Western Norway
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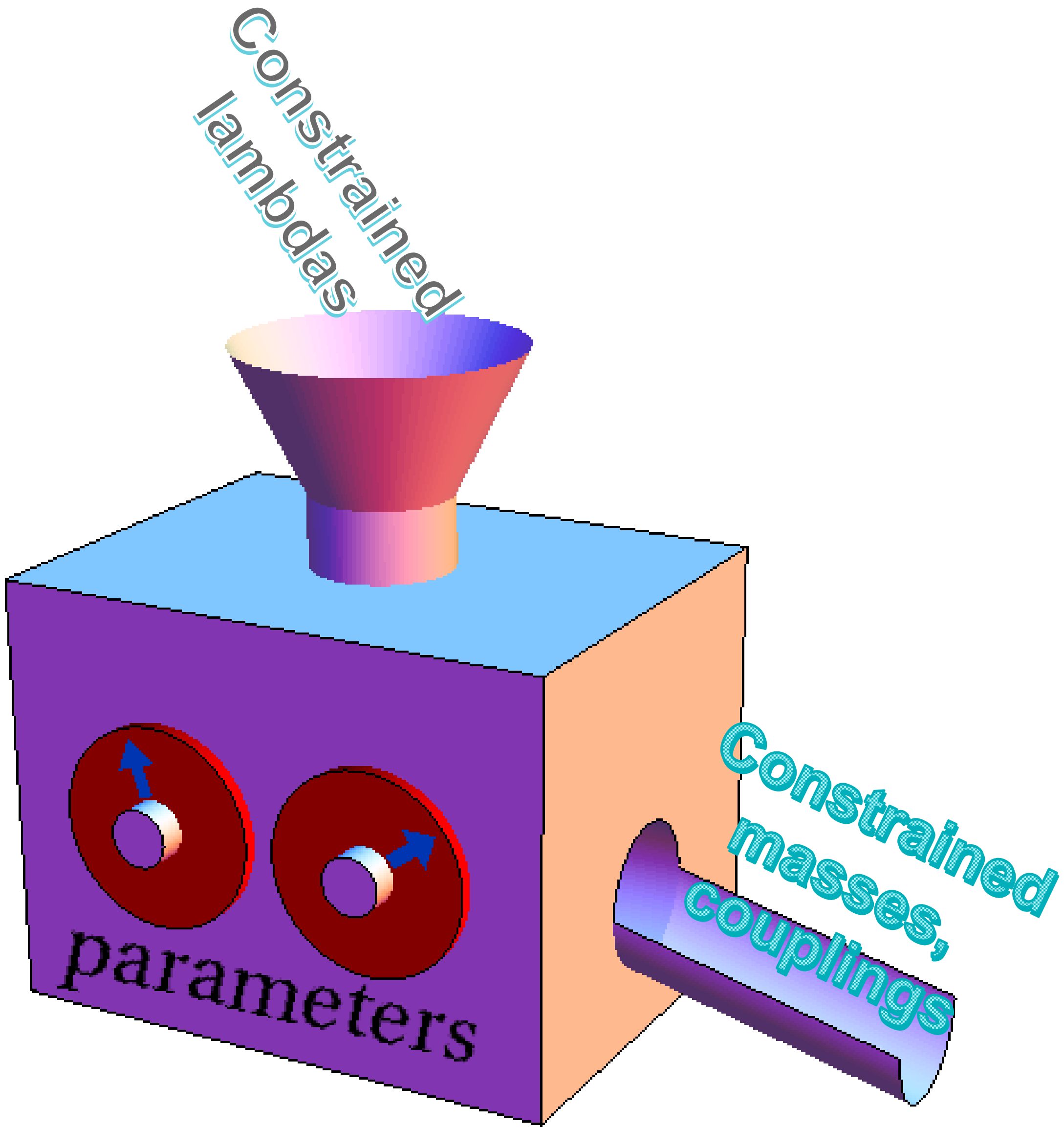
Physical implications of different CP symmetries in the bosonic sector of the 2HDM.

Talk given at
“DISCRETE 2018”,

Wien, November 27, 2018



Odd Magne Ogreid
Based on work with B. Grzadkowski and P. Osland

Outline of talk

Preliminaries:

- Traditional parametrization of 2HDM
- VEVs and basis changes
- Counting of parameters
- Choosing the Higgs basis
- The physical fields
- Inpendent couplings, and the introduction of the physical parameter set $\mathcal{P}$
- Translation from standard parameters to the parameter set $\mathcal{P}$

Applications:

- CP-symmetric 2HDM (potential + VEV)
- CP-symmetric 2HDM potential only (spontaneous CP-violation)
- CP1
- CP2
- CP3
- Physical processes
- Alignment limit

Summary

Traditional parametrization(s) of the 2HDM potential

$$
\begin{aligned}
V(\Phi_1,\Phi_2) &= -\frac{1}{2}\left\{m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 + \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right]\right\} \\
&\quad + \frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
&\quad + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right](\Phi_1^\dagger\Phi_2) + \text{h.c.}\right\} \\
&\equiv Y_{ab}\Phi_a^\dagger\Phi_b + \frac{1}{2}Z_{abcd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d)
\end{aligned}
$$

Second form useful in the study of invariants.

A total of 14 parameters.

$$
Y_{11} = -\frac{m_{11}^2}{2}, \quad Y_{12} = -\frac{m_{12}^2}{2},
$$
$$
Y_{21} = -\frac{(m_{12}^2)^*}{2}, \quad Y_{22} = -\frac{m_{22}^2}{2},
$$

$$
\begin{aligned}
&Z_{1111} = \lambda_1, \quad Z_{2222} = \lambda_2, \quad Z_{1122} = Z_{2211} = \lambda_3, \\
&Z_{1221} = Z_{2112} = \lambda_4, \quad Z_{1212} = \lambda_5, \quad Z_{2121} = (\lambda_5)^*, \\
&Z_{1112} = Z_{1211} = \lambda_6, \quad Z_{1121} = Z_{2111} = (\lambda_6)^*, \\
&Z_{1222} = Z_{2212} = \lambda_7, \quad Z_{2122} = Z_{2221} = (\lambda_7)^*.
\end{aligned}
$$

Vacuum expectation values (VEVs) and choice of basis

Most general form that conserves electric charge:

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 e^{i\xi_1}\end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi_2}\end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 = (246\,\mathrm{GeV})^2$$

- We demand that the VEVs should represent a minimum of the potential
- Electroweak Symmetry Breaking:
  Work out stationary-point equations by differentiating the potential with respect to the fields and put these to zero.
  [Ref: Grzadkowski, Ogreid, Osland,
  JHEP11 (2014) 084].
- Minimum enforced by demanding all physical scalars have positive squared masses (later).

- Initial expression of potential is defined with respect to doublets Φ_1 and Φ_2.
- We may rotate to a new basis by

  $$\bar{\Phi}_i = U_{ij}\Phi_j$$

  where *U* is any U(2) matrix.
- Potential parameters change under change of basis.
- Physics is the same regardless of our choice of basis.
- Observables (constructed from masses and couplings) cannot depend on choice of basis – they are **invariant** under a change of basis.

Counting parameters and choosing the Higgs basis

- Potential has initially 14 parameters
- Exploit the freedom to change basis and reduce to 11 independent parameters.
- Traditional approach:
  Work out masses and couplings expressed in terms of the initial parameters of the potential.
- Our approach:
  Work the other way around. Pick a set of 11 independent physical masses and couplings (all invariants) and express the initial parameters in terms of these.

- In the Higgs-basis only one doublet has non-zero VEV.

$$\langle\Phi_1\rangle_{\rm HB} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle\Phi_2\rangle_{\rm HB} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Not unique, as one may still perform a U(1) transformation on Φ_2 without giving Φ_2 a non-zero VEV.
- Algebra much simpler in the Higgs-basis than in a general basis.
- Stationary-point equations:

$$\begin{aligned} m_{11}^2 &= v^2\lambda_1, \\ \mathrm{Re}\, m_{12}^2 &= v^2 \mathrm{Re}\,\lambda_6, \\ \mathrm{Im}\, m_{12}^2 &= v^2 \mathrm{Im}\,\lambda_6, \end{aligned}$$

Parametrization of the doublets and the physical masses in the Higgs basis

- Doublets are parametrized as:

$$\Phi_1 = \begin{pmatrix} G^+ \\ (v + \eta_1 + iG^0)/\sqrt{2} \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} H^+ \\ (\eta_2 + i\eta_3)/\sqrt{2} \end{pmatrix}$$

We work out the mass of the charged scalars:

$$M_{H^\pm}^2 = -\frac{m_{22}^2}{2} + \frac{v^2}{2}\lambda_3$$

Neutral sector mass terms given by

$$\frac{1}{2}\begin{pmatrix} \eta_1 & \eta_2 & \eta_3 \end{pmatrix} \mathcal{M}^2 \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}.$$

- With the neutral sector mass matrix

$$\mathcal{M}^2 = v^2 \begin{pmatrix} \lambda_1 & \mathrm{Re}\,\lambda_6 & -\mathrm{Im}\,\lambda_6 \\ \mathrm{Re}\,\lambda_6 & \frac{1}{2}(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5 - \frac{m_{22}^2}{v^2}) & -\frac{1}{2}\mathrm{Im}\,\lambda_5 \\ -\mathrm{Im}\,\lambda_6 & -\frac{1}{2}\mathrm{Im}\,\lambda_5 & \frac{1}{2}(\lambda_3 + \lambda_4 - \mathrm{Re}\,\lambda_5 - \frac{m_{22}^2}{v^2}) \end{pmatrix}$$

- Is diagonalied by an orthogonal 3x3-matrix *R*

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}$$

as

$$R\mathcal{M}^2 R^{\mathrm{T}} = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$$

- Physical neutral fields are now given as

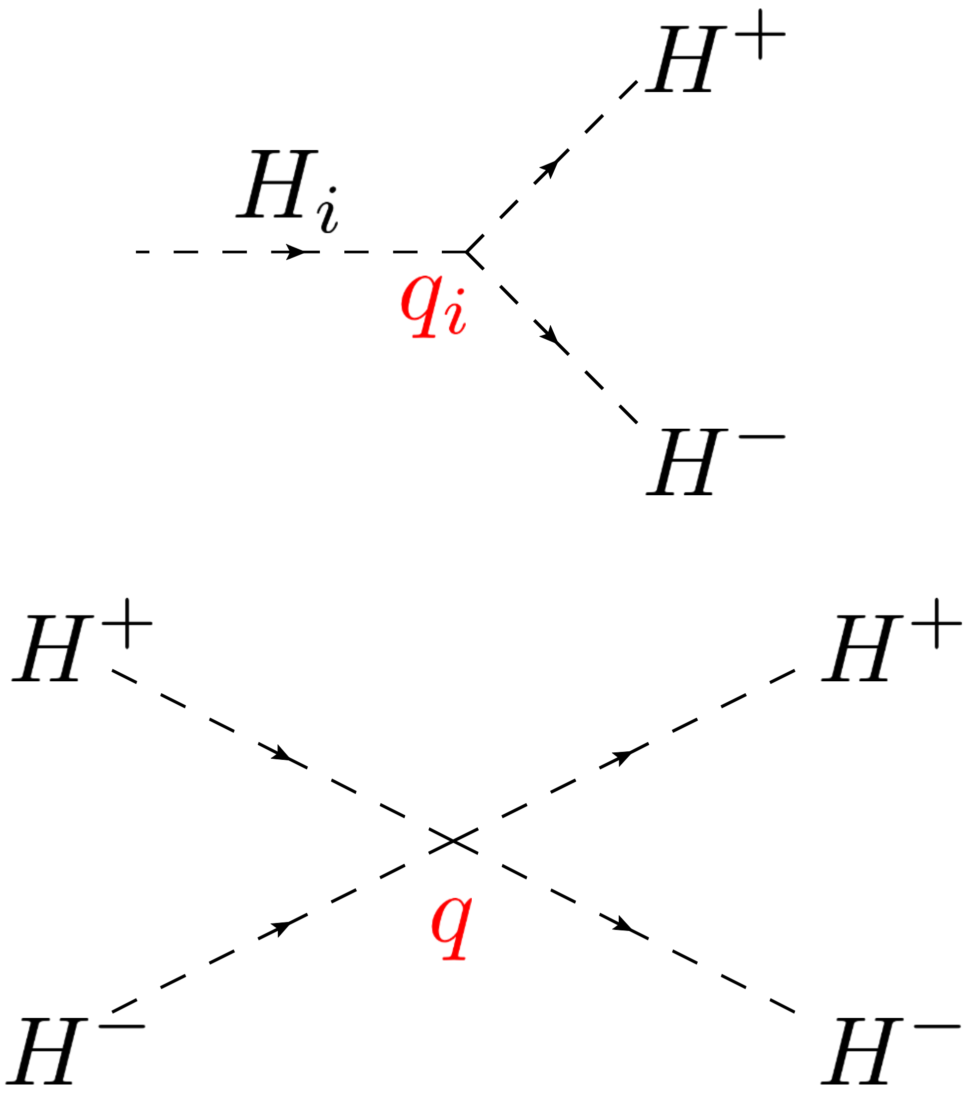
$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}$$

Some tree-level scalar couplings

- Some important scalar couplings expressed in the Higgs-basis

$$
\begin{aligned}
q_i &\equiv \text{Coefficient}(V, H_i H^- H^+) \\
&= v(R_{i1}\lambda_3 + R_{i2}\text{Re}\,\lambda_7 - R_{i3}\text{Im}\,\lambda_7), \\
q &\equiv \text{Coefficient}(V, H^- H^- H^+ H^+) \\
&= \frac{1}{2}\lambda_2.
\end{aligned}
$$

- If calculated in a general basis, we can explicitly verify that these couplings are basis invariant, hence observables.



Some tree-level gauge couplings

- Gauge couplings from the Lagrangian

$$(H_i \overleftrightarrow{\partial}_\mu H_j) Z^\mu : \quad -\frac{g}{2v\cos\theta_W}\epsilon_{ijk}e_k,$$

$$H_i Z_\mu Z^\mu : \quad \frac{g^2}{4\cos^2\theta_W}e_i,$$

$$H_i W_\mu^+ W^{-\mu} : \quad \frac{g^2}{2}e_i,$$

$$(H^+ \overleftrightarrow{\partial}_\mu H_i) W^{-\mu} : \quad i\frac{g}{2v}f_i,$$

$$(H^- \overleftrightarrow{\partial}_\mu H_i) W^{+\mu} : \quad -i\frac{g}{2v}f_i^*.$$

where $e_i \equiv v_1 R_{i1} + v_2 R_{i2}$

$$f_i \equiv v_1 R_{i2} - v_2 R_{i1} - ivR_{i3}$$

- Satisfies

$$e_1^2 + e_2^2 + e_3^2 = v^2 = (246\,\mathrm{GeV})^2$$

$$f_i f_j^* = v^2\delta_{ij} - e_i e_j + iv\epsilon_{ijk}e_k$$

- In a general basis we can show that e_i is invariant under a change of basis, hence an observable, whereas f_i is a pseudo-observable (its absolute value is invariant).

- Simpler form in the Higgs-basis:

$$e_i = vR_{i1}$$

$$f_i = v(R_{i2} - iR_{i3})$$

The physical parameter set $\mathcal{P}$

- We now choose our set of 11 independent parameters to consist of:
  - Four squared masses
  - Three gauge couplings
  - Four scalar couplings

$$\mathcal{P} \equiv \{M_{H\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

- All observables (invariants) expressible through these.
- All trilinear and quadrilinear scalar couplings expressible through these.

In the Higgs-basis we obtain

$$
\begin{aligned}
m_{11}^2 &= \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^2}, \\
m_{12}^2 &= \frac{e_1 f_1 M_1^2 + e_2 f_2 M_2^2 + e_3 f_3 M_3^2}{v^2}, \\
m_{22}^2 &= -2M_{H\pm}^2 + e_1 q_1 + e_2 q_2 + e_3 q_3, \\
\lambda_1 &= \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^4}, \\
\lambda_2 &= 2q, \\
\lambda_3 &= \frac{e_1 q_1 + e_2 q_2 + e_3 q_3}{v^2}, \\
\lambda_4 &= \frac{M_1^2 + M_2^2 + M_3^2 - 2M_{H\pm}^2}{v^2} - \frac{e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2}{v^4}, \\
\lambda_5 &= \frac{f_1^2 M_1^2 + f_2^2 M_2^2 + f_3^2 M_3^2}{v^4}, \\
\lambda_6 &= \frac{e_1 f_1 M_1^2 + e_2 f_2 M_2^2 + e_3 f_3 M_3^2}{v^4}, \\
\lambda_7 &= \frac{f_1 q_1 + f_2 q_2 + f_3 q_3}{v^2}.
\end{aligned}
$$

CP-symmetries of the 2HDM potential and the VEV

Whenever there exists a U(2) matrix X_{ij} so that both the 2HDM potential and the VEV is symmetric under the transformation

$$\Phi_i \to X_{ij}\Phi_j^*$$

the 2HDM is CP-symmetric, or
CP-conserving.

Equivalently:
If a basis exists in which all the parameters of the 2HDM potential and the VEV is simultaneously real, then the 2HDM is CP-symmetric.

Challenge: How to find the correct basis?

Whenever a basis exists in which the 2HDM potential is CP-symmetric, but no basis exists in which both the potential and the VEV can simultaneously be made real, then the 2HDM
violates CP spontaneously.

Challenge: How to deal with the basis issue?

Solution: Basis-invariant descriptions of the CP-properties of the 2HDM

CP-symmetry of the 2HDM

- CP-properties determined by three CP-odd invariants, first discovered by Lavoura and Silva. Re-expressed by Gunion and Haber as:

$$\text{Im}\, J_1 = -\frac{2}{v^2}\text{Im}\left[V_{da}Y_{ab}Z_{bccd}\right],$$
$$\text{Im}\, J_2 = \frac{4}{v^4}\text{Im}\left[V_{ab}V_{dc}Y_{be}Y_{cf}Z_{eafd}\right],$$
$$\text{Im}\, J_3 = \text{Im}\left[V_{ab}V_{dc}Z_{bgge}Z_{chhf}Z_{eafd}\right].$$

- Here, $V_{ab} = \frac{v_a v_b^*}{v^2}$

- In Higgs-basis: $V_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

- We translate these to $\mathcal{P}$ to find

$$\begin{aligned}
\text{Im}\, J_1 &= \frac{1}{v^5}[e_1e_3q_2(M_1^2-M_3^2)+e_2e_1q_3(M_2^2-M_1^2) \\
&\qquad +e_3e_2q_1(M_3^2-M_2^2)], \\
\text{Im}\, J_2 &= 2\frac{e_1e_2e_3}{v^9}(M_1^2-M_2^2)(M_2^2-M_3^2)(M_3^2-M_1^2), \\
\text{Im}\, J_3 &= \frac{2}{v^4}\left[(e_1^2M_1^2+e_2^2M_2^2+e_3^2M_3^2)\right. \\
&\qquad \left. +v^2(e_1q_1+e_2q_2+e_3q_3)+2v^2M_{H^\pm}^2\right]\text{Im}\, J_1 \\
&\quad +\text{Im}\, J_2 + \frac{2}{v^7}\sum_{i,j,k}\epsilon_{ijk}(v^2q_i+2e_iM_i^2)M_i^2e_jq_k
\end{aligned}$$

- Put Im J_1 = Im J_2 = Im J_3 = 0 and solve

Cases of CP-conservation:

Case 1: $M_1 = M_2 = M_3$.

Case 2: $M_i = M_j$ and $e_iq_j - e_jq_i = 0$.

Case 3: $e_k = q_k = 0$.

[Ref: Grzadkowski, Ogreid, Osland, JHEP11 (2014) 084]

Spontaneous CP-violation

- Nature of CP-violation determined by four invariants presented by Gunion and Haber:

$$I_{Y3Z} = \mathrm{Im}\left[Z^{(1)}_{a\bar{c}} Z^{(1)}_{e\bar{b}} Z_{b\bar{e}c\bar{d}} Y_{d\bar{a}}\right],$$

$$I_{2Y2Z} = \mathrm{Im}\left[Y_{a\bar{b}} Y_{c\bar{d}} Z_{b\bar{a}d\bar{f}} Z^{(1)}_{f\bar{c}}\right],$$

$$I_{3Y3Z} = \mathrm{Im}\left[Z_{a\bar{c}b\bar{d}} Z_{c\bar{e}d\bar{g}} Z_{e\bar{h}f\bar{q}} Y_{g\bar{a}} Y_{h\bar{b}} Y_{q\bar{f}}\right],$$

$$I_{6Z} = \mathrm{Im}\left[Z_{a\bar{b}c\bar{d}} Z^{(1)}_{b\bar{f}} Z^{(1)}_{d\bar{h}} Z_{f\bar{a}j\bar{k}} Z_{k\bar{j}m\bar{n}} Z_{n\bar{m}h\bar{c}}\right].$$

- We translate all these to $\mathcal{P}$, demand that they should all vanish, and obtain:

Theorem. Let us assume that the quantity

$$D = e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2$$

is non-zero. Then, in a charge-conserving general 2HDM, CP is violated spontaneously if and only if the following three statements are satisfied simultaneously:

- At least one of the three invariants $\mathrm{Im}\, J_1$, $\mathrm{Im}\, J_2$, $\mathrm{Im}\, J_3$ is nonzero.
- $$M^2_{H^\pm} = \frac{v^2}{2D}[e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2],$$
- $$q = \frac{1}{2D}[(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2].$$

[Ref: Grzadkowski, Ogreid, Osland, Phys. Rev. D 94, 115002 (2016)]

CP1, CP2 and CP3

- Ferreira, Haber and Silva discovered that CP-symmetries of the 2HDM-potential can be classified into three different classes according to the form the U(2) matrix X_{ij} can have.
- CP1: $X_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- CP2: $X_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
- CP3: $X_{ij} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$0 < \theta < \pi/2$$

[Ref: Ferreira et.al, PhysRevD.79.116004 (2009)].

- Ferreira, Haber, Maniatis, Nachtmann and Silva put up basis-invariant conditions for CP1, CP2 and CP3 in terms of the properties of two vectors, $\vec{\xi}$ and $\vec{\eta}$ and a 3x3-matrix *E*.
- Using our technique we manage to translate these into restrictions among the parameters in our physical parameter set $\mathcal{P}$.
- CP1 is equivalent to the conditions we already found for a CP-conserving potential.
- Also, CP1 > CP2 > CP3.

[Ref: Ferreira et.al, Int.J.Mod.Phys.A26:769 (2011)].

CP1

We have CP1-symmetry (CP-symmetric potential) in four cases:

$$\begin{aligned}
&\text{Case 1:} && M_1 = M_2 = M_3, \\
&\text{Case 2:} && M_i = M_j,\ e_i q_j - e_j q_i = 0, \\
&\text{Case 3:} && e_k = q_k = 0, \\
&\text{Case 4:} && D M_{H^\pm}^2 = \frac{v^2}{2}[e_1 q_1 M_2^2 M_3^2 + e_2 q_2 M_3^2 M_1^2 + e_3 q_3 M_1^2 M_2^2 - M_1^2 M_2^2 M_3^2], \\
& && Dq = \frac{1}{2}[(e_2 q_3 - e_3 q_2)^2 M_1^2 + (e_3 q_1 - e_1 q_3)^2 M_2^2 + (e_1 q_2 - e_2 q_1)^2 M_3^2 + M_1^2 M_2^2 M_3^2].
\end{aligned}$$

where $D = e_1^2 M_2^2 M_3^2 + e_2^2 M_3^2 M_1^2 + e_3^2 M_1^2 M_2^2$.

CP2 and CP3

Full mass degeneracy:

$$M_1 = M_2 = M_3 \equiv M$$

If in addition we have

$$\begin{aligned} M_{H^\pm}^2 &= \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3 - M^2), \\ q &= \frac{M^2}{2v^2}, \\ (e_1 q_2 - e_2 q_1) &= 0, \\ (e_1 q_3 - e_3 q_1) &= 0, \\ (e_2 q_3 - e_3 q_2) &= 0. \end{aligned}$$

Then the potential has both the CP2 and the CP3 symmetry.

No mass degeneracy:

If we have

$$\begin{aligned} e_j &= e_k = 0, \\ M_{H^\pm}^2 &= \frac{1}{2}(e_i q_i - M_i^2), \\ q &= \frac{M_i^2}{2v^2}, \\ q_j &= q_k = 0. \end{aligned}$$

Then the potential has the CP2 symmetry. If either M_j or M_k vanish, then the potential also has the CP3 symmetry.

CP2 and CP3

- Partial mass degeneracy:

$$M_i \neq M_j = M_k \equiv M$$

- Case A: If in addition we have

$$\begin{aligned} e_i &= 0, \\ M_{H^\pm}^2 &= \frac{1}{2}(e_j q_j + e_k q_k - M^2), \\ q &= \frac{M^2}{2v^2}, \\ (e_j q_k - e_k q_j) &= 0, \\ q_i &= 0. \end{aligned}$$

Then the potential has the CP2 symmetry. If also one of the scalar masses vanish then the potential has the CP3 symmetry.

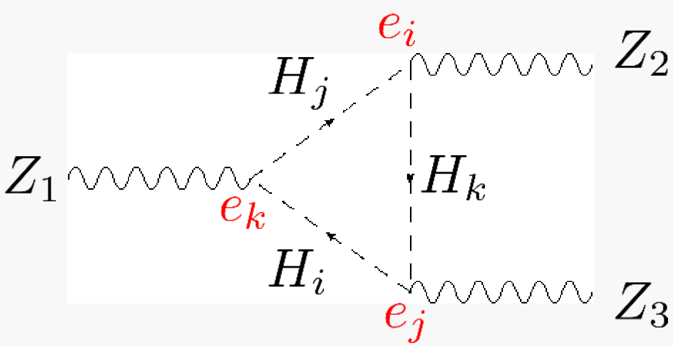
- Case B: If instead we have:

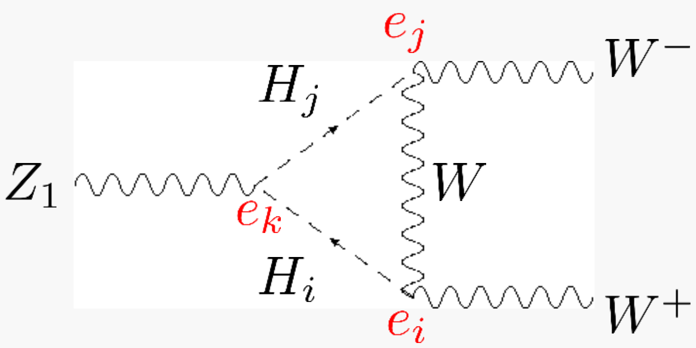
$$\begin{aligned} e_j &= e_k = 0, \\ M_{H^\pm}^2 &= \frac{1}{2}(e_i q_i - M_i^2), \\ q &= \frac{M_i^2}{2v^2}, \\ q_j &= q_k = 0. \end{aligned}$$

- Then the potential has both the CP2 and the CP3 symmetry.

ZZZ and ZWW vertices contain Im J_2:

ZZZ vertex and ZWW vertex





- Summing over all possible combinations of *i,j,k*, we find

$$\mathcal{M} \quad \propto \quad \mathrm{Im}\, J_2$$

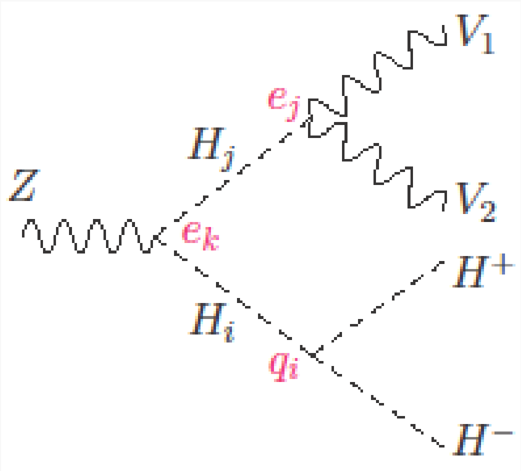
$$\mathrm{Im}\, J_2 \quad = \quad 2\frac{e_1 e_2 e_3}{v^9}(M_1^2 - M_2^2)(M_2^2 - M_3^2)(M_3^2 - M_1^2)$$

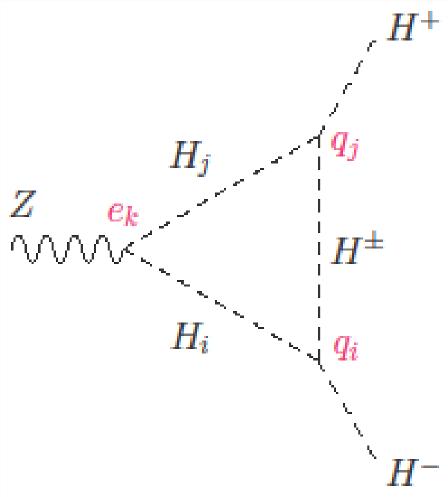
[Ref: Grzadkowski, Ogreid, Osland, JHEP05 (2016) 025]

Processes containing Im J_1 and Im J_3 :

$Z \rightarrow VVH^{+}H^{-}$ or $Z \rightarrow H^{+}H^{-}$

Summing over all possible combinations of *i,j,k*, we find





$\mathcal{M}$ contains $\mathrm{Im}\, J_1$ $\mathcal{M}$ contains $\mathrm{Im}\, J_3$

Alignment Limit (AL)

- 2HDM is aligned if H_1 couples to the gauge-bosons in the same way as the Higgs of the Standard Model.
- Alignment expressed in terms of $\mathcal{P}$ simply become

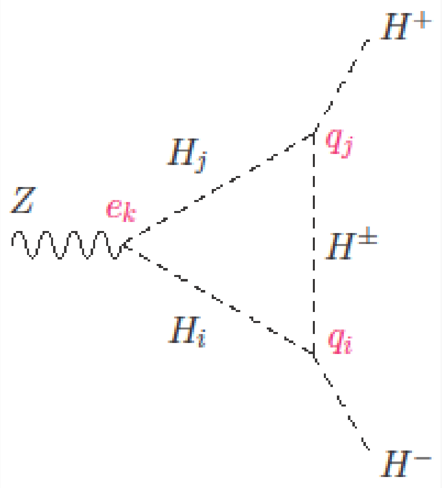
$$e_1 = v, \qquad e_2 = e_3 = 0$$

- Also possible to study "near-alignment" by expanding in the small parameters

e_2 and e_3 .

CP-violation in AL:

$$\begin{aligned} \mathrm{Im}\, J_1 &= 0, \\ \mathrm{Im}\, J_2 &= 0, \\ \mathrm{Im}\, J_3 &= \frac{2q_2q_3}{v^2}(M_3^2 - M_2^2). \end{aligned}$$



$$\mathcal{M} \quad \propto \quad \mathrm{Im}\, J_3$$

Spontaneous CP violation in the alignment limit

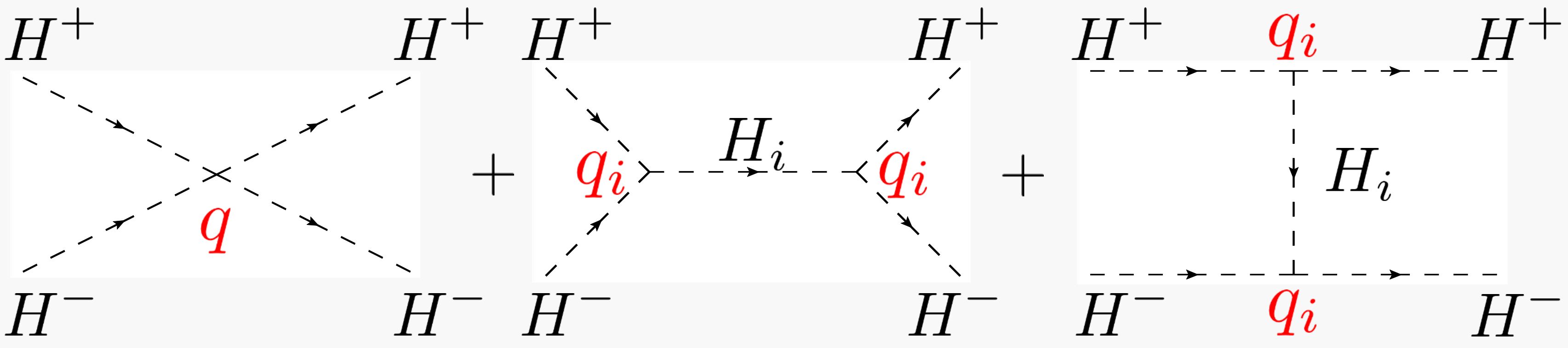
$$M_{H^\pm}^2 = \frac{vq_1 - M_1^2}{2},$$
$$q = \frac{1}{2}\left(\frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2} + \frac{M_1^2}{v^2}\right).$$

Rewriting constraints to

$$q_1 = \frac{1}{v}\left(2M_{H^\pm}^2 + M_1^2\right),$$
$$q - \frac{1}{2}\left(\frac{q_1^2}{M_1^2} + \frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2}\right) = \frac{-2M_{H^\pm}^2(M_1^2 + M_{H^\pm}^2)}{v^2 M_1^2}.$$

- First constraint testable by measuring q_1 and compare to $M_{H^\pm}^2$ and M_1^2.
- What about the second constraint???

Elastic H^+H^- scattering



Combined amplitude:

$$\mathcal{M} \propto -4iq - \frac{iq_1^2}{s-M_1^2} - \frac{iq_2^2}{s-M_2^2} - \frac{iq_3^2}{s-M_3^2} - \frac{iq_1^2}{t-M_1^2} - \frac{iq_2^2}{t-M_2^2} - \frac{iq_3^2}{t-M_3^2}.$$

Limit: $s \to 0$ and $t \to 0$ yields $\mathcal{M} \propto -4i\left[q - \frac{1}{2}\left(\frac{q_1^2}{M_1^2} + \frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2}\right)\right]$

Lhs. of second condition

Challenge 1: Need colliding beams of charged scalars.

Challenge 2: Limit: $s \to 0$ and $t \to 0$ is impossible to achieve.

Extrapolation and fits needed.

CP2 and CP3 in the Alignment Limit (AL) $e_1 = v, \quad e_2 = e_3 = 0$

If we also have

$$M_{H^\pm}^2 = \frac{1}{2}(vq_1 - M_1^2),$$
$$q = \frac{M_1^2}{2v^2},$$
$$q_2 = q_3 = 0.$$

Then the potential has the CP2 symmetry. Note that both H_2 and H_3 behave as if they were CP-odd.

If in addition H_2 and H_3 are mass degenerate $(M_2 = M_3)$, or one of them is massless, $(M_2 = 0 \text{ or } M_3 = 0)$, then the potential also has the CP3 symmetry.

Note that the two constraints

$$M_{H^\pm}^2 = \frac{1}{2}(vq_1 - M_1^2),$$
$$q = \frac{1}{2}\left(\frac{q_2^2}{M_2^2} + \frac{q_3^2}{M_3^2} + \frac{M_1^2}{v^2}\right).$$

are enough to guarantee a CP conserving 2HDM-potential in the AL.

Furthermore. either condition $e_2 = q_2 = 0$ or $e_3 = q_3 = 0$ is enough to guarantee a CP conserving 2HDM (potential + VEV)

In some (unprecise) sense we may say that these symmetries are stacked on top of each other in the case of CP2.

Summary

- CP properties of the 2HDM are expressible in terms of the physical parameter set $\mathcal{P}$.
- Invariants directly related to physical processes containing CPV form factors
- Experiments: If 2HDM, then we are close to the alignment limit.
- All cases found for CP2 and CP3 combines the conditions for a CP-symmetric potential (in a CP-violating model) with at least two separate different conditions for a CP-conserving model.

- Couplings/masses provide direct connection to experiments and tell us what measurements to make in order to test for CP-violation or CP-conservation of the 2HDM. In the case of CP-conservation we are able to test for the degree of CP-symmetry, CP1, CP2 or CP3.

- Thank you for your attention!