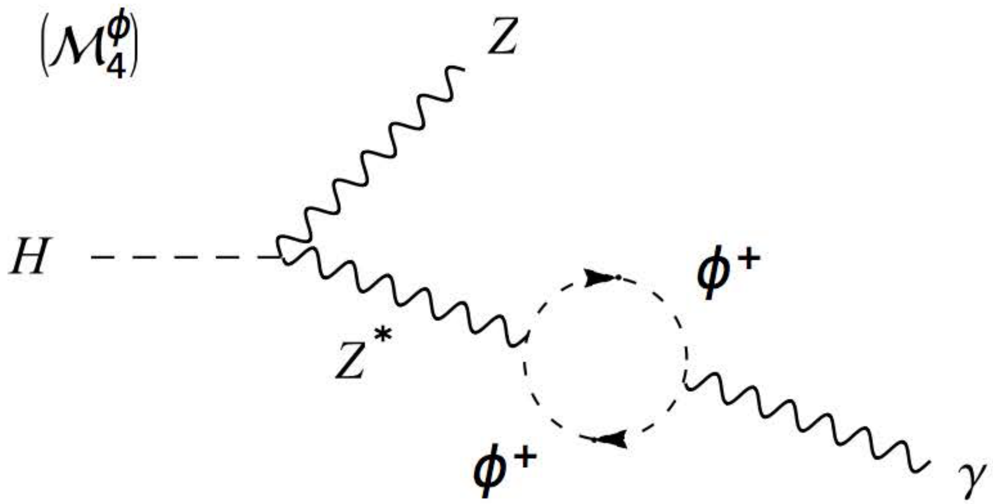


$(\mathcal{M}_3^{\phi})$

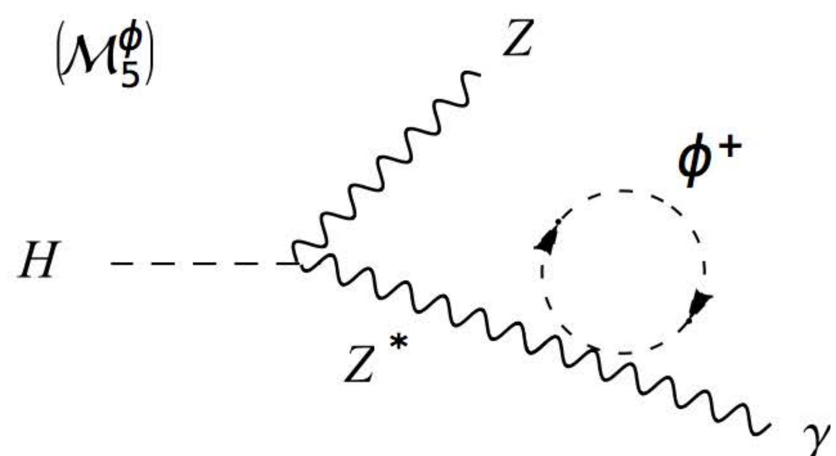


The inclined lines indicate the cuts.

$(\mathcal{M}_4^{\phi})$

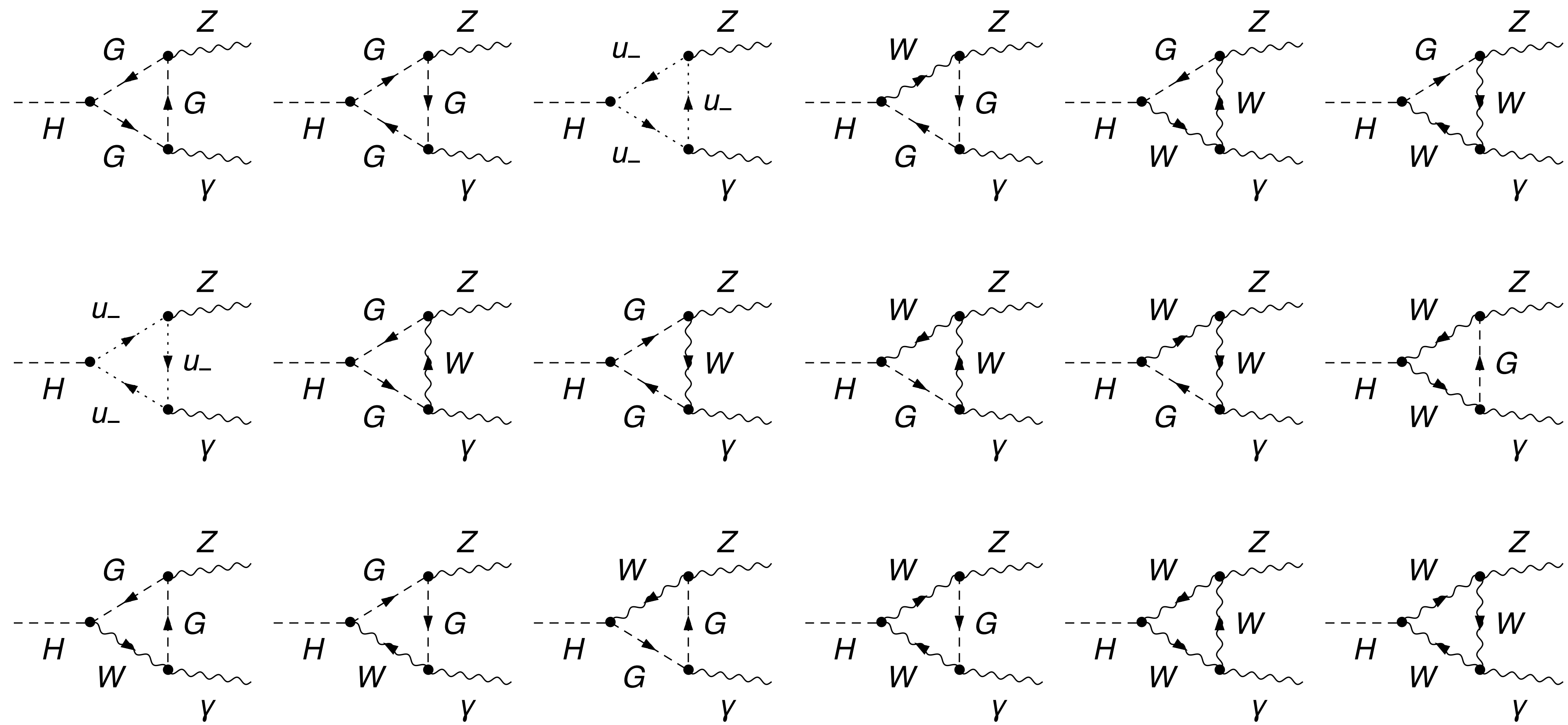


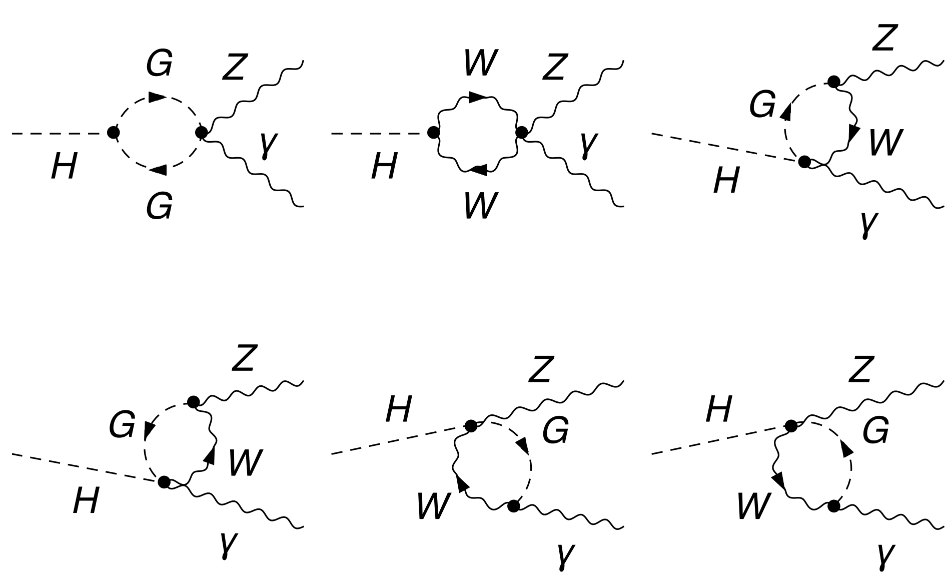
$(\mathcal{M}_5^{\phi})$



These two selfenergy graphs do not contributed to the imaginary part of the amplitude.

The Feynman diagrams, in R_ξ gauge





+ 10 selfenergy graphs

24 genuine vertex graphs and 10 with $Z - \gamma$ selfenergy transitions in the R_ξ gauge.

Calculation in R_ξ gauge

R_ξ gauge calculation done with the Mathematica Packages `FeynArts` and `FormCalc`.
ξ independence of total amplitude checked.

Here dimensional regularization (DimReg) is used.

The result $\mathcal{F}_{\rm DimReg}(\tau, a)$ coincides with the "classical" one, see e.g. [6] L. Bergström and G. Hulth (1985)

In the limit $a \to 0$ we get the result for $H \to \gamma\gamma$ [2] J. Ellis, M. K. Gaillard, D. V. Nanopoulos (1976)

We get the relation

$$2\pi\mathcal{F}_{\rm DimReg}(\tau, a) = 2\pi\mathcal{F}_{un}(\tau, a) + \underbrace{2(1-2a)}_{\text{C(a)}} = 2\pi\,\mathcal{F}(\tau, a)$$

We see that both calculations agree, obeying the GBET.

The decay width

Approximating the total width by top and W-boson loop we get

$$\Gamma(H \to Z+\gamma) = \frac{M_H^3}{32\pi}\left(1-\frac{M_Z^2}{M_H^2}\right)^3\left[\frac{eg^2}{(4\pi)^2 M}\right]^2\left|-\cos\theta_W[2\pi\mathcal{F}_W(\tau)] + \frac{2\left(3-8\sin^2\theta_W\right)}{3\cos\theta_W}[2\pi\mathcal{F}_t(\tau_t)]\right|^2$$

$\mathcal{F}_t(\tau_t)$ stands for the sum of the t-quark one-loop diagrams and
$\mathcal{F}_W(\tau)$ stands for the sum of the W-boson one-loop diagrams.

correct result

$$\Gamma(H \to Z+\gamma) = 8.1\,\mathrm{KeV} \text{ using } \mathcal{F}_W(\tau) = \mathcal{F}(\tau,a) = \mathcal{F}_{\mathrm{DimReg}}(\tau,a)$$

$$\Gamma(H \to Z+\gamma) = 6.6\,\mathrm{KeV} \text{ using } \mathcal{F}_W(\tau) = \mathcal{F}_{\mathrm{un}}(\tau,a)$$

Note, for $\Gamma(H \to \gamma\gamma)$ even 52% reduction!

wrong result, 20% smaller

Working with regularisation:

$$I_{\mu\nu} = \int \frac{\mathrm{d}^n k}{(2\pi)^n}\frac{4k_\mu k_\nu - k^2 g_{\mu\nu}}{(k^2-M_W^2)^3}$$

$I_{\mu\nu} = i\frac{\pi^2}{2}g_{\mu\nu}$ for $n = D$

$I_{\mu\nu} = 0$ for $n = 4$

Some references

$H \to \gamma\gamma$

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$H \to Z\gamma$

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correct results

Some references, continued

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$H \to \gamma\gamma$ and $H \to Z\gamma$

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Concluding remarks

- W-boson induced corrections to the decay $H \to Z + \gamma$ in the Standard Model calculated in the unitary gauge using the dispersion-relation approach.

- Decay $H \to \gamma\gamma$ automatically included.

- Plus: – Only finite quantities and thus does not involve any uncertainties related to regularization.
  – simpler, working in the unitary gauge effectively we deal with only 2 Feynman diagrams, while in the R_ξ-gauge one has 24 graphs.

  Minus: The dispersion method determines the amplitude merely up to an additive subtraction constant.

- Subtraction constant fixed by using the Goldstone Boson Equivalence Theorem.

- As a cross-check we also calculated the amplitude in the commonly used R_ξ-gauge class with dimensional regularization as regularization scheme. Same result as in the dispersion method.

Thank you!